**Course on Momentum Transfer in Process Engineering**
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**Lecture 27**
**Module 6**
**Flow through nozzle-1**

Yeah, in continuation to that compressible fluid flow now we would go to because we have done the relations of compressible fluid flow that what is the relation between pressure and velocity that how you can find out the pressure drop right from the Bernoulli's equation we have done it. So we will now go to the compressible fluid flow which we think so we go to the compressible fluid flow and there we go to now flow through nozzle, right?

Flow through nozzle the best example is that you are whistling, right? You do whistle this whistle you do, so whenever you are whistling this compressible flow through nozzles that also come into if you are in if you are in industrial area then obviously you have seen or you have witnessed you have also come across that at certain interval maybe 2 o clock in the afternoon or 10 o clock in the night or 6 o clock in the morning depending on the shift changes the industrial bells they are also they ring some siren and that sound you also have heard.

Unfortunately, or fortunately it should be both fortunately there is no such word and during war (())(2:22) we have seen that to alert people there is a big siren that blows. So these are all example of compressible fluid flow through nozzle, right? So then and this not only this is for air and as we said you remember that for milk homogenization for milk homogenization that high pressure milk is passed through a nozzle and that slit orifice through that when it is moving, then there is also that high pressure to low pressure that pressure drop occurs.

So that is also one such example the flow through small hole orifice or slit things like that explicitly we have done also the flow through slit now we also take care of the flow through nozzle, right?

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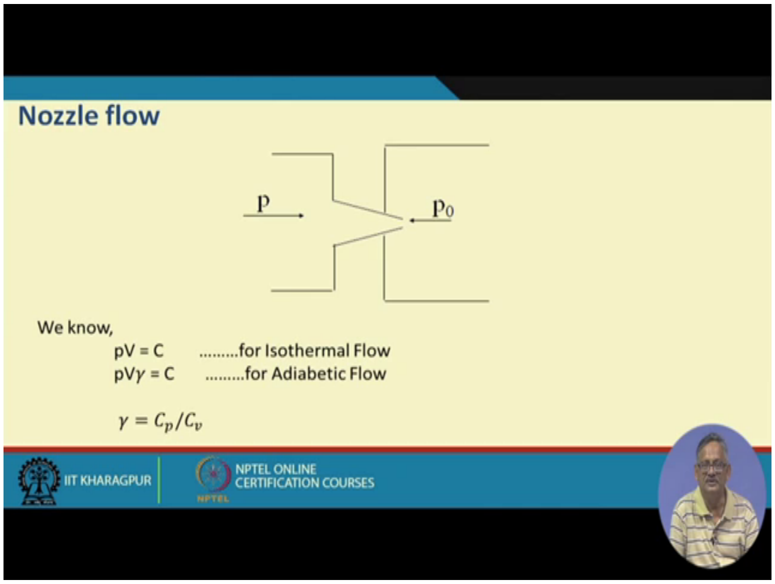


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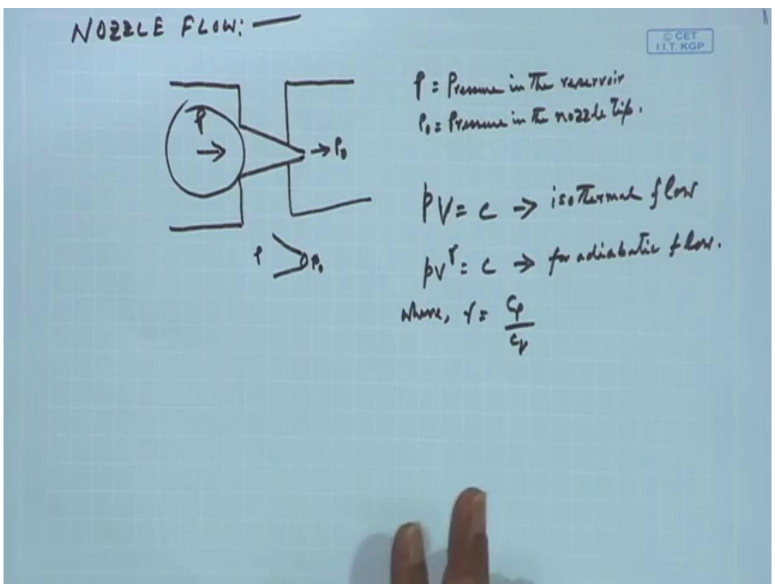


Now nozzle if you see it looks like this as you see from this slide that nozzle is like this that from a high pressure, right? Through a variable area a tip like this, right? And this is one and another could be like this, right? So here the pressure is P the this zone and in this zone at the tip pressure is P0 or P outlet or P at the tip or P at the nozzle whatever you call, right? So this P is the pressure in the reservoir and Po or P0 or P outlet whatever you call that is pressure in the nozzle tip, right?

So tip of the nozzle is like that so we name it to be nozzle flow, right? Now we know two relations between pressure volume, 1 is for isothermal case that pV is equals to say okay pV is equals to constant, right? This is for isothermal case for isothermal flow and for adiabatic flow we know pV gamma is equals to constant, right? So where this is applicable for adiabatic flow where this gamma is equals to the heat capacity ratios that is Cp over Cv or specific heat at constant pressure and specific heat at constant volume this ratio of that fluid, right? Ratio of that fluid Cp is the gamma, right?

Now this situation that is high pressure P where it is moving from this big reservoir through a you see variable area going to the tip, right? So if this tip if we look at this is nothing but like this, right? So through this tip when it is passing from this P to P0 this pressure is dropping, right?

And I do not know whether you have gone through refrigeration or not there also or how so what best example household refrigerator which you come across every now and then there you have seen that in that household refrigerator after compressor there is a condenser and if you have the old refrigerator then you might have seen that on the back side of this there is there is net kind of thing and that is called the condenser and this is of course extended surface condenser finned tube rather and from that condenser it goes to the expansion valve and this expansion valve is also a very small orifice through which from the condenser high pressure it goes to the evaporator through this nozzle and when it is going from this high pressure to the through the nozzle to the evaporator, then because of the pressure drop this process is known as throttling that the temperature drops substantially depending on from which pressure to which pressure you are allowing to drop and also on the refrigerant you are using.

But this is again a very hand on experience which you come across every now and then at your even residence. So that is what so high pressure to low pressure it is being expanded or it is (()) (9:18) because of the nozzle it is becoming the pressure drop. Now, we will find out what is the relation between this pressure and the velocity of the fluid through it. Now again this fluid can be gas or it can any liquid, right? So depending on what you are using obviously your system also will be and also we has we have told that there could be two processes one could be isothermal that is that constant temperature or there could be adiabatic that is there is no heat flow that is adiabatic.

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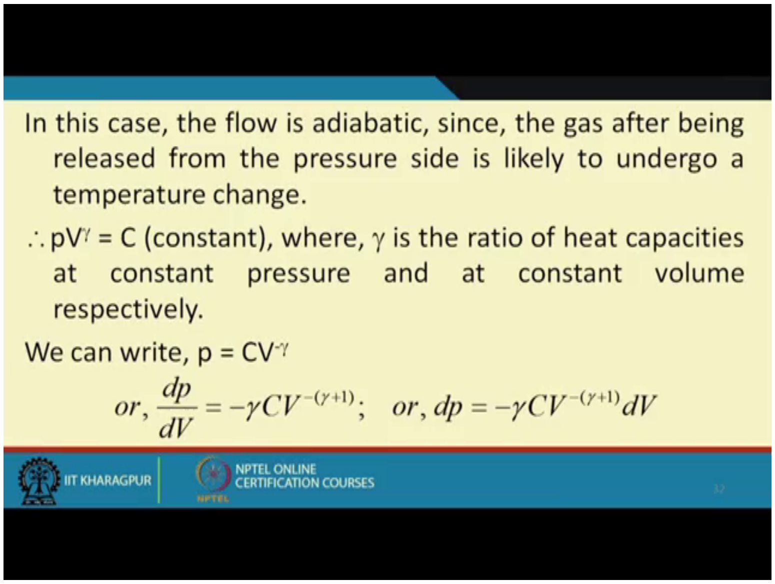


So these two conditions can also come up, right? So let us look into how we can develop this, right? So if the flow is adiabatic so gas after being released from the pressure side high pressure side is likely to undergo a temperature change we said that throttling we gave the example of household refrigerator, right? So this high pressure to low pressure so that undergoes a drop in temperature and this is called throttling and this throttling brings down the refrigerant temperature from condenser temperature to the evaporator temperature, right?

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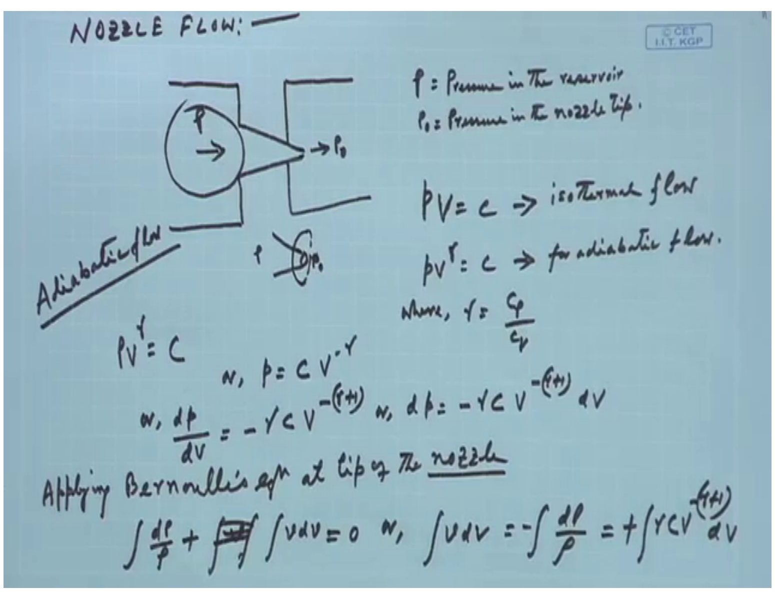

And in that case if it is adiabatic if the flow is adiabatic then let us see that pV gamma is equals to constant, right? Where gamma is the heat capacity ratios at constant pressure and constant volume, right? So if it is pV gamma is constant then we can write that p is equals to C constant V to the power minus gamma, right? Or we also can write dp over dV is equals to minus gamma C V to the power minus gamma plus one, right? Gamma the C minus gamma dp over dV is minus gamma C V to the power minus gamma plus 1 or we can write dp is equals to minus gamma C V to the power minus gamma plus 1 into dV, right?

So this dp over dv this relation we can write. Now if we apply Bernoulli's equation at the tip that is here, so if we apply Bernoulli's equation here so applying Bernoulli's equation at the tip of the nozzle and the tip of the nozzle if we apply Bernoulli's equation, then we can write dp over rho integral plus integral v dv, okay integral v dv, right? Is equals to 0 assuming that there is no other and since that you are considering at the tip so the dz factor if you remember in the earlier classes we had shown the Bernoulli's equation and there we have said that there are many factors like f that is the frictional head loss if there we consider or all losses we came into the terminology as f, right?

So we neglect them we also neglect that there is since at the tip so there is no vertical or z factor also so we can simple write that Bernoulli's equation at the tip of the nozzle is dp over rho plus vdV integral of this two, right? Or we can also write that vdV, right? vdV integral of that is equals to integral of dp over rho minus because this has changed, right? So dp over rho this we can write again as this minus is there and earlier one minus was there so if we substitute this dp from here, then it is integral of gamma C V to the power minus gamma plus 1, right? Into dV, right?

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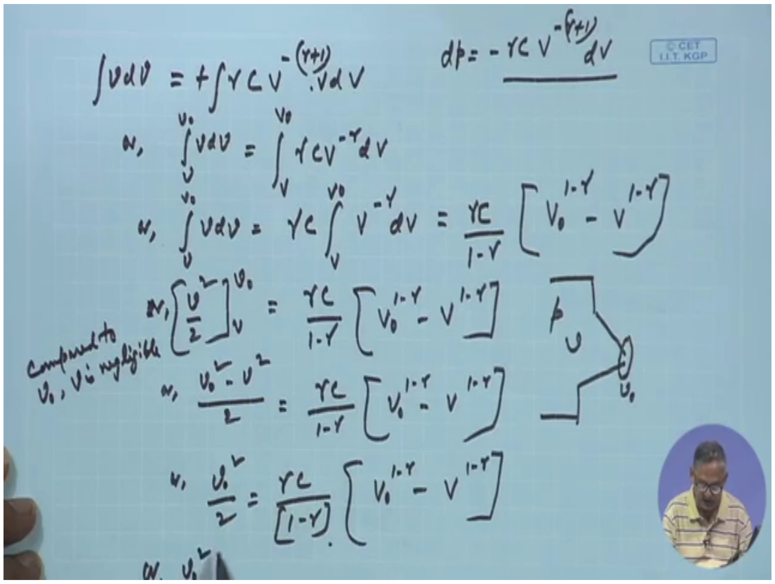


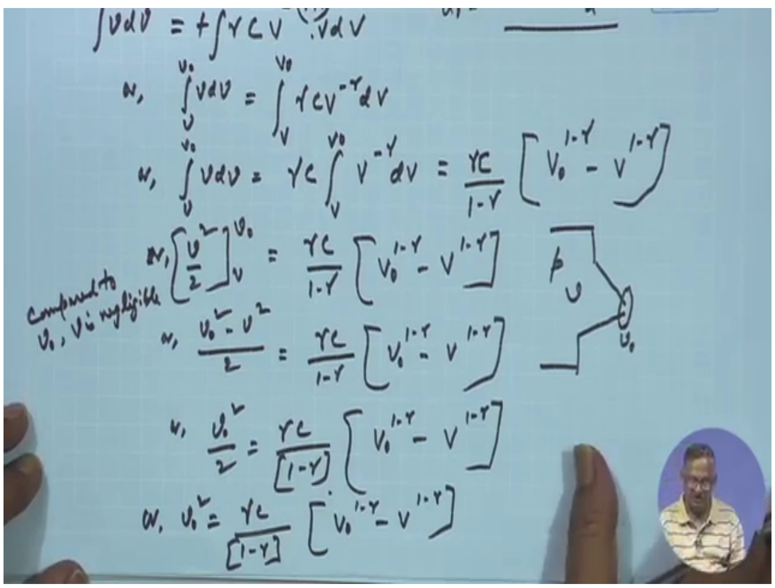


Then, we can say that we have now integral of vdV that is equals to plus gamma C V to the power minus gamma plus one dV, right? So if now integrate within limit, so integral of vdV this is between v is equals to v and v is equals to v0 corresponding to capital V is V and this is v0 gamma C V to the power minus gamma dV, right? So then this is velocity, right? This v is velocity and as in seen as we have told earlier it has that this capital V is specific volume, okay.

Then we can write that this v is equals to this, then we also can write that if this is integrated this vdV v v0 vdV is equals to that this gamma and C are constant so gamma C between v to v0, right? Gamma C v to v0 V to the power minus gamma dV, right?

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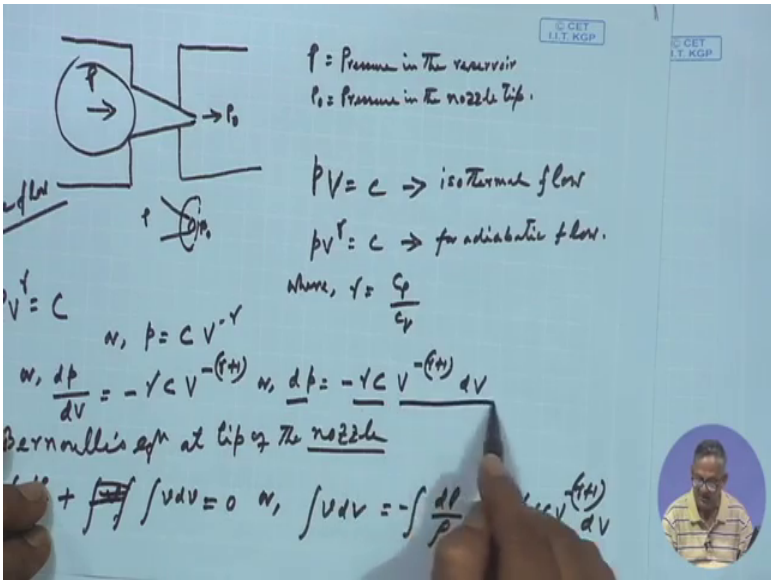


So we can write our one this let us look into this dp was minus gamma C V to the power minus gamma plus 1 dV, right? So when we wrote dp over rho when we wrote so we had let us look into that dp was equals to minus gamma C V to the power minus gamma plus 1 dV, right?

So when we wrote this vdV okay 0 to that and we substituted dp with this because it was vdV if we remember it was integral of vdV minus dp over rho, right? So that minus dp over rho this came to minus gamma C V to the power minus gamma plus 1 dv and because of this 1 by rho 1 v was also there which we have omitted. So 1 v was also there so this comes to equal to gamma C V to the power 1 (minus) plus 1 minus 1 so this comes to equals to gamma C V to the power minus gamma dV, right? So we substitute this in this, then here C V to the power minus gamma plus 1, then we should also write 1 v that was due to rho so that comes gamma C V to the power minus gamma dV, right?

So vdV v to v rho gamma C v to v0 V to the power minus gamma dV gamma C being constant it is out of integration, right? So this on integration we can write gamma C divided by 1 minus gamma, right? Into v0 to the power 1 minus gamma minus v to the power 1 minus gamma, right? This is on definite integral v0 minus gamma and v to the power 1 minus gamma by 1 minus gamma, right? So gamma C is that. So we can write this is or is v square by 2 between v to v0 this is equals to gamma C by 1 minus gamma into v0 to the power 1 minus gamma minus V to the power 1 minus gamma, right?

Or this is nothing but v0 square minus v square by 2 is equals to gamma C by 1 minus gamma v0 to the power 1 minus gamma minus v to the power 1 minus gamma, right? This we know, right? So now compare to v0, what is v0? v0 is the velocity at the tip here, right? What is the velocity at the tip v0 and what is v? v is the velocity in the interior in the your reservoir. So in the reservoir where p pressure was p and velocity is v this velocity is much much negligible that to compare to v0 because v0 area is very very small compare to this area.

So here the velocity is very negligible so compare to v0 v being very very small, right? We compare to v0 v is negligible, so we can write v0 square by 2 is equals to gamma C by 1 minus gamma into v0 to the power 1 minus gamma minus v to the power 1 minus gamma, right? Or v0 square is equals to gamma C by 1 minus gamma into v0 to the power 1 minus gamma minus v to the power 1 minus gamma, right?

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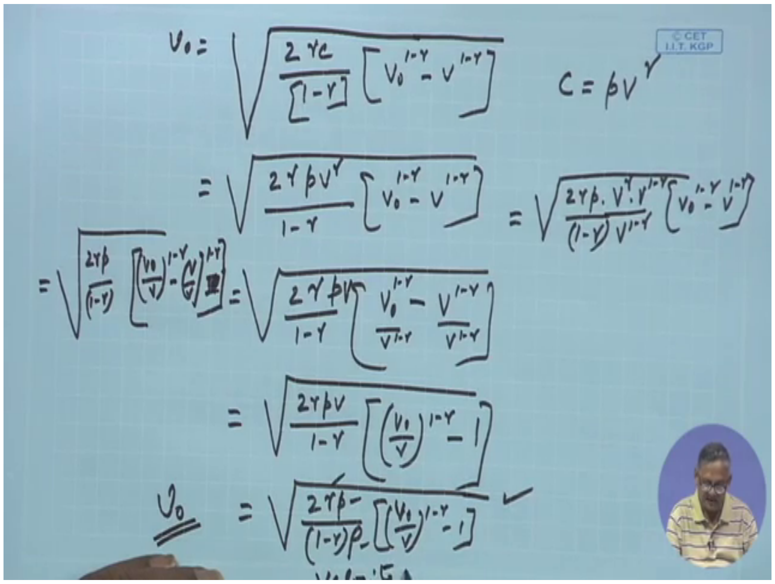

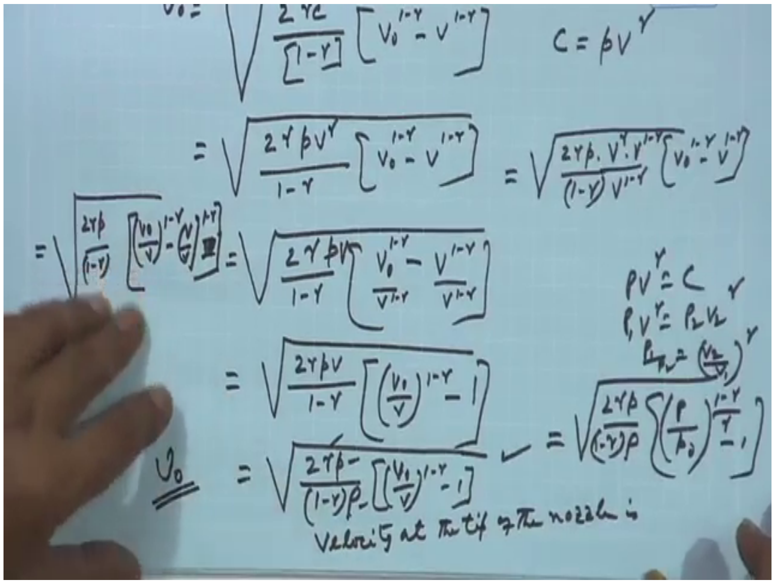


So we also can write that v0 is equals to under root it was v0 by 2, right?

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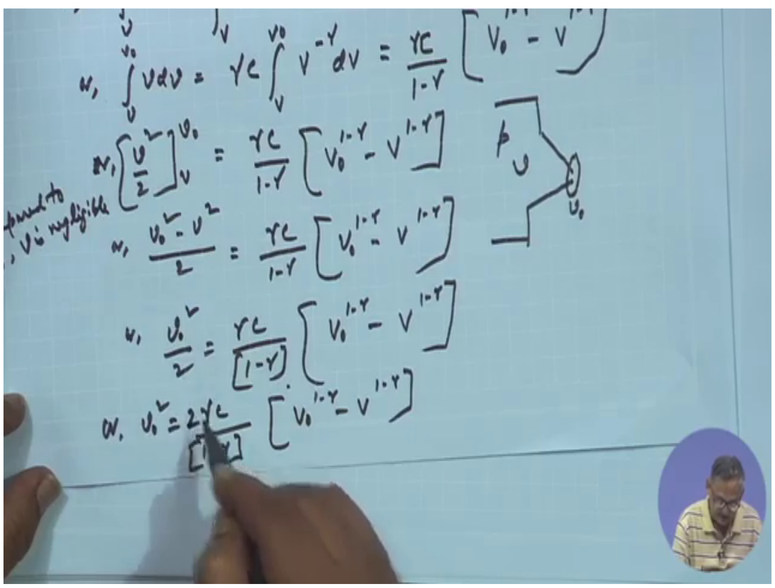

So v0 is 2 gamma C, right? v0 square by 2 so it is v0 square is 2 gamma C 1 minus gamma v0 to the power minus gamma v to the power 1 minus gamma, right? So we can also write that v0 is under root 2 gamma C over 1 minus gamma into v0 to the power 1 minus gamma minus v to the power 1 minus gamma, right? This we can write, therefore we can also write that this is true this is true we can also write this C instead of C C was equals to pV gamma, right? So we can write under root 2 gamma C is pV gamma divided by 1 minus gamma into v0 minus v 1 minus gamma 1 minus gamma, right?

So if we take this v inside, then we can write under root 2 gamma divided by 1 minus gamma p and this V when it is going inside then it becomes gamma plus 1 minus gamma, so v0 only, right? So this was (())(24:43) gamma so you can write that this become equals to this v0 gamma 1 plus plus gamma plus 1 minus gamma so v0 minus V, right? This when when we are putting it inside when we are putting it inside then obviously then this has to be 1 p, right? So v to the power gamma if we take inside then it becomes v0 to the power 1 minus gamma by v, right?

And this becomes v to the power 1 minus gamma by v to the power 1 minus gamma to the power 1 minus gamma, right? Because this we can write under root 2 gamma p divided by 1 minus gamma, right? So we are taking it inside so that means we are introducing divided by v to the power 1 minus gamma if we write v to the power 1 minus gamma v to the power 1 minus gamma here, right? v to the power 1 minus gamma here, then here also we have to write v to the power gamma into v to the power 1 minus gamma, right? Into v0 to the power 1 minus gamma minus v to the power 1 minus gamma, right?

So if that be true, then this v to the power 1 minus gamma can go inside so that we can write under root 2 gamma p by 1 minus gamma this let 1 minus gamma and inside of it is v0 by v to the power 1 minus gamma minus this is v (())(27:16) by v to the power 1 minus gamma, right? 1 minus gamma, right? And in outside we had v to the power gamma into v to the power 1 minus gamma so this gamma plus gamma minus gamma goes out so it remains only v. So that is why it is pV into v0 by v to the power 1 minus gamma by v to the power 1 minus gamma v to the power 1 minus gamma by v to the power 1 minus gamma.

So this is equals to under root 2 gamma pV by 1 minus gamma into v0 by v to the power 1 minus gamma minus this is 1, right? So 1 minus gamma by 1. So this also we can write under root 2 gamma pV or instead of v gamma p by 1 minus gamma over rho p by rho into v0 by v to the power 1 minus gamma minus 1, right? So this is the velocity at the tip, so if we can find out the velocity at the tip, now for that what we need? We need to know the pressure, we need to know the density, we need to know the gamma that is coefficient specific heat to at constant pressure to constant volume heat ratio heat capacity ratio and v0 and v that is the specific volume at the tip and specific volume at the inlet or somewhere inside.

So if we know this, then we can tell that the velocity at the tip v0 can be determined, right? Knowing the of course these parameters which we are talking about, right? So this is the velocity at the tip. So velocity at the tip of the nozzle is this, right? Two gamma p by 1 minus gamma by into rho rather into v0 by v to the power 1 minus gamma minus 1, right? Now of course this all we can convert in terms of in terms of pressure also this can be written as 2 gamma p over 1 minus gamma into rho, right?

Now v0 by pV gamma is constant, right? So p1 v1 gamma is equals to p2 v2 gamma, right? So that means p1 by p2 is equals to v2 by v1 to the power gamma, right? So that means we also can write that v0 by v to the power 1 minus gamma is nothing but p p0 by p p0 by p to the power 1 minus or this is gamma minus 1 by gamma, right? This can be written as gamma minus 1 by gamma, okay 1 minus this can be written as 2 gamma p by 1 minus gamma into rho so this is converted into P by p0 to the power 1 minus gamma by gamma because this is that so P by p0 to the power 1 minus gamma by gamma is that.

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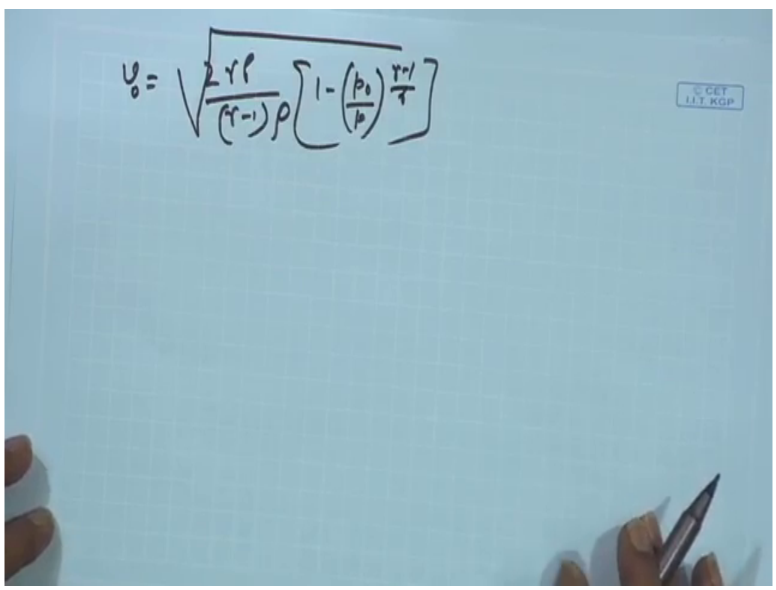

So we can write this minus 1 was there so we can write that v0 is equals to 2 gamma P by gamma minus 1 into rho under root 1 minus p0 by p to the power gamma minus 1 minus by gamma, right? So the this velocity at the tip is this, right? Okay thank you we will do in the next class, thank you.