Course on Momentum Transfer in Process Engineering
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Lecture 26
Module 6
Compressible fluid flow

Till now whatever we have done that was for incompressible fluid you you go through your previous classes everywhere we had said specifically the fluid is incompressible and the problems which you had also taken were also given in such way that the fluid becomes incompressible so that the density is constant, right? But that may not be all the time for all the fluid there are some fluids which we can assume to be incompressible, for example mostly we take water to be incompressible as a liquid, right?

But same it may not be able to take all the time with air or some other gases, so they also come under the fluid. So for that we need to know something about the nature of the incompressibility, right? So if it is not incompressible if it is compressible fluid and we said the difference between compressible and incompressible is that in in incompressible fluid the density change with pressure is not significant, right? It is not significantly changing but if the density and generally if it is even 10 percent it is taken to be incompressible if the change is even within 10 percent this taken to be incompressible for all practical purposes.

But if it is not within 10 percent or it is beyond that then we cannot you can no longer say that the density is constant which is not getting affecting by the application of the pressure, right? So in that case we have to say that okay this is incompressible fluid and where the density is not changing and constant we can achieve in density etcetera. But when it is not that it is changing with the pressure, then we need to say no the pressure is getting changed and that way we can say that density is also changing so it is no longer (com) incompressible.

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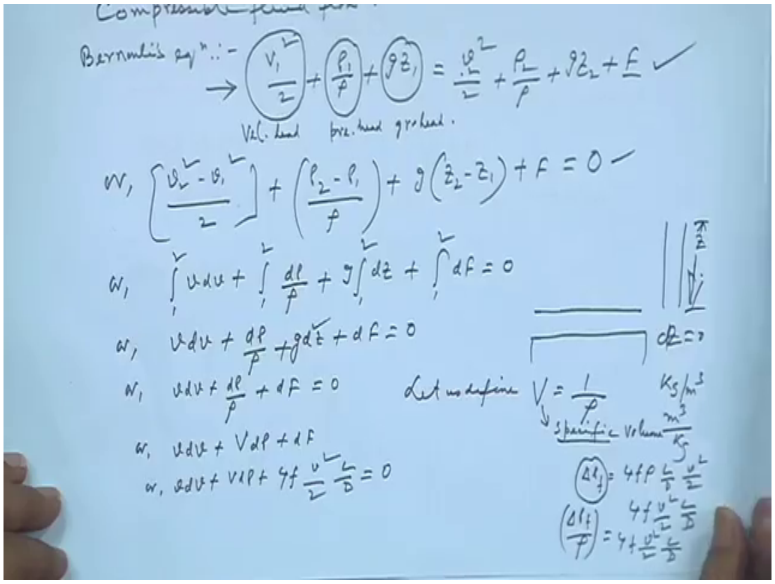

So then it becomes compressible, so now let us come to compressible fluid flow, right? Now Bernoulli's equation I hope you remember, right? So Bernoulli's equation this was also taught in higher secondary level that is known and we what there it is, it is said that say v1 square by 2 plus P1 by rho plus gz1 this is equals to v2 square by 2 plus rho or P2 P1 by rho and P2 by rho plus gz2 plus f if any changes during due to friction and other causes that all comes under f, right?

So this is what we know as Bernoulli's equation where it is called velocity head, this is called pressure head, this is called gravitational head, right? So this is velocity head, this is pressure head and this is called gravitational head, right? So in that case once we know then we can write the Bernoulli's equation in this form. Now if we rearrange this equation and write like this v2 square minus v1 square by 2, right? This is one term plus P2 minus P1 by rho this is another term plus gz2 minus z1, right? Plus say all these factors come into frictional or another losses equal to f is equals to 0, right?

So this rearranging now this we can also re-write as integration between 1 to 2 vdv, right? Plus integration between 1 to 2 dt by rho, right? Plus integration 1 to 2 g dz plus integration between 1 to 2 df this is equals to 0, right? How did you write? So this is nothing but the expanded or integral (fo) integration of this vdv and v2 square minus d1 square by 2. Similarly this is P2 by rho minus P1 by rho, similarly this is g into z2 minus z1 and this is f as df, right?

So this is called definite integration within the boundary 1 to 2 (())(6:37), right? Now if we remove the boundary and if we make it generalize differential form then we can write vdv plus dP over rho plus dz g of course g dz plus df is equals to 0, right? Now if assume that the flow is occurring in horizontal pipe, right? Then this dz term goes out if it is horizontal if it is added in vertical, then it should have been that what is the value of z, right?

(Wha) what is the value of z that would have come, but if it is not vertical, then if it is horizontal there is no z, so z is equals to 0 or vz is equals to 0 we can neglect this term. So we can write vdv plus dP by rho plus df is equals to 0, right? So this is true, now let us define let us define that capital V is equals to 1 by rho, right? rho is kg per meter cube, so 1 by rho would be meter cube per kg so then it is called specific volume, right? This capital V is nothing but specific volume.

So anything per unit mask is becomes specific like heat specific heat, specific gravity per of course specific gravity is (())(8:44) to but that has density you may it is less so specific heat then specific energy consumption all these are per unit mask, so when it is per unit mask then it is called the specific of that thing. In this case since meter cube per kg so it is a specific volume, right? So once we know that and once we substitute rho with V, then you can write vdv plus 1 by rho is v so capital V dv plus dx.

Now this f if you remember we had in our earlier class also said that delta Pf is equals to 4 f rho L by g 3 square by 2, right? This was there, now if we say that whatever (wha) (wha) whatever f value that put on f losses is due to the frictional loss then you can substitute this to with the with the df, right? As 4 f, right? And v square by 2 and 1 by D, right? So this if we substitute there then you get vdv plus vdp plus 4 f v square by 2 L by D this is equals to 0, right?

So here we are assuming that all the (fric) losses rather spatial or friction loss all the losses are are combined in df and in that case delta f by rho this is said delta Pf by rho that is the (())(11:26) is equals to 4 f v square by 2 L by D, right? So this is known.

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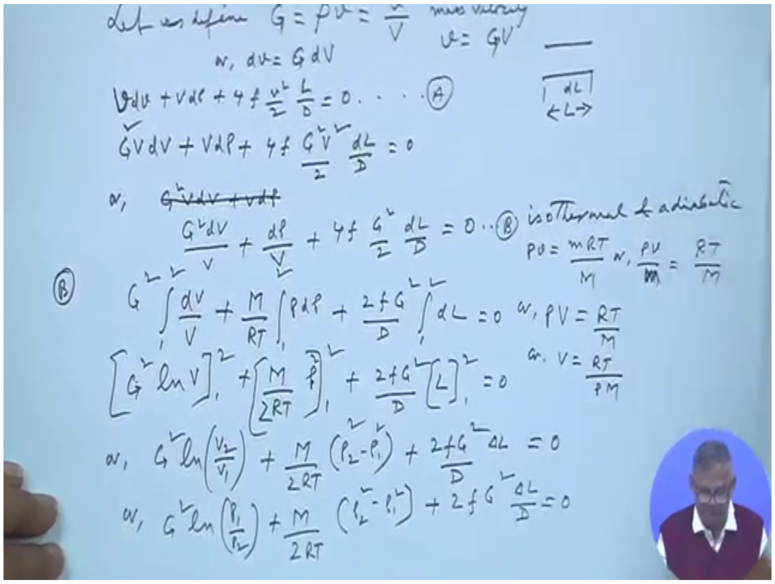

Now let us also define another term called capital G, right? This capital G is equals to rho into v that means rho is 1 by capital V is v by V this capital V is called mass velocity, right? So this capital V is called mass velocity.

So if we know G is v by V, then you can write dv is equals to G dv because this mass velocity is a constant, right? If rho increases v decreases if v decreases rho increases, right? rho increases v decreases rho decreases v increases, right? Then the product remains constant and in that case this is dv is G dv, right? So if we substitute in the previous equation now let us write V dv plus v dt plus 4 f v square by 2 L by D is equals to 0 that was the equation.

So in this if we substitute dv G dv, then we will get okay this was v dv small, okay. So v so that v we have substituted v is equals to G into capital V, right? So this GV then you can write GV and dv already we have seen is also G dv, right? So then it becomes G square V dv, right? Plus v dP, right? Plus 4 f v square, so G square capital G square, right? Divided by 2 and this L by D you can write as dL by ultimately this L has come on integration of upon dL, right?

So this is there because L is variable how much is the length of the pipe so that is a variable. So if had not been taken dL so it should be ultimately L, right? So that we can also write in terms of dL by D, right? So we had written this as G square v dv plus v dP, right? Or this we can also write dP plus okay now now if we if we divide all the terms with v square so that this v square goes out, then we can re-write as this was G square so this v goes out so G square dv over v, right? Plus this v square comes in so V by v square so dP by v, right? Plus 4 f G square by 2 dL by D, right?

This is equals to 0, right? Now for isothermal and adiabatic process so isothermal and adiabatic processes so we know the relation between the test that is pressure volume. So we know Pv is equals to mRt by capital M, right? This is for isothermal case or we can write Pv by M is equals to Pv by m small m is equals to RT by M, right? Pv by small m is equals to RT by m this is small m, right? Or we can also write Pv by small m this v by small m is nothing but capital this is volume, this is mass so this is capital V, right? So PV is equals to RT by M, right? Or V is equals to RT by PM, right? So if we substitute this value of V in this equation say it has equation named B, right?

So earlier if we (())(17:20) A so if it was A, then then the equation B if we substitute we can write that this G square, right? G square dv by v and integrate of course substituting and then integrating, so between 1 to 2 this is dv by v, right? dv by v plus this we write in terms of in terms in terms of dP by V, okay.

This is dP and M by RT this means dv that is PV is equals to this, so so PV is equals to RT by PM so okay we can write M by RT, right? Between 1 to 2 PdP, right? Plus 2 f G square by D between 1 to 2 dL, right? So G square by 2 2 this two goes off 2 f G square by D dL 1 to 2 this is dP by V that this dP we have kept but capital V we have substituted as in the form of dP in the form of P that V is equals to RT by PM. So RT by PM in that case RT by M comes out of the integration P dP remains and this is since it is dv by V note it remain as dv by V because both are in the (())(19:52).

So in then if we integrate this we can write this to be equals to G square ln of dv by V though it is ln V between 1 to 2, right? G square ln V this is between 1 to 2 plus this is M by RT, right? So this is again between 1 to 2 so P 2 minus P1 so that we can write so M T R N P square by 2, right? So 2 RT MRT this is within the this limit plus 2 f G square by D L between 1 to 2, right? So this we can write this is equals to 0 this was equals to 0 this is also equals to 0. So you can write G square, right? ln this is V2 minus V1 means V2 by V1.

So this we can write plus this is M by 2RT this is P2 minus P1, right? Plus 2 f G square by D, right? And this we can write to be equals to delta L or whatever, right? L2 minus L1 delta L whatever we like, right? This is equals to 0. So we can also write this as G square ln, right? V2 by V2 by V1, right? So it is P1 V1 by T1 is T2 V2 by T2 from there we can write V2 by V1 is equals to T1 by T2. So ln of P1 by P2 plus this is in terms of P M by RT or 2 RT this is in terms of P P 2 square, okay it was square this square, right? That we have given P square by 2.

So this was P1 P2 square, right? P2 square minus P1 square, right? And plus plus P2 square by P1 square P2 minus P1 square (())(23:04) plus 2 f G 2 f G square delta L by D. So this is equals to 0, right?

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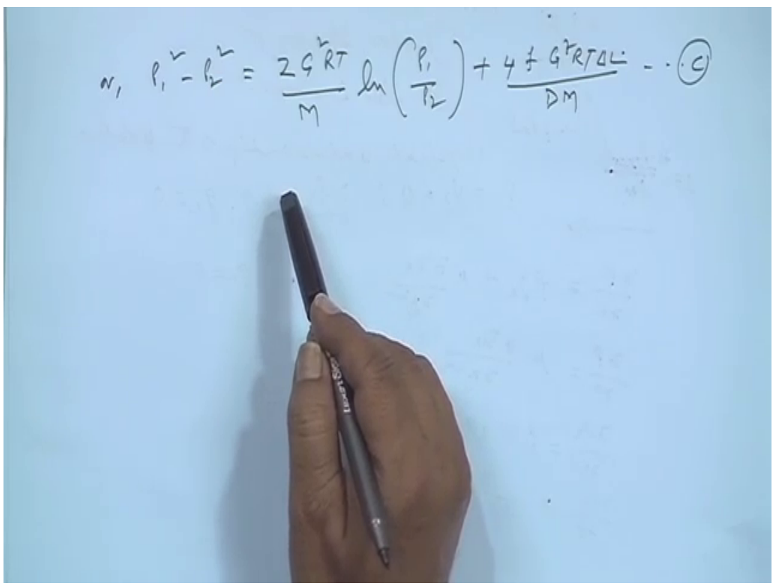

This on simplification we can write or this on simplification we can write P1 square minus P2 square this is equals to 2 G square RT over M into ln of P1 by P2, right? Plus 4 f G square RT delta L over DM, right?

Where of course we know the values where of course we know the cases that this G is the mass velocity, right? G is the mass velocity capital V is the specific volume, right? M is the molecular rate of the gas and and P1, P2 are the pressures at the two points, delta L is the length through which it is flowing or moving, R is the universal gas constant, D is the diameter through which the fluid is flowing, right? So if we know some of them or most of them we can know one unknown, right? Because this is one equation so we can find out from one equation only one unknown it cannot be that multiple unknown can be found out.

But in many cases if it is more than 1, then by trial and error we do not know the solution. So by trial and error the solutions also can be found out and then the right it is just find out root like that finding out root of an equation, similarly what is the finding it is nothing but finding out the solution and the root of the equation that means that what is what what is the value exact value which is satisfying the equation. So if that can be done, then we can say yes for the compressible fluid flow we can now find out the pressure drop or we can find out the individual pressure if one is known the other can be found out.

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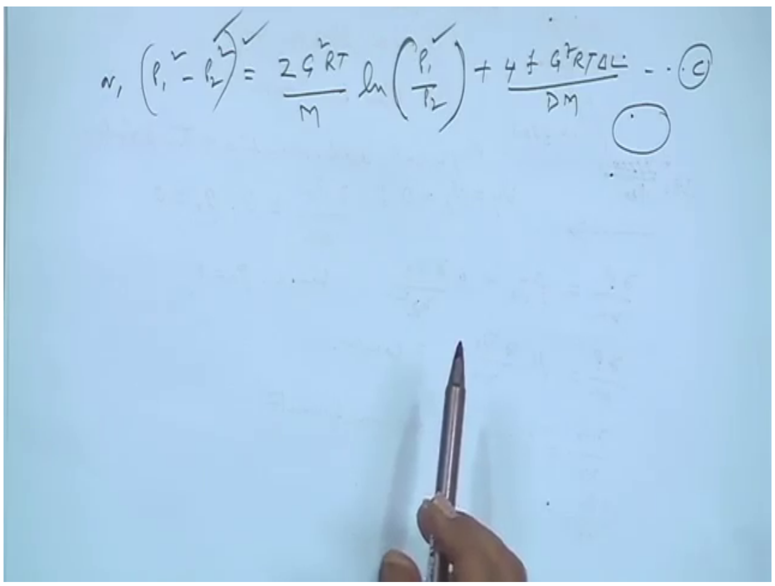

But it is not that easy because P1 and P2, right? Both are in the square term P1 square and P2 square. So once if it is known, then we can find out the other one, right? And this value is that 2 G square RT by M ln of P1 by P2, right?

The problem is that when you are handling that time since P1 square and P2 square they are different, right? And since you do not have any relation between P1 and P2 so you cannot know the ln value of that, right? If you do not know the ln value of that, how can you find out this value also. So what normally is done what you can do or what we can do we can assume one P1 value say P2 value is give we assume one P1 value then we know this ln value of this P1 P2 and all other in this term delta L, T, R, G, D, (())(27:14) f all are known.

So this quantity is known and if we see that assuming this P1 whatever be the value, this value becoming much more smaller compare to this, right? Then, for the first this is again finding out the roots of an of an equation not solution of the equation, so once we find out that this value total is much much smaller this value, then we neglect this value find out from there what is the value of P1 new, right? So that new P1 can be then taken as substituting this P1 again as the initial (in) (in) initial guess for this just as you have done or if if you will be doing somewhere that finding of roots or you have to you have to initial initialize or you have to assume some initial value.

So that assumption can be made from here that we assume it and then we find out this ratio and in that case we find out this total. Again till they are becoming more or less closer both are not given out then we go on finding out this value of P1 and gradually in 1, 2, 3, 4 trials we will find out to the decimal first or second or third position accuracy. So this way by trial and error it maybe (())(29:10) we can find out that solution, okay So next time we will do this problem and solution and see how this (())(29:20) can be done, thank you.