Course on Momentum Transfer in Process Engineering
By Professor Tridib Kumar Goswami
Department of Agricultural & Food Engineering
Indian Institute of Technology, Kharagpur
Lecture 32
Module 7
Sonic velocity-Mach number

In continuation to the previous class where we could not end up with the relation of the Mach number and the velocity because the time was out so we had to leave and recapitulation that we had said that sonic velocity, right?

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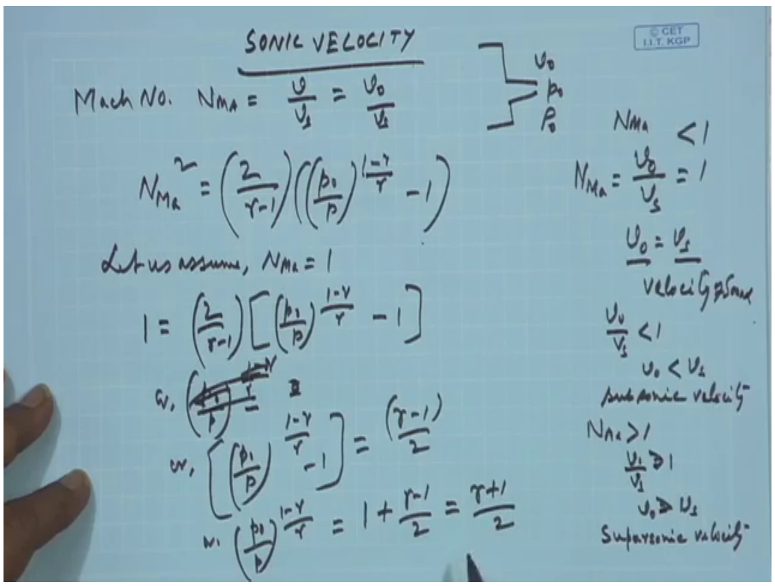

So we had said that sonic velocity and from that sonic velocity we have defined that Mach number so Mach number was Nma, right? And that was the velocity of that to that velocity of sound and ultimately we bring in we brought it to be rather that tip velocity of the nozzle, right? Tip velocity at the nozzle v0, p0, rho 0 at the nozzle, right? Tip velocity at the nozzle so v0, p0, rho 0 at the nozzle, right? From there what I remember that he had come to the level Nma square is equals to 2 by gamma minus 1 into p0 by p to the power 1 minus gamma by gamma minus 1, right?

This we had done in the previous class, right? Now let us take let us assume that Mach number is Nma is equals to 1. What does Mach number is equals to 1 mean? That Mach number is equals to 1 mean that from the definition of Mach number Nma was v by Vs, right? Or v0 at the tip and by Vs where this is equals to 1 that means v0 is equals to Vs which means that when Mach number is 1, the velocity at the tip is velocity of the sound, right? Velocity of the sound that is what we get, right?

And if it is less than 1, if Nma is less than 1 then we call to be this is velocity of sound and if we say that Mach number is less than 1 that means v0 by Vs is less than 1 that means v0 is less than Vs, right? That means velocity at the tip is less than the velocity of sound it is called sub sonic velocity and if Nma is greater than 1, then this means v by or v0 by Vs v0 by Vs is equals to is greater than 1 that means v0 is greater than Vs that is velocity at the tip is greater than velocity of sound and this situation is called supersonic velocity, right? So that is what you remember in the previous class which I said that if you are staying in area where there are air force region and you might have seen that lots of air force planes are moving at very very high-speed, right?

And when that speed is the speed of the sound then we hear that, but if it is less than that or greater than that then we do not hear it, right? And I said also one thing that sometimes it makes ”boom”, right? Some sound like a “Boom” and this booming and it is so seviour that your if you are staying there that your windows and other things may shatter or it may have some vibrations seviour vibration, right? Depending on how close it is, then when this is changing is happening that is supersonic to sonic or sonic to subsonic or vice versa then this booming takes place, right?

So this is a very great example of this supersonic subsonic and sonic velocity, right? So we have now assumed that if the Mach number is 1, then what happens, so if we take Mach number here is equals to 1 that is Nma square is 1 so this is equals to 2 by gamma minus 1, right? Into p0 by p to the power 1 minus gamma by gamma minus 1, right? Or p0 by p to the power 1 by minus gamma by gamma this is 1 minus gamma by gamma this is equals to p0 by p this is equals to this 1 goes there, right? So this is that means okay p0 let us write let us again write again or p0 by p to the power 1 minus gamma by gamma minus 1 this is equals to gamma minus 1 by 2, right?

So from this relation or p0 by p to the power 1 minus gamma by gamma 1 plus gamma minus 1 by 2, right? That is this is equals to gamma plus 1 by 2.

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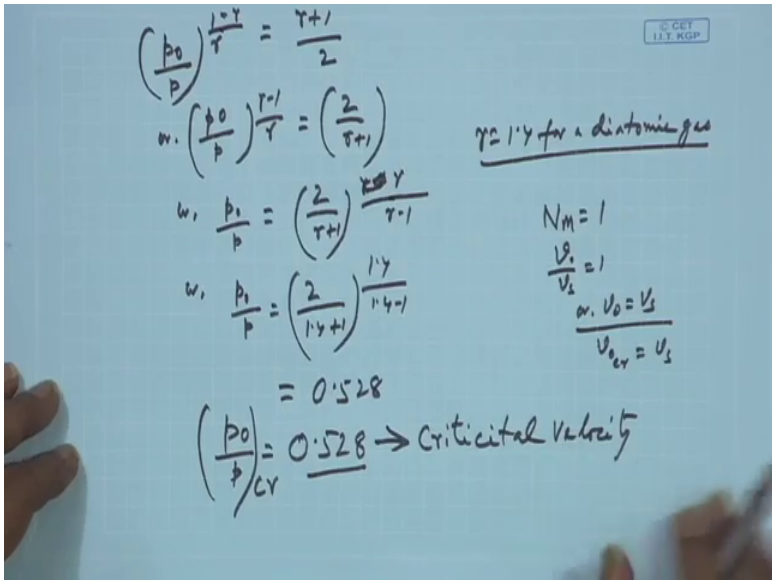

So we can rewrite that p0 over p to the power 1 minus gamma by gamma is equals to gamma plus 1 by 2, right? Or we can write p0 by p to the power gamma minus 1 by gamma is equals to 2 by gamma plus 1, right? This we can write easily. So we can also write p0 over p is equals to 2 by gamma plus 1 to the power gamma minus 1 in this case it will be inverse so it will be gamma by gamma minus 1, right? This will be inverse so gamma by gamma minus 1.

So p0 by p is equals to 2 by gamma plus 1 to the power gamma by gamma minus 1, right? That if now we assume that gamma is equals to 1.4 for a diatomic gases for a diatomic gas if gamma is 1.4, then we can write p0 over p is equals to 2 by 1.4 plus 1 to the power 1.4 by 1.4 minus 1, right? And let us see what it makes so we have 2 by 2.4 2 by 1.4 plus 1 so this is 2.4 so this is this to the power 1.4 divided by 1.4 minus 1, right? So this one is two and this three.

So this is equals to 0.528, so this is equals to 0.582 that means p0 over p is equals to 0.528 which is nothing but the critical velocity, right? So this means this is nothing but critical velocity and that tells that when the velocity is under critical condition that is p0 by p becomes under critical condition, then the ratio of tip velocity to the inside or inlet velocity that becomes equals to 0.521, right? So we have seen that means when Mach number is equals to 1, right?

When Mach number is equal to 1, then the pressure ratio becomes critical and the value is 0.528 when this value becomes 528 we can say that this velocity has become the velocity of the sound, right? Because this is what we showed earlier that Mach number is equals to 1 is equals to v0 by

Vs equals to 1 or v0 is equals to Vs, right? Now we have shown that p0 by p becomes critical when it is 0.528 so corresponding velocity is v0 and that becomes critical velocity v0 critical so that becomes the velocity of sound.

So under critical condition the velocity at the tip when it is 0.528 of the pressure ratio of the outlet to inlet or tip velocity to inside velocity that becomes 0.528 then it becomes the velocity of sound, right? This is great thing which was developed earlier by the greatest scientists like Mach and others, right? So we said that someday we will try to bring their photographs and some biography of the some of the people who were really contributed to this field, okay.

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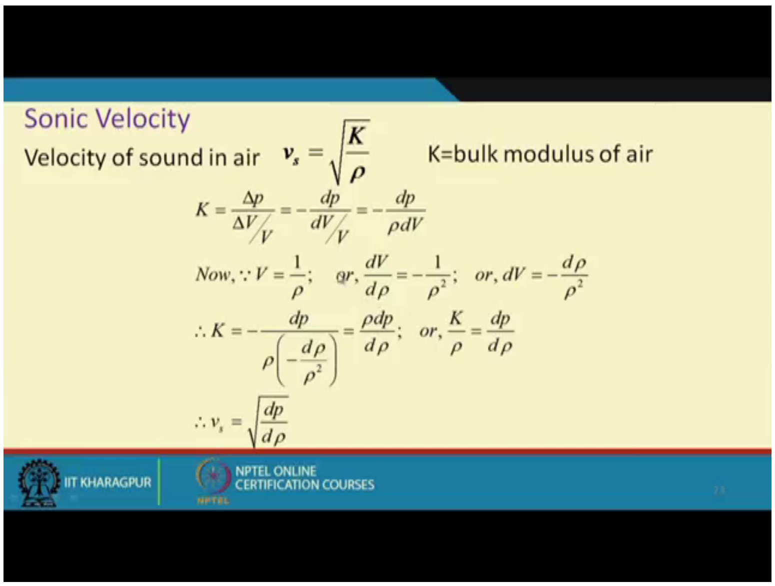

So far recapitulation, let us look into that whole thing for the sonic velocity let us looking into that we had started with this that sonic velocity is Vs velocity of sound in air is under root K by rho, where K is the bulk modulus of the medium so if it is air than it is the bulk modulus of air, right? And also be said that K K is bulk modulus that is by definition can be said del p over del v by v that is the change in volume per unit volume with respect to the change in pressure, right?

Now it has been seen that change in volume per unit volume is decreasing as the change in pressure is increasing, right? So as delta p is increasing the change in volume per unit volume for the air that is the bulk modulus that is decreasing. So for to account for this the negative sign comes in and that if we introduce then we can say that del p over del v by v is equals to minus del p over del v by v, right? And this is equals to del p over so this v 1 by v can substitute as rho dv, right? So this we have substituted and we have made minus dp over rho dv.

So this is possible because v is nothing but 1 by rho so we can write from there that if v is 1 by rho, therefore dv over d rho dv over d rho or dv d rho is equals to minus 1 by rho square that is minus rho to the power minus 2 so that means minus 1 by rho square or dv can be written as if we substitute that dv can be written as d rho minus d rho over rho square, right? So therefore that bulk modulus K can be substituted with minus dp over rho times minus d rho by rho square, so this minus this minus goes off this rho this rho square 1 rho goes off so this rho comes to the top so that is rho dp over d rho.

So K becomes equals to rho dp over d rho or K by rho is equals to dp over d rho, right? Therefore we can write from this definition Vs, Vs is under root K by rho so that can be written as under root dp over d rho, right? dp over d rho because dp over d rho that has become from this K is this so K by rho is dp over d rho, so Vs is dp over d rho.

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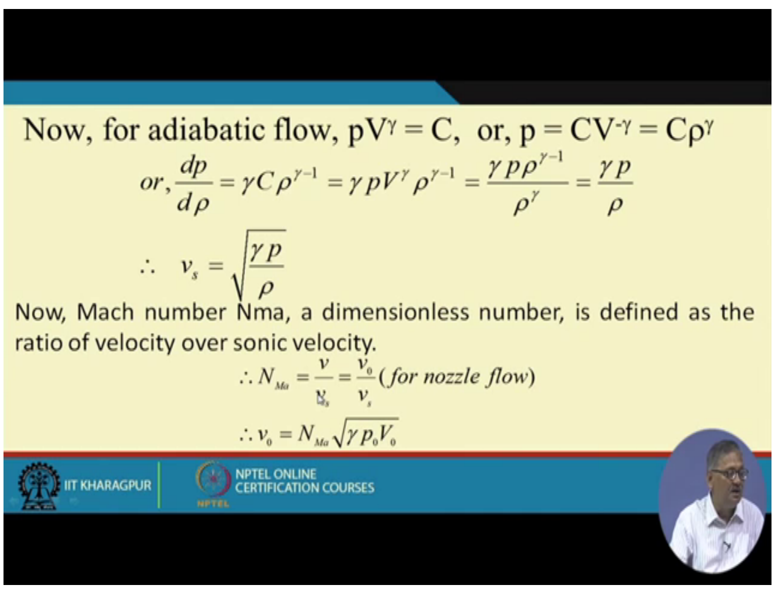

Now we also said that if the flow is taking place through adiabatic flow that is there is no heat loss or heat gain, right? There is no heat loss or heat gain if the flow is under adiabatic condition, then we can also write that for adiabatic flow pV gamma is equals to constant or we can also write that p is equals to CV to power minus gamma that is equals to C rho to the power gamma, right? C v to the power minus gamma or C rho to the power gamma, right?

So we can write that dp over d rho is equals to gamma C rho to the power gamma minus 1 that is equals to gamma p V to the power gamma into rho to the power gamma minus 1 so that is equal to gamma p rho to the power gamma minus 1 by rho to the power gamma or this can be simplified as gamma p by rho. So we write Vs as under root gamma p by rho when the flow is under adiabatic condition, so it is a must that flows under adiabatic condition that is there is no heat gain or heat loss or there is no heat flow into the medium.

So that condition pV gamma is constant this we have to keep in mind, right? So Vs has become under root gamma p by rho, right? Now be define a new number called Mach number, so Mach number defined as if it is written as Nma so this is a dimensionless number and that is defined as the ratio of velocity over the sonic velocity ratio of the velocity of that point or of that system over the velocity of sound Vs, right?

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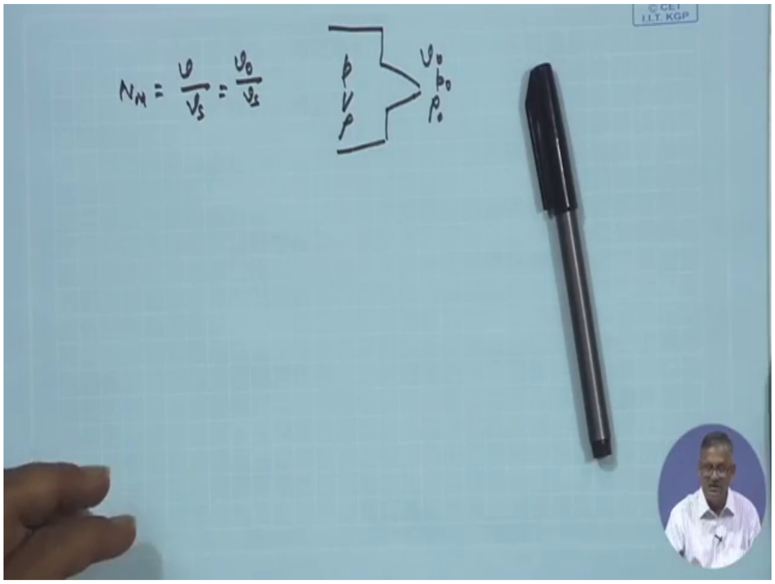

So we wrote that Mach number Nma is v by Vs is v0 by Vs that is if the flow is also acquiring through the nozzle so this we said repeatedly but again that if the flow is occurring through the nozzle and if the tip velocity of the nozzle is v0 corresponding pressure is p0 and corresponding density is rho.

Similarly inside is p, v and rho, then we can write that for the nozzle velocity Nma is equals to v over Vs is equals to v0 over Vs, right? So from there we also wrote that the velocity of the tip is v0 equals to Mach number times under root gamma p0 v0 assuming that the flow is occurring at the tip of the nozzle, right?

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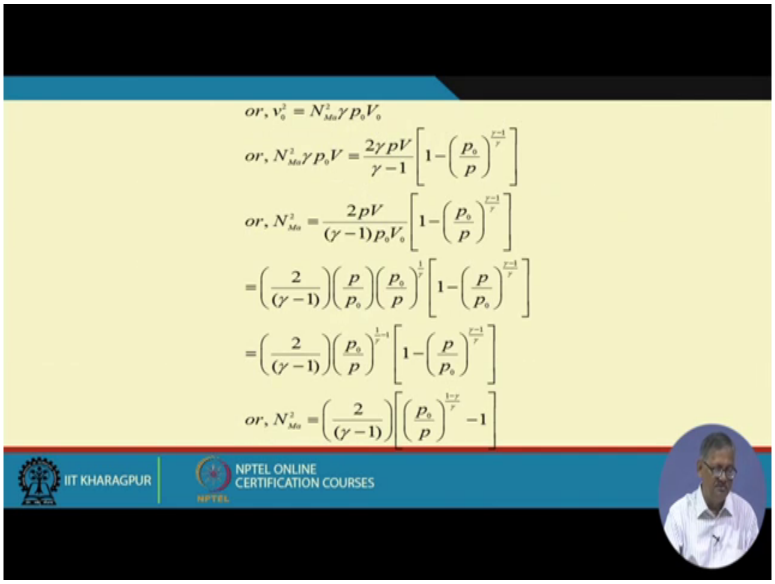

So this we finished in the previous class and that is a recapitulation and then we squared up this v0 square is equals to Nma square gamma p0 v0.

So we can write Nma square gamma p0 v0, p0 and this should be v0, right? Gamma p0 v0 is equals to this v0 square v0 we had written many times that is under root of this so n square is there so that under root goes out. So gamma pV into gamma minus 1 or in many cases we had written gamma p by gamma minus 1 into rho. So that means this rho goes out v comes on the top times 1 minus p0 by p to the power gamma minus 1 by gamma, right?

So this on simplification can be written that Nma square is equals to 2 pV, right? By gamma minus 1 into p0 v0 times 1 minus p0 by p to the power gamma minus 1 by gamma, right? So this on simplification we can write Nma square is equals to 2 by gamma minus 1 this one into p by p0 p by p0 into v by v0 which can be substituted as p0 by p to the power 1 by gamma because pV gamma is equals to constant, so pV is equals to p0 pV gamma is equals to p0 v0 gamma, right? From there we can write that p0 by p is equals to v by v0 is equals to p0 by p to the power 1 by gamma, right? So this we substituted and then the remaining one was 1 minus p0 by p to the power gamma minus 1 by gamma, right?

So since it is there, so we can rewrite or we can rearrange as 2 by gamma minus 1 times this p0 by p, right? This is p by p0 so if we make it is 1 inverse so it becomes 1 by gamma minus 1 so p0 by p 1 by gamma minus 1, this can be written p0 by p to the power minus 1, so that minus 1 goes into there so this is 1 by gamma minus 1 p0 by p to the power 1 by gamma minus 1. So into 1 minus p by p0 to the power gamma minus 1 by gamma so then Mach number square this we can write to be equals to 2 by gamma minus 1 into p0 by p to the power 1 minus gamma by gamma minus 1, right?

So this was also done in the previous class for recapitulation we have taken it, then we said if Mach number is equals to 1, right? When v0 becomes equals to Vs, then Mach number becomes equals to 1, right? So because Nma we said it is v by Vs, right?

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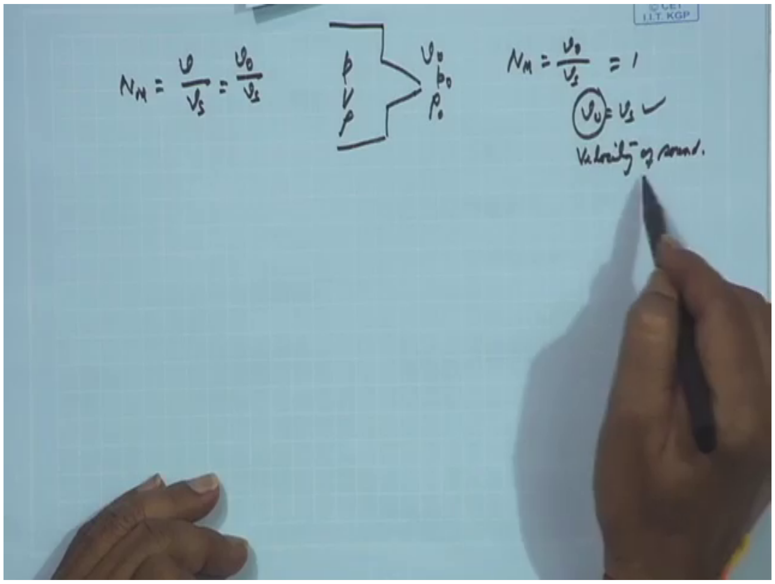

v0 by Vs so when it is equals to 1 then v0 is Vs that is the tip velocity becomes the velocity of sound, right? So if velocity of sound so if it is that when v0 becomes the velocity of sound, then Mach number becomes equals to 1.

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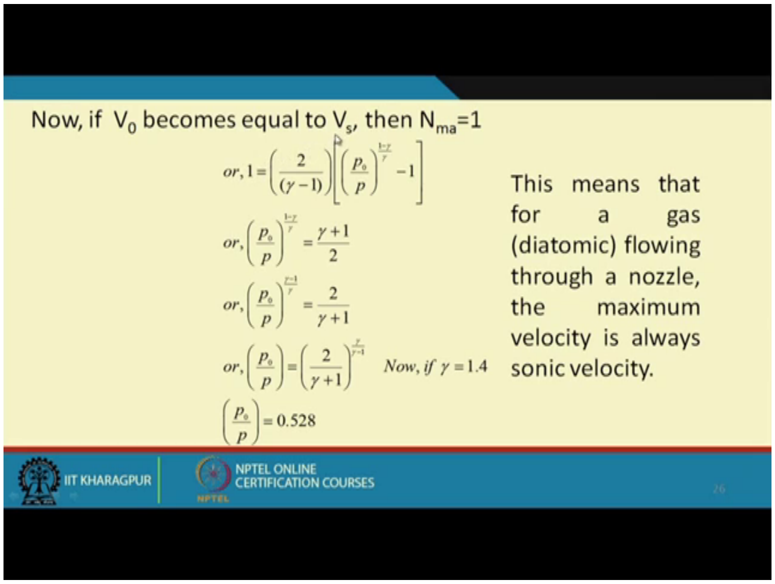

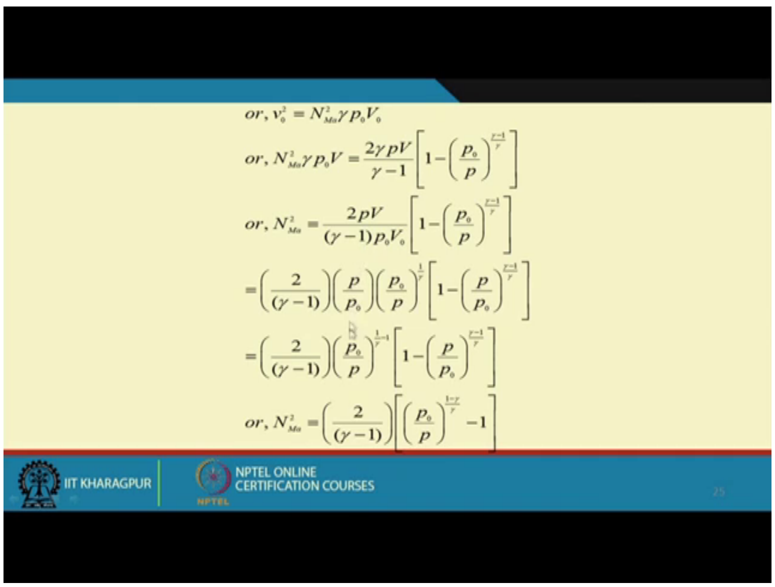

So if Mach number becomes equals to 1, then we can write that Nma is 1 so Nma square here it was so it is 1 times 2 by gamma minus 1 times p0 by p to the power 1 by gamma minus 1 if you remember we had this, okay p0 by p to the power 1 minus gamma by gamma minus 1, 1 minus gamma by gamma minus 1 minus 1, right? So this is then we can rearrange, right? So we can rearrange that this we had shown today that this gamma minus 1 by 2 this becomes gamma minus 1 by 2, right? Minus 1 so that means gamma minus 1 plus 1 that means becomes gamma plus 1 by 2, right?

So that is what it has become gamma plus 1 by 2 that is equals to p0 by p to the power 1 minus gamma by gamma. So we can also write p0 by p to the power gamma minus 1 by gamma that is equals to 2 by gamma plus 1, so p0 by p equals to 2 by gamma plus 1 it will be inverse of this, so gamma by gamma minus 1. So 2 by gamma plus 1 to the power gamma by gamma minus 1. Now if we say that the fluid is diatomic gas, then the value of gamma that is heat capacity ratio we can take as 1.4 and if we take 1.4, then this ratio p0 by p that becomes equals to 0.528, right?

Now the implication of this is that the pressure ratio when it becomes 0.528 earlier we also have shown and established that at that situation the pressure ratio is called critical pressure ratio and also we have established earlier that at this pressure ratio of 0.528 the velocity becomes maximum that is Vs is maximum or rather v0 that is tip velocity becomes maximum. And we also have shown that when this pressure ratio critical becomes 0.528, then the velocity of the tip that becomes a velocity of the sound that means that velocity is equal to the velocity of sound and when it is you can infer that when the pressure ratio becomes 0.528 then it becomes the velocity of sound and it attains the velocity that velocity at the tip attains the velocity of the sound when the pressure ratio becomes critical and we also have shown earlier at this point the discharge also become the maximum, right?

So when the velocity is maximum when the discharge is maximum, then the velocity which it attains in air, then it is the velocity of the sound. So this we can infer that when the velocity maximum when velocity becomes max discharge becomes maximum, then the pressure ratio becomes, pressure ratio means outlet to inlet or the tip velocity to the inlet velocity that becomes 0.528 and if this 0.528 pressure ratio is attained then all corresponding velocity or discharge becomes maximum and the velocity attains the velocity of sound and this is what is the sonic velocity, right? So this we have given the example with the aero plane also, right? So hope you have understood this, thank you.