

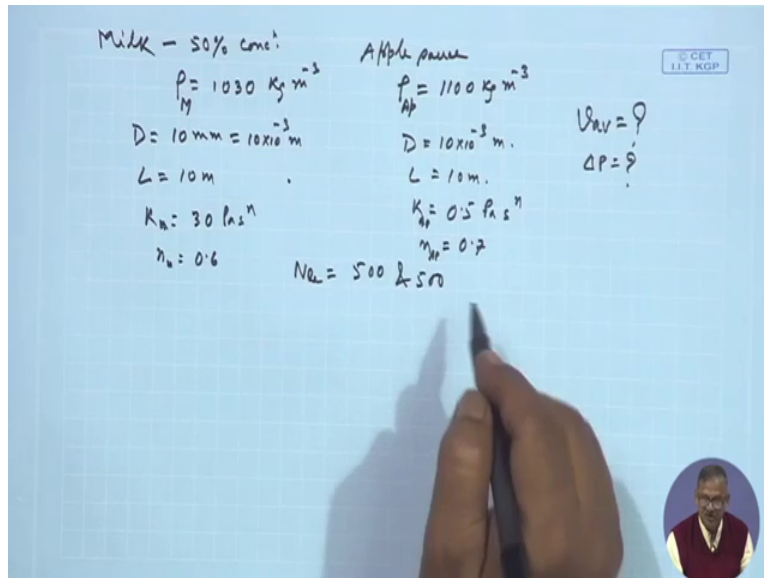
Course on Momentum Transfer in Process Engineering
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Lecture 44
Module 9
Problems and solution of Non Newtonian fluid-Part-2

Okay, I hope you have tried with at home that problem which was given, right? And if you could have solved it extremely well, but for all students who tried or could not do it for the benefit of the those students let us also do this in this class, right? So let us see what has been given and what we have to find out, right?

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Prob. Two non Newtonian fluids such as 50% concentrated milk having density of 1030 kg m^{-3} and applesauce having density of 1100 kg m^{-3} are being pumped through a pipe having a diameter of 10 mm and length of 10 m. Consistency and flow behavior indices for applesauce are assumed to be $0.5 \text{ Pa}\cdot\text{s}^n$ and 0.7 and those of concentrated milk are $30 \text{ Pa}\cdot\text{s}^n$ and 0.6 respectively. If the Reynolds numbers are 500 and 5000 for both the liquids, calculate the average velocities and pressure drops.





So for that we read that problem that two Non Newtonian fluids such as 50 percent concentrated milk so milk we have been given 50 percent concentrated and having a density of rho is equals to 1030 kg per meter cube, right? And applesauce having a density of 1100 kg per meter cube, right? So let us write rho milk and rho apple, right? Then, are being pumped through a pipe having a diameter 10 meter 10 millimeter. So diameter of the pipe is 10 millimeter or is equals to 10 into 10 to the power minus 3 meter, right?

So D is 10 into 10 to the power minus 3 meter, right? And the length of the pipe is L equals to 10 meter, consistency and flow behavior indices for applesauce are that is consistency coefficient is 0.5 Pascal second to the power n and flow behavior index n is equals to 0.7, right? And for milk this k say apple n apple k milk is equals to 30 Pascal second to the power n and flow behavior indices nm is 0.6 Reynolds number for the two cases are Reynolds numbers are 500 and 5000 for both the liquids, so Nre equals to 500 and 5000 for both the liquids. Calculate the average velocity, so what we have to find out is v average how much the pressure drop delta P is how much, right?

So if we again read out, two Non Newtonian fluids such as 50 percent concentrated milk that is one Non Newtonian fluid having density of 1030 kg per meter cube and applesauce having density of 1100 kg per meter cube are being pumped through a pipe having a diameter of 10 millimeter and length of 10 meter. Consistency and flow behavior indices for applesauce are assumed to be 0.5 Pascal second to the power n and 0.7 and those of concentrated milk are 30

Pascal second to the power n and 0.6 respectively. If the Reynolds numbers are 500 and 5000 for both the liquids, calculate the average velocities and pressure drops.

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$\rho = 1030 \text{ kg m}^{-3}$ $\rho = 1100 \text{ kg m}^{-3}$
 $D = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$ $D = 10 \times 10^{-3} \text{ m}$
 $L = 10 \text{ m}$ $L = 10 \text{ m}$
 $K_p = 30 \text{ Pa s}^n$ $K_p = 0.5 \text{ Pa s}^n$
 $\eta = 0.6$ $\eta = 0.7$

for concentrated milk generalized coefficient of viscosity $\gamma = K' B^{n'-1}$
 $\gamma = K \left(\frac{3n+1}{4n} \right)^n$ $\gamma = \frac{K'}{8^{1-n}}$ $K = K'$
 $\frac{30 \times \left(\frac{3 \times 0.6 + 1}{4 \times 0.6} \right)^{0.6}}{8^{1-0.6}} = 14.32 \text{ Pa s}^n$ $n = 0.6$

$N_{Re} = 500$; $V_w = \left(\frac{N_{Re} \gamma}{D^n \rho} \right)^{\frac{1}{2-n}} = \left(\frac{500 \times 14.32}{(10 \times 10^{-3})^{0.4} \times 1030} \right)^{\frac{1}{2-0.6}} = 28.75 \text{ m/s}$

$AP = 32 \sqrt{\frac{L}{D}} \left(\frac{V_w}{D} \right)^n = 32 \times 14.32 \times \frac{10}{1 \times 10^{-2}} \left(\frac{28.75}{1 \times 10^{-2}} \right)^{0.6} = 54484863 = 54484.86 \text{ bar}$

So this we have to find out, right? Now if we do this we have already write it down that what has been given and what we have to find out, right? And for first let us try for concentrated milk the generalized coefficient of viscosity gamma this is k to the power k prime 8 to the power n prime minus 1, right? So this is from the definition, now if we substitute k prime n prime with k and n, then we can write gamma is equals to k into 3 n plus 1 by 4 n to the power n and this is 8 to the power 1 minus n, right? Because this n prime minus 1 this can also be written as k prime divided by 8 to the power 1 minus n prime, right?

So that is what exactly we have written here, so we have k prime and k relation and n prime and n relation this two we have utilized which we had given you earlier, right? So it is in terms of k 3 n plus 1 by 4 k into 3 n plus 1 by 4 to the power n 8 to the power 1 minus n. Now given k for milk concentrated milk it is k is equals to 30, right? Into 3 n, n is given as 0.6, so 3 into 0.6 plus 1, right? Divided by 4 n 4 into 0.6, right? This to the power n that is 0.6 divided by 8 to the power 1 minus n 8 1 minus 0.6, right? So now let us see how much it comes, so it is 30 or 3 into 0.6 equal to so much plus 1 is equal to so much divided by 4 divided by 0.6 is equal to so much to the power 0.6 is equal to so much into 30 is equal to so much divided by 8 to the power 1 minus 0.6 so much is equal to so much 14.32, right?

This is for gamma, right? Now N_{re} general this is given as 500 so it is 14.32 Pascal second to the power n , right? So if N_{re} general is given for 500, then we can write the average velocity v average, right? This is equal to N_{re} general, right? Into gamma divided by D to the power n rho this to the power $1/2 - n$, right? So for that N_{re} general is 500 into this gamma has been found out to be 14.32, right? D has been found out to be 10 millimeter that is 10 into 10 to the power minus 3 whole to the power n , n is given as 0.6 and rho given is 1030, right?

So this whole to the power $1/2 - 0.6$, right? So let us also find out how much it is, right? So 500 into 14.32, right? Is equal to so much divided by 10 into 10 to the power minus 3 that is 1 into 10 to the power minus 2 that is 10 to the power minus 2 minus 2 that is here plus minus 2 whole to the power 0.6 divided by 1030, so x to the power $1/2 - 0.6$ 1 divided by 2 minus 0.6 not 8 0.6 so 1.4 this much so should be this much 28.749 or 75 so much meter per second, right?

So v average for concentrated milk is 28.75, right? So that is true, now let us also find out the (sa) for this we have to also find out not only this delta P , right? So delta P we can write $32 \gamma L$ by $D v$ average by D to the power n , right? Is equal to 32 into gamma we have found out 14.32 into L by D , right? L is 10 meter so into 10 divided by D is 10 millimeter so 1 into 10 to the power minus 2, right? Into v average we have just found out to be 28.75 over D is 1 10 to the power minus 2 whole to the power 0.6 n , right? So this comes equal to let us look into that 28.75 divided by 10 to the power minus 2 whole to power 0.6 into this is 10 by 10 to the power minus 2, so into 10 to the power minus 3 ((16:43) to the power 3, right? Into 10 to the power 3 into 14.32 into 32 is equal to 54484843, right?

So this is equal to 1, 2, 3, 4, 5, right? So equals to 554.84 kilo Pascal, no this is 1, 2, 3, 4, 1, 2, 3, 4, 5 so 554 so much Pascal, so it is 554 and okay divided by bar so this we can write 55484 bar, right? Because 1 bar is 101325 kilo Pascal or Pascal so that is roughly this 554.84 bar or is equals 544 1, 2, 3, 1, 2, 3 so 54484 kilo Pascal, right? So if we convert it into bar then it is around 554.84 bar, right?

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for Applesauce for $N_{re} = 5000$

$$v_{av} = \left(\frac{N_{re} \gamma}{D^{\rho \rho}} \right)^{\frac{1}{2-n}} = \left[\frac{5000 \times 14.32}{(1 \times 10^{-3})^{0.8} \times 11030} \right]^{\frac{1}{2-0.6}} = \left[1101.73 \right]^{\frac{1}{2-0.6}}$$

$$\Delta P = 32 \gamma \left(\frac{L}{D} \right) \left(\frac{v_{av}}{D} \right)^{\rho} = 148.9 \text{ m/s.}$$

$$= 32 \times 14.32 \times \frac{10}{1 \times 10^{-2}} \left(\frac{148.9}{1 \times 10^{-2}} \right)^{0.6}$$

$$= 14160.430 \text{ Pa}$$

$$= 14160.43 \text{ kPa}$$

$$= 176.26 \text{ bar.}$$

for Applesauce

$$\gamma = K \left(\frac{3n+1}{4n} \right)^{\rho}$$

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$$= 176.26 \text{ bar.}$$

for Applesauce

$$\gamma = K \left(\frac{3n+1}{4n} \right)^{\rho} = 0.5 \left(\frac{3 \times 0.2 + 1}{4 \times 0.2} \right)^{0.7} = 1.073 \checkmark$$

$N_{re} = 500$

$$v_{av} = \left[\frac{500 \times 1.073}{(1 \times 10^{-3})^{0.7} \times 110} \right]^{\frac{1}{2-0.7}} = 42.46 \text{ m/s}$$

$$\Delta P = 32 \times 1.073 \times \frac{10}{1 \times 10^{-2}} \left(\frac{42.46}{1 \times 10^{-2}} \right)^{0.7} = 12089.614 \text{ Pa}$$

$$= 12089.614 \text{ kPa}$$

$$= 120.896 \text{ bar.}$$

Now this is for milk we also have to do the same thing for the applesauce, so for applesauce the same thing we have to do for 500 and 5000, so for N_{re} general is equals to 5000 so v average is equals to N_{re} general γ by D to the power n rho to the power 1 by 2 minus n , so this is equals to 5000 into 14.32 divided by 10 or 1 into 10 to the power minus 2 to the power 0.6 into 1030 this becomes equals to 5000 into 14.32 divided by 1 into 10 to the power minus 2 to the power minus 0.6, so divided by 1 into 10 to the power minus 2 to the power 0.6 is equal to so much divided by 1030 is equal to 1101.73, right? So much meter per second, right? 1101.3, we have not done another thing 11.01 this to the power 1 by 2 minus n that is we have to do 1 by 2

minus n that is $1 \text{ by } 2 \text{ minus } 0.6$, right? So if we do this to the power x to the power y to the power $1 \text{ divided by } 2 \text{ minus } 0.6$, right? So this is done this is done so this is equals to 148.9 , so this is $148.9 \text{ meter per second}$, okay.

So ΔP then that comes equals to, right? ΔP that then comes equals to $32 \text{ gamma } L \text{ by } D \text{ v average by } D \text{ to the power } n$, so that is equals to 32 gamma we found out to be into 14.32 , right? Into L , L was 10 , right? And D was $1 \text{ into } 10 \text{ to the power minus } 2$ or $11 \text{ } 0 \text{ millimeter into } v$ average is $148.9 \text{ by } D \text{ is } 1 \text{ } 10 \text{ to the power minus } 2 \text{ to the power } n$, right? n is 0.6 , right? So if we do this in the calculator this becomes $148.9 \text{ divided by } 10 \text{ to the power minus } 2$, right? Whole to the power 0.6 , right? Into this is $\text{minus } 2$ and this is $1 \text{ minus plus } 3$, so into $10 \text{ x to the power } 3$ into 14.32 is equal to into 32 , right? 14160432 is equal to 14160430 so much Pascal is equal to $14160.43 \text{ kilo Pascal}$ is equal sorry $146160430 \text{ } 146160430$.

So this so much 1461 or 2 say rounded of 1462 bar , right? So this is what is the ΔP . Now we need to also find out this is for milk, so for applesauce we can write that gamma is equals to $k \text{ into } 3 \text{ n plus } 1 \text{ by } 4 \text{ n to the power } n$ is equals to for applesauce we had if you remember we had ρ is 1100 D remain same, n remain same, k is 0.5 n is 0.7 , right? So k is $0.5 \text{ into } 3 \text{ into } 0.7 \text{ plus } 1 \text{ by } 4 \text{ into } 0.7 \text{ to the power } 0.7$ this much equals to we write $3 \text{ into } 0.7$ is equal to so much plus 1 yeah equal to this divided by $4 \text{ into } 0.7$, right? Is equal to this whole to the power 0.7 is equal to 1.07 , so earlier it was 14 . now it is 1.073 because earlier value of k was 30 now the value of k is 0.5 .

So if gamma is so much, so for given v general N_{re} general equals to 500 so v average is equals to we can write that $500 \text{ into } 1.073 \text{ divided by } 1 \text{ into } 10 \text{ to the power minus } 2$ whole to the power 0.6 so here it is $0.7 \text{ into } 1100$, right? 1100 whole to the power $1 \text{ by } 2 \text{ minus } 0.7$, so if we do it under calculator it becomes $500 \text{ into } 1.073$, right? So much divided by 1100 is equals to so much I hope we have done rightly divided by $10 \text{ to the power minus } 2 \text{ to the power } 0.7$ equal to this whole to the power $1 \text{ by } 2 \text{ minus } 0.7$ 43 . So $43.46 \text{ meter per second}$ is the average velocity which was 28.75 for that, so gamma value is less, there gamma value was more so that is why viscosity was less there and viscosity velocity is less there and velocity is more there, right?

This is for gamma and earlier gamma value was 28.75 so viscosity was viscosity was 28.75 gamma was 14.32 , right? So for 500 this we have done, now ΔP is equals to $32 \text{ into } \text{gamma}$,

gamma is found out to be 1.073, right? Into L by D is 10 by 1 into 10 to the power minus 2 into v average that is 43.46 divided by D that is 1 into 10 to the power minus 2 whole to the power 0.7, right? So this is equals to let us do quickly the this time is very short, so 43.46 divided by 10 to the power minus 2 sorry 43.46 divided by 10 x to the power 2 minus equal to this much to the power 0.7 is this much into this 10 and this so 1000, right? Into 1.073 can we additionally did 10, so into 32 is equals to 12089615.9 or 16 we can rounded of so much Pascal is 12089.616 kilo Pascal is equal to 120.89 bar. So one we have done for 500 both velocity and delta P so you please find out the velocity and delta P for 5000 Nre general, okay time is out thank you.