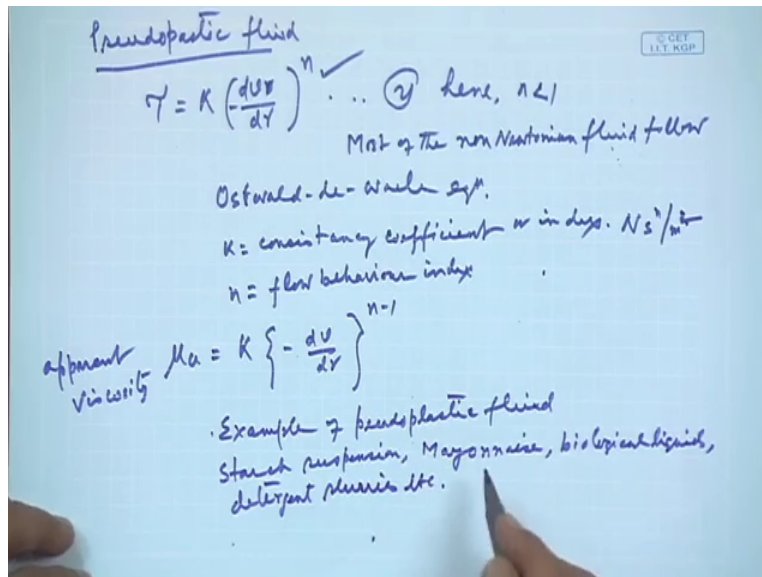


Course on Momentum Transfer in Process Engineering
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Lecture 40
Module 8
Non Newtonian fluid flow part-2

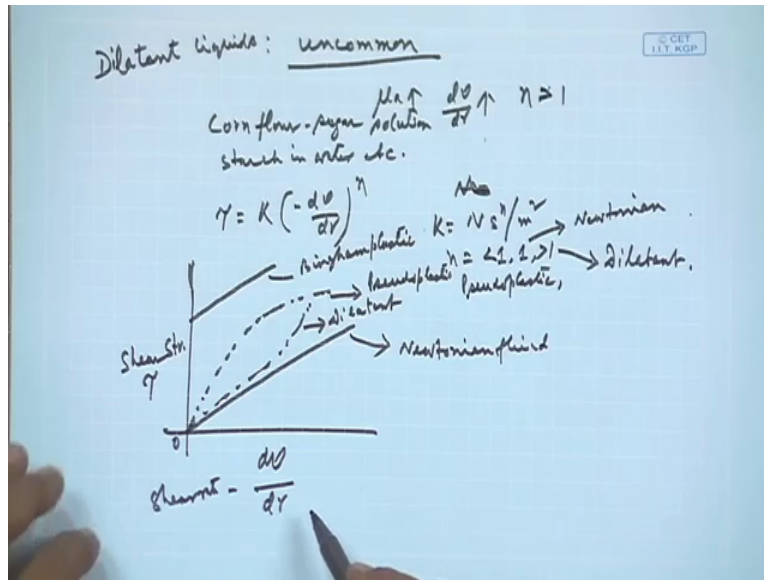
Yeah, so we said in the previous class that what is shear stress, how shear stress is related to shear rate for Newtonian or Non-Newtonian fluid, right? And we came up to the pseudo plastic fluid, right?

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Now, example of pseudo plastic fluids are starch suspension, mayonnaise, biological liquids, detergents slurries etcetera, right? So this follow this Non-Newtonian behavior where n is less than 1, right?

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So we will plot all of them in one so now the third one is the dilatant type, right? As we said the third one is dilatant fluid, right? So this is very much uncommon this type of liquid exhibit and increase in apparent viscosity with increase in shear rate, right? So μ_a increases as shear rate increases, right? So this is very very uncommon and the constant n of the power law equation is not 1 is not less than 1 is greater than 1, right?

So example of this kind of liquid is corn flour, sugar solution one starch in water etcetera very very uncommon kind of fluid this is the dilatant one where n is greater than 1 in the power law equation you remember we had said the power law equation was k into minus dv/dr to the power n where k was consistency coefficient and n was flow behavior index, right? And the unit of this k was Newton second to the power n per meter square, right?

So this was the unit of k and n is constant it varies between less than 1, 1 and greater than 1, right? So these three so this is when less than 1 is pseudo plastic is equal to 1 is Newtonian and greater than 1 is dilatant, right? So if we now plot them and see that if we plot shear stress versus shear rate, than dv/dr of course with a negative is shear stress that is τ , right? For Newtonian fluid it follows like this if this be the 0, right?

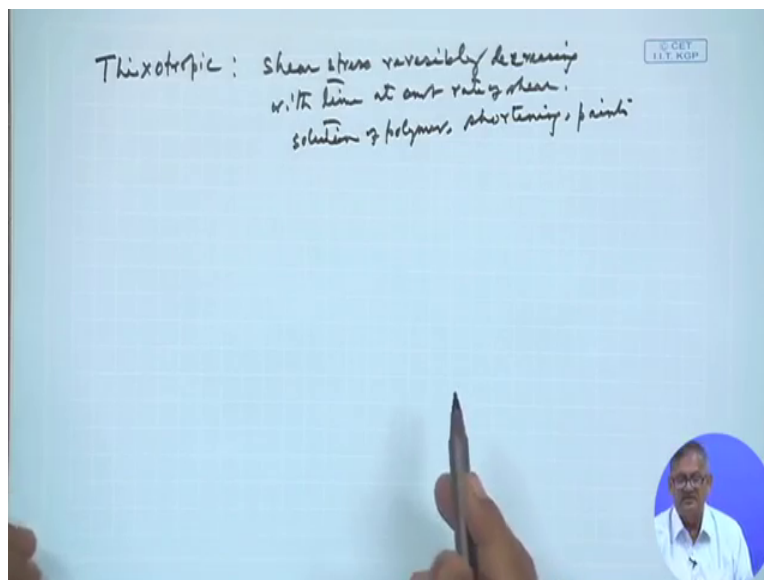
For Bingham plastic it is like that, so this we call to be Newtonian fluid this is Bingham plastic, then for pseudo plastic this is like this for dilatant it is like this, right? Right? So if we plot shear rate versus shear stress so the Newtonian liquid is directly proportional to shear stress is

proportional to shear rate and in that case it follows it passes through the origin also, right? Other two that is pseudo plastic and dilatant they also pass through the origin, but dilatant n is greater than 1 so it is the that is shear rate is increasing as the n is shear rate is increasing when we said it to be, right?

In the previous one we had said that, yeah here we had said that as here, right? That r was increasing that is the rate, okay shear rate and we also said that for dilatant, yes that as μ_a apparent viscosity is increasing with the increase in the shear rate, it is very very uncommon, right? So this is the dilatant and for pseudo plastic which is less than 1 it fast increases and then decreases, right?

So this type of flow behavior is there for the time independent fluids, right? We also said time independent fluid means that the value say viscosity it does not change with time, at 11 o'clock, 12 o'clock, 1 o'clock all the where same, right? But there are we said another type of fluid which is time dependent and when it is time dependent, then there are two types, okay.

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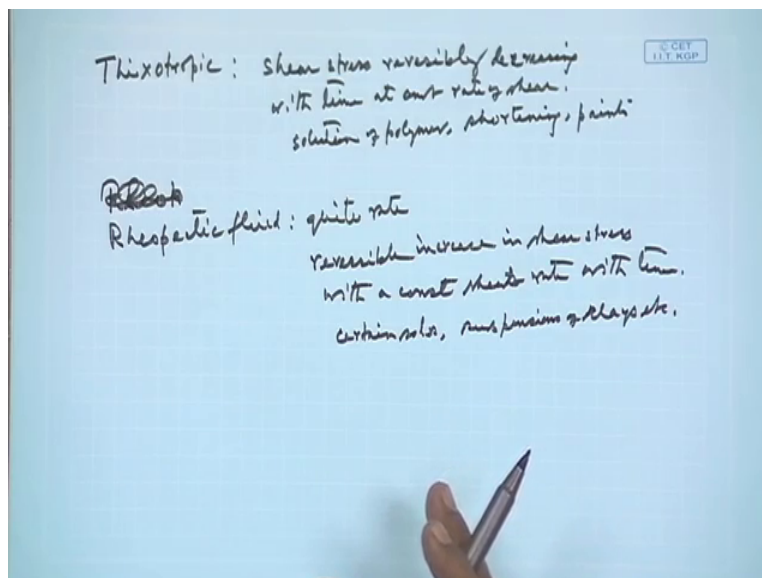
So one is called thixotropic and this thixotropic fluid the shear stress is reversibly decreasing, right? With time at a constant rate of shear, so shear stress is reversibly decreasing with time at a constant rate of shear. Example is some polymer solution some shortenings and also maybe some paints, right? Why it is happening, that is shear stress is reversibly decreasing with time for a given or constant shear rate this theory is not yet fully developed or is not developed to that

extent which can be substantiated, right? So that is why till now this is not known how it is and why it is getting changed.

So this is still now people are working on it and maybe sometime someday the solution will come out, right? And similar to this earlier we also said do you remember I said that the kinetic theory is also people are working on that the way kinetic theory of gases is established, right? We have seen kinetic theory of gases many people have worked or are working also and it is substantially established that the kinetic theory, but the same kinetic theory is not yet substantiated or defined for liquids.

So and it said then most of the cases the kinetic theory of liquid not being there, so it is assumed the similar expressions for gases will also hold good in the liquid and the explanations have been given along with that and in most of the cases it is assumed the deviations where not very very significant or very very high, right? So in there, same is true for this type of thixotropic type of where till now people could not understand why the shear stress with increase of shear stress or reversibly the decrease of shear stress with the shear rate for a given shear rate why it is shear stress is reversibly decreasing that is not known, right?

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So another type thixotropic and rheopectic fluid, right? And these are also quite I mean quite rare, these are also quite rare and this exhibit or reversible increase in shear stress with a constant shear rate with time. Example, again certain sols certain suspensions of clays etcetera, right? So

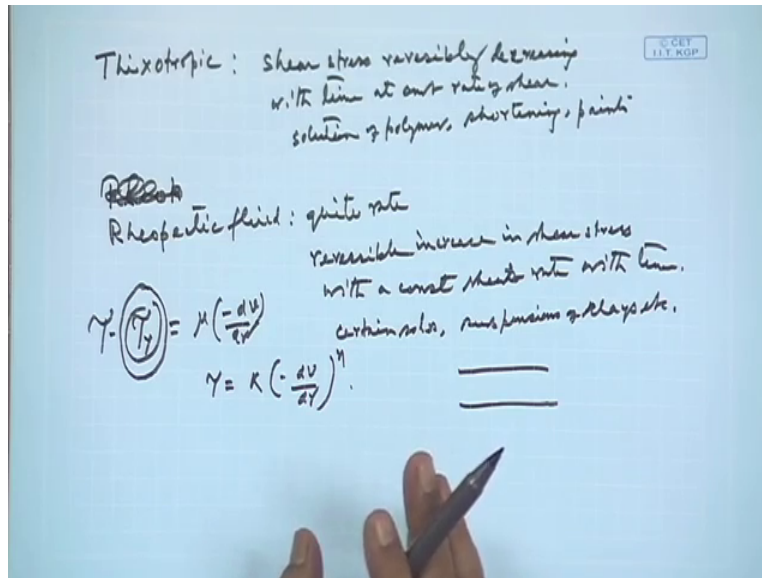
in one case for a given shear rate shear stress is in reversible increasing with time and the other case for a given shear rate the shear stress is reversibly decreasing. So these both and both are rare very rare occurrence in nature, but still they are it is not that even rare but it is not there, right?

We have given example certain polymeric solutions or shortenings or say certain sols, sols of course are you have seen that there are many types of sprays are there available for different purposes, right? So they are sols and maybe some certain suspensions of say clays they maybe there which examples of both thixotropic and rheopectic, right? So with this let us now say that or summarize that there are two types of fluid, one is time independent and another is time dependent.

Time independent fluids are three types and time dependent fluids are two types, right? So time independent fluids are generally of course time independent fluids are not three types it should be typically four types because (17:06) fluid is also time independent, so there it is one pseudo plastic then another Bingham plastic and third one dilatant and Newtonian, right? So these are four types which are time independent, but time dependent are also there, there are two types one is thixotropic and another is rheopectic, right?

And examples also we have given, right? So and we also said that time dependent fluids the behavior is not yet known maybe someday it will come up and that time obviously we will discuss or maybe in future if it is coming up you will come across and discuss, right? So we for all practical purposes for engineering purposes let us keep aside the time dependent fluids, right? So we call time independent fluid we start with that, right?

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Now, as we said for Bingham plastic fluids we need initial stress that is yield stress τ_y that is why the relation was like that, right? Minus this minus $\mu \frac{dv}{dr}$, right? This was there for the Bingham plastic and these are similar to the Newtonian fluid only expectation is that this needs some yield stress or initial thrust to put so that it start moving, right? And pseudo plastic and dilatant they were this τ_y is equals to k minus $\frac{dv}{dr}$ to the power n we said k is the flow behavior index with the coefficient as Newton second to the power n per meter square and n is the flow behavior index and it is dimension less, right?

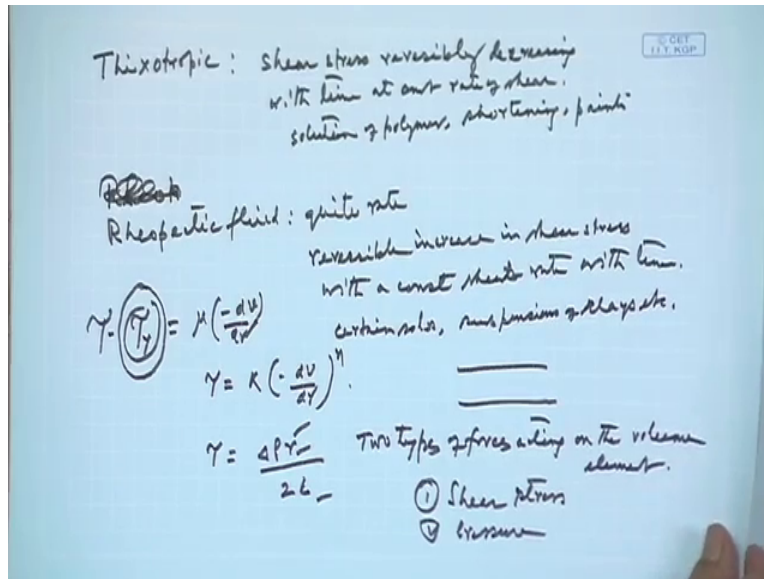
For this let us now say that we said while fluid flow we where we began that time said that in many cases we will try if the flow is through a through a pipe or conduit when it is happening that time we may not be starting all the time right from the governing equation, right? Because again the same thing will come up that flow is steady there is no end effect, the flow is laminar then all these conditions when we apply then the momentum transfer due to the convective flow that because of this steadiness of the fluid it is going off that terms are deleted and the other one that is the momentum transfer due to the molecular transport that comes up, right?

So if we start from there, then without going into the beginning that is when a fluid is flowing be it Newtonian or Non-Newtonian the general governing equation remains same that some of the forces acting on the volume element is equals to 0, right? So that all the forces if we do that

momentum balance on that we said that momentum transfer takes place in two ways, one by the bulk flow of the fluid another by the molecular transport of the fluid.

So since it is steady bulk flow goes off so we have the due to the molecular transport whatever momentum is being transferred.

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And from there after the initial solutions we came to this relation that tau is equals to delta Pr by 2L, right? So in the volume element there are two types, one is shear stress another is pressure, right? So two types of forces acting on the volume element if we take and there one is shear and another is pressure.

So these two terms is there along with the length of the tube and the radius of the tube or pipe through which it is being flown, right? So we come delta tau is equals to delta Pr by 2L.

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The whiteboard contains the following handwritten equations:

$$\tau = k \left(\frac{dv}{dr} \right)^n \quad \tau = \frac{\Delta P r}{2L}$$

$$\frac{\Delta P r}{2L} = k \left(\frac{dv}{dr} \right)^n \quad \text{or} \quad -dv = \left(\frac{\Delta P r}{2kL} \right)^{\frac{1}{n}}$$

$$\text{or} \quad -\int_0^v dv = \int_R^r \left(\frac{\Delta P r}{2kL} \right)^{\frac{1}{n}} dr$$

$$\text{or} \quad \int_0^v dv = \left(\frac{\Delta P}{2kL} \right)^{\frac{1}{n}} \int_r^R r^{\frac{1}{n}} dr$$

$$\text{or} \quad v = \left(\frac{\Delta P}{2kL} \right)^{\frac{1}{n}} \left[\frac{r^{\frac{1}{n}+1}}{\frac{1}{n}+1} \right]_r^R = \frac{n}{n+1} \left(\frac{\Delta P}{2kL} \right)^{\frac{1}{n}} \left[R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}} \right]$$

$$= \left(\frac{n}{n+1} \right) \left(\frac{\Delta P}{2kL} \right)^{\frac{1}{n}} R^{\frac{n+1}{n}} \left[1 - \left(\frac{r}{R} \right)^{\frac{n+1}{n}} \right]$$

$$v = \left(\frac{n}{n+1} \right) \left(\frac{\Delta P}{2kL} \right)^{\frac{1}{n}} R^{\frac{n+1}{n}} \left[1 - \left(\frac{r}{R} \right)^{\frac{n+1}{n}} \right]$$

So we begin with this, then substituting the value of tau, tau we have given tau is equals to k minus dv dr to the power n, right? If we substitute this value to the relation tau is equals to delta Pr by 2L in this if we substitute, then we can write that delta Pr by 2L this is equals to k minus dv dr to the power n, right? Or we can write minus dv dr is equals to delta Pr by 2kL whole to the power 1 by n, right?

Minus dv dr so delta P by 2kL whole to the power 1 by n, right? Or we can write that integrating minus dv between 0 to V, right? This is equals to integrating between r is equals to capital R to small r, right? As delta Pr by 2kL to the power 1 by n into dr, right? So this is because at we know that V if this is r, if this is r so at r is equals to r V is equals to some value. Whereas, at r is equals to capital R, V becomes equals to 0. So that is why we have put this limit that when dv that this velocity is 0 is corresponding to capital R velocity is V corresponding to r is equals to r, right?

So if take this we can write further that this negative we can take care of 0 to V dv and with just change the we just change this limit and delta P by 2kL whole to the power 1 by n delta P is constant k is constant, 2 is constant, L is constant, n is constant independent of r. So we can write this to be r to R and this r to the power 1 by n dr, right? So if this is true, we can write or on integration this is v is equals to that is delta P by 2kL whole to the power 1 by n, right? And this

we can write on integration r to the power $\frac{1}{n} + 1$ by $\frac{1}{n} + 1$ this for r is equals to r to capital R .

So this we can write to be equals to, right? This becomes n by $n + 1$, okay. This becomes n by $n + 1$ $n + 1$ by n , okay so $n + 1$ n by $n + 1$ times ΔP by $2kL$ whole to the power $\frac{1}{n}$ by n , right? Times if we put the boundary for r , it becomes R to the power $\frac{1}{n} + 1$ minus r to the power $\frac{1}{n} + 1$, right? So this can be rewritten as n by $n + 1$ this ΔP by $2kL$ whole to the power $\frac{1}{n}$ by n , right? Times R to the power $\frac{1}{n} + 1$ into $1 - r$ by R whole to the power $\frac{1}{n} + 1$ means this is equals to n by $n + 1$ times ΔP by $2kL$ to the power $\frac{1}{n}$ R to the power $n + 1$ by n times $1 - r$ by R whole to the power $n + 1$ by n , right?

So this is the v , right? So v means at any instance, v at any instance is a related in the power law equation as we have said that τ is k minus dv/dr to the power n when we apply that this relation comes into, okay. Now today is out of time so let us stop it here today, thank you.