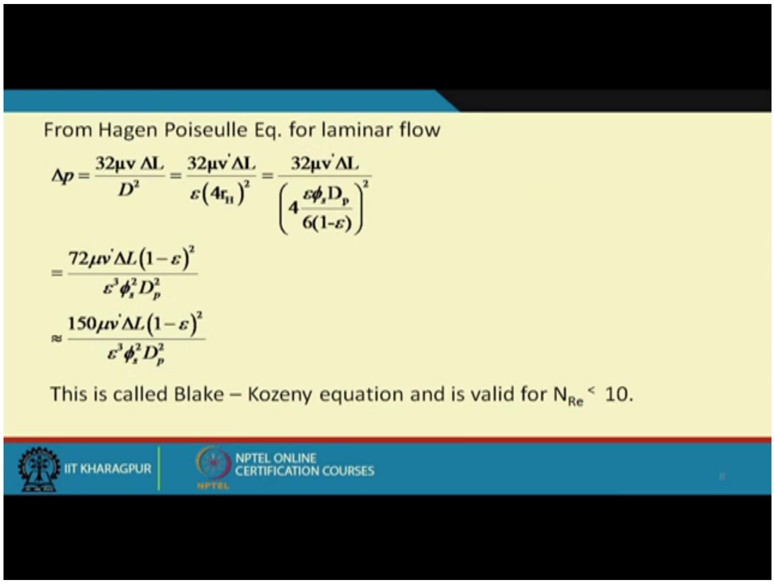
Course on Momentum Transfer in Process Engineering

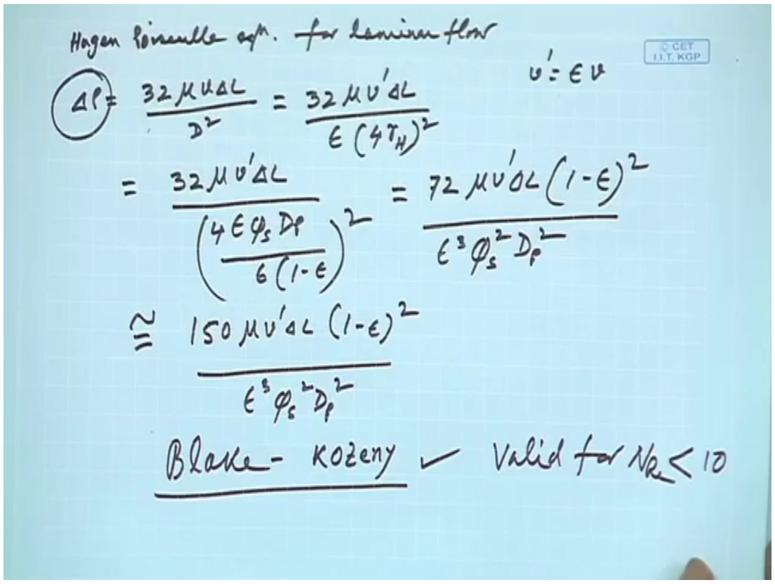
By Professor Tridib Kumar Goswami
Department of Agricultural & Food Engineering
Indian Institute of Technology, Kharagpur

Lecture 48
Module 10
Ergun's equation-derivation (Part-1)

Hello, we are doing packed bed, okay. So in the packed bed we have already said what is the utility of the packed bed? What is the application? Particularly in process engineering, but we also have shown there were some definitions a or many others like hydraulic radius or sphericity this things we have already defined we have also found out Reynolds number, right? So now we will go from that packed bed Reynolds number, then yeah.

(Refer Slide Time: 1:08)





Now from the Hagen Poiseulle equation for laminar flow we have seen that delta P was equal to 32 mu v delta L or mu v L since it is L so we can take it as delta P for delta L for delta P over D square, right? Now this we can rewrite as 32 mu v prime delta L over epsilon into 4 rh where rh is hydraulic radius square, right? This we have substituted D, right? v as v p prime and v prime by epsilon and D as 4 rh whole square, right? Because your v prime was equals to epsilon into v, right? So this v prime is the flow through the packed bed.

So if that be true, then we can also write this is equal to 32 mu v prime delta L over this we can rewrite as 4 epsilon phi s Dp over 6 into 1 minus epsilon this square, right? This rh we have done it earlier in the previous class, right? rh hydraulic radius and then to from there to hydraulic diameter that we have done. So it is 4 epsilon phi s Dp by 6 into 1 minus epsilon square, right? This on rearrangement we can write this is equals to 72 mu v prime delta L to 1 minus epsilon whole square over epsilon cube into phi s square into Dp square, right?

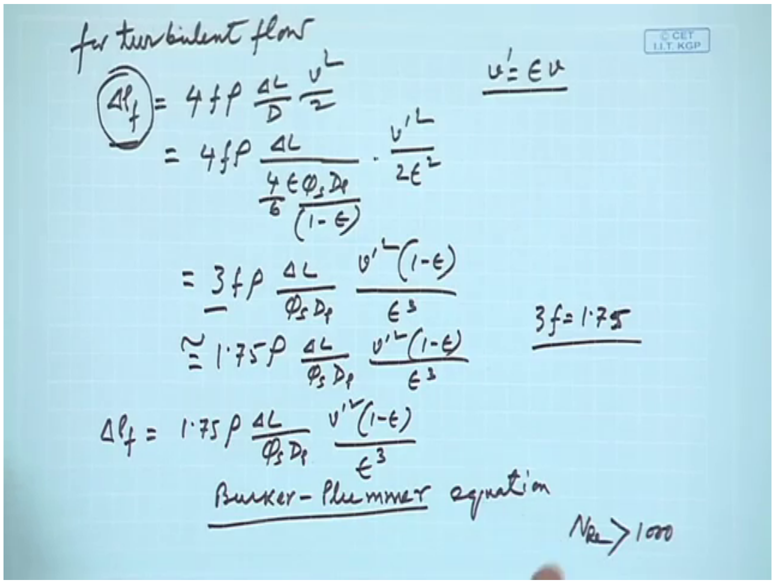
Now this can roughly be written as equals to 150 mu v prime delta L into 1 minus epsilon square divided by epsilon cube phi s square into Dp square, right? Why? Because this has been seen this has been found out that whenever the value of delta P predicted this is a relation, so that means you can predict the delta P for a laminar flow using Hagen Poiseulle's equation this you can at (())(5:20) at this state using epsilon phi everything, right? So this is for packed bed that what is the delta P when your there is a sphericity of phi s or there is a void space of epsilon.

So when all there are known, then you can write it in terms of 72 mu v prime delta L into 1 minus epsilon square by epsilon cube into phi s square Dp square. So that means if you know all these terms then you can tell that what is the delta P, but the problem is that when you do this the actual case the it has been found that the actual case will be much better if this 72 is replaced by 150 that is the predation is much closer to the actual delta P that is why this 72 has been replaced with 150, right?

This was done by Blake and Kozeny and according to their name this equation is known as Blake Kozeny equation for flow through packed bed and this is for the laminar part, so for laminar flow we have started with Hagen Poiseulle equation from there the delta P relation whatever we had we have taken that and we have modified that only in terms of putting the actual parameters like epsilon phi s Dp and the velocity v prime, right? So these by rearranging we have now got one relation where delta P is instead of 72 it is 150, then mu v prime delta (()) (7:59) v prime delta L 1 minus epsilon square by epsilon cube into phi s square into Dp square, this we call it to be the Blake Kozeny equation, right?

And this is valid for Nre less than 10, this is valid for Nre less than 10, right? Now let us look into the other that is the other part when the flow is not laminar.

(Refer Slide Time: 8:45)



For turbulent flow we can use that delta Pf is equals to 4 f rho delta L by D into v square by 2 this we know for turbulent flow for laminar flow it was there using fanning friction factor we have reduced it that delta Pf is 4 f rho delta L by D or L by D to v square by 2, right?

This we have established long back. Now if we start from that point and rewrite, right? Now this we can also write as 4 f, right? rho then this delta L remains there, this D we can write 4 by 6 epsilon phi s Dp over 1 minus epsilon, right? Times that v has now become v prime square divided by 2 epsilon square so v square by 2 so v prime square by 2 epsilon square again the same thing that v prime is equals to epsilon to v, right?

From that relation we got this v prime square by 2 epsilon square, right? This D we have replaced with Dp and that we have shown earlier that this is equal to that 4 by 6 epsilon phi s Dp by 1 minus epsilon, right? This we can rearrange rewrite as equal to 3 f rho delta L by phi s Dp times v prime square times 1 minus epsilon over epsilon cube, right? This we can rewrite like this because this 4 that 4 goes up this 2 and this 6 goes up and 3 remains so 3 comes 4 rho delta L phi s remains there Dp remains there 1 minus epsilon goes up v prime square and this remains epsilon and 1 epsilon so epsilon cube, right? This epsilon square epsilon remains and it becomes epsilon cube, right?

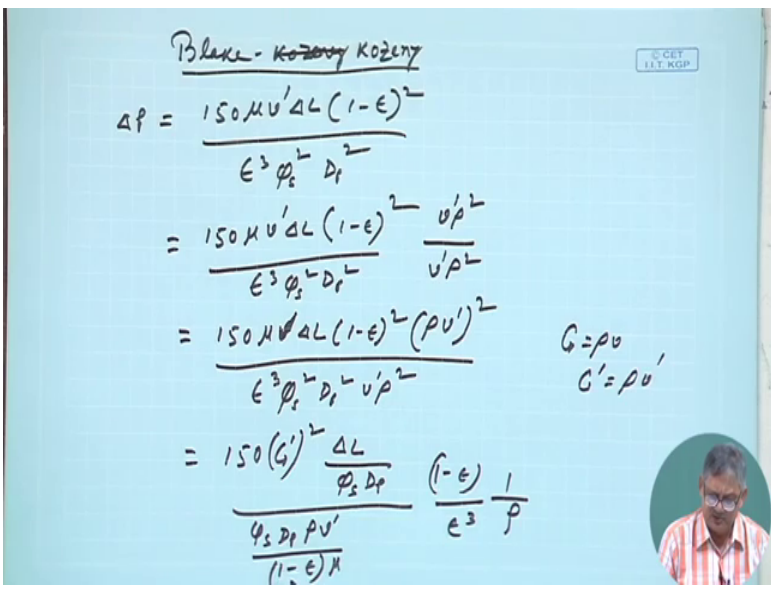
So here also the researchers have seen that the delta P which is predicted with this equation with this relation the delta P which is delta Pf for turbulent flow for turbulent flow the pressure drop under friction factor delta Pf that can be predicted through this relation where we also have the particle size as Dp we also have the void space as epsilon and we also have this velocity through that packed bed that is v prime I think we had said that v prime is empty cross section and v is through that packed bed, right?

So if there is epsilon void space, then that v that becomes equals to v prime over epsilon, right? That we have earlier established and shown. Now so that is what that delta Pf when it is equal to this 3 f rho delta L by phi s into Dp v prime square into 1 minus epsilon by epsilon cube this has been seen that the predation of delta P with this relation is not so close. So it was changed to roughly equal to 1.75 rho delta L by phi s Dp v prime square into 1 minus epsilon divided by epsilon cube, right?

So roughly 3 f is equals to 1.75 sorry 1.75, right? So 3 f is 1.75 we have substituted, right? Like the previous one we had substituted 150 72 with 150, right? Here we have substituted 3 f with 1.75, right? Then delta Pf that is equals to 1.75 rho delta L by phi s Dp v prime square into 1 minus epsilon by epsilon cube, right? So this is called this was proposed or this relation was derived by the another pair of scientist or another pair of researchers there name is Burker Plummer and this equation is known as Burker Plummer equation, right?

So this like the previous one that was valid for Nre less than 10, this is valid for Nre greater than 1000, right? So in one case we have seen that we can predict delta P for flow through packed bed when Nre is less than 10 and in another case for turbulent flow when Nre is greater than 1000. Than the question automatically comes that what about if Nre is between 10 and 1000, is not it.

(Refer Slide Time: 17:08)



So for that another scientist who did a clumbing I mean punching of these two that he has predicted he has used both of these two relations and then what it did it from the Burker Plummer equation or Blake Kozeny equation first was Blake Kozeny, right? It was Blake Kozeny equation from Blake Kozeny equation we have seen delta P was 150 mu v prime delta L 1 minus epsilon whole square divided by epsilon cube phi s square Dp square, right?

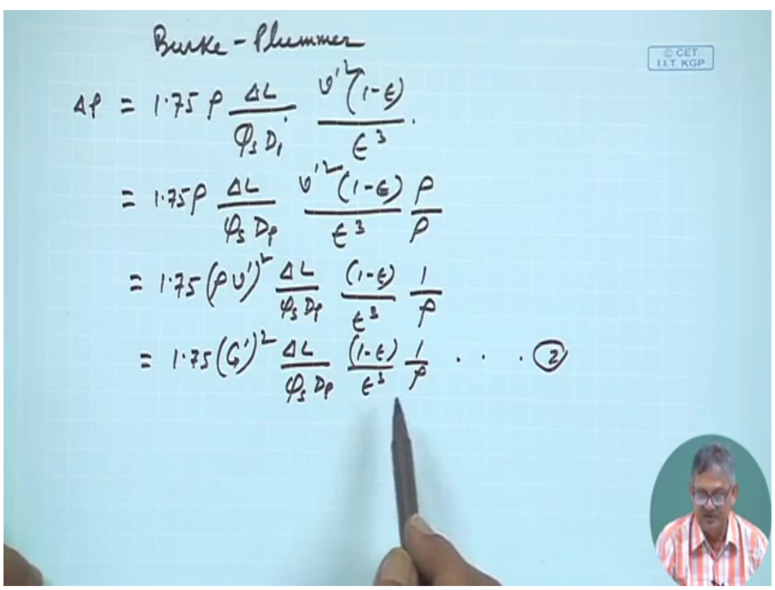
So this can be rearranged as 150 mu v prime delta L 1 minus epsilon whole square by epsilon cube phi s square Dp square. Now if we multiply both numerator and denominator with v prime rho square by v prime rho square, then we can rewrite and rearrange it as 150 mu v prime, right?

mu v prime, v prime we can use afterwards, okay 150 mu say delta L mu delta L because this v prime we can use inside. So 150 mu delta L into 1 minus epsilon whole square now this v prime we can take there and write rho v prime square, right?

And in denominator we have epsilon cube phi s square, right? And Dp square and this side it was v prime rho square, right? So this again we can rewrite as equals to 150 G prime square, right? Because G is rho v so G prime is equals to rho v prime, right? So if that be true then we can write 150 G prime square delta L by phi s Dp, right? Over phi s Dp rho v prime over 1 minus epsilon into mu times 1 minus epsilon over epsilon cube times 1 by rho because we have taken 1 additional rho and that rho came out here, right?

So this we can rearrange rewrite 150 G prime square delta L by phi s Dp into 1 minus epsilon by epsilon cube into 1 by rho, right? Into phi s into Dp into rho v prime over 1 minus epsilon into mu, right? This v prime was there and this rho is taken only here it was rho square so 1 rho is taken here and 1 rho remains here, right? So this rearrangement is required because we would like to proceed to one relation because this was developed by the scientist we will name him afterwards and during that process he has done all these.

(Refer Slide Time: 22:03)



So this equation we name it to be say equation 1, right? So let us keep it aside and now from the other one that is Burke Plummer I think the previous one where we so far remember it was Burke not er BURKE only, right? B, U, R, K, E Burke Plummer P, L, U, M, M, E, R, right? Burke

Plummer, okay. So next form the equation Burke Plummer we can write delta P is 1.75 rho into delta L over phi s Dp square Dp rather into v prime square into 1 minus epsilon over epsilon cube this was the original equation which was Burke Plummer, right? This we just have done here, it was like that 1.75 rho delta L over phi s Dp v prime square into 1 minus epsilon by epsilon cube, right?

So this if we know, then we can rewrite it as 1.75 rho, right? Delta L by phi s Dp into v prime square into 1 minus epsilon by epsilon cube into both numerator and denominator let us multiply and divide with rho. So in that case we can rewrite it as 1.75, right? This as rho into v prime this as rho into v prime, right? rho v prime and we also can write this delta L by phi s Dp, right? Into 1 minus epsilon by epsilon cube, right? Divided by into 1 by rho because this rho v prime we have gathered together, so 1 by rho remains. So this we can write that this is equals to 1.75, right? G prime, right? And this was v prime square, right? So it was v prime square so it should be square here this rho this rho is this rho v prime square.

So this is rho v prime square so we can write G prime square into delta L by phi s Dp, right? Into 1 minus epsilon by epsilon cube into 1 by rho, right? So this is if we tell that this is equation number 2 so that means till now what we have done we have taken in one hand that Blake Kozeny equation that we have taken and we have come to equation number 1 with a relation, right? So delta P is equals to this this this this with a relation we have found out what is the delta P.

And in another with the Burke Plummer equation which was for the turbulent flow we have taken that equation and we have come to this relation that 1.75 G prime square delta L by phi s Dp into 1 minus epsilon by epsilon cube to 1 by rho, right? This we have done purposefully so that we can utilize ultimately our target is we have said we have gotten 1 equation that is Blake Kozeny which is valid for Nre less than 10, right? And we have also taken another equation which was and that we started with laminar flow, right? And now in this case we have taken for turbulent flow and in turbulent flow we have said that this is valid for Nre greater than 1000, right?

And in both the cases in terms of G prime that is mass velocity we have taken that okay now in terms of G prime and epsilon Dp rho all these terms we have done in one form as delta P for prediction in terms of G prime and we named them as say equation 1 and equation 2, right? So perhaps this class we cannot proceed further, when we again come to the next class then we will see how beautifully that scientist has utilized and made another what I should say another relation by which you can use both the for laminar as well turbulent flow you can use the generalized equation and for the intermediate also, okay thank you let us go for the next class thank you.