Course on Momentum Transfer in Process Engineering

By Professor Tridib Kumar Goswami
Department of Agricultural & Food Engineering
Indian Institute of Technology, Kharagpur

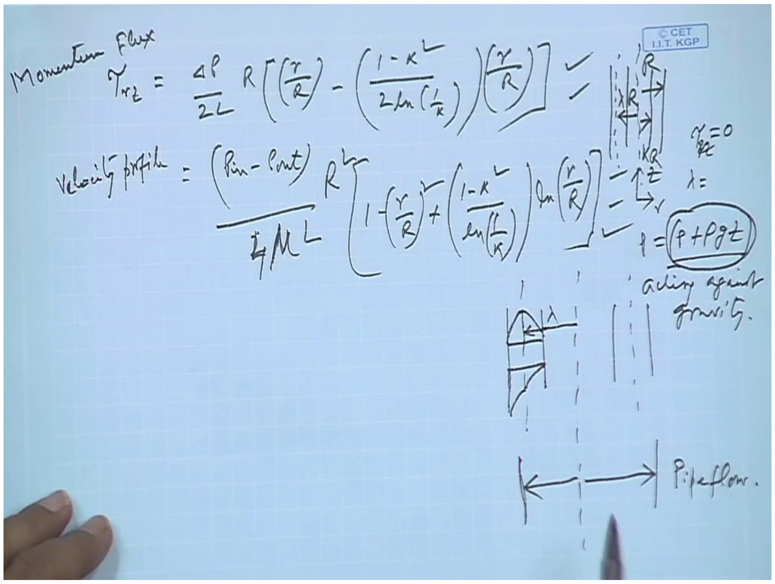
Lecture 22

Module 4

Flow through annulus (Part 2)

Okay, then we had developed the flow characteristics flow behavior when we had seen that the flow is taking place in a annular space, right?

(Refer Slide Time: 0:45)



So that annular space if we remember that we had we had develop the equation that momentum flux tau rz is equals to delta P over 2 L R into into r over R minus 1 minus k square by 2 ln of 1 by k in this times r over R that was for the tau rz or momentum flux, right? When we had the two annular space this was our central this was our kR and this was our R, right? And we also assume one imaginary one which was lambda R, right? And there we assume that this tau rz is equals to 0 that was one imaginary axis where this tau rz was 0 we found out this value of lambda and then we substitute those values to the momentum flux.

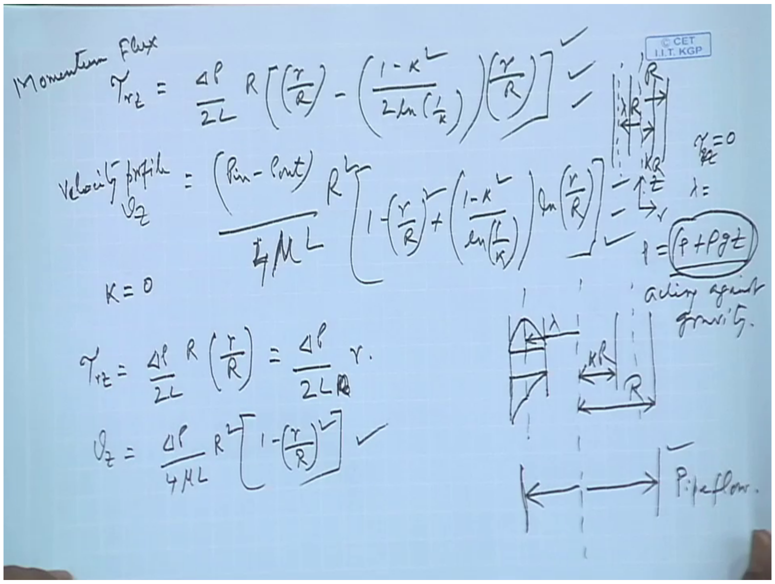
Similarly, the velocity profile can be this can be written as delta P or say P in minus P out and if you remember that we had said this was our r, this was our z, right? And so we said the total P is P flow plus rho g z, right? So this was total which is acting against the gravity, right? This we said and now that means this P in P out it includes this total P plus P rho g z, right? Or the the gravity term also, so this divided by 2 this divided by 4 mu L, right? 4 mu L was our this times R square times 1 minus r over R whole square plus 1 minus k square divided by ln of 1 by k and this times ln of r by R this was our velocity profile or the velocity distribution, right?

And we saw in the annular space, see if it is the annular space, right? Okay to avoid the avoid the this is our central axis, right? We saw this velocity profile was like this where this is corresponding to lambda, right? This is corresponding to lambda the imaginary site, right? And we had said some value here, right? (())(4:37) at this point, so this is some other value, okay then then we can say that if this is true we also said that there are some ways by which whether this derivation is correct or not we can determine, how?

Now imagine that if this inner if this inner one inner inner pipe or inner tube is taken out, then this becomes only r, right? Then instead of this full we have this instead of that full one this entire r that becomes, so then instead of annulus if we take out this one, then we have one pipe so then this under that situation constitute correspond to the pipe flow, right? So this we have to check whether this is really happening or not, if it is happening then we can say that, yes the limiting condition is also 0 is coming correct.

Now limiting condition then it can come what?

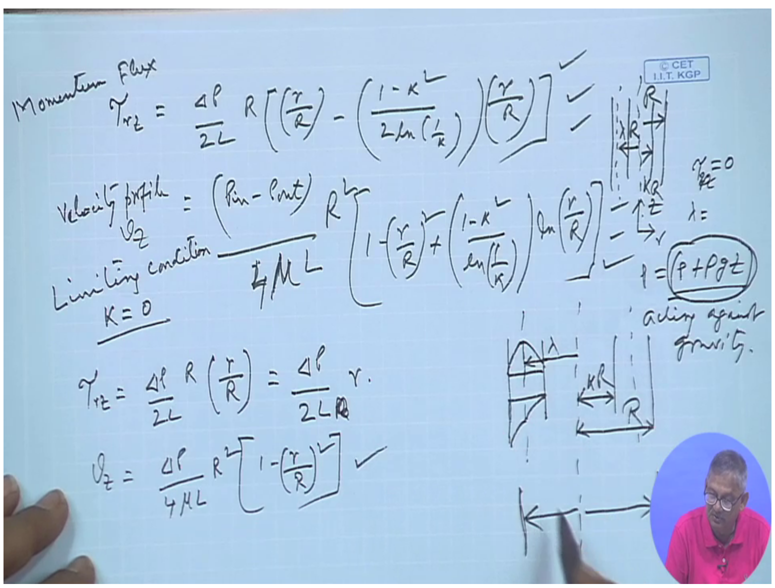
(Refer Slide Time: 6:17)



That here we had taken kR if you remember here we had taken here it was kR and here it was R, right? So if if your if your k becomes equals to 0, then this whole thing vanishes this goes out this goes out then it becomes one single R. So under this situation if we substitute the value of k is equals to 0, then that must correspond to the equations for the pipe flow, so let us look into that tau rz is equals to delta P by 2 L into R, right? And this k is 0 and this becomes r divided R, right? This becomes r divided R and and here this is k is 0, right?

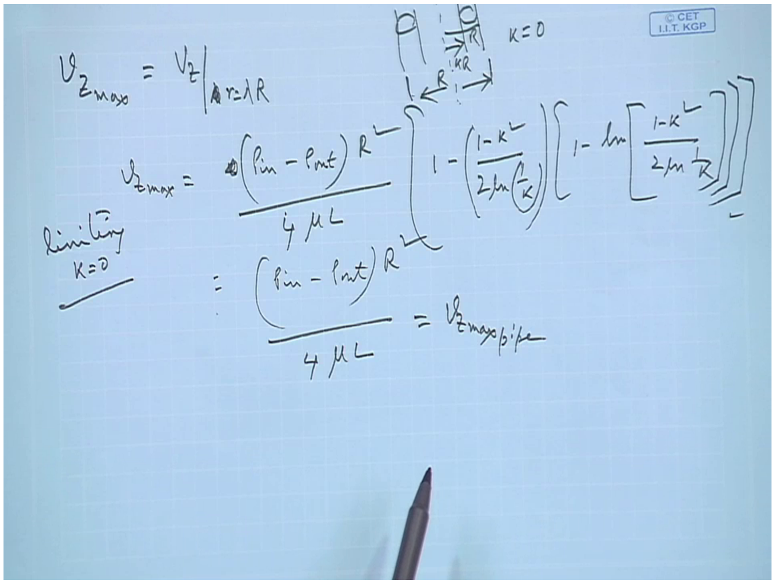
The entire thing goes out entire things goes out and we can write we can write that this is delta P over 2 L R sorry this delta P over 2 L into R for the pipe flow, right? And and the velocity profile that we had this was vz vz is equals to the same thing, right? This entire thing goes out and we get delta P divided by 4 mu L this R square into 1 minus r by R whole square. So this corresponds to the velocity profile for the pipe flow, right?

(Refer Slide Time: 9:05)



So another thing which is needed that that under that limiting condition then we have said okay this is come k is equals to 0 is the limiting condition k is equals to 0 is the limiting condition. So under limiting condition, then we will see that this this (th) (th) (th) this was flow through annulus so flow through annulus is becoming the flow through the pipe, right? Under limiting condition.

(Refer Slide Time: 9:44)



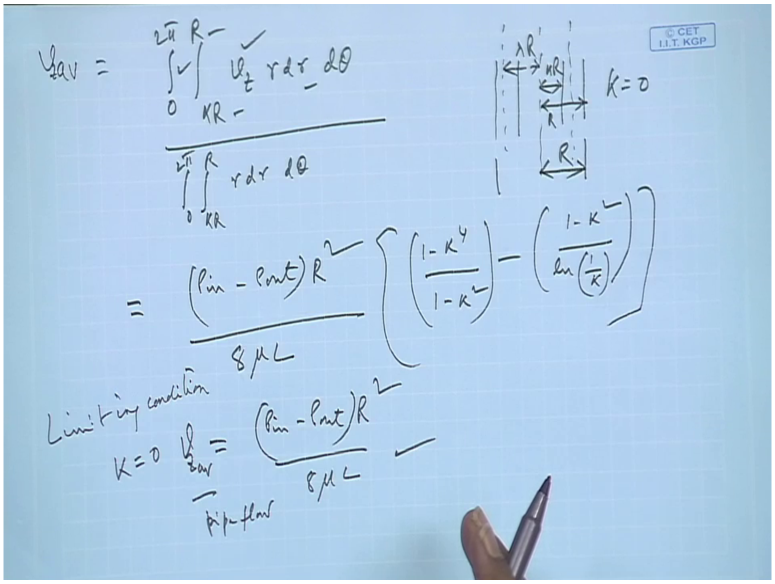
So for the annular flow one more thing we must also find out what is the value of vz max, right? So vz max should be equals to vz at lambda is equals R is equals to lambda r is equals to lambda R, if r is equals to lambda in the vz expression earlier we can write that vz max this becomes equals to delta P or P in minus P out that is much better P in minus P out, right? Times R square divided by 4 mu L, right? Times 1 minus 1 minus k square divided by 2 ln of 1 by k, right? Times this times 1 minus ln of ln of 1 minus k square divided by 2 ln 1 by k, right?

So this becomes equal to this, right? So vz max is like that, now again under limiting condition when we have k is equals to 0 that is the that is our this annulus will become 1, right? This annulus will become 1 and in that case this was our kR and in this case this was our R, right?

Now it will become 1 R, right? When when k is equals to 0. So this limiting condition if we if you put it here, then you see what is the value which is coming so we have we can write this is equals to this is equals to delta P or P in minus P out, right? Times R square divided by 4 mu L, right? So all these terms goes put so it remains only 1. So delta P R square by 4 mu L that is the vz max or pipe flow, right? So this matching with the limiting condition that means we have shown that under limiting condition that is when k becomes equals to 0 this (lamina) this this flow through the annular space that is become equal to the flow through the pipe, right?

So this we have seen, okay now another thing remains for the laminar flow what is the average velocity or vz average what is the average velocity.

(Refer Slide Time: 13:21)



So average velocity as we know vz average this was integral of 0 to 2 pi 0 or kR to R, right? In this case kR to R vz into r dr into d theta r dr is one arc and d theta is the other, so this is the total area through which it is flowing and this time the total area through which it is flowing 0 to pi 2 pi kR to R, right? r dr d theta, right?

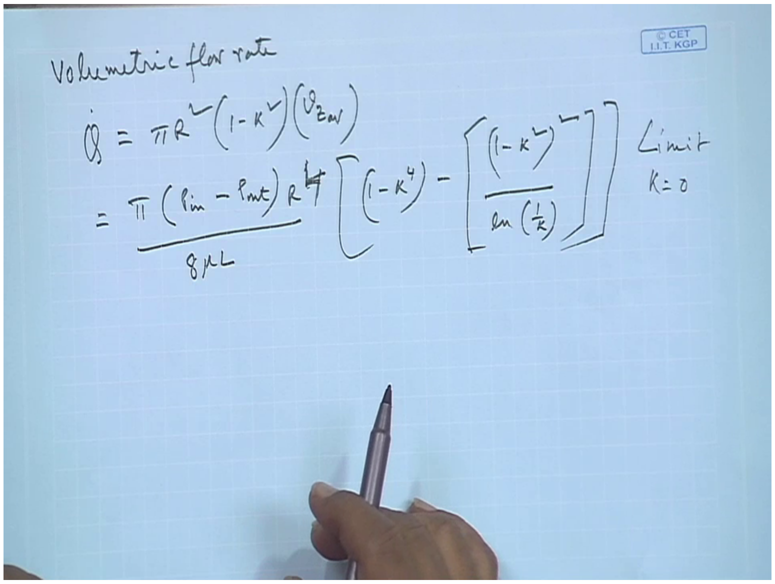
So if now if we substitute the value of vz and then integrate them with respect to both r and d theta, then 0 to 2 pi will go out and for r so it will be between kR and R after putting the integration and the and the limit. So this limit kR and R by substituting that we can write that this becomes equal to P in minus P out into R square divided by 8 mu L times 1 minus k to the power 4 divided by 1 minus k square minus 1 minus k square divided by ln of 1 by k, right? This over this, so vz average is like that.

Now here also if you put the limiting condition, now what was again limiting condition? Our again and again I am showing the same so that it is remembered by you, right? This was our boundary for imaginary and this is our kR this was our R, right? And this was our lambda R, right? So we said that if k becomes equals to 0, then this inner one goes off then the entire thing becomes R, right? So entire thing becomes R so that means when k be is limiting condition is k is equals to 0, so the entire thing goes off and and it becomes equals to 1, right?

And we can write this vz average is equals to P in minus P out into R square by 8 mu L, so that is the vz average for pipe flow, right? So that means this limiting condition is valid, so if it is the limiting condition valid then we can say that our derivations where correct. So this is how we normally find out that if the limiting condition is existing and if it is valid that means the derivation which you have done is also valid. Subsequently also in many other cases you will see that similar conditions you can apply limiting conditions and then we can say that if this limiting condition is valid, that means we have rightly followed and we derived the equations correctly, right?

So these we have to keep in mind, okay.

(Refer Slide Time: 18:25)

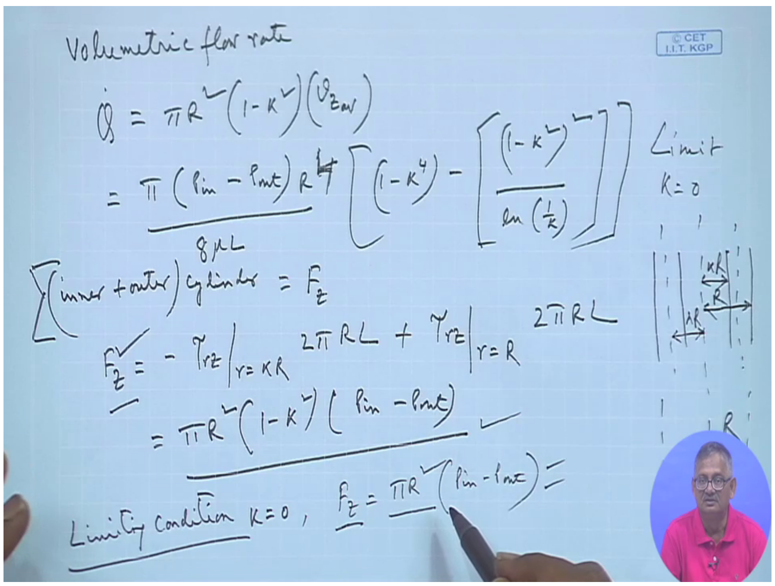


Now let us find out some other part that is now let us also see that what is the volumetric flow rate, what is the volumetric flow rate? So volumetric flow rate Q is equals to pi R square that is the area into 1 minus k square this is the area through which it is flowing is vz average, right? So if we multiply then final fall comes to delta P pi into delta P that is P in minus P out, right? Over over 8 mu L into R square into 1 minus k to the power 4 minus 1 minus k square divided by ln k square whole square divided by ln of 1 by k, right?

So this another expression for the volumetric flow, now if we remember that for limiting condition then this is pi R square, okay that is the area, right? pi R square that is the area and and this R square and from here this will becomes R to the power 4, right? This will become R to the power 4 this is pi R square already there and from vz 1 R square will get, so you will get pi R to the power 4, right? So for similar volumetric flow rate for k is limiting condition limit is k is equals to 0 if we put there, then this term becomes equal to 1 and then you can write that this is also equivalent to that, okay.

Then, we have to also find out some other properties (oth) (oth) other parameters like how much force is associated with that as we have done also in the case of in the case of flow through this flow through that film or film film flow flim flow also we have also said that yes this is what the force is acting on it.

(Refer Slide Time: 21:35)



So if we look at that forces and if we do the force analysis, then we can find out that the force exerted by the fluid on the solid that is equal to the sum of the forces acting on the inner cylinder and the outer cylinder.

So inner plus outer cylinder the sum of the forces acting on the inner and outer cylinder is the forces acting on it this is Fz. So then we can write Fz is equals to minus tau rz at r is equals to kR into 2 pi R L plus tau rz at r is equals to capital R 2 pi R L, right? So this on simplification gives pi R square into 1 minus k square into delta P that is P in minus P out, right? So this is the total force which reacting on that and once we determine this, then we can say that this is the total force acting on the on the cylinder on on the annular space, right?

Where both the inner and outer cylinders this is the center and this is your imaginary boundary, right? It was our kR it was our R and this was our lambda R, right? Lambda was the imaginary point or imaginary vertical axis where we can say that it was acting on the it was acting on the plane where the the tau rz that is your shear stress that is becoming equal to 0, right? So and we also found out the limiting conditions and once the limiting conditions are also proved that yes this is the one which we have developed in under under similar conditions, then this derivations are absolutely okay for the the pipe flow.

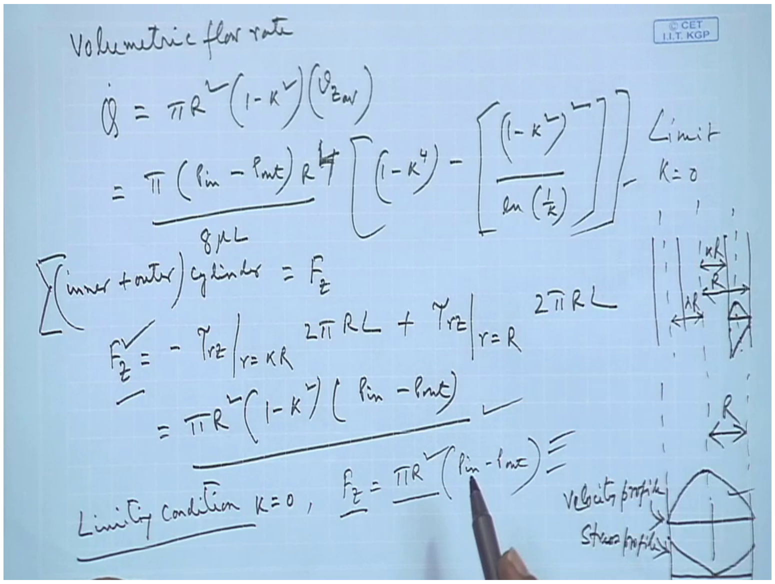
For example, here that limiting conditions example limiting condition is k that is equals to 0, right? So (())(25:18) k that is equals to 0 means this pipe has now become one pipe and that is R, right? And the moment it is so from here also we see that the total force acting on that Fz is equals to pi R square k is 0 so it is 1 so into P in minus P out, right?

So this is the area through which it is flowing and this is the pressure force. So this total force which is acting on the pipe is same when we put the limiting condition. So under limiting condition k is equals to 0 we have come to this condition that sorry that this is Fz that is equals to the limiting condition of the force acting on the pipe, right?

So that means if we look at the flow through annular space that is nothing but the we though we cannot say that this annular space this flow if we take out that inner inner pipe and the outer pipe also will be behaving like the inner one or single it is not so, it is always that the outer pipe and the inner pipe they together because we do not know the we do not know the actual place where in the inner pipe or single pipe when it is a single pipe then we know that the boundaries which are available like for example that for a single pipe we know that at the wall at the other wall which are fixed so there the velocities are 0, right?

And and the shear stress is maximum, right?

(Refer Slide Time: 28:11)



And that the center of the single pipe the velocity is maximum and shear stress is 0 that is what we know and in that case we have shown that the velocity profile would have been for this like that, right? This was the velocity profile, right? And and the and the stress profile or or momentum flux profile would have been like this that this is that this is maximum at this point and this is minimum at this point, so this was our velocity stress profile so this is a velocity profile and this was stress profile for the single pipe and we also have shown that for for this is the imaginary one so the velocity profile was like this and the stress profile was 0 here the maximum there like this, right?

So this we have to keep in mind that when we had removed the inner pipe, then this pipe became equal to the one pipe through which the flow is occurring and that limiting condition should prevail and you must be able to substantiate that the derivation which you have made is equal to the one under limiting condition becoming similar, right?

So this till you attain to you can confirm you will not be able to find out that thing, okay if you will not be able to become confident or become say that yes the derivation done was correct, right? Okay thank you.