Course on Momentum Transfer in Process Engineering
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Lecture 31
Module 7
Sonic velocity

Now we come to another very important topic, right? You remember the day we were seeing compressible fluid flow and flow through nozzle we had said that you are whistling phuu like that, right? So when you are whistling there is a sound when there is a pressure cooker there is a sound when (pla) aero plane is moving there is a sound but I do not know whether you are staying near to any airports area we have nearby one so we also hear in many days that there is a sound boom that boom sound is not because of the aero plane on movement of the aero plane it is that of course it is movement of the aero plane but that booming is because of that from that one phase top another phase of the sound it was crossing or coming up or down.

So because of that this booming happens, right? So that means when we are talking when we are listening any sound, then that sound unless it is some mechanical or some other. In normal if it is with a fluid, then definitely that fluid has to undergo a velocity which is known as sonic velocity, right? That is the velocity of sound, now you also know that in the whole universe there are so many things every now and then happening, but from the earth you are not listening anything, why? Because for the propagation of the sound of course this you have come across in higher secondary also in first year or some other class or from general knowledge you have upgraded yourself.

Then you have seen that this velocity of sound or the sound what is propagation it depends on the medium, right? That is why when you are swimming and if somebody is shouting you listen one (())(2:50) of, whereas the same thing when you are not swimming you are on the bank, then you are hearing a proper sound, right? So far we have talked about proper sound means when it is in the air, right? When the medium is air. So same is happening if you are in the vacuum under some vacuum level and then if you are hearing you will find that distortion of the sound, right?

So this is because the propagation of the sound needs some medium and in all our normal cases the medium is air, right? And that is why in the universe where things are happening we do not receive any sound only because there is no medium from there to bring to the earth and then we can hear, right? So whatever is happening on the periphery of the earth we are hearing, but not be on that till the atmosphere is there, right?

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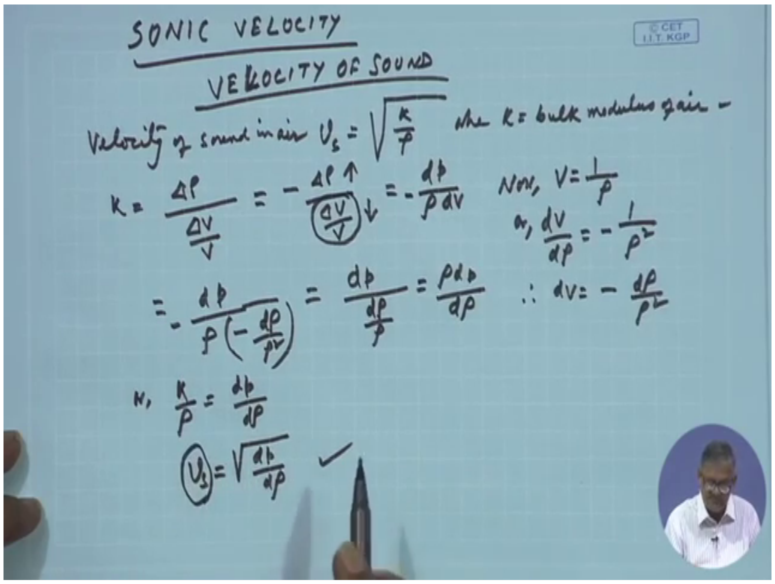

So that we will do we will discuss today that is the sonic velocity are velocity of sound v, e, l sorry v, l, o, right? So velocity of sound by definition velocity of sound in air is defined as Vs is equals to under root k by rho, where k is the bulk modulus of air, right? And this bulk modulus of air k is defined as delta p over delta v by v that is change in pressure or change in velocity per unit velocity for the change in pressure change in velocity per unit velocity for the sorry delta p by delta v by v, so this is specific this is specific volume.

So change in specific volume per unit volume of the air for a change in pressure, right? So this is the specific volume, so if delta p over delta v by v is k, then we can write this is equal to negative of delta p over delta v by v, why negative? The negative term is because that with the change in delta v per unit volume v per unit specific volume of obviously is decreasing with the increase of the pressure, right? So that is why the negative value is there. So change in this is decreasing as delta p is increasing, right? So that is why this negative indicates that. So this means we can also write that this is nothing but dp over delta p by v so 1 by v can be written as rho and this is dv, right? Now we know that v is equals to 1 by rho, right? Or we can also write that dv d rho this is nothing but minus 1 by rho square, right?

So we can write that dv is equals to minus d rho over rho square, right? So now if we substitute this value into the value of k, then we can write that this is dp over rho dv so rho we have seen that is nothing but 1 by v or let us write it to be rho and this is rho dv, right? dv we have already changed in the form of d rho and here we had of course one negative which you have not given. So this negative should have been because here it was there so there it is there. So this negative and this negative or let us write rather otherwise there is a chance of mistake.

So rho into dv dv is nothing but minus d rho over rho square, right? So this means this is dp over this negative this negative goes off and this square this square goes off so d rho over rho, right? So that means it is nothing but equal to rho dp over d rho, right? So we can write k by rho is equals to dp over d rho, right? So if that is true, then we can also write that Vs is nothing but it was k by rho so that can be written as under root dp over d rho. So velocity of sound is can be expressed then in terms of dp over d rho dp is the change in pressure d rho is the change in density corresponding change in density, right? this is under root.

This we defined as the velocity of sound, right?

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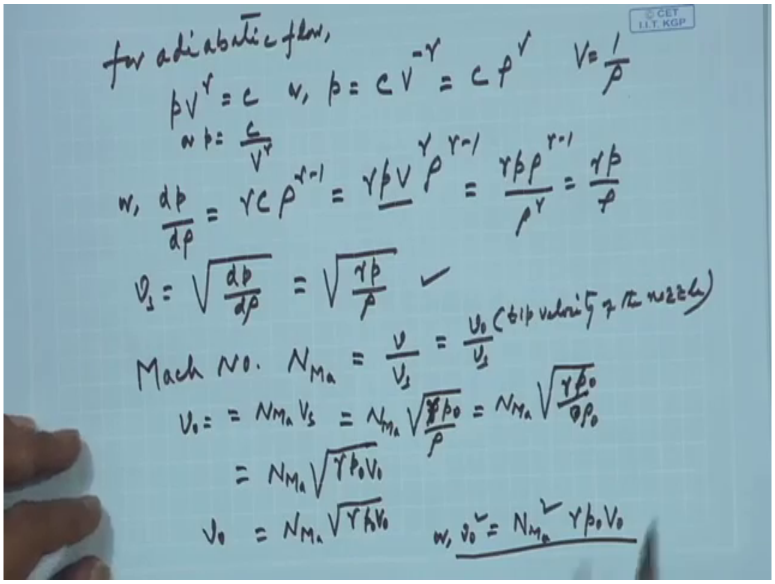

now if we look at the other relation sorry, if we look at the other relation that for adiabatic process we know pv gamma is equals to constant, right? Or we also can write that p is equals to c into v to the power minus gamma because this comes down or v is equals to c by v to the power gamma so p is equals to c v to the power minus gamma, right? Or this can also be written in terms of rho as c rho to the power gamma because v is equals to 1 by rho, right? So v is equals to 1 by rho, so we can write that c rho to the power gamma, right?

So we can write our dp is equals to our dp over d rho this is equals to gamma c rho to the power gamma minus 1, right? So this is nothing but gamma p v to the power gamma rho to the power gamma minus 1, right? Because we have substituted c as pV gamma, right? So this we can write is equals to gamma p rho to the power gamma minus 1 over rho to the power gamma, right? So if that be true so this is rho to the power gamma so minus gamma plus gamma it goes out so it becomes rho to the power minus 1, so gamma p by rho, right?

So this becomes gamma p by rho, now we have seen earlier that velocity of sound Vs was nothing but under root dp over d rho, so now dp over d rho we have found out to be gamma p by rho. So velocity of sound we can write nothing but equals to gamma p by rho, right? So velocity of sound is gamma p by rho, right? Then now if we define a new number which we have not come across till now that is called Mach number, so if possible someday I will introduce this kind of legendary people who have done a lot in the science of fluid flow so at least if I can collect there photographs and names so that really they are remembered whenever you are studying fluid flow, right? So they have done at least century back or even earlier they have done a lot when the facility was not so much, but they have done lots of miracle.

So hope that they also should be remembered or they also should be known very little if possible if time permits someday I will try to make that maybe not whole class but a little so that we come across, however one such similar scientist was Mach and in his name this number is written as Mach number, right? This is M, A, C, H Mach and the number is called Mach number like Reynolds number this is written Nma, right? This Mach number is defined as the velocity of sound in the medium that is v over velocity of sound in that, right?

So this is the dimensionless velocity of that particular thing over velocity of sound, right? So this is v over Vs, right? Now if it is under nozzle because our concern is on in the nozzle we started with nozzle flow we are still continuing so if we look at the nozzle and at the nozzle we said that tip velocity of the nozzle is v0. So we can write it to be v0 over Vs where v0 is the tip velocity of the nozzle, right? So if we write, then we can say Mach number is v0 by Vs, right? And then we can write this v0 that is the tip velocity equals to nothing but Mach number Nma into Vs, right?

Now Nma Vs already we have found out is gamma p by rho under root gamma p by rho under root which can also be rewritten in terms of v specific volume as under root this is p this was gamma p so not rho p gamma p by rho. So this we can write gamma into this corresponding to this rho is the tip velocity corresponding rho so gamma rho okay gamma p by let us write here gamma p by first rho 0 then is equals to Nma gamma p into this rho 0 that is v0, right?

So gamma p and this p is also corresponding to the p at the tip, so we can also write it to be gamma p0 by rho 0, right? So the Mach number velocity at the tip v0 is Nma under root gamma p0 v0, right? Or we can also write v0 square is equals to Nma Mach number square into gamma p0 v0, right? So this is velocity at the tip and this is Nma square gamma 0 v0, right? So if we know this and we already have found out what is the velocity at the tip if you remember that that was 2 gamma p by gamma minus 1 into rho into 1 minus p0 by p to the power gamma minus 1 by gamma whole under root if we remember that.

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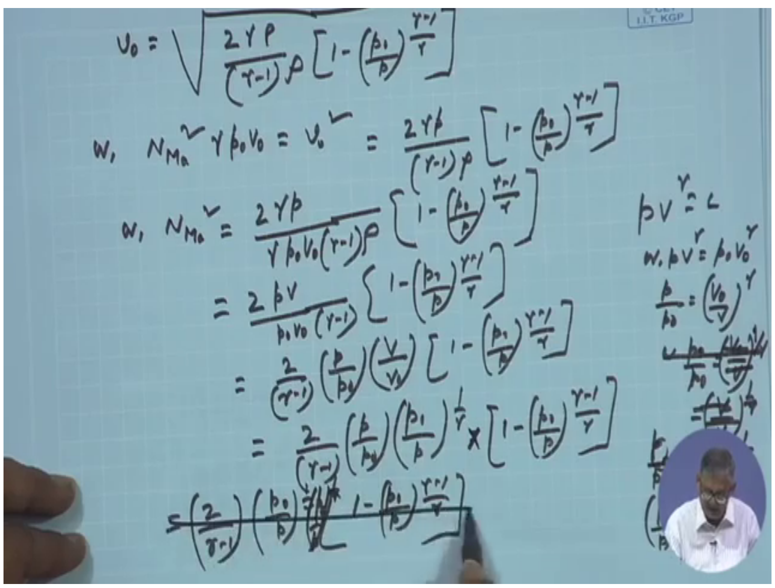

So you can substitute that value of p here, right? So substituting that value of p here, we can write that value of v0 so v0 which we have already found out to be equals to v0 equals to under root 2 gamma p by gamma minus 1 into rho into 1 minus p0 by p to the power gamma minus 1 by gamma this under root, right? So we can write from this previous expression that Nma square is into gamma p0 v0 this was equals to v0 square, right? So this on substitution of v0 we can write is equals to 2 gamma p by gamma minus 1 into rho, right? Into 1 minus p0 over p to the power gamma minus 1 by gamma, right?

So this is Nma square, now if we divide Nma square if we divide both side with gamma p0 v0, then we can write 2 gamma p over gamma p0 v0 into gamma minus 1 into rho into 1 minus p0 by p to the power gamma minus 1 over gamma, right? Then, this we can write again equals to 2 pV because this rho we can take it to the top 2 and this gamma this gamma goes out, so 2 pV by p0 v0 into gamma minus 1 this times 1 minus p0 by p to the power gamma minus 1 by gamma, right?

Now this pV by p0 v0 this we can again segregate like 2 by gamma minus 1 so p by p0 1 v by v0 another into 1 minus p0 over p to the power gamma minus 1 by gamma, right? So this again we can write 2 by gamma minus 1, right? p0 by p is okay, let it remain or p by p0 it was p by p0 and this v by v0 is nothing but p0 by p to the power 1 by gamma p0 by p to the power 1 by gamma because we know pV gamma is equals to constant or pV is equals to p0 v0, right?

So pV gamma is equals to constant or pV gamma is equals to p0 v0 gamma, right? So we can write p by p0 is equals to v0 by v to the power gamma or p0 by p is equals to v0 by v to the power minus gamma, right? This is 1 by gamma because that is inverted, right? So v0 by v v by v0 sorry this is then v by v0 p by p0 is so p by p0 is v0 by v to the gamma, fine so that can be written as v by v0 that can be written as v by v0, right? To the power 1 by gamma p by p0, okay.

Then, so p by p0 is v0 by v to the power gamma and p0 by and here we have v by v0, right? Here we have here v by v0 so v by v0 we can write equals to p0 by p, right? okay. So v0 by v to the power gamma or is equals to v by v0 to the power 1 by gamma, right? p by p0 is so much so it is p0 by p to the power 1 by gamma is v by v0, right? p by p0 remains p0 by p, okay that this is from here we can write this that p0 by p is equals to v by v0 to the power 1 by gamma that is there.

So we write p0 by p to the power 1 by gamma p by p0 this is equals to not equals to into sorry into 1 minus p0 by p to the power gamma minus 1 by gamma, right? So we can then write this we can then take it inside p0 by p, right? p by p0 it was in the inside p0 by p, okay. So p0 by p gamma minus 1 by gamma, right? And if we put 1 by gamma inside p0 by p we can write 2 by gamma minus 1this into p0 by p to the power 1 by gamma into if we take it inside, right? p by p0 if we take it inside, then this is if we take it inside then p by p0 we can write 1 minus gamma 1 minus p0 by p to the power gamma minus 1 by gamma and here we also had this p0 by p to the power 1 by gamma, right?

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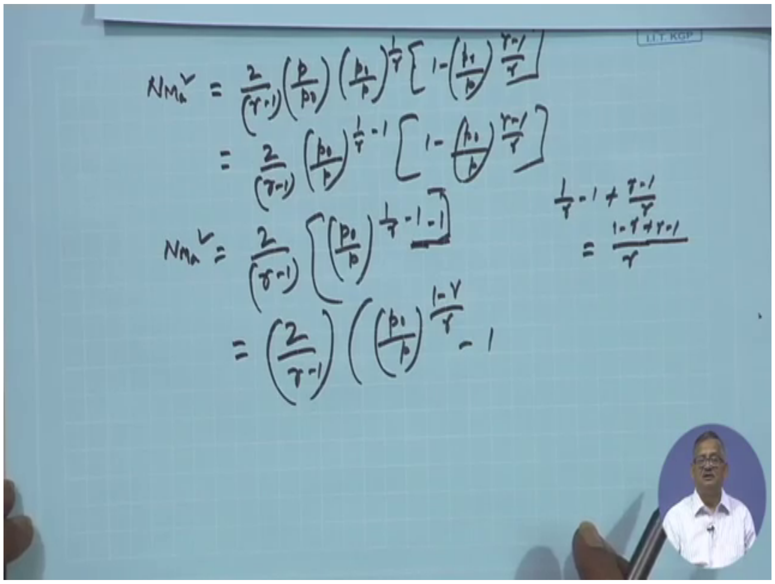

So rather let us rewrite here, Nma square is equals to 2 by gamma minus 1, right? And p by p0 into p0 by p to the power 1 by gamma into 1 minus p0 by p to the power gamma minus 1 by gamma, right? So this is equals to 2 by gamma minus 1, so this is 1 by inverse, right? p0 by p and 1 by gamma, so this is 1 by gamma minus 1. So it comes p0 by p to the power 1 by gamma minus 1, right? Into 1 minus p0 by p to the power gamma minus 1 by gamma, right?

So if we take it inside, then we can write that Nma square is equals to 2 by gamma minus 1, right? Into if we multiply this p0 by p inside, then it becomes p0 by p to the power 1 by gamma minus 1 by gamma minus 1 that is 1 minus gamma by gamma and minus this was p0 by p to the power gamma minus 1 by gamma and this is p0 by p to the power 1 by gamma minus 1 that is 1 by gamma minus 1 plus gamma minus 1 by gamma is equals to gamma so 1 minus gamma plus gamma minus 1, right? So it becomes then gamma gamma goes out this 1, 1 goes out, then it becomes equal to 1, right?

So p0 by p that goes out this becomes equals to 1. So p0 by p to the power 0 so that becomes 1, so this is equals to 2 by gamma minus 1, right? Into p0 by p to the power 1 minus gamma by gamma minus 1, right? So this is what we get and subsequently I think our time is up, so in the next class we will do that from here what could be the relation between Mach number and the velocity, right? Thank you.