Course on Momentum Transfer in Process Engineering
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Lecture 24
Module 5
Flow through flat plates or slits

Yeah, today we will do on flow through flat planes, right? Flow through slit or flat plates those as we mentioned in the previous class previous class that the flow through the homogenizing valve this homogenizing valve can be assumed to be say slit, right? A very small hole through which it is moving and during that process as we said earlier that the fat globules get homogenized or the bigger fat particles becomes smaller according to the size which you have given according to the space through which the flow is occurring in then that will indicate that at what ultimate size of the fat globules you will get, right?

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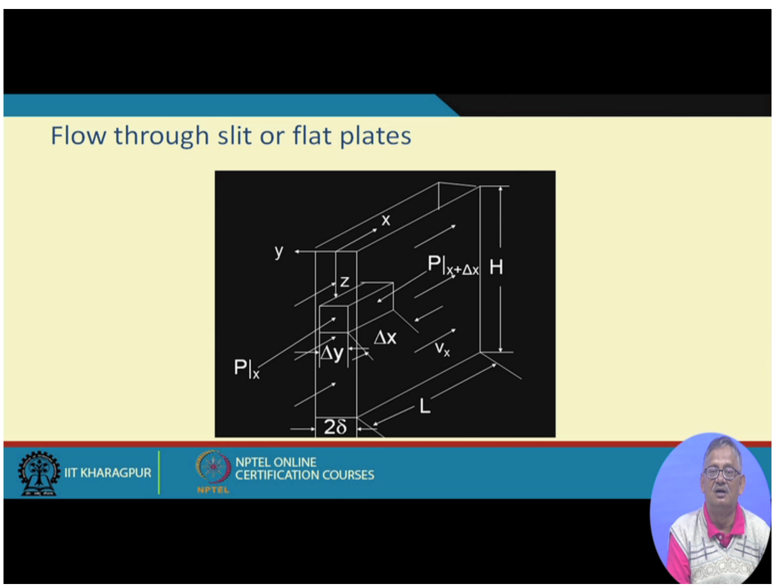

So let us look into that today, so now we have drawn one similar thing as it is here flow through slit or flat plates, right? So we have taken this to be enlarged (value) figure of the slit, right? This is the enlarge view and in which we have taken a small volume element, right? We have taken a small volume element and this volume element has the dimensions of delta x, delta y and delta z, right? Or say if the third dimension be delta z or H and this width or one dimension could be 2 delta so that the center is at plus minus del, right? 0 plus del minus del and this is say L and this is I, right?

So we will do the similar force balances as we have done earlier and we will find out what is the velocity profile and what is the stress profile in the volume element and then integrate over the entire range right?

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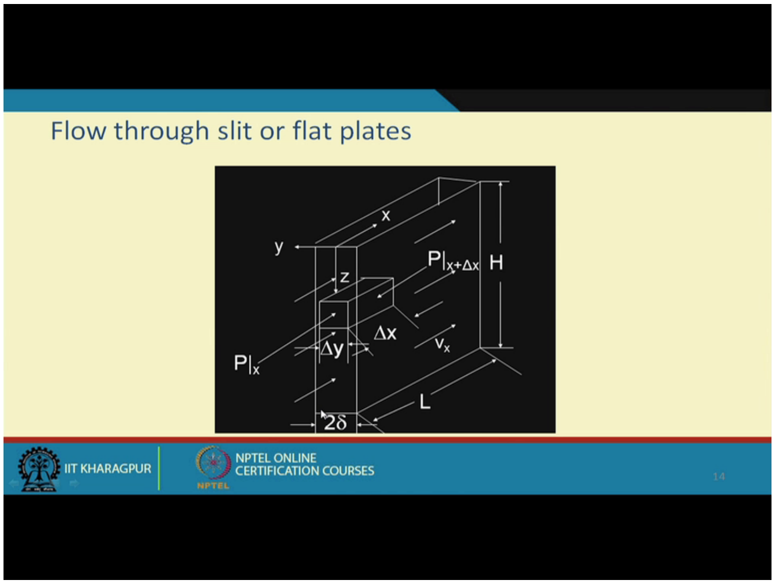

So again this we had taken a 2 del, so the axis is at the center so this is plus del and minus del this is L length and height H, right? So what is acting P at the (fo) pressure inside our inlet at P at x and at the outlet at the phase x plus delta x it is P at x plus delta x, right? So let us look into that so similar to the flow problems earlier we have done, right?

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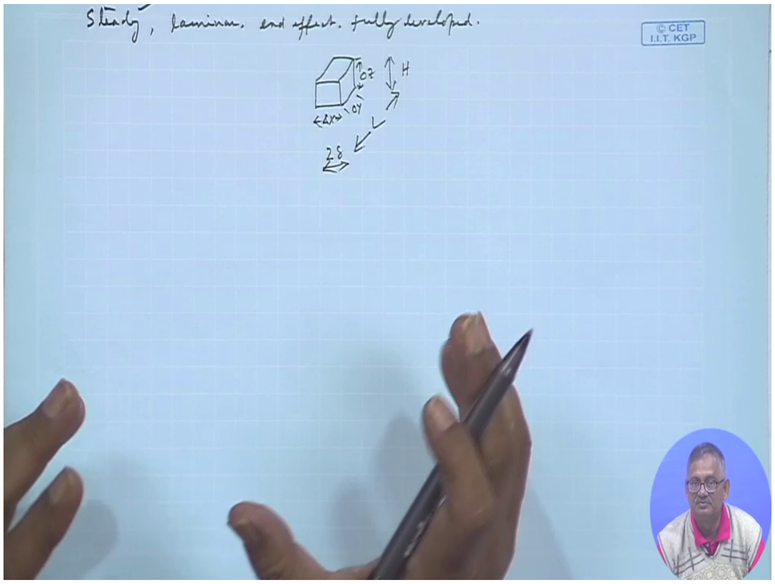

Here, also we assume some of the things that one is the flow is steady flow is stead then the flow is laminar there is no end effect rather it is fully developed, right? Fully developed flow and we will take a small volume and that small volume is having del x del y del z, right? This del x del y and del z, right? Where we have shown this is the (vol) small volume and and the entire slit we had shown to be equals to 2 del then L to del in this way L in this way and H in this way, right?

That was the entire slit, right? Now, since we are saying that it is a steady state, then the then the from the general equation which we start with that the general equation or governing equation that some of the forces acting on to it is equals to 0, right? And that also we will do here, but since we said that the momentum transfer takes place in two ways, one by the bulk molecule and one by the molecular transport so this bulk force bulk fluid that term goes off, right?

Then we are neglecting that so unnecessarily we again writing everything momentum in minus momentum out for the bulk fluid that we are skipping.

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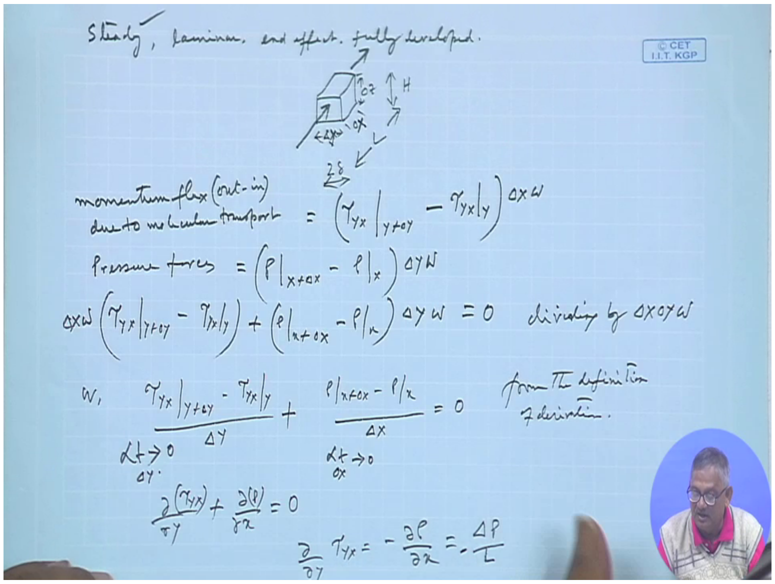

So we are starting with that the momentum flux, right? Due to molecular transport momentum flux due to molecular transport that we take as tau yx at the phase y plus delta y minus this is okay instead of flux we call it to be out minus in, right? tau y (pl) at the phase tau y plus delta y times the area that or rather tau yx at the phase y times the area that is del that is del del at the at the phase del x into w, right?

So this is area del x into w this is acting flow is occurring through this, right? We said the flow is occurring like that, okay. So there is the area del x into width that is w or H ultimately it will come, right? This is the momentum flux, so pressure drop or pressure force acting on this volume element net is P at the phase x plus delta x, right? And minus P at the phase x, right? Times the area that is delta y times w, right? So if this be true, then some of the forces acting we can write tau yx at y plus delta y minus tau yx at y, right? Plus P at x plus delta x minus P at x this times delta y w this times delta x w, right?

So now if we divide this is equals to 0, if we divide both sides with del x w and y, then next stage we can write tau yx at y plus delta y minus tau yx at y over over this we can say delta y minus P at x plus delta x minus P at x over or this is of course plus divided by delta x is equals to 0 dividing dividing by del x del y w, right? Then we get this, and now if put that limit condition limit delta y tends to 0, right? And the limit delta x tends to 0 if we put and then from the definition of derivative we can write that del del y of tau yx plus del del x of P del P del x rather is equals to 0, right?

So we can write del del y of tau yx is equals to minus del P del x, right? So this del P del x you can write delta P over del x that is delta P actually actually this should have been written del y this should have been written del x, right? So this is del y this is del x exactly, so yeah this is del y this is del x otherwise this is not coming, okay del x del y this area okay. So this is 2 del which we ultimately said, so delta P over this is del x if it is written like that so minus delta P over L is del del y of tau yx, right?

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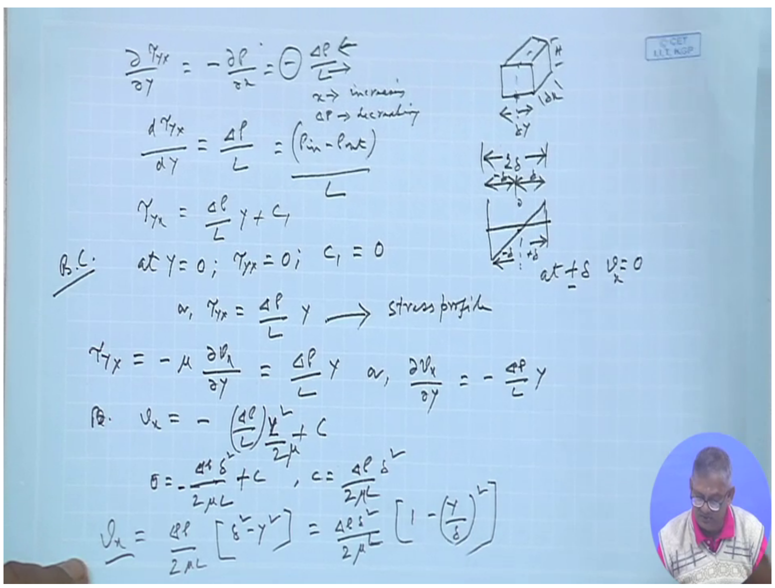

Then, we can now say that this negative sign or del del y of tau yx this is equals to minus del P del x is equals to minus delta P over L this indicates that as the as x increasing, right? As (incres) at x increasing value of delta P is decreasing the negative sign says that, alright as L is increasing delta P is decreasing, right? So we can write that del tau yx again that del tau yx then we can say that d tau yx dy or delta yx del y is equals to delta P over L in this delta P takes care of is equals to P in minus P out, right? Over L.

So this negative sign is taken into delta P, right? Now if we integrate then what we get that tau yx is equals to, right? delta P over L into y plus C1, right? So what we can write at boundary conditions if we put boundary condition that is what at y is equals to 0 y is equals to 0 if you remember our thing like this, right? And this was like this, right? We said that this is 2 del okay del y it was this was del x and this was H or del z whatever you call, right?

So at y is equals to 0, that means this point at y is equals to 0 tau yx that is the (())(14:57), right? Not the one the center. So tau at y is equals to 0 tau yx is equals to 0, right? Then we can write C1 is also equal to 0 or tau yx is equals to delta P over L into y that is the this is the stress profile, right? If you plot this stress profile that as y is increasing tau yx is also increasing, right? So under limiting condition that is when y is del, then tau yx will have one plus del when y is equals to minus del tau yx will have another because this we have told ultimately the bigger one is 2 del that is the thickness or width or whatever we call, right?

Where the axis is at the center, so it is minus del plus del say this point and minus del this point, right? So here it is 0 so we write this as y is increasing tau yx is increasing true but it is (lim) putting to the limit when it is plus del then yx is plus and where is minus del tau yx is also some minus, that means some if this is the point and if this is the point this is 0, so one will have one and another will have this, right? This is the stress profile if we take this to be the stress profile, okay fine.

Then substituting tau yx as minus tau yx is equals to minus mu del vx del y, right? We can write that this is equal to delta P over L into y, right? And if this is true, then again putting the second boundary or or now if we if re-write this del vx del y is equals to minus delta P over L, right? Or L into y rather, okay or we on integration you can write that vx is equals to this minus delta P over L into C or rather y y square by 2, right? y square by 2 plus C y square by 2 plus C.

So if we put that boundary, what is the boundary? Boundary is at plus del or minus del v is equals to 0 or vx is equals to 0 at plus del or minus del because plus del is at the valve and this minus del is also at the valve and this minus del is also at the valve, right? So in that case we can say that this plus del or minus del at plus minus del this is equals to 0 vx. So we can write 0 is equals to minus 0 is equals to minus say plus minus del we have taken, so we can write delta P over 2 mu L this mu comes here or here it should have been 2 mu.

So 2 mu L, right? 0 is equals to is equals to this is plus C so if we take minus then C is equals to delta P over 2 mu L, right? And and we can write vx is equals to delta P by 2 mu L if we take it to be common, right? Then we have said delta P this was y C and of course y square. So it is del square, right? This was del square so C is equals to this into del square, right? So if we take del square common or del square minus y square is the vx, right? So or we can also write this is also equals to delta P del square over 2 mu L into into 1 minus y by del whole square, right?

So this is the vx at any instant velocity is like that at any instant velocity vx that is delta P del square by 2 mu L into 1 minus y by delta whole square, okay y by del whole square.

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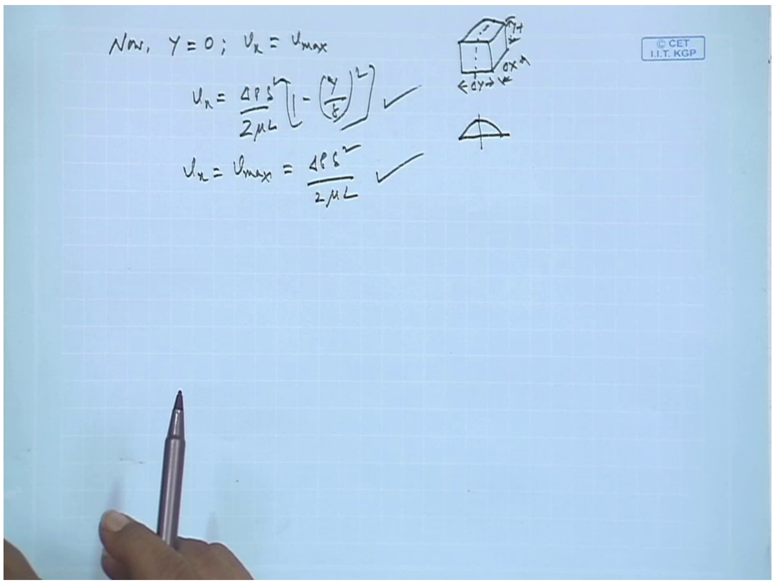

Now to (understa) for the next one that is now at y is equals to 0, what is the value of vx? vx should become v max, right? I again for understanding put this we had this, we had this delta x we had this delta y, right? We had this as H or height or delta z whatever we call, right?

Then v at y is equals to 0 that is this, right? At y is equals to 0 we can say vx is v max, right? What we had this one equation vx is equals to delta P by 2 L 2 mu L into 1 minus y by del whole square, right? del square, right? This is nothing but parabolic equation, right? That is why the the profile we always see to be parabolic like that, right? If we see the profile of velocity it is always parabolic like this, right? So this is also parabolic in nature, so at y is equals to 0, then v max vx becomes v max and that it becomes equals to delta P into del square by 2 mu L because this becomes 0 1 minus this so that is 1.

So delta p del square by 2 mu L is the v max, right? Next we go to the other one that is the average velocity.

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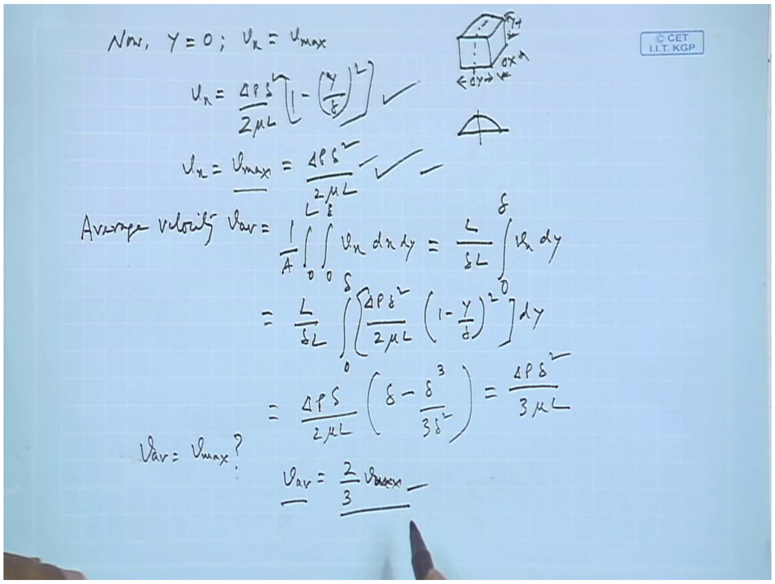

So v max you have found out v we have found out, then from there we have found out v max you have also seen the tau and now tau is that is the momentum flux, now let us go to find out average velocity. So average velocity v average that can be said to be equals to integration of 1 by A 0 to l 0 to del or plus del to minus del or 0 to del half of that vx dx dy, right? So this is nothing but L over delta L, right? Because one integration of dvx dx so that that goes into this del.

So equal to L, right? 0 to L 0 to del (())(25:09) y is there, so vx dy we have L by del L, right? 0 to del vx dy we have and this we can write L by del L, right? And vx if we substitute with this delta P del square by 2 mu L into 1 minus y by del whole square, right? This was between 0 to del, right? This into dy, right? So this on simplification and integration and simplification we can write to be delta P we missed out delta P is there so that comes out delta P into del by by L L goes out 2 mu L this comes there and on integration dy is y putting the limit is del minus this becomes del cube by 3 del square, right?

Del cube by 3 del square or this is becoming nothing but equals to delta P del square delta P del square by 3 mu L because this del cube this del square goes out it becomes del by 3, so 3 del minus del that is 2 del by 3, right? 2 del this 2 and this 2 goes out this del becomes del square and this 3 remains, right? So this is 3 mu L delta P del square by 3 mu L, right? And this if you remember we had made v max v max was delta P del square by 2 mu L. Then what is the relation between v average and v max?

v average and v max that relation becomes equals to v average is equals to 2 third of v max, right? 2 third of v max is the v average, right? So this way if we can find out that what is the average velocity in terms of maximum velocity it is 2 third.

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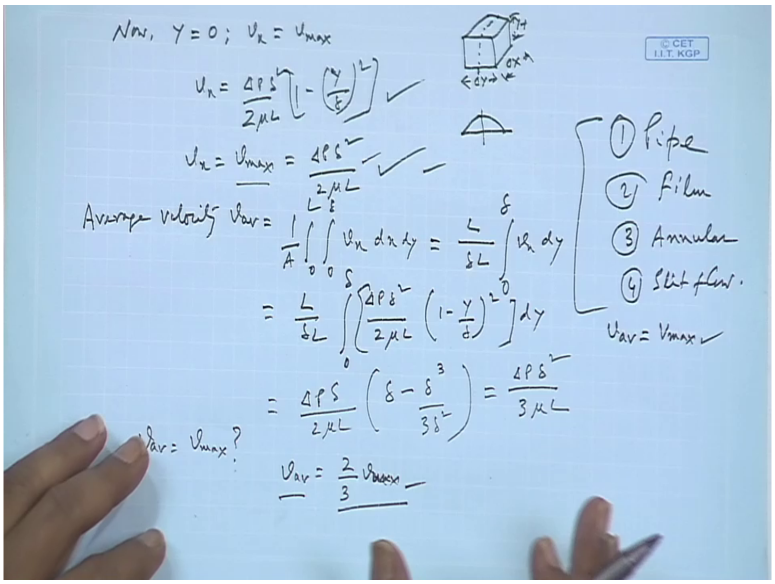

Now if you if you remember what it was for the pipe flow average velocity with maximum velocity what it was for for the flow through or or for film flow and this is the slit flow then this 3 you put together and and see and see that there will be a distinct difference between the three, right?

In one case it is pipe flow, in other case it is film flow, in third case it is annular flow and in the fourth case it is that slit flow. So all these four cases you just see what is the relation between v average and v max in all 4 cases you will see that they are distinctly different and I tell I tell you that you find out and make a tabular form so that you remember them for future activities, right? For future work or for in future so that you can memorize it, if required that what is the velocity and maximum velocity relation in pipe or in annulus or in film or maybe in case of slit, right?

Then you will see they are not same, they are different, okay.

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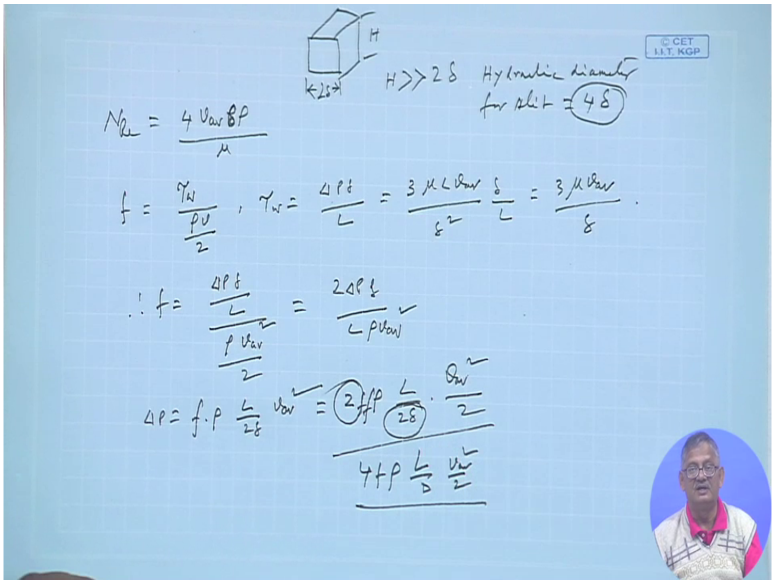

Now let us look into that H we have said to be the height again we draw that H to be equals to the height, right? This was our H and this H is quite quite bigger, right? H is much much greater than to delta, right? So in that case we can say that the hydraulic diameter for the slit for the slit that becomes equals to 4 del, right? So so we can say NRe this becomes 4 because we have taken hydraulic diameter to be 4 del because H is much much greater than 2 del, so hydraulic diameter becomes equal to 4 del.

So we can write 4 v average del rho, sorry del rho divided by mu to be the Reynolds number 4 v average del rho by mu, right? And fanning friction factor f we can write that is tau at valve w over rho into v by 2, right? Or tau at valve is equals to delta P del by L, right? That is equals to 3 mu L v average by del square into del by L, right? That is equals to 3 mu v average by del, right? Therefore, f we can write delta P del over L over rho v average square by 2, so this is 2 delta P del by L rho v average square.

Therefore, delta P can be written 4 rho L by 2 del v average square, right? So this can be also written 2 f rho L by 2 del into v average square by 2, right? So if you here it is 2 del here it is 2, right? So so if we remember for the pipe flow what was delta P, right? That with f that was 4 f rho L by D v square by 2, right? So that was for the pipe flow so this is also analogous, so that is why these four as I said that if you find out all 4 you will find it to be so different and then you should have all these together in one page and then it will be easy for you to remember, okay thank you.