Course on Momentum Transfer in Process Engineering
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Lecture 42
Module 9
Average velocity for Non Newtonian fluid

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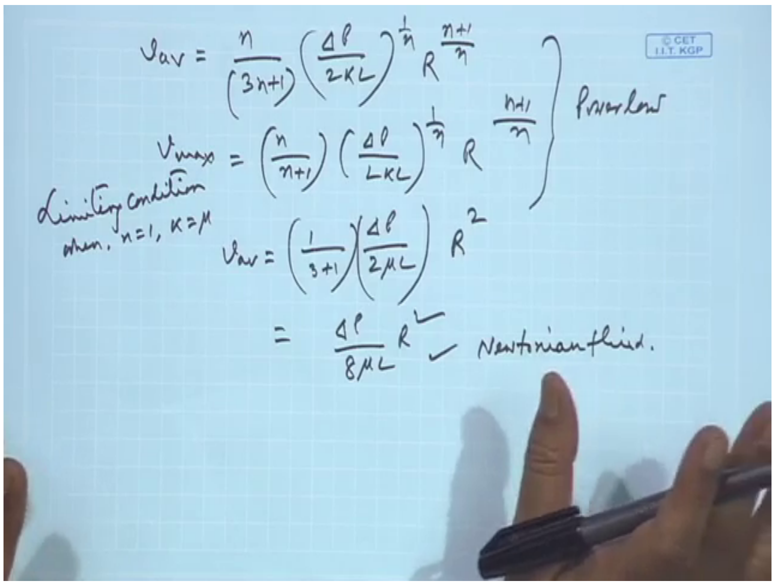

So we have done in the previous class that average velocity that came out to be n by 3 n plus 1 into delta p by 2kL to the power 1 by n R to the power n plus 1 by n, right? This we saw, also we saw V max, right? V max was equal to just a minute V max was equal to n by n plus 1 into delta p by 2kL to the power 1 by n and R to the power n plus 1 by n, right? This is true for any fluid following the power law equation, right? And we said for V max that if we add an limiting condition v for V max we saw that when n becomes equals to 1 and k becomes equals to mu, then under limiting condition this has become the same expression as we obtained for Newtonian fluid flowing through a conduit or pipe, right?

Having radius r, this we had established. Now let us also look into that limiting situation, so under limiting condition when n becomes equals to 1 and k becomes equals to mu the average velocity is 1 by 3 plus 1, right? Divided by delta p by 2 mu L to the power 1 by n is 1 R to the power n plus 1 that is 1 plus 1 2 by 1 R square, right? So this we can write is delta p by 2 into 3 plus 1 4, 8 8 mu L R square, right? If you see this is the same as that of the Newtonian fluid. So this is true for Newtonian fluid, right? So that means the thing which we have developed which we have derived is correct because it is also having the same value when we are putting limiting condition, right?

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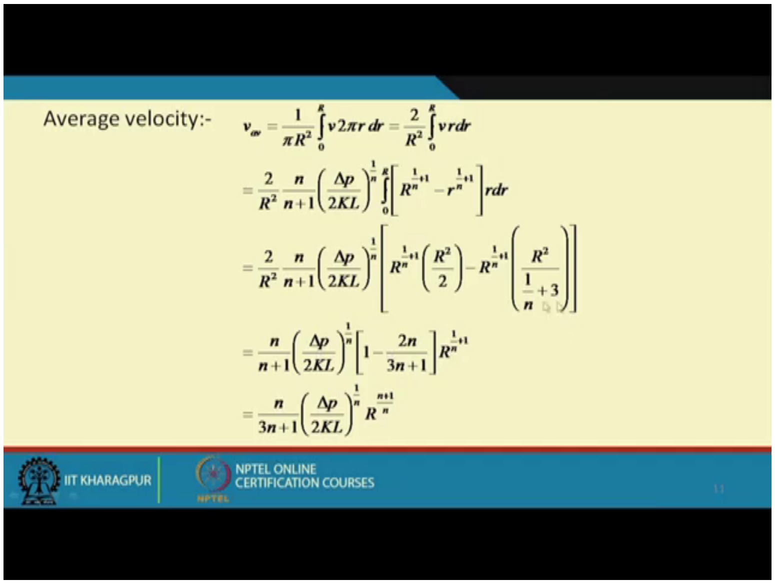

So let us look into that our slide also because this you will get you can see online that v average is 1 by 2 pi r square 0 to r v into 2 pi r we have done all in detail we take every steps, but here we might have omitted some, so v into 2 pi r into dr so that becomes 2 by r square 0 to r vr dr substituting the values of v, it becomes 2 by r square n by n plus 1 delta p by 2kL to the power 1 by n the integration between 0 to r R to the power 1 by n plus 1 minus R to the power 1 by n plus 1 times r dr, so this becomes 2 by R square into n by n plus 1 into delta p by 2kL to the power 1 by n times capital R to the power 1 by n plus 1 times R square by 2 minus capital R to the power 1 by n plus 1 times R square by 1 by n plus 3, right?

So this on simplification becomes n by n plus 1 into delta p by 2kL to the power 1 by n into 1 minus 2 n by 3 n plus 1 into R to the power 1 by n plus 1, so that becomes n by 3 n plus 1 into delta p by 2kL whole to the power 1 by n, right? Whole to the power 1 by n into R to the power n plus n by n, right? So this is the average velocity, right?

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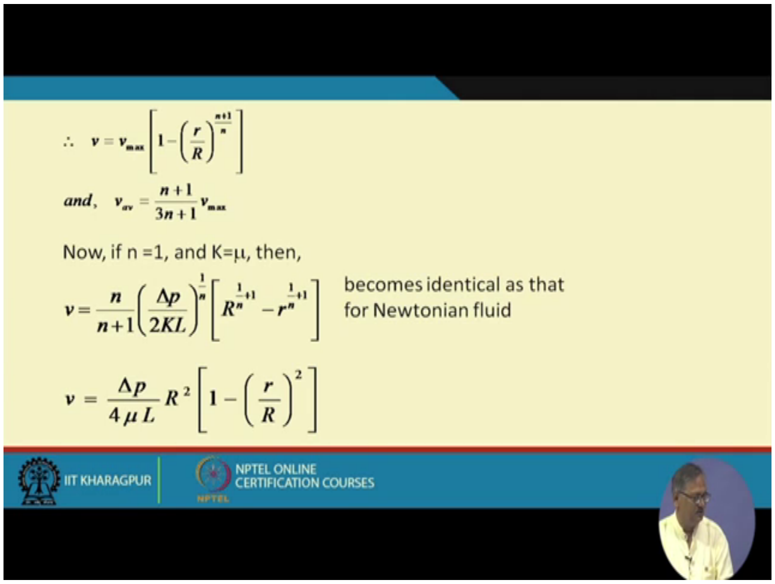

And we also have seen v is V max that was 1 minus r by R to the power n plus 1, v was V max that is this V max was n by n plus 1 delta pl by LkL, right? And v we had shown that v was one second v was yes v was this, right? n by n plus 1 delta p by 2kL or R to the power n plus 1 by n into 1 minus r by R to the power n plus 1, right?

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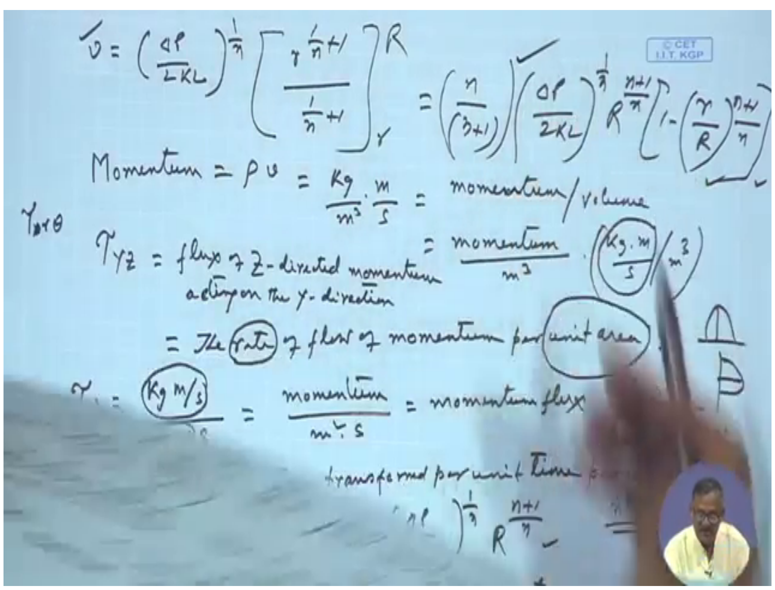

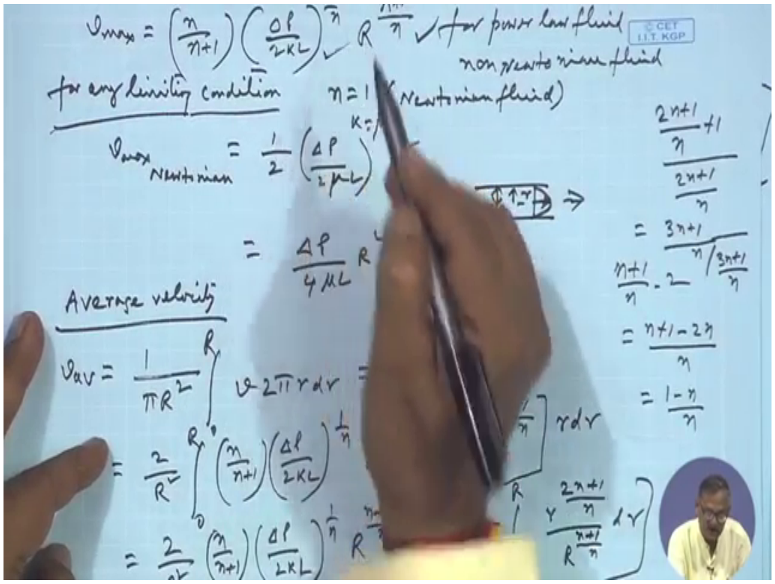

If this is true, then we can write that v is equals to and V max we have shown that V max was this is the average or this is the instantaneous velocity, this was v is V max, right?

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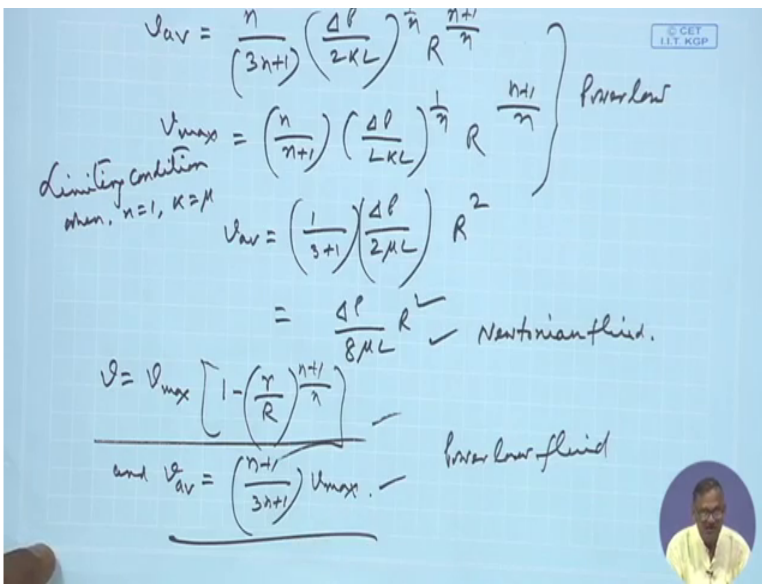

So this is V max so we can now write that v is equals to V max we can now write v is equals to V max into 1 minus r by R to the power n plus 1 by n, right? And also we can write v average is equals to n plus 1 by 3 n plus 1 into V max, right? This we can write for any power law fluid, right? So for power law fluid we saw v is V max into 1 minus r by R to the power n plus 1 and v average is n plus 1 by 3 n plus 1 into V max, right?

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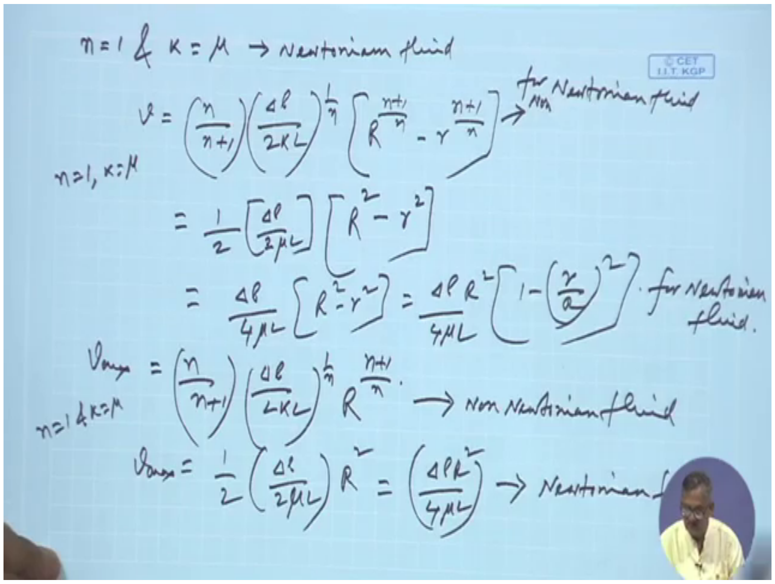

Now as we said that under limiting condition here also we put if n is equals to 1 and k is equals to mu, right? That is that Newtonian fluid it becomes Newtonian fluid form Non-Newtonian n is equals to 1 and k is equals to mu, we can then write v is equals to n by n plus 1 into delta p by this we have also shown 2kL to the power 1 by n into R to the power n plus 1 by n minus small r to the power n plus 1 by n, right?

So it becomes the same for Newtonian fluid of course this is for Non-Newtonian, otherwise you will be misled so this is for Non-Newtonian. So this becomes if we put n is equals to 1 and k is equals to mu in this, then n is equals to 1 that is 1 by 2 this is delta p by 2 mu L this to the power 1 and this is R to the power n is 1, 1 plus 1 by n that is 2 R square minus r to the power n plus 1 by n r square, right? So that means this is delta p by 4 mu L into R square minus r square that is equals to delta p by 4 mu L into R square into 1 minus r by R whole square so for Newtonian fluid which we had developed, right?

So this is v and similarly we can also say that V max is equals to n by n plus 1 into delta p by 2kL to the power 1 by n into R to the power n plus 1 by n this was for V max when and this is for Non-Newtonian fluid and when we are writing n is equals to 1 and k is equals to mu, then V max becomes equals to this is 1 by 2 this is delta p by 2 mu L, right? To the power 1 and this is R to the power square, right? So V max is then equals to delta p by 4 mu L R square, right? And this is for Newtonian fluid under limiting condition, right?

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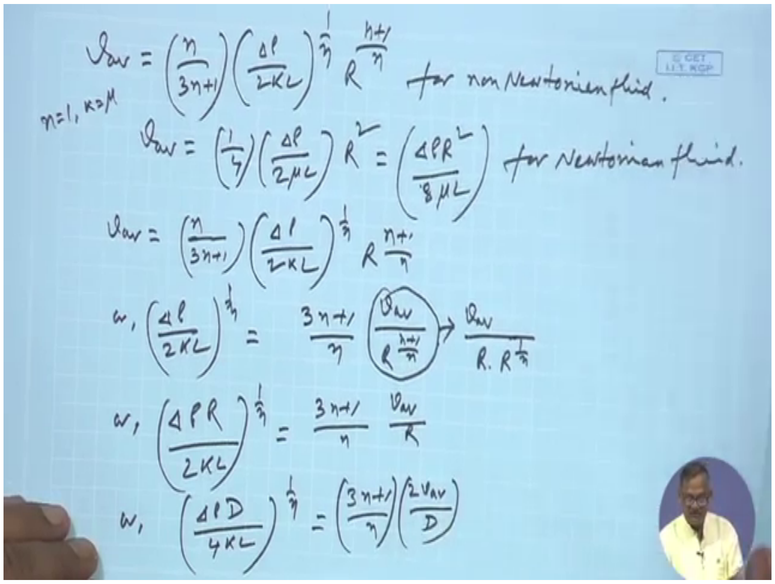

And average velocity v average we have seen it is n by 3 n plus 1 into delta p by 2kL whole to the power 1 by n R to the power 1 by n or n plus 1 by n, right? This is for Non-Newtonian fluid, now if we put the limiting condition n is equals to 1 and k is equals to mu, then we can write average v average is this is 1 by 4, right? Into delta p by 2 mu L this is 1 R to the 1 plus 1 by n so R square, so that means this is delta p R square by 8 mu L, right? So this is for Newtonian fluid. For all Newtonian fluids for all these expressions we had earlier, if you do a back track you can definitely get it that we have done these things for the Newtonian fluids, right?

So that means the expressions are correct under limiting conditions they are showing that what is the value for the Newtonian fluid when limiting condition is n is 1 and k becomes mu for the expression developed for power law fluids that is Non-Newtonian fluids, right? So when Non-Newtonian fluid becomes Newtonian, then the expressions which we developed earlier for instantaneous velocity for maximum velocity and for the average velocity the expressions are identical, right?

This concludes that the derivations which we had done are true, right? So this you try. Now we say that the average velocity equation we can write that v average is equals to n by 3 n plus 1, right? Into delta p by 2kL to the power 1 by n R to the power n plus 1 by n, right? So this we can rewrite delta p by 2kL this to the power 1 by n, right? This is equals to 3 n plus 1 by n into v average by R to the power n plus 1 by n, right? This also we can rewrite as delta p by delta p R okay by 2kL to the power 1 by n is equals to 3 n plus 1 by n, right? Into this means this term we can rewrite as v average, right? Divided by R into R to the power 1 by n, right? This we can rewrite as R into R to the power 1 by n. So when we have taken out this 1 by n here, so remaining is 3 n plus 1 by n v average by R, right?

So that becomes this, so this we can rewrite or delta p r, r is say R by 2 or it becomes 2 if we write in terms of D that is R by 2 that is 4kL whole to the power 1 by n this is equals to 3 n plus 1 by n and if we convert it to R by 2 is D that is 2 v average divided by D, right? This is true.

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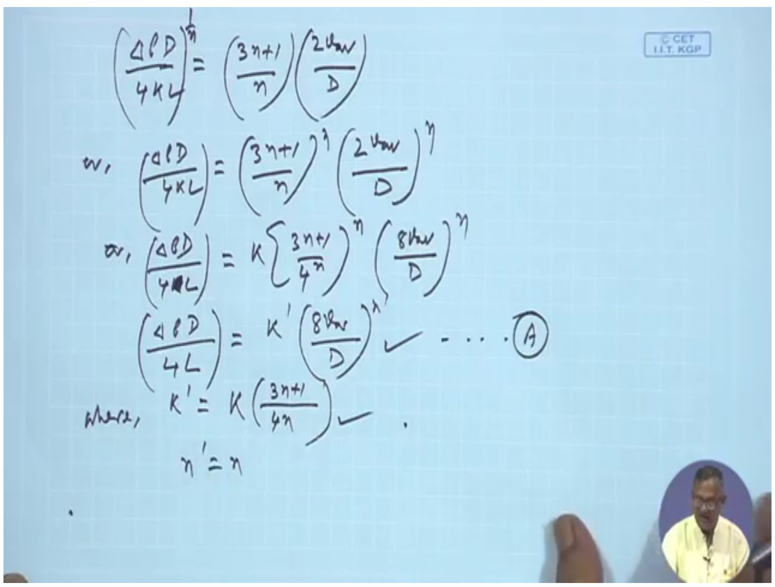

Now this we can rewrite as delta p D by 4kL this is equals to 3 n plus 1 by n this 1 by n, okay let us write again otherwise it is out of that thing this is this was 2 v average over D, right? So if we write delta p D by 4kL this is equals to 3 n plus 1 by n whole to the power n because this when goes there is inverse, so to the power n and this also becomes equals to 2 v average by D to the power n, right? So we can write delta p D by 4kL, right? This is equals to delta p D by 4L so if we take k out, k is here into 3 n plus 1 by n to the power n into this if we write like this okay if we write like this 4 n, right? If we can write 3 n plus by 4 n so here we can write 8 v average divided by D to the power n because this 4 to the power n we have put in the denominator so we have put here in the numerator, so 2 into 4 8 v average by D, right?

So this we can rewrite that delta p D by 4L this is equals to a new constant k prime into 8 v average by D to the power n prime, right? Where, k prime is equals to k into 3 n plus 1 divided by 4 n and n prime is equals to n, right? So if this is true, then we can write delta p D by 4L this is equals to like that, okay. k prime new constant k prime is k into 3 n plus 1 by 4 n and n is equals to n prime, right? Now in this equation that is delta p D by 4L is equals to k prime 8 v average D prime, right? So say this is equation A, right?

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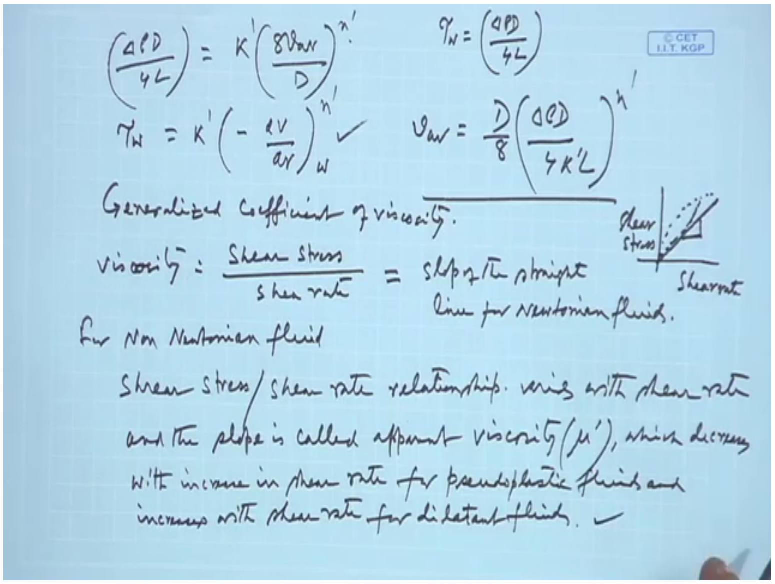

So now in equation A so it was delta p D by 4L this is equals to k prime 8 v average by D to the power n prime, right? So if this equation we rewrite that this is nothing but tau w, right? Because this tau at the valve is delta p D by 4L, right? Tau w that is tau at the valve is delta p D by 4L, right? So tau w that becomes equals to k prime into that okay if we say that this into minus dv dr to the power n prime at the valve, right? dv dr at the valve this we have shown earlier, right? This we have shown earlier also in earlier expression if you go back trace that tau w that is shear stress at the valve is delta p D by 4L and that is equals to k prime and dv dr negative at the valve that becomes equals to 8 v average by D, right?

So we can write where v average is equals to D by 8 delta p D by 4 k prime L whole to the power n prime, right? So v average we can rewrite D by D, D by 8 delta p capital D by 4 k prime L, right? So this is called generalized coefficient of viscosity generalized coefficient of viscosity, right? And this now viscosity means shear stress by shear rate, right? So shear stress by shear rate is the viscosity, so this is equals to the slope of the straight line for Newtonian liquids if we had a plot this is shear rate and this is shear stress so this was the Newtonian fluid and that is the slope of the line is this, right?

Slope of the line is that slope of the straight line for Newtonian fluids, right? For Non-Newtonian fluids we can say that shear stress over shear rate this relationship varies with shear rate, right? This relationship varies with shear rate, and the slope is called apparent viscosity or this can be say termed as mu prime, right? Which decreases with increase in shear rate for pseudo plastic liquids and increases with shear rate for dilatant fluids, right? So if you remember it was like this for pseudo plastic and for dilatant it was like this, right?

So if we say that for dilatant fluids with the increase in shear rate the increase in the fluid increases the shear rate for shear stress is to shear rate this relationship varies with shear rate and the slope is called apparent viscosity which decreases that is apparent viscosity decreases with increase in shear rate for pseudo plastic fluids and increases with shear rate for dilatant fluids, right? So this is true, so till now this we have done and apparent viscosity we have shown and next time we will also talk about the apparent viscosity and the Reynolds number of the Non-Newtonian fluids, right? Thank you.