

Course on Momentum Transfer in Process Engineering
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Lecture 46
Module 10
Generalized coefficient of Reynolds number

Yeah, in the previous class we also discussed about the flow through slits, right? For Non-Newtonian fluid and if we remember that we had come to this point, yeah (0:51) flow through slits it was so we came we found out average velocity and yes we came up to this that is average velocity is like that and it was $n + 1$ by $2n + 1$ v_{max} , right?

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$$v_{av} = \frac{n}{(2n+1)} \left(\frac{dp}{dx} \right)^{\frac{1}{n}} \left(\frac{\delta}{n} \right)^{\frac{n+1}{n}}$$

$$v_{max} = \frac{n+1}{2n+1} v_{max}$$

$$v_x = \frac{n}{(2n+1)} \left(\frac{dp}{dx} \right)^{\frac{1}{n}} \left(\frac{\delta}{n} \right)^{\frac{n+1}{n}} \left[1 - \left(\frac{y}{\delta} \right)^{\frac{n+1}{n}} \right]$$

reduces to

$$v_x = \left(\frac{dp}{dx} \right)^{\frac{1}{n}} \left(\frac{\delta}{2n} \right)^{\frac{n+1}{n}} \left[1 - \left(\frac{y}{\delta} \right)^{\frac{n+1}{n}} \right]$$

and

$$v_{av} = \frac{n}{(2n+1)} \left(\frac{dp}{dx} \right)^{\frac{1}{n}} \left(\frac{\delta}{n} \right)^{\frac{n+1}{n}}$$

reduces to

$$v_{av} = \left(\frac{dp}{dx} \right)^{\frac{1}{n}} \left(\frac{\delta}{3n} \right)^{\frac{n+1}{n}}$$

Now if we look into this that v average was for the n by $n + 1$ for the Newtonian fluid Non-Newtonian fluid by δP into rather δP by kL to the power 1 by n δ to the power $n + 1$ by n δ to the power $1 + n$ this was v average, right? And v_{max} we had like this n by $n + 1$ into δP by kL to the power 1 by n δ to the power $n + 1$ by n , right? So this was for v_{max} and this was for v average, right? And from there we can write v average is equals to $n + 1$ by $2n + 1$ this was n by $2n + 1$, no (2:42) not this v average $2n + 1$ that is what I am thinking.

So $n + 1$ by $2n + 1$ v_{max} is v average, right? So if this be true, then we can say that if we put now that limit or limiting condition that is at n is equals to 0 , k is equals to μ this flow is

Newtonian, right? Flow is Newtonian in that case v_x is equals to n by n plus 1 into ΔP by kL to the power 1 by n del to the power n plus 1 by n into 1 minus y by del this to the power n plus 1 by n this was general velocity and this reduces to v_x is equals to ΔP by $2 \mu L$ to del square into 1 minus y by del whole square this.

And v average which was n by $2n$ plus 1 into ΔP by kL to the power 1 by n , right? Into del to the power n plus 1 by del and this reduces to v average is equals to ΔP by $3 \mu L$ del square, right? So now these are really the proof that the limiting conditions do exist and do consider with the equations derived earlier, right?

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Newtonian fluid

$$\tau_w = \frac{\Delta P \delta}{L} = k \left(\frac{3 v_{av}}{\delta} \right)$$

$$v_{av} = \left(\frac{\eta}{2n+1} \right) \left(\frac{\Delta P}{kL} \right)^{\frac{1}{n}} \delta^{\frac{n+1}{n}}$$

$$\left(\frac{\Delta P}{kL} \right)^{\frac{1}{n}} = \left(\frac{2n+1}{\eta} \right) \frac{v_{av}}{\delta^{\frac{n+1}{n}}}$$

$$w. \left(\frac{\Delta P \delta}{L} \right)^{\frac{1}{n}} = k^{\frac{1}{n}} \left(\frac{2n+1}{3n} \right) \left(\frac{3 v_{av}}{\delta} \right)$$

$$k. \frac{\Delta P \delta}{L} = k \left(\frac{2n+1}{3n} \right)^n \left(\frac{3 v_{av}}{\delta} \right)^n$$

If we define,

$$k'' = k \left(\frac{2n+1}{3n} \right)^n$$

$$v_{av} = \frac{\delta}{3} \left(\frac{\Delta P}{k'' L} \right)^{\frac{1}{n}}$$

Now for Newtonian fluid what we have seen earlier τ_w is equals to ΔP del by L is equals to μ into $3 v$ average by del, right? Now re-writing this equation if we say that v average is equals to n by $2n$ plus 1 into ΔP by kL to the power 1 by n del to the power n plus 1 by n , right?

So this we can write that ΔP by kL to the power 1 by n is equals to $2n$ plus 1 by n into v average by del to the power n plus 1 by n , right? Or we can write ΔP del, right? ΔP del by L to the power 1 by n this is equals to k to the power 1 by n into $2n$ plus 1 by $3n$, right? Into $3 v$ average by del, right? Because this one del we have taken out 1 by n del to the power 1 by n so 1 del remains, right? So del so here it is del 1 by n so that is n when it is inverse there and yeah it goes to this side and inverse so it is okay.

So that means we can write that $\Delta P \delta / L$ is equals to this was 1 by n so k that goes out then $2n$ plus 1 by $3v$ average over δ to the power n and $3v$ average over δ to the power n , right? Now if we define k double prime is equals to k into $2n$ plus 1 by $3v$ average over δ to the power n , right? Then v average can be written as δ by $3 \Delta P \delta / L$ into rather to the power 1 by n k double prime L to the power 1 by n , right?

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Handwritten mathematical derivation on a whiteboard:

$$\mu = \frac{\Delta P \delta / L}{3 v_{av} / \delta}$$

$$\mu'' = \frac{\Delta P \delta / L}{3 v_{av} / \delta} = K'' \left(\frac{3 v_{av}}{\delta} \right)^{n-1}$$

$n = n' = n$
 $K' = 0 K = 0 K''$

and $\gamma'' = K'' 3^{n-1}$

$$N_{agen} = \frac{4 v_{av} \delta P}{\mu''} = \frac{4 v_{av} \delta P}{K'' \left(\frac{3 v_{av}}{\delta} \right)^{n-1}}$$

$$N_{agen} = \frac{4 v_{av}^{2-n} \delta^{n+1} P}{K'' 3^{n-1}} = \frac{4 v_{av}^{2-n} \delta^{n+1} P}{\gamma''}$$

from the definition of fanning friction factor

$$f = \frac{\tau_w}{\rho v_{av}^2 / 2} = \frac{2 \Delta P \delta / L}{\rho v_{av}^2} = \frac{2 \Delta P \delta}{L \rho v_{av}^2}$$

$$\mu, \delta = f \rho \frac{L}{2} \frac{v_{av}^2}{2}$$

So we established earlier that for Non-Newtonian liquids through a slit if the we established that μ is equals to $\Delta P \delta / L$ over $3 v$ average by δ , right?

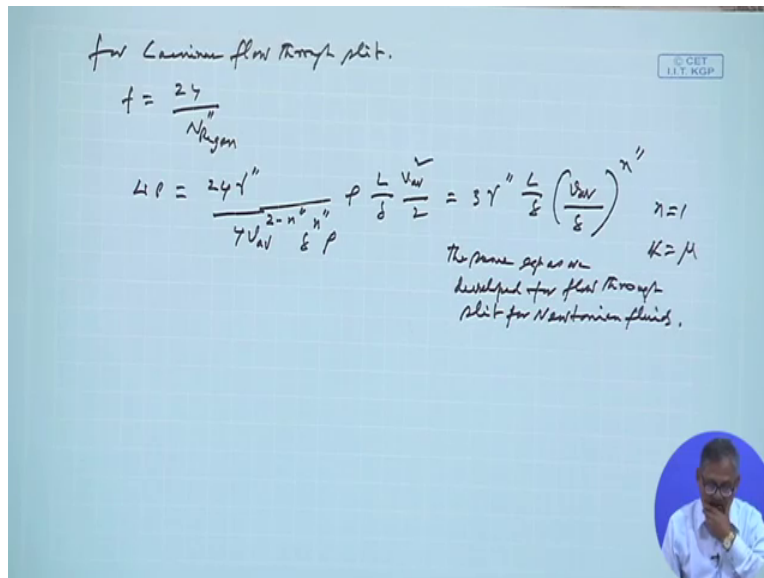
So keeping the same ratio we can write μ double prime ((9:39)) is equals to $\Delta P \delta / L$ over $3 v$ average over δ , right? So this is k double prime $3 v$ average over δ , right? This to the power n minus 1 and γ double prime is equals to k double prime 3 to the power n double prime minus 1 , right? So it should be since it is k double prime so v ((10:19)) should be double prime, right? So because n is equals to n prime is equals to n double prime so there is no change, but k is equals to something into k prime is something into k is equals to again another thing into k double prime.

So it is not that all the time this n has to be, but it is rightly that instead of n prime if we write n , then it may misleading that is why when k is k prime n is n prime when k is k , n is n sorry and when k is kept double prime n is also in the double prime that should be written, right? So if γ is so much, then N_{re} general we can write to be $4 v$ average δ rho divided by μ

double prime which is 4ν average into ΔP divided by $k \nu$ average by ΔP whole to the power n double prime minus 1, right?

So if this is true, then we can write N_{re} general is equals to 4ν average to the power $2 - n$ double prime ΔP to the power n double prime ρ by k to the power rather k double prime 3 to the power n double prime minus 1 that is equals to 4ν average to the power $2 - n$ double prime ΔP to the power n double prime ρ divided by γ double prime, right? So from the definition of fanning friction factor we can write f is equals to τ at the wall divided by $\rho \nu$ average square divided by 2 that is $2 \Delta P \Delta P$ by L by $\rho \nu$ average square, right? This is equals to $2 \Delta P \Delta P$ over $L \rho \nu$ average square, right?

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Or ΔP is equals to $f \rho L$ by ΔP ν average square divided by 2, right? Now for Laminar flow through slit we write f is equals to 24 by N_{re} general double prime, right? Then ΔP is equals to 24γ double prime by 4ν average to the power $2 - n$ double prime into ΔP to the power n double prime into ρL by ΔP , then ν average square by 2 so that is equals to 3γ double prime L by ΔP to the power ν average by ΔP to the power n double prime. Now this if we substitute n is equals to 1 and k is equals to μ , then we get the same equation as we developed for flow through slit for Newtonian fluids, right?

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Prob. A non Newtonian fluid say 50% concⁿ milk having $\rho = 1030 \text{ kg/m}^3$ is pumped through a slit, $x = 10 \text{ mm}$ & $y = 5 \text{ m}$, $L = 10 \text{ m}$

$k = 0.5 \text{ Pa}\cdot\text{s}^n$, $n = 0.6$, $N_{Re} = 500$, cp , v_{av} ?

Solⁿ. $\gamma'' = K'' \dot{\gamma}^{n-1} = K \left(\frac{2n+1}{3n} \right)^n \dot{\gamma}^{n-1} = 30 \times \left(\frac{2 \times 0.6 + 1}{3 \times 0.6} \right)^n$

$= 23.63 \text{ Pa}\cdot\text{s}^n$

$N_{Re} = 500$ $v_{av} = \left[\frac{N_{Re} \gamma''}{4 \dot{\gamma}^n} \right]^{\frac{1}{2-n}}$

$= \left[\frac{500 \times 23.63}{4 \times (5 \times 10^{-3})^{0.6} \times 1030} \right]^{\frac{1}{2-0.6}} = (68.88)^{\frac{1}{1.4}}$

$= 20.56 \text{ m/s}$

$\Delta P = 3 \gamma'' \frac{L}{S} \left(\frac{v_{av}}{b} \right)^n = 3 \times 23.63 \times \frac{10}{0.01/2} \left(\frac{20.56}{0.005} \right)^{0.6} = 208.956 \text{ Pa}$

So this is what we can say that this is okay, now if we do a similar problem we have done earlier that is say if we formulate the problem like this that Non-Newtonian fluid, right? Say 50 percent concentrated milk having density rho is equals to 1030 kg per meter cube, right? And is pumped through a slit that dimension of the slit is like this that dimension is like this x is equals to 10 millimeter and y is equals to 5 meter, right? Of a rectangular slit (18:02) through a rectangular slit that slit is x is 10 millimeter that is the width and the other I mean thickness and the other one is 5 meter, right?

And rest of the things are like this that the consistency coefficient k is equals to say 0.5 and milk it was say not 0.5 30 Pascal second to the power n and n is 0.6 so if the Reynolds number is Nre is 500, right? So what is the pressure drop and v average? So that is if we formulate the problem like this, then how can we proceed, right? So let us see the solution what can be the solution if you remember that what we did earlier a similar thing here gamma prime is equals to gamma double prime is equals to k double prime 3 to the power n double prime minus 1 is equals to k into 2 n plus 1 by 3 n to the power n into 3 to the power n minus 1. So this is equal to 30 into 2 into 0.6 plus 1 divided by 3 into 0.6, right? Divided by 3 to the power 3 n, right? So k is 30, okay n is 0.6, fine and this is so much so this to the power 1 minus 0.6 because 3 to the power n minus 1 so if goes divided by this then 3 to the power 1 minus n.

So this is like that, so this comes to equal to let us look into that calculator so it is $2 \times 0.6 + 1$ divided by 3×0.6 , right? Is so much divided by x to the power $1 - 0.6$ that is 0.4 that is 0.7875 , right? This into 30 is there, 23.63 , right? So much Pascal second to the power n , right? So this is γ that is 23.63 Pascal second to the power n , right? Now if γ is that, then for N_{re} general given is N_{re} general that is equals to 500 , right? So that is what we have given that for N_{re} general 500 so we can write $v_{average}$ is equal to N_{re} general double prime γ double prime by $4 \times \rho$ to the power n double prime ρ to the power $1 - n$ double prime, right?

So this we can write that this is equals to 500 N_{re} general γ double prime has been found 23.63 , right? And this is 4 into this Δ is given as 10 millimeter so 10 millimeter is this thickness, right? So it is $2 \times \Delta$, so only Δ is 5 millimeter, right? So that is 5 into 10 to the power minus 3 meter, right? So we can write that 4 into 5 into 10 to the power minus 3 to the power 0.6 into 1030 , right? So this is equal to if we look at this is equal to 500 into 23.63 $(\rho)^{(25:12)}$ is equal to divided by 4 divided by 5 into 10 to the power minus 3 to the power 0.6 is equal to this divided by 1030 68.889 so 9 68 . this is also there to the power $1 - 0.6$, right? 68.889 to the power $1 - 0.6$, so that is to the power y , right? 1 divided by $2 - 0.6$, right?

So this comes to 20.55 , right? So that came to be 20.55 say 56 meter per second, therefore ΔP can be written as ΔP is 3γ double prime L by $\Delta v_{average}$ by Δn double prime, so this is 3 into 23 . this is 23.63 that we found out as γ double prime 3 into 10 , right? This is 10 by 0.1 , L is that is length other side so L remains same this is y and L is equals to 10 meter, right? 10 divide by D , D is 0.01 , right? $2 \times \Delta$ so this is 0.01 divided by 2 , right? And $v_{average}$ is 20.56 divided by 0.005 , right? To the power 0.6 .

So this comes on calculation 208.95 bar, right? So this way we can find out that what is the pressure drop and velocity and we can do the similar thing in our system, right? So this we can do and say there is no time left so we tell you that thank you for this class.