

Course on Momentum Transfer in Process Engineering
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Lecture 34
Module 7
Variable fluid flow-problems and solutions

So we have seen that what is a variable flow and how much time it takes for the pressure to drop from an initial pressure to a final given pressure. And we also said that this will be under critical pressure ratio condition, right? p_0 by p will be 0.528 till the pressure p by p_a increases to the level that p equals to 1 by 0.528 that is 1.894 atmosphere it reaches the 1.894 atmosphere till it reaches that value it will be under critical pressure ratio condition, right?

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Problem:- A nozzle of 20 mm diameter releases air from an air reservoir to the atmosphere. Initial pressure and temperature of air are 6 atm and 20 °C respectively. Volume of the receiver is 20 cubic cm. If C_D is 0.98, M is 28.97, γ is 1.3, how long it will take to lower the pressure to 3 atm?



Solution:-

$$K_1 = C_D A_0 \sqrt{\frac{2\gamma}{(\gamma-1)} \left[\left(\frac{p_0}{P} \right)^{\frac{2}{\gamma}} - \left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}} \right]}$$

$$= 0.98 \times \frac{\pi}{4} \times (20 \times 10^{-3})^2 \times \sqrt{\frac{2 \times 1.3}{(1.3-1)} \left[(0.528)^{\frac{2}{1.3}} - (0.528)^{\frac{2 \times 1.3}{1.3}} \right]} = 2.0528 \times 10^{-4}$$

$$K_2 = \frac{M}{RT} = \frac{28.97}{8314.34 \times 293} = 1.18519 \times 10^{-5}$$

and, $t = \frac{V \sqrt{K_2}}{K_1} \ln \frac{p_1}{p_2} = \frac{15 \times \sqrt{1.18519 \times 10^{-5}}}{2.0528 \times 10^{-4}} \ln \frac{6}{3}$
 $= 174.3 \text{ sec}$

So if we do a problem on it like this that a nozzle of 20 millimeter diameter a nozzle 20 millimeter diameter releases air from an air reservoir to the atmosphere initial pressure and temperature of air are 6 atmosphere and 20 degree centigrade respectively initial pressure and temperature volume of the reservoir is 20 cubic centimeter. If C_D is 0.928 molecular weight is 820 8.97 this should be capital M molecular weight is 28.97, gamma is 1.3, how long will it take to lower the pressure to 3 atmosphere? Right?

So this is our problem, so from 6 atmosphere to 3 atmosphere how long it will take? So I repeat for benefit of yours, the A nozzle of 20 millimeter diameter decreases air sorry releases air from an air reservoir to the atmosphere. Initial pressure and temperature of air are 6 atmosphere and 20 degree centigrade respectively. Volume of the reservoir is 20 cubic centimeter. If coefficient of discharge Cd is 0.98 molecular weight of air is 28.97, heat capacity ratio is 1.3 or gamma is 1.3, how long it will take to lower the pressure to 3 atmosphere? Right?

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Handwritten derivation on a blue background:

$$t = \frac{V_c \sqrt{K_2}}{K_1} \ln \left(\frac{p_1}{p_2} \right)$$

$$K_1 = C_D A_0 \sqrt{\frac{2\gamma}{\gamma-1}} \left[\left(\frac{p_1}{p_2} \right)^{\frac{2\gamma}{\gamma-1}} - \left(\frac{p_1}{p_2} \right)^{\frac{2\gamma}{\gamma-1}} \right]$$

$$= 0.98 \times \frac{\pi}{4} (20 \times 10^{-3})^2 \sqrt{\frac{2 \times 1.3}{1.3-1}} \left[(0.526)^{\frac{2 \times 1.3}{1.3-1}} - (0.526)^{\frac{2 \times 1.3}{1.3-1}} \right]$$

$$= 1.366 \times 10^{-5}$$

$$K_2 = \frac{M}{RT} = \frac{28.97}{8314 \times 293} = 1.18526 \times 10^{-5}$$

$$t = \frac{20 \times 10^{-6} \times 1.185 \times 10^{-5}}{1.366 \times 10^{-5}} \ln \left(\frac{6}{3} \right) = 174.3 \text{ sec.}$$

Parameters listed on the right:

- $C_D = 0.98$
- $M = 28.97$
- $\gamma = 1.3$
- $p_1 = 6 \text{ atm}$
- $p_2 = 3 \text{ atm}$
- $V_c = 20 \text{ cm}^3$
- $D_n = 20 \text{ mm}$
- $= 20 \times 10^{-3} \text{ m}$
- $T = 20^\circ\text{C}$
- $= 293 \text{ K}$

Now this we can solve very easily because we have seen in the previous class that the relation was $t = \frac{V_c \sqrt{K_2}}{K_1} \ln \left(\frac{p_1}{p_2} \right)$, right? So if p_1 is the initial and p_2 is the final, right? So if that be true, we also said that K_1 is equals to K_1 rather it is equals to we had said to be equal to $K_1 K_2$ that was in the other page, yeah K_1 was $C_D A_0$ under root 2γ by γ minus 1 times p_0 or p_1 over p_2 initial is p_1 and final is p_2 , so in this case we should write that p okay p_1 by p_2 as we are writing here to the power this is under critical condition, right?

e to the power point 2 by γ or rather 2 by γ minus p_1 over p_2 to the power γ plus 1 by γ under critical condition this was K_1 , right? And since the values which are given to us are like this, C_d is 0.95 0.98 not 95, then molecular weight M is given 28.97, then we are given γ to be 1.3, we are given p_1 is equals to 6 atmosphere, you are given p_2 is equals

to 3 atmosphere and also we are given the volume of the container V_c is equals to 20 centimeter cube, right?

So these are the values we are given, then also we were given a nozzle of diameter that is D nozzle is equals to 20 millimeter, right? So 20 millimeter diameter is equals to 20 into 10 to the power minus 3 meter, right? So if we substitute, then C_d is 0.98 A_0 is π by 4 D nozzle square that is 20 into 10 to the power minus 3 whole square into under root 2 gamma 2 into 1.3 by 1.3 minus 1 times this ratio is under critical, so 0.528 to the power 2 by 1.3 minus 0.528 to the power 1.3 plus 1 divided by 1.3, right?

This was our K , now if we see this in the calculator what we can see that, yeah so we have 0.98 into π divided by 4 into 20 into 10 to the power minus 3 whole square into 20 into 10 to the power minus 3 this my mistake, okay let us do this first 20 into 10 to the power minus 3 okay whole square this is the square, right? Into 0.98 into π into say 1 is equal to this much by 4 is this side, right? Now what about under root? So let me also write it otherwise 15393804002 how come it is so, somewhere again something has been made wrong, how come it is 10 to the power minus 3 whole square, so it cannot be that.

So it is 20 into 10 x to the power y 3 plus minus is equal to this, now this square is this into π by 4 into π into (2.98) 0.98 is equals to this divided by 4 is equals to this, right? And now this into so this we can write this was 3.0787 into 10 to the power minus 4, right? Times this factor say this is A , right? So now what is A ? Already we have made into, so we can write here 1, 2 in the second we can write or it can be third 2 into 1.3, right? Divided by 1.3 minus 1 that is 0.3 divided by 0.3, right? So this into first bracket that is 0.528 x to the power 2 divided by 1.3, right? Minus again 0.528 x to the power 1.3 plus 1 that is 2.3 divided by 1.3 something wrong 1.3 hopefully we have done wrong 9.4 which is not true.

So let us see this second part second part was 0.528 to the power x, y to the power 2 by 1.3 2 by 1.3 so this was this, right? Minus 0.528 x to the power 2.3 divided by 1.3 this 1 this 2, right? Into or we can make is equal to, my goodness we are making something wrong repeatedly, so it is 0.528 x to the power 2 by 3 so that becomes 2 by 1.3 pardon me for this, x to the power 2 by 1.3 is this is equal to this, right? Minus again 0.528 x to the power what 2.3 divided by 1.3 this again second bracket and this is equal to, right? 0.05 that is what should have been, fine. Into 2 into 1.3

divided by 1.3 minus 1 that is 0.3 this so this is this, right? Into we have 3.0787 into 3.0787 into 10 to the power minus 4 this is coming 1.3 10 to the power minus 4 1.368 10 to the power minus 4 this check whether this is correct or not, okay.

K_2 is coming to be equals to M by RT M is given here as 28.97 R is 8314 and T is 20 degree centigrade which we have not written T is 20 degree centigrade that is equals to 273 so 293 Kelvin. So 293 Kelvin, so this is equals to 28.97 divided by 8314 divided by 293 so this is equals to 1.18, right? This is equals to 1.18924 10 to the power minus 5 K_2 , right? And therefore the time T we can write from here time t is equals to V_c is given this is 20 centimeter cube 20 into 10 to the power minus 2, right? Minus 3 somewhere this we made wrong here, okay square that was square but this V_c 20 into 10 to the power minus 3 minus 2 meter cube, right?

So into 3 that is it that what it should be, right? And this is that T , okay K_2 is 1.185 10 to the power minus 5 185 not 9, okay 5 10 to the power minus 5 by K_1 , K_1 we got 1.368 10 to the power minus 4, right? \ln of p_1 by p_2 , p_1 is 6 initial final is 3, right? So if this is true, than we can write that V_c is 10 20 centimeter cube so 10 to the power minus 2 or minus 6, so rightly we write it to be 10 to the power minus 6 meter cube. So 20 10 to the power minus 6 meter cube that if we write if we put it here, then we get 10 to the power minus 6 into 20, right?

So this much meter cube is volume V_c , K_1 is 1.185 into 1.185 into 10 to the power minus 5, right? So this divided by K_2 that was K_2 and this is K_1 1.368 into 10 to the power minus 4 this is divided by so is equal to this divided by 10 to the power minus 4 so this is equal to this much, right? Times 6 by 3 that is 2 \ln of 2 times 1.73 times 2 \ln what is \ln , yeah 1.7324 10 to the power minus 6, right? Times 2 \ln , right? I again think we have done wrong 10 to the power minus 6 yeah we again done wrong perhaps, so it should be 1.713 10 to the power minus 6 plus minus, right? actually we are not so much used 1.713 into 10 to the power minus 6 plus minus this, right? Into let it be like this 2 of \ln , \ln of 2 is this so into this is 1.1873 10 to the power minus 6 somewhere we have done something perhaps wrong.

However, this comes to 174.3 second, right? So this comes to 174.3 seconds, so when we make it here we see that the thing where we have done wrong is K_1 is given at $C_d \pi$ by 4 D square, okay. Into under root 2 gamma by gamma minus 1 p by p_0 p_0 by p to the power 2 by gamma

minus p_0 by p to the power $\gamma + 1$ by γ so this comes to 2.0528×10 to the power minus 4, please check whether it is correct or not, right?

Then K_2 as it is M by RT which we also got the same 28.97 by 8314 , right? 8314.34 that you can take into 293 that comes to be 1.18519×10 to the power minus 5, right? And there by we get time t is V_c under root V_c into under root K_2 by K_1 times \ln of p_1 by p_2 , right? Now V_c here the difference is not 15 and 20 this 15 this 15 was perhaps 20 cubic centimeter so that is why our thing was coming so low, 20 cubic centimeter we had taken but here in the problem actually we have taken 15 meter cube, right? 15 meter cube into this root K_2 that under root 1.18519×10 to the power minus 5 it was so divided by K_1 that is 2.0528×10 to the power minus 4 \ln of 6 by 3 $(\ln(6/3))$ p_2 or p_0 by p this should not be p_0 by p this is p_1 by p_2 in that t expression which we got earlier, right? Here perhaps this p_1 by p_2 , okay.

So got \ln of 6 by 3 this on simplification gives 174.3 so the change is here 15 meter cube instead of 20 cubic centimeter if we take 20 cubic centimeter perhaps whatever we were getting that should come, right?

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Problem:- A nozzle of 20 mm diameter releases air from an air reservoir to the atmosphere. Initial pressure and temperature of air are 6 atm and 20 °C respectively. Volume of the receiver is 15 cubic m. If C_D is 0.98, m is 28.97, γ is 1.3, how long it will take to lower the pressure to 3 atm?

Solution:-



$$K_1 = C_D A_0 \sqrt{\frac{2\gamma}{(\gamma-1)} \left[\left(\frac{p_0}{p} \right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{p_0}{p} \right)^{\frac{\gamma+1}{\gamma-1}} \right]}$$

$$= 0.98 \times \frac{\pi}{4} \times (20 \times 10^{-3})^2 \times \sqrt{\frac{2 \times 1.3}{(1.3-1)} \left[(0.528)^{\frac{1.3}{0.3}} - (0.528)^{\frac{1.3+1}{0.3}} \right]} = 2.0528 \times 10^{-4}$$

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$$\text{and, } t = \frac{V_c \sqrt{K_2} \ln \frac{p_1}{p_2}}{K_1} = \frac{15 \times \sqrt{1.18519 \times 10^{-5}}}{2.0528 \times 10^{-4}} \ln \frac{6}{3}$$

$$= 174.3 \text{ sec}$$

However, this is a this is only a calculation which you can do it with your calculator not like this online where lot many other difficulties are there please check and here in the problem let me also change here that will be better that instead of 20 centimeter cube let me write 15 cubic meter, right? So then only we can say that this is what is coming that cubic 15 cubic meter then

only this is coming 15 times V_c is 15 15 times this over this K_2 by K root K_2 by K_1 ln of 6 by 3, then it is 174.3, okay this also you practice and hopefully we can do as many problems we can come across and you also do some practice with other problems, okay. Thank you.