Course on Momentum Transfer in Process Engineering

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Mod 01 Lecture 05

Equation of motion Part-1

Okay, so we have done equation of continuity in the three co-ordinates, Cartesian co-ordinate, cylindrical co-ordinate and spherical co-ordinate. Of course out of these three we have derived 2 that is cartesian and cylindrical and spherical co-ordinate we had shown you that what is the final form of the equation of continuity and we also said that these relation, so right from that partial derivative, time, total derivative, substantial time derivative. These will be very much useful whenever also progressing towards the development of the equation of motion.

So when you are deriving those equations will be very much required and of course even in more complicated. Now in earlier case, what we have done in the equation of motion we had done the mass balance, but here when you are doing the equation of motion you have to do the momentum balance and this momentum can be transferred in not only one way, but two ways this momentum can be transferred, right. How? One is the bulk flow, right for example, if this is the say 1 fluid, of course this is solid, but assume it to be if it is coming, so this bulk is converting or transporting this momentum to another if this (())(2:14) another, but this is one and another way is say, these are the 2 or if we assume that this fluid to be of this kind of multilayers, right.

So one layer like this can transfer to its neighboring layer and that can transfer to its another neighboring layer. So like that this momentum can be transferred from one layer to the other if you exactly the way if we have seen that playing cards, playing cards if you keep that bunch of playing card and if you just given one push at the button move of that card then what happens, the it is note that you have to give very high , I mean force to be very high, just give a small force and then see that the top one has come out bottom and one has (())(3:20) in and there is distribution of the velocity or that cards that coming like a like angular one, right.

In a in an angle not angular in an angle that those cards are making an angle with the base, right. So this is how there this cards we can assume to be each card to be one layer and they are passing one layer to the other layer the force, which you have given a say horizontal force and that force is also translated to the vertical one that is why the top one as remain there, whereas bottom one has gone a little away and they have made one like this.

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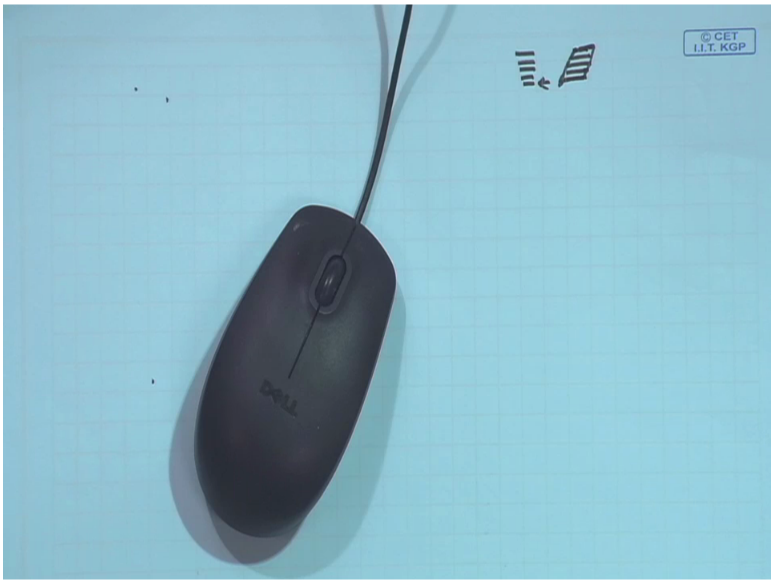

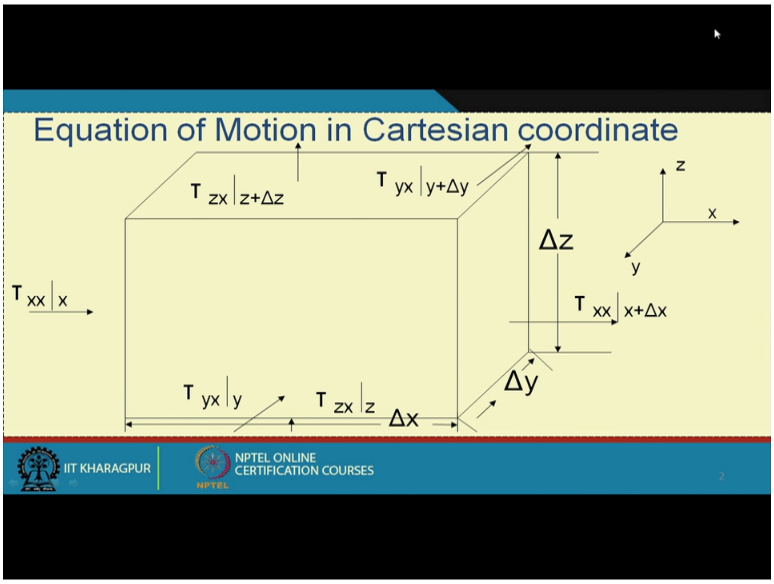

So if we take this, so these are the cards, right and you have given a force at this place then what we will see? This card, which we have given a force has gone a little away whereas its adjacent is like this, sorry it should have been like that, okay and this adjacent is like that this adjacent is like that, this adjacent remain like that, right. So there is distribution, there is this kind of change you can see there now. So that if you see in reality, in the in the liquid also, in the fluid also the same thing happens, if you consider the fluid to be of multilayers and when the force is a given, this is happening to that. So how they are changing? How they are transporting the energy? This energy is bring one we said (())(5:22) that it was through the bulk that this is a flow of the fluid by the bulk flow, so that is being transported and another, when you are doing this that shear the one layer to the other layer that is by you called shear.

So that force is called shear force so that shear force is acting on the layer and thereby as it is like this, so adjacent to this, it is being pulled this it is being pulled by that, that is being pulled by that, that is being pulled by that whether the top one remain may be there or a sight moment. So it is there is definite way of deflection, you can see in all the cards.

So in similarly the layers scientific way what we say that, we know that in the fluid particularly gaseous, right in the fluid particularly say gaseous, so right, there what we are showing that (())(6:42), okay. So what is in that what we have seen that when you are looking at these layers in the you have read kinetic theory of gases, right? There you have seen that gas molecules can in the kinetic theory can move in all directions, right of course they can collide also, but they can also move in all directions and thereby, they are importing (())(7:20) the energy from one to the other, may the kinetic theory of gases is established. So many many people have given many theories on that, but they till now there is no such kinetic of in fluids or liquids, right.

So gases also a fluid, but okay in liquid there is no such kinetic theory , but more or less (()) (7:50) we assume that this may be applicable for the deviation, which you will get will not give so much that a prediction will be absolutely different. So in that case a there this molecules you imagine (())(8:11) that one layer molecule and next layer molecule. So this molecules are by (())(8:18) in a, so they in that vibration they are exchanging the energy one to the other, right.

So one to this energy is exchange (())(8:29) whichever is having more is (())(8:31) giving to the less one work that energy is exchange and thereby this kind of vibration we call it to be the shear ultimately, one is dragging the pulling down the other and that is cause the drag force or when we are calling it to be shear force. So this shear force is also acting on that. These two types of these two types of transport mechanism is there, when you are considering the momentum equation or equation for motion on the basis of the momentum transfer, right.

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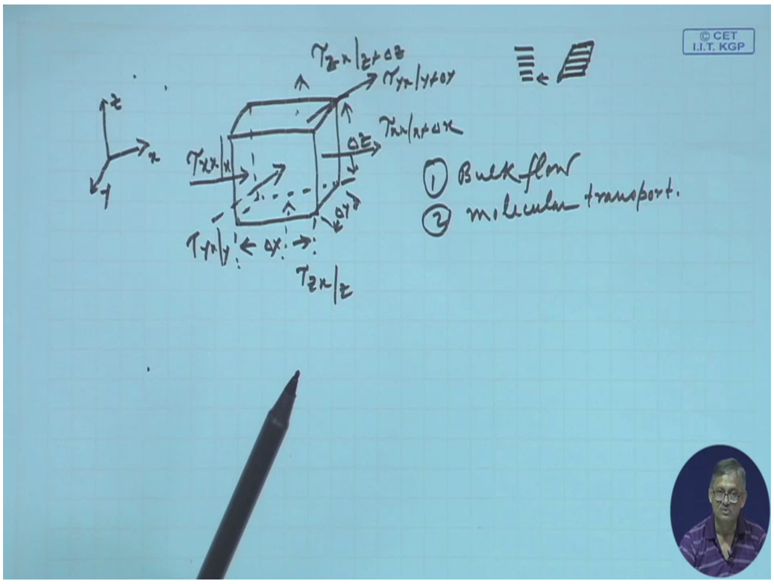

So here simply we take the simple, one that is the cartesian co-ordinate. So you take one volume element. So take one volume element, right and as earlier we have said it to be del x, del y and del z, right. So if that to be there then we said that the momentum will be transferred by the one by the bulk flow and another by the molecular transport, right. So this molecular transport if we consider that to be say, this is normally represented shear as the Tau, this symbol Tau is used and of unfortunately may be in the in our slides that tau might not have come properly, it might be in place of Tau T, so but it is exactly the Tau. So Tau xx at the face x and this is Tau xx at the face x plus delta x, right.

Similarly one can be like this and like that right. So which we can write to be Tau at the face zx at z and Tau here, it is zx right at the face z plus delta z, right whereas the third one is perpendicular to this. So there we can write to be like this and one like that, okay then that can be said to be Tau yx at the face y and Tau yx at the face y plus delta y, right. So other than and this is our xyz, this is z, this is x and this is y. So other than xyz sorry, (other than the molecular) other than the bulk transport, which we have shown that other than the bulk transport, this additional one is the molecular transport which has to be also taken care of, right.

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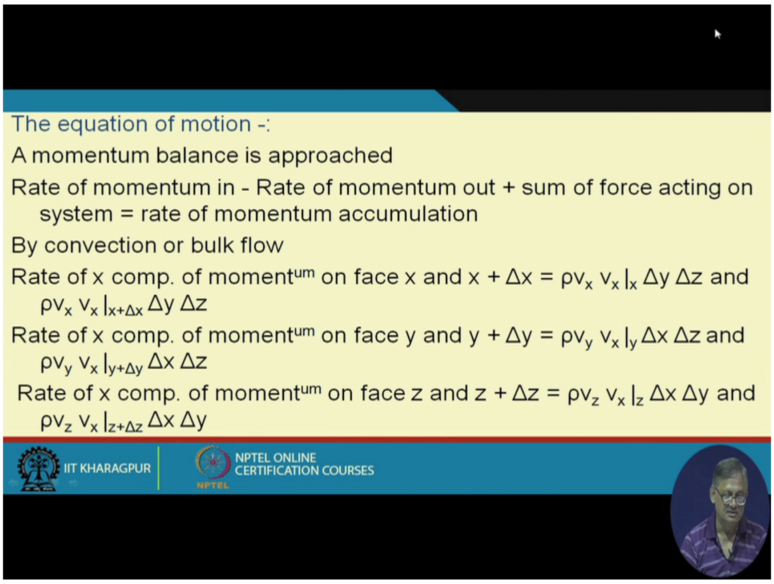

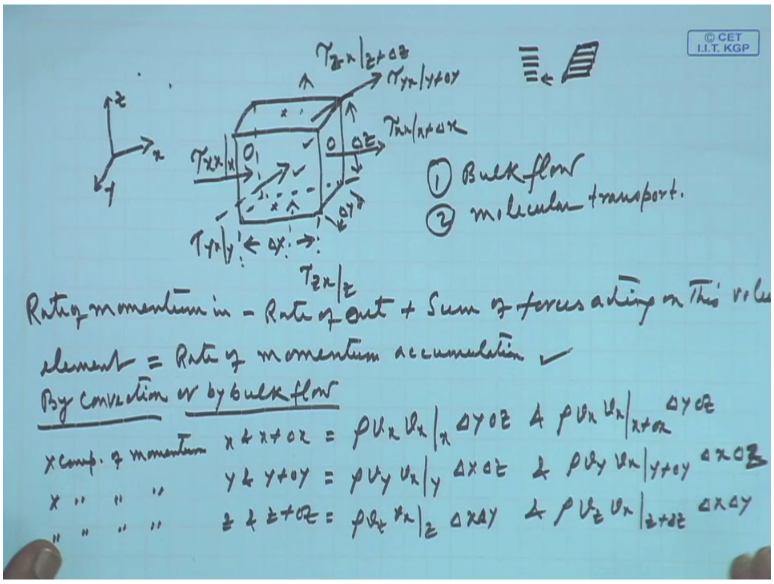

So when we are doing it then we do the general equation for motion is like that momentum balance as we have said in the beginning that momentum balance is done, so in that case we can say that the rate of momentum in minus rate of momentum out plus sum of all the forces acting on this volume element, right. This must be equal to the rate of momentum accumulation, right. So this is the basic general equation for the momentum transfer that is based on the momentum the rate of momentum in minus rate of momentum out plus all the forces acting on the volume element, this must be equals to the rate of momentum accumulation, right.

So this based on this the entire development of the equation of motion is based on, so here we can write that by convection what is coming and going out by convection or we can also say by bulk flow, the work we have coming that rate of x component of momentum on face x and x plus delta x, right. So what is x, so this was the face x, this was the face x plus delta x, right and this was the face y and the other one was this, y plus delta y and this was the face z and the other one is the z plus delta z, right.

So if we look at that by convection, now there are three, right x component momentum acting on the x, x component of the momentum acting on the y, x component of the momentum acting on the z. all these three are to be considered, right. Similarly for the other, so x component now first let us then look into that x component of the momentum that means momentum derived from the velocity the V which is acting on the x direction as Vx.

So momentum arising out of that in the acting on the x direction is that is the x component of the momentum acting on the x. This we will be first finding out and then we will go to x component of the momentum acting on the y and z, right. So then what (we) you can write that x component of the momentum or rate of x component of momentum. So which is acting on the face x and x plus delta x, right. So they are Rho Vx times Vx at the face x times its area del y del z, this is for face x and the other one Rho Vx right into Vx at x plus delta x into del y del of z, right.

So similarly we can also write that x component of the momentum right acting on the face y and y plus delta y, this can be written as Rho Vy due to the x component at the face y this times the area what is area del x del z, this is for the face y. similarly for the face y plus delta y it will be Rho Vy Vx at the face y plus delta y into the area del x del z sorry, del x del z, right. So similarly the x component of the momentum acting on the z direction is, so z and z plus delta z that can be written as Rho Vz right into the x component that is Vx at the face z into area del x del y and similarly Rho Vz due to x component Vx at the face z plus delta z into area del x del y, right.

So these are then all three x component momentum acting in the all three, right what we have given? So fluid was flowing, right fluid was flowing, so if the moment it has velocity V, it will have three components, one acting in the x direction, one acting in the y direction and another acting in the z direction. So all these three directions we have taken care of due to the velocity component Vx or the Vx component that. How much momentum is being transported to the all three directions that is x, y and z that we have taken care of, right. So both at the inlet and also at the outlet, right.

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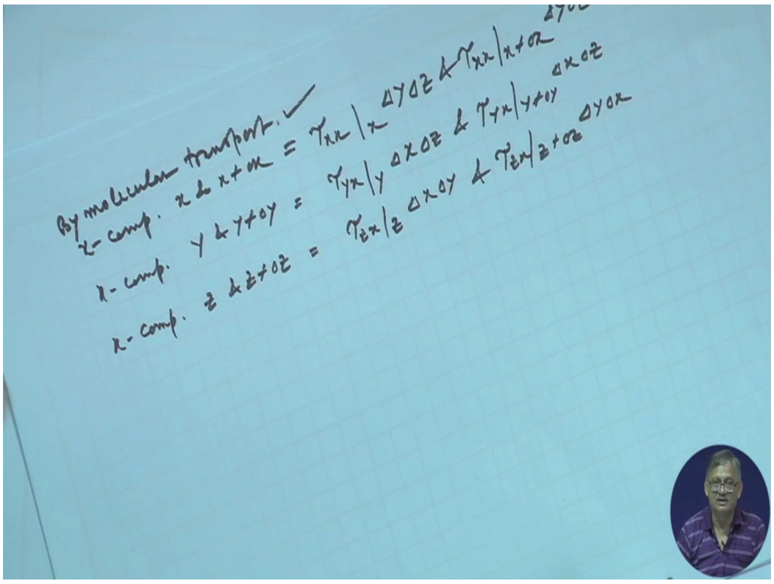

So then we can say that this was by the bulk flow. So by molecular transport, this molecular transport again for understanding, I am repeating that molecular transport means that one (()) (21:03) we assume that in the face (in). This is one plane or we also showed that this is one plane, right and that is another plane. So these two layers when they are there, so if we assume that this to be one molecule and if we assume that this to be another molecule, these two molecules, because of these vibration of the molecule within that plane. It is note that they are going like kinetic theory of gases what we have seen that it is going a till it is colliding, right and to the boundary. It is not like that it is on that plane not going the molecules and not going away, but it is just like vibration. It is oscillating, right in that case, this molecule one molecule is importing energy to the other molecule, obviously the one which is having more will give to the lower one, right.

So but this is happening for the flow of the fluid. Now we are considering in the x direction, right. So that if we look at by molecular transport if we see similar way that how it is being rate (())(22:33) how we write it that rate of x component of momentum on the face x and delta x we can write that x component of momentum acting on the face x and x plus delta x, right. So this is by molecular transport we can write to be equals to Tau xx at the face x into its area del y del z, this was for the x face and for the x plus delta x, we can write Tau xx at the face x plus delta x to del y del z, right. Similarly x component of the momentum acting on the face y and y plus delta y that should be equals to Tau yx at y into the area del x del z and Tau yx at the face y plus delta y into area del x del z.

Similarly, we can write x component of the momentum acting on the z and z plus delta z direction is Tau zx at z into the area del x del y and for the other face Tau zx at z plus delta z into del y del x, right. So if we see this we **we** have taken all both for the molecular transport as well for the bulk transport the total component for the bulk transport the total components of the x component or rather total molecular (())(25:07) total momentum transport for the x component of the momentum in all the three directions say x, y and z, right.

So velocity component Vx that is importing the momentum and this momentum is in acting on all three directions and in two ways one by the bulk transport another by the molecular transport. So all these 6 right three for the bulk flow and three for the molecular transport that we have seen how they are acting for one component that is velocity component Vx, right.

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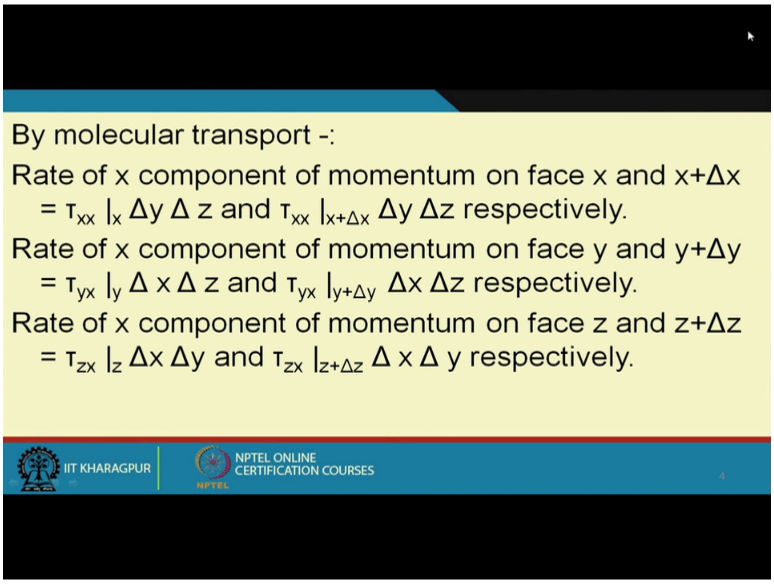

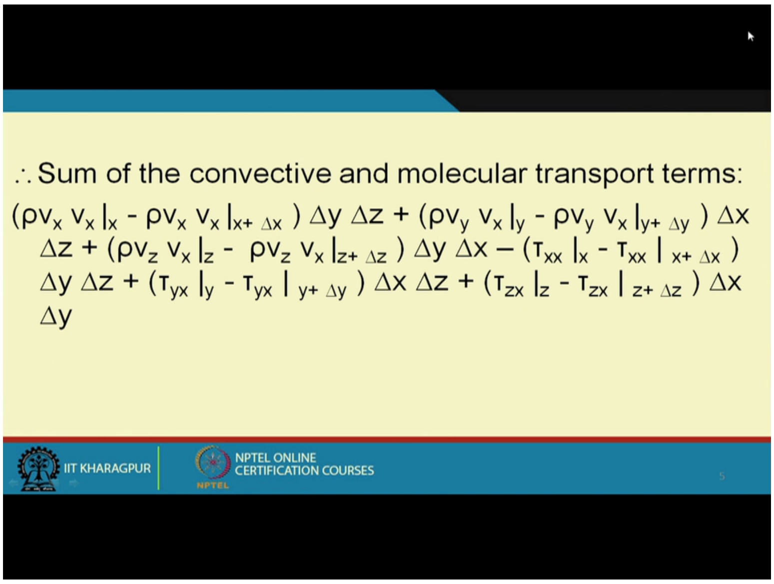

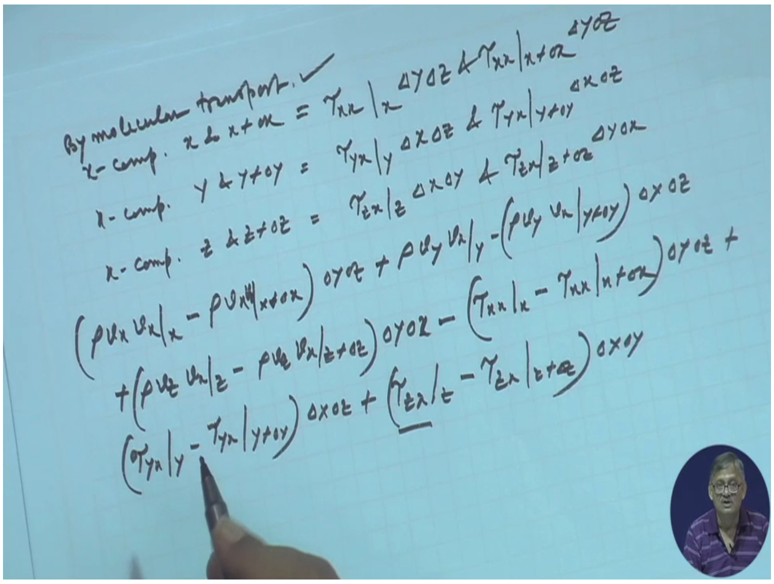

We can then say that sum of the convective and molecular terms if we write they will be rho Vx into Vx at x plus (())(26:17) minus rho Vx at into Vx at x plus delta x into the area del y del z plus Rho Vy into Vx at y minus Rho Vy into Vx at y plus delta y times the area that is del x del z, right plus Rho Vz into Vx at z minus Rho Vz into Vx at z plus delta z, this, right plus when this times the area, area is del y del x, right del y del x and this is for the all three terms of the bulk transport and for molecular transport we can write that Tau at xx at the base face x minus Tau at xx at the face x plus delta x times the area that is del y del z plus Tau at the face yx at y minus Tau yx at the face y plus del y times the area that is del x del z plus Tau zx at the face z minus Tau zx at the face z plus delta z into del x del y. So into del x del y okay, right. So these are all the forces acting not all the forces rather this is for bulk and molecular transport, right. Bulk flow and molecular transport.

So this is said the Tau z is that is x component of the molecular transport acting on the z direction right, x component of the molecular transport acting on the z direction, x component of the molecular transport acting on the x direction, x component of the molecular transport acting on the y direction, this is how? This is interpreted, this we have to keep in mind subsequently when we will (())(29:10) also find out that this Tau can also be expressed in terms of viscosity, right from the basic definition of viscosity.

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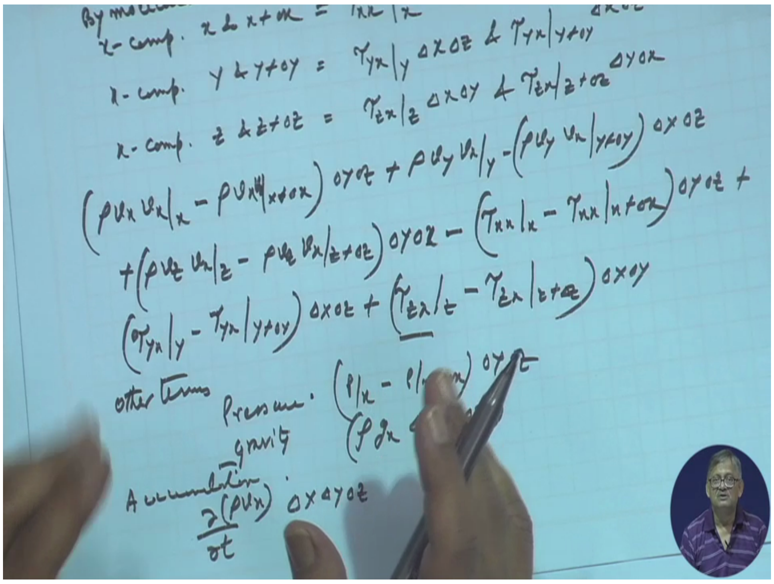

Then we also we will see that there the velocity component and the direction both are acting together. So here as we said at Tau yx Tau zx Tau xx these are x component of the molecular transport acting on the respective directions of x, y and z, right. So other terms could be pressure force that can be written as p at the face x minus P at the face x plus delta x into the area that is del y by del z then this is all for x component and gravity forces acting, which is Rho times gx times del x del y del z, right Rho is kg per meter cube, this is a meter cube, so that why this is come and accumulation, we can write accumulation of the x component of the momentum that we can write del del t of Rho Vx right, del del t of Rho Vx into the volume element del x del y del z, right. So we have done in this class till (())(31:00) till now that how the molecular transport and how the bulk transport we can formulate it, in terms of the components of the rather velocity components individually acting on the different different co-ordinates that is x, y and z, of carouse you have taken Cartesian co-ordinates right.

So as we said that this will take long time this is not small one and that is why you have taken the easiest one or the one which you can handle easily in the cut there is cartesian co-ordinate and we have done up to that accumulation of x component momentum accumulation, right. So next we will summit up sum means we will go for the other two that is y and z component and then look into the total and finish the entire thing as the equation of motion known as the which we said, Neivers stokes (())(32:20) equation, right. Thank you.