Course on Momentum Transfer in Process Engineering
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Lecture 41
Module 9
Velocity profile for Non Newtonian fluid

Hello, you remember in the last class we were discussing about the Non Newtonian fluid, right? And we had also developed velocity at any instance the expression for the velocity that we had done, right?

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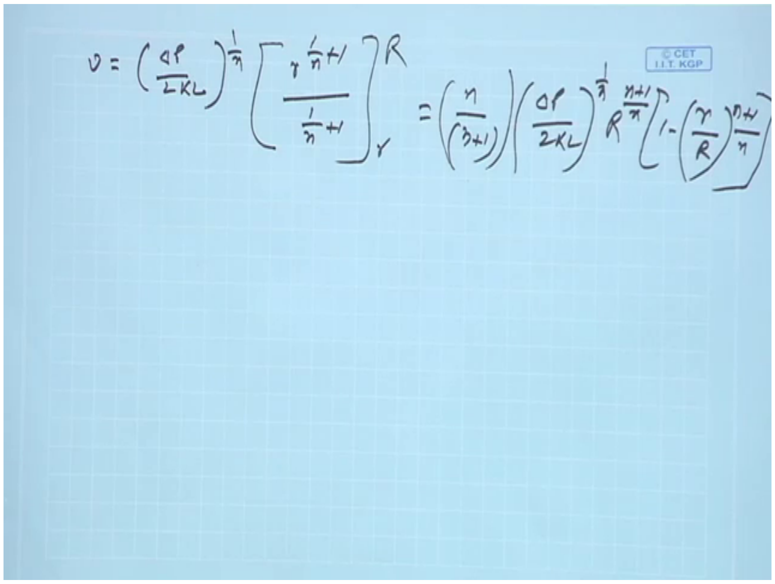

Now if we go to that, yeah this we had developed that velocity was at any instance v was delta p, right? By 2kL, right? To the power 1 by n, right? And r to the power 1 by n plus 1 divided by 1 by n plus 1 within that limit of r to R and this was n by n plus 1, right? Into delta p by 2kL to the power 1 by n and R to the power n plus 1 by n into 1 minus r by R to the power n plus 1 by n, right? This we had developed in the previous class, right?

So this is the instantaneous velocity, now I would like to go back to a little as recapitulation because I got some information from the students side that if I explain a little more about the momentum relation between momentum and the shear stress, right?

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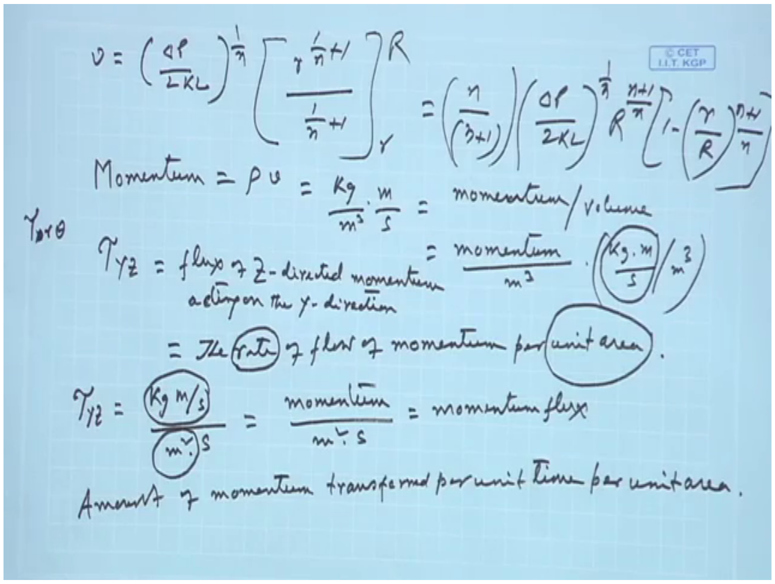

So a relation between momentum and shear stress if we look at the definition momentum we said is equals to rho v, right? rho into v, right? So rho v if this is the momentum, then this is rho is kg per meter cube into meter per second, right? So this is called momentum per unit volume, right? Or can also write as momentum, right? Per meter cube, right? Where this is kg meter per second per meter cube, right?

So this is momentum, now this also be a recapitulation that shear stress when we write we write with the plane, right? Two dimensions rather two coordinates that is tau yz or tau xr or rather r theta, right? Whatever there, right? Depending on the coordinate system you are employing. Now this tau yz this we can interpret as this is equals to flux of z directed momentum acting on the y direction, right? z directed momentum acting on the y direction, right? This can also we written as the rate of flow of momentum per unit area, right? This can also be written as this and unit of momentum obviously we have said that unit of momentum is kg meter per second, yes kg meter per second.

And shear stress then this can written as tau yz is equals to kg meter per second per meter square into second, right? The moment rate is there that is per unit time, right? Any rate is per unit time and per unit area we have said, so this is per unit area and this is the momentum. So this we can say momentum per meter square per second, right? That is why this is called momentum flux, right? So any flux is that thing per unit area per unit time that is the flux of that thing if it is momentum, momentum, if it is heat, heat, if it is mass, mass whatever it be that flux of that thing means per unit time per unit area is the flux of that thing, right?

So this is true whether it is for momentum or for heat or for mass, so depending on what variable you are taking that will be the flux of that thing that per unit time per unit area. So and rate means anything per unit time is the rate, right? So how you are at what rate you are moving is as it is being said in terms of velocity that is meter per second. So this is the rate at which you are moving, right? Your velocity is like that, similarly any other thing per unit time is the rate of that thing right? So this we can say that this gives us an amount this is what is the amount of momentum transferred per unit time per unit area, right? So this we have done a recapitulation, right? Whenever this comes into so please bring to our notice we will definitely bring back again and we will try to explain as much as we can, right?

Now from this expression that v is which we developed v is equals to delta n by n plus 1 delta p by 2kL to the power 1 by n R to the power n plus 1 by n into 1 minus r by R to the power n plus 1 by n. So this tells that the velocity profile is something like parabolic, right?

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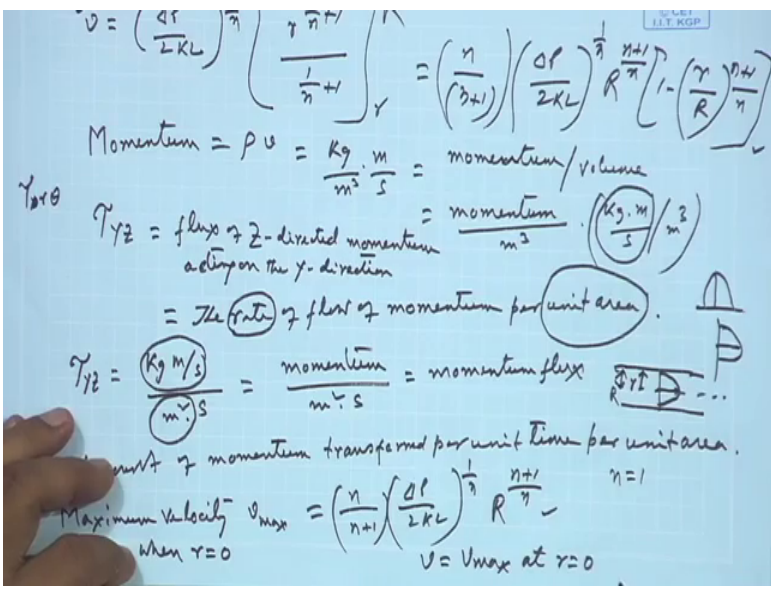

So velocity profile is something like parabolic, so that parabolic in nature depending on in which direction you are taking so it can be like this, it can be like this. So we need to know that means it has a maximum, right?

So if we again draw that pipe this is the pipe this is the diameter this is the axis, right? Then the velocity profile looks like this, right? That means this 0 at the surface and maximum at the center, right? So if this is r, right? If this is r, then and if this is capital R, right? So the maximum velocity we can write this to be V max, right? And that is true when r is equals to 0. So in this expression if we put r is equals to 0, then v becomes V max and then we can write n by n plus 1, right? Into delta p by 2kL this remains unchanged 1 by n, right?

And since this is 0, right? So this is 1, so that means into R to the power n plus 1 by n. So the maximum velocity is this, right? So v is equals to V max that means v is equals to V max at r is equals to 0, right? So this we know and we have seen earlier in pipe flow also. Now here also we can see we had told you that limiting condition, right? This is the Non Newtonian, right? And now if we put n is equals to 1 that is if it is Newtonian, then it becomes 1 by 1 plus 1, okay let me write in another page.

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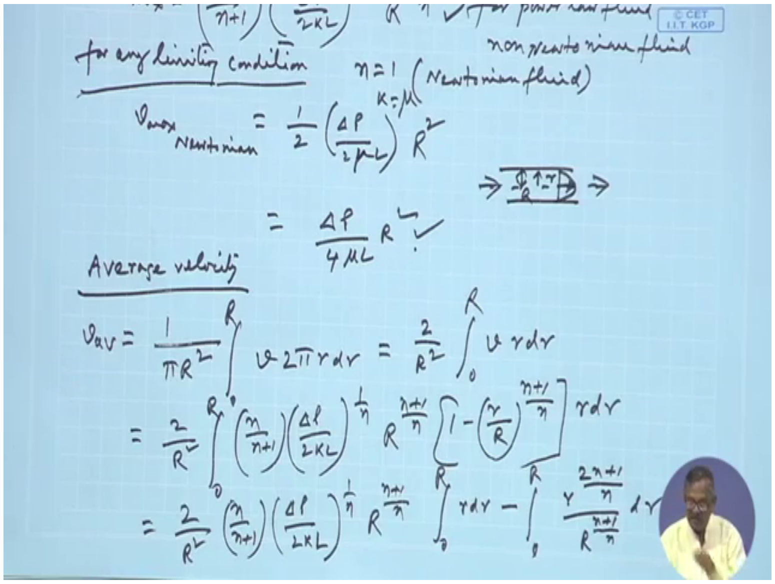

So V max equal to n by n plus 1, right? Into delta p by 2kL, right? To the power 1 by n R to the power n plus 1 by n, right? This is for any power law fluid or conventional we call Non Newtonian for any Non Newtonian fluid, right? So if this be true, then we said that for an limiting condition if this holds good, right? So what can be the limiting condition? Limiting condition is n is equals to 1 that is Newtonian fluid that is for Newtonian fluid. So V max we can write for Newtonian fluid here we write Newtonian that is equals to n is 1, so 1 by 2, right? Delta p by 2 k is okay here n is 1 and k is equals to mu, right?

So delta p by 2 mu L, right? 1 by n is equals to 1 R to the power n plus 1 by n that is n is 1 R square, right? So this is nothing but delta p by 4 mu L R square, so that is corresponding to the Newtonian fluid flowing through a pipe this we have done earlier so this is r and this is capital R, right? And flow is taking place in this direction and velocity profile is like this and the maximum velocity is this, this we have done earlier and you can check that the one which we have said this for power law fluid that is fluid following the power law where in can be any value which earlier we said n is what for what kind of fluid and generally it is called Non Newtonian fluids, right? And typically it is Newtonian when n becomes 1 and k becomes mu. So under that situation this turns out to be the expression developed for Newtonian fluid flowing through the pipe, right? That means the development or the proceedings which have we have done is correct and it is following the limiting condition, right? Very good.

Now we look into when we have seen velocity at any instance at any point that is v, right? Instantaneous velocity, so we have also seen the maximum velocity that is at r is equals to 0, right? Now next is what is the average velocity, right? What is the average velocity, right? Now average if you remember we had said that for average velocity you need 1 by area that is pi R square, right? And 0 to R so we had shown earlier in many times number of times that how area and how the velocity they are both together is coming into picture, right?

So here we are taking directly 1 by pi R square that is the area, so 0 to R and this is v 2 pi r dr, right? So that is there, so this means we can write this 2 pi r, right? dr so 2 comes out pi goes out, so it is 2 by R square, right? 2 by R square we can write 0 to R, right? v r dr, okay now this v if we substitute with okay vr dr, right? Now substituting the values of v we can write 0 to R the value of v was this, right? So if we substitute this value of v that it is n by n plus 1, right? Delta p by 2kL to the power 1 by n R to the power n plus 1 by n, right? Times it was 1 minus r by R to the power n plus 1 by n, right? Times this is v already 2 we have taken out, pi is out into r dr, right?

So this on simplification we or on integration, okay before that again we simplify a little 0 to 2 by R square this is n by n plus 1 because that is also not within the integration or we can call to be constant 2kL to the power 1 by n and R to the power n plus 1 by n and remaining is 0 to R, right? This is r dr minus 0 to R r into r to the power n plus 1 by n that is 1, so 2 n plus 1. So r to the power 2 n plus 1 by n divided by R to the power n plus 1 by n dr, right?

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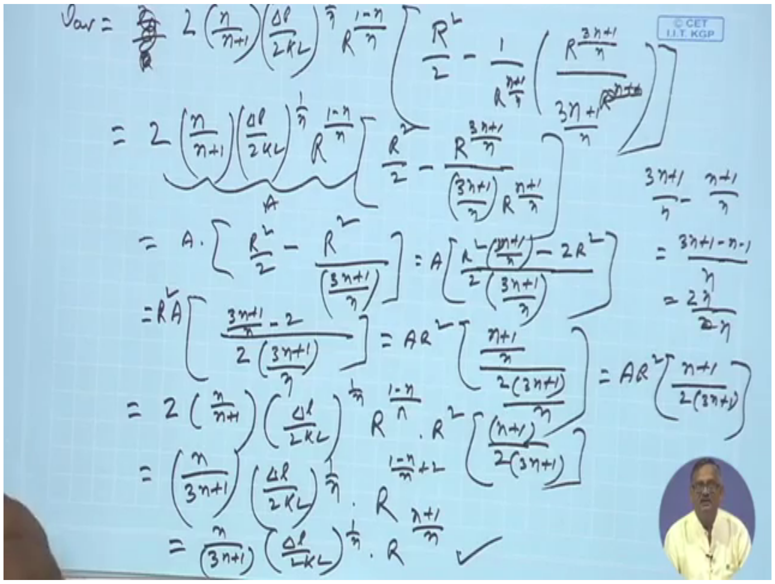

So this on simplification we can write that this is equal to so this r if it goes okay this is beyond, so if this R square goes with this so it is R to the power minus 2, right? So minus 2 n, right? So that is 1 minus n, right? By n R to the power 1 minus n by n. So we can write v average is equals to 2 by R square or okay 2 into n by n plus 1, right? Delta p by 2kL to the power 1 by n and this R becomes 1 minus n by n because this again this R square goes in up as R to the power minus 2, so n plus 1 by n minus 2 so n plus 1 by n minus 2 this is equal to n plus 1 minus 2 n divided by n is equal to 1 minus n by n, right?

So this is R to the power 1 minus n by n, right? And here we write this is under bracket, okay that should have been. So it is r dr with a boundary of 0 to R so R square by 2 that means R square by 2, right? Minus this is again r that is 1 by R to the power n plus 1 by n this remains common and we write at R is equals to r this is R to the power 2 n plus 1 by n again 2 n plus 1 by n plus 1 divided by 2 n plus 1 by n, that means this becomes equal to 2 n plus n so 3 n plus 1 divided by n divided by 3 n plus 1 by n, right?

So we can write this to be equals to R to the power 3 n plus 1 by n, right? Divided by R to the power n plus 1 this we have already taken out, right? Okay, R to the power this we have taken out, so 3 n plus 1 by n, right? So if we now rewrite this is equals to 2 into n by n plus 1 delta p by 2kL to the power 1 by n R to the power 1 minus n by n and let us rewrite R square by 2 minus 1 by or we write R to the power 3 n plus 1 by n divided by 3 n plus 1 by n, right? 3 n plus 1 by n into r to the power n plus 1 by n, right? So on simplification we can write this term remains say same A, A into so this we can write so R to the power 3 n plus 1 by n minus n plus 1 by n this becomes n and 3 n plus 1 minus n minus 1, so this becomes 2n by n, right?

So 1, 1 goes out so this 1 2 n, right? So 2 n that is R, so here it is R square by 2 minus this is R to the power 2 n by n so that is R square again by 3 n plus 1 by n, right? So if we again simplify this is A into R square by 2, right? So this is 2 into 3 n plus 1 by n, right? So square into 3n plus 1 by n this minus this is 2 R square, right? This is equals to A times, right? On the denominator 2 3n plus 1 divided by n on the numerator if we take r square as common, then we can write R square into A, R square we take common then it becomes 3 n plus 1 by n minus 2, right? So we can write A R square into this becomes 2 n 3 n minus 2 n that is n plus 1 by n whole divided by 2 into 3 n plus 1 divided by n.

So this n that n goes out, so ultimately it comes equal to A R square into n plus 1 by 2 into 3 n plus 1, right? Now if we substitute the value of A which we were doing that is 2 into n by n plus 1 into delta p by 2kL to the power 1 by n R to the power 1 minus n by n, right? Into R square, right? A square into n plus 1 by 2 into 3 n plus 1. So this is that so which we can write this 2 this 2 goes out, right? This n plus 1, this n plus 1 goes out. So n by 3 n plus 1 this becomes that this is delta p by 2kL to the power 1 by n and R to the power 1 minus n by n plus 2, so that means this is equal to n by 3 n plus 1 into delta p by 2kL to the power 1 by n into R to the power n this is 2 n minus n that is n plus 1 by n, right?

So if you see that the final average velocity is like that, okay. Now time is out, thank you.