Course on Momentum Transfer in Process Engineering
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Lecture 21
Module 4
Flow through annulus (Part 1)

Yeah, so today we shall do some other like your we have done the film and now we will do such a system like we have seen the flow through a pipe, right?

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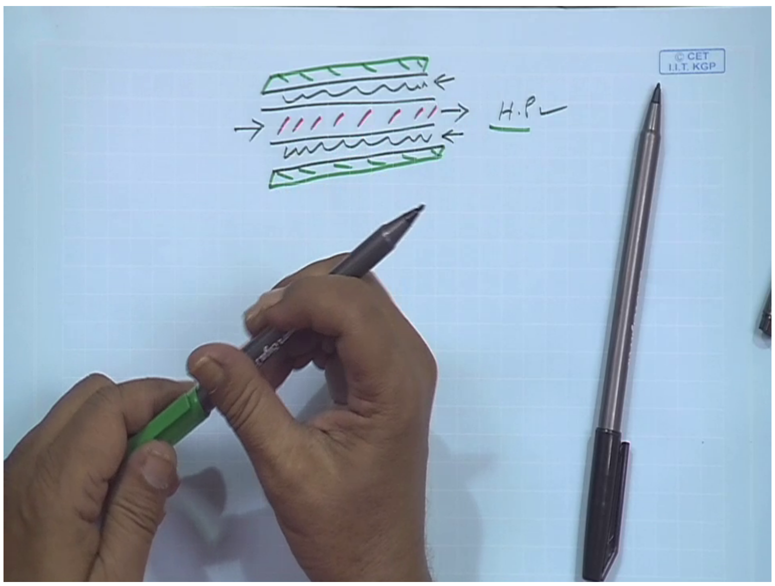

If a fluid is flowing through that we have found out and with that Hagen–Poiseuille's equations we developed, right? Now, if the situation is like that in many cases you will see you have a double tube heat exchanger, right? So you have a double tube heat exchanger one fluid is flowing through this so that you can find out with this Hagen–Poiseuille.

Another fluid say in that maybe co-current or counter current depending on the situation is flowing over this, so this could be some medium by which is being heated this could be that heating the thing which is being heated and this could be another which is the medium which is to be this heating medium, right? So if this is the heating medium and if this is to be heated say this is mail can this is steam, right? So in that case your of course the will be insulated in reality that will be insulated like here there you will have one insulation, similarly here there you will have one insulation so that is different that is the engineering part.

But what we are concerned? We are concern is if this flow we can find out with Hagen–Poiseuille, the same Hagen–Poiseuille cannot be utilized for this kind of fluid and in that case this is called flow through annular medium, right? In that case if we know that flow through annular medium how we can get it, then okay.

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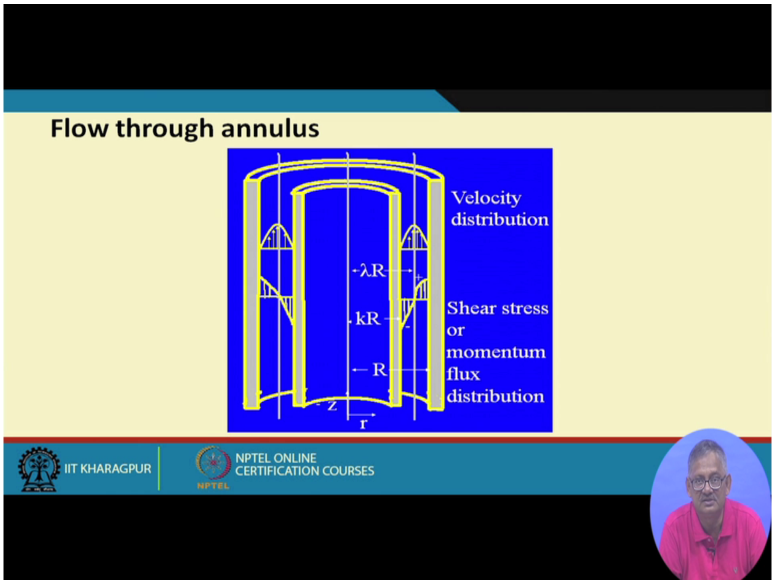

So flow through annular medium this is the one which we are going to determine, right? Now, if you remember in earlier cases we had taken the in many cases horizontal.

So in this case let us also take it to be vertical, right? The difference between horizontal and vertical will be obviously the gravity. So when gravity is also taken care of and the gravity when it is taken care of obviously that will come under the pressure force. So (gra) gravity the entire thing whether it is again towards the gravity or average from the gravity that depends on the flow of the fluid, right?

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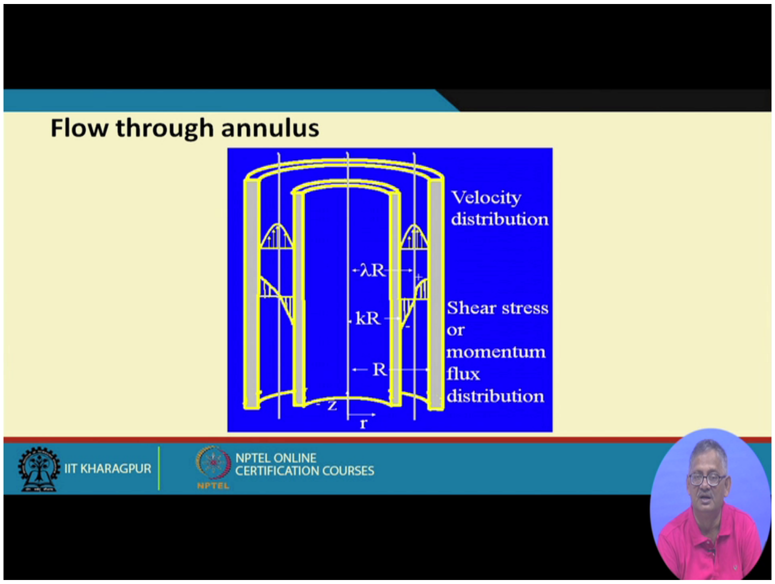

So in any case let us take one such here you see we have neatly drawn that this is the one pipe and this is a another this is the section of the pipe which is having annular flow, right?

So we are not concerned about the fluid which is flowing through this, but we are concerned about the fluid which is flowing through this annular space. So this annulus flow or flow through annulus is in many cases are very useful particularly for designing your designing engineering equipments like say heating equipments, cooling equipments those cases this heat exchangers they are very (use) useful. So there you need to know the flow behavior how flow behavior characteristics, how how the flow is determine, how the velocity can be determine what is will be the what will be the momentum flux in in the inside the system.

So those things if we are do not determine, then we take this, right?

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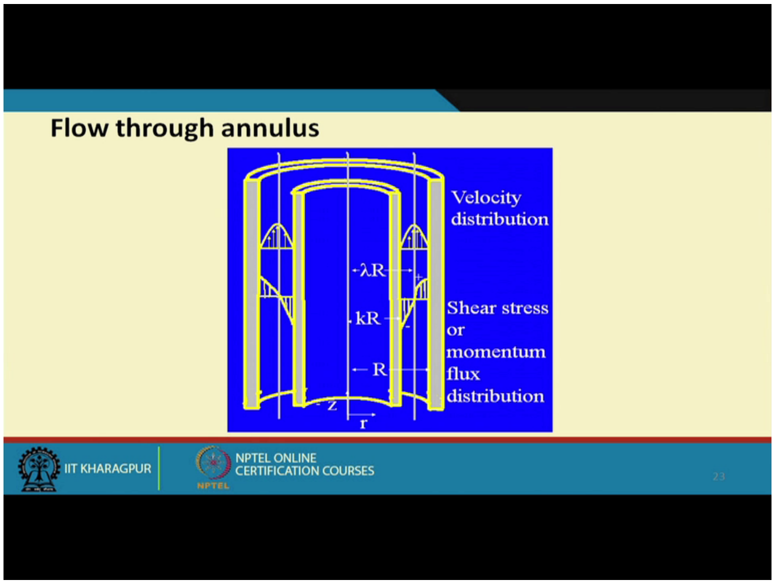

So we had taken then this as the annular flow and in this let us take the at the center the axis and we have one radius outer radius R and inner radius say multiple of R say kR, right? Obviously this k from the physical understanding it appears that k has to be less than 1, otherwise it will not be less than R, right? R is the outer and kR is the inner, right? So if we take that inner and outer of course this arrow should have been here, right? This arrow is up to this perhaps by some by some mistake it has gone up to that, so it is from the inner one, right? This is the thickness of the wall.

Though if we take this, then we have to find out what is the velocity distribution in this annular space and what is the shear stress or momentum flux distribution in this, right? So our our this thing is coordinate system is r, z r is the radial one, z is the vertical and third one is the theta, so this being cylindrical so r theta z that coordinate we are flowing, right?

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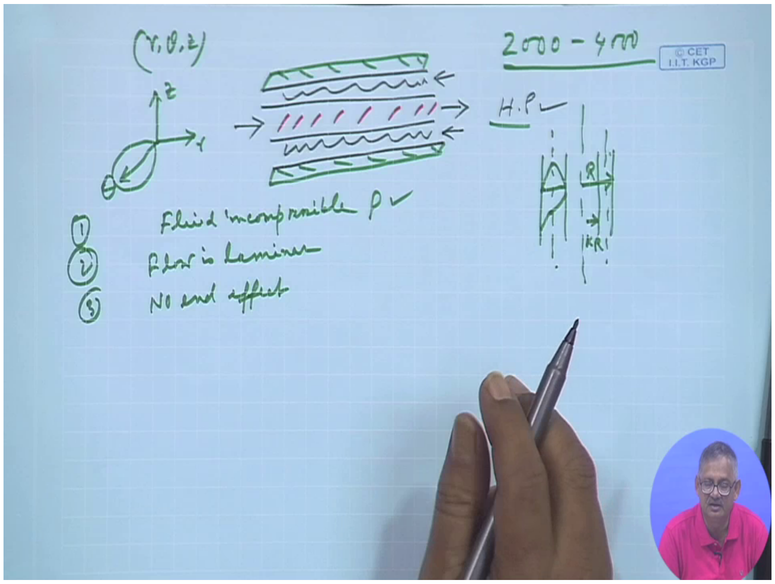

So if we are following that r theta z coordinate of course of course here only one thing which we are saying that r is this and z is the vertical and theta that (non) other dimension is there. Now we assume in the beginning there will be obviously some assumptions so those assumptions in that first which say that the fluid is incompressible, right? Fluid is incompressible, that means density does not vary with pressure significantly that we assume that this density remains constant, right?

And then some other assumptions we assume say the second one is flow is laminar, the momentum flow is laminar that is why this third dimension theta is coming out going out, right? You remember we had this vertical cylinders so we are not taking the thickness, right? And this was our axis and we had said this is to be kR and we also said that this is to be R, right? And we have to find out what is the velocity profile what is the velocity profile in this and what is the stress profile in this, right? We said like this.

Okay, so kR and R are the radie so flow is laminar that is why the third dimension so when the flow is occurring so it is like this, right? So in this theta direction there is no no rippling no no turbulence, so they are having stream line flow that that is why we are not keeping theta in this, right? Then there is no end effect no end effect and and the fluid is flowing between two vertical coaxial cylinders, right? With a laminar flow and vertical cylinders radie are kR and R inner and outer radie respectively, right?

And the laminar turbulence transition occurs in the (())(10:07) of somewhere 2000 to 4000, right? That we know that this is the transition zone between laminar and and turbulence, right? And and this if we take a momentum balance in this, right? We can start with as we said earlier you remember that in many cases we will say that instead of instead of doing every time from the beginning doing all the balances force balances and using the using the governing equation or rate of momentum in minus rate of momentum out plus sum of the forces acting on it is equals to rate of momentum accumulation, then steady state or ever if it is not steady, then that unsteady del del rho del t or the variable that comes in as the time is not independent time is rather rather in this case that property is not independent of time.

So in that case you need to take, but in here we have said fluid is incompressible flow is laminar and steady, right? right?

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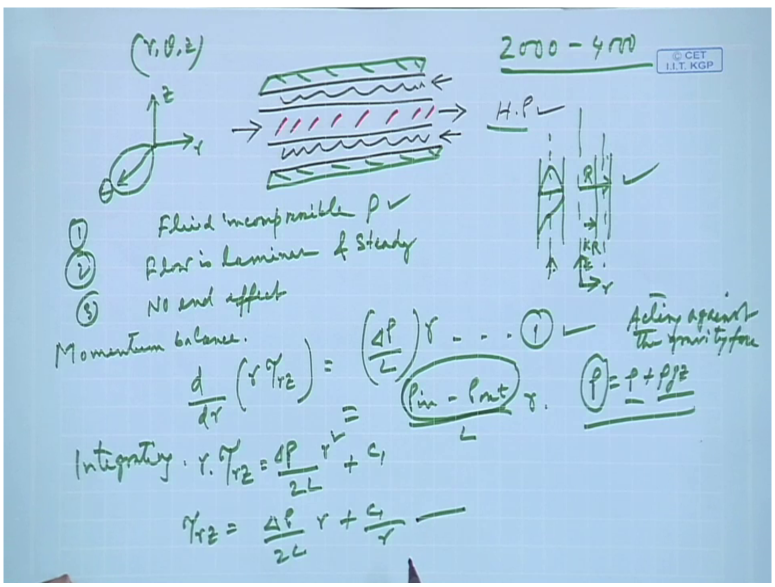

And and we are finding out doing the momentum balance so if we do momentum balance and if we look back the past where we had done for the pipe flow, right? So we we came to a point where we could re-write that d dr of tau r tau rz r tau rz that was equals to delta P over L times r if you remember this within the pipe flow we had said that in many cases subsequently we may start from here because other things remains same because the the flow bulk flow is not coming into picture bulk flow because it is steady so bulk flow is cancelling out, so only remaining the molecular transport.

And in that molecular transport if we do the mass balance momentum balance, then that bulk flow terms they will go out and remaining that molecular transport that we will land up to this. So that is why we referred earlier and told also earlier specifically that in future we may start with this if required, right? So here also we are taking that as the starting point that let this be already derived d dr of r tau r rz del P over L times r, right?

And this del P we can also write as P in minus P out over L times r, right? Now here this P as I said the moment we are taking vertical so depending on the flow and we if you remember that we said that the flow will be in the in the z direction. So this was our rz, right? So it is in the vertical direction, so we can (sa) tell that pressure P is equals to total pressure P is equals to P plus rho g and z, right? That vertical pressure due to the gravity and the pressure which is for the flow this two together will be causing that total pressure P which we are taking it at this P in minus P out, right? Acting in opposite direction the gravity versus if it is if it is going like this no, so it is acting this gravity is like so the total pressure required is P plus this rho g z so it is acting against the gravity force, right?

So it is acting against the gravity force, okay so that means this P is equals to P plus rho g z, right? Now if we integrate it we get tau rz and re-write of course if we integrate and re-write, so we had r tau rz delta P over over this was r, so r square by 2 L plus C and on re-writing we can say r tau rz is equals to delta P by 2 L into r plus C1 by r, right? So this means that we have one equation where this constant C1 has to be found out and to find out this constant C1 we need to know the boundary conditions.

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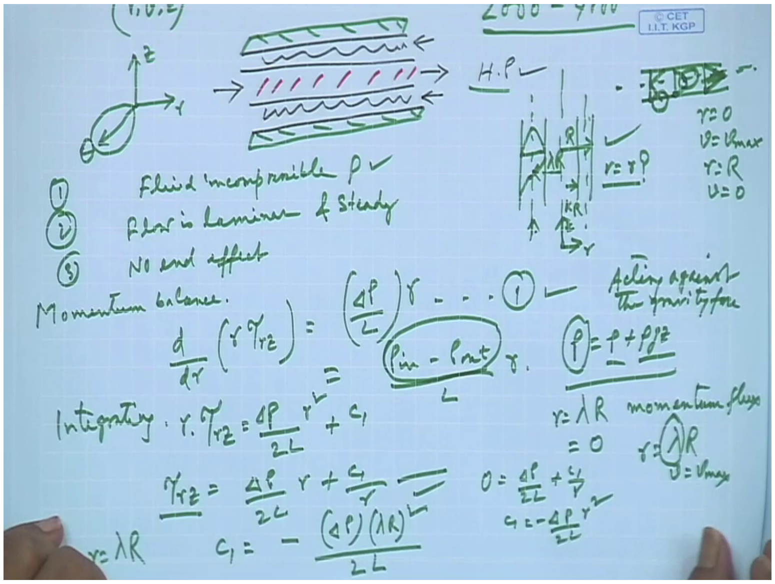

So boundary conditions which will be, right? The boundary conditions now one thing we can we can refer it back that unlike this pipe flow unlike this pipe flow where the boundary condition was very much defined.

Very much defined means, we could say that since so if we take this is as the pipe and we could and this is the axis then we could say that velocity this is the solid wall, so this is where the velocity is 0 and this is where your velocity is maximum, right? And this is where the momentum flux is maximum and this is where the momentum flux was minimum. So if we had plotted we had seen that the plot of the momentum flux was like this and this other one the plot of the velocity was like this, right?

So if you remember that we had made like this and the other way other way sorry, so it is other way this comes like this. So this is minimum and maximum is there maximum is there minimum is there, so it was like this like a K, right? So this was like that and in that case we knew that at the center the velocity is maximum and at the surface the velocity is minimum, this we knew, right? And but here when we are talking here since we have no such R and that means here at r is equals to 0, v is v max and at r is equals to R, v is equals to 0.

So this boundary was known, but here we do not know where at r is equals to what r what is the velocity where it will be maximum. So at which r the velocity will be maximum that is not known, so in order to solve this kind of problem what is done we are assuming and imaginary r say lambda r, right? An imaginary r we are assuming say lambda r where this velocity becomes maximum, right? If this velocity becomes maximum at the point lambda R, then we can put the boundary, right? So here we can say at r is equals to lambda R this this momentum flux will be 0 at r is equals to lambda R momentum flux will be equals to 0, right?

Similarly obviously r is equals to lambda R that time we will also we will see that velocity becomes v max, right? But what is the value of lambda that we do not know, we are assuming at a distance at a radius where this lambda is equal to some value and that we have to determine. So in order to find out this value of C1 let us now say that tau rz that momentum flux is 0 at r is equals to lambda R, right? So if that be true, then C1 becomes equals to minus delta P, right? Into lambda R square lambda R square divided by 2 L, right?

So this lambda R becomes this is 0 equals to delta P over 2 L plus C1 by r so C1 becomes equals to delta P minus by 2 L r square, right? So this r we have put as lambda R square and C1 value of C1 is like that.

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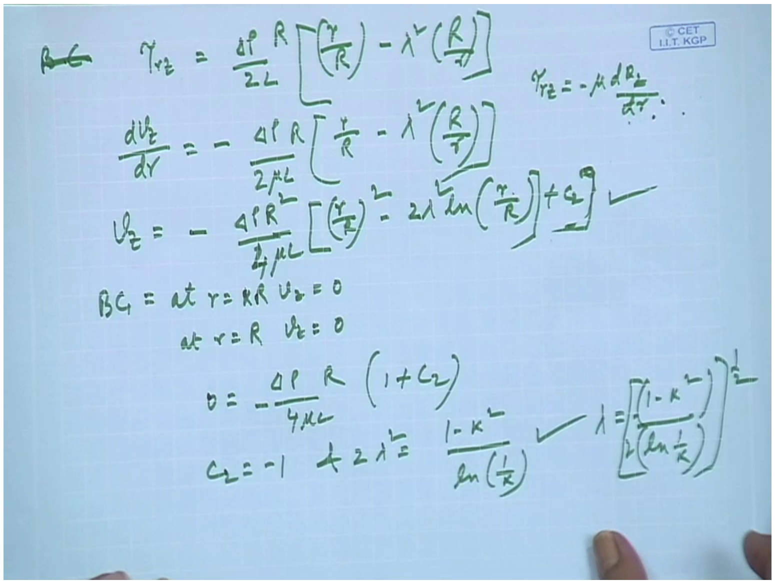

Then we now say that boundary condition we have already said. So then we can now say tau rz this is equals to delta P, right? into by delta P by 2 L into R if we can write as R as common, then r by R this minus lambda square capital R by small r this on simplification we can write, right?

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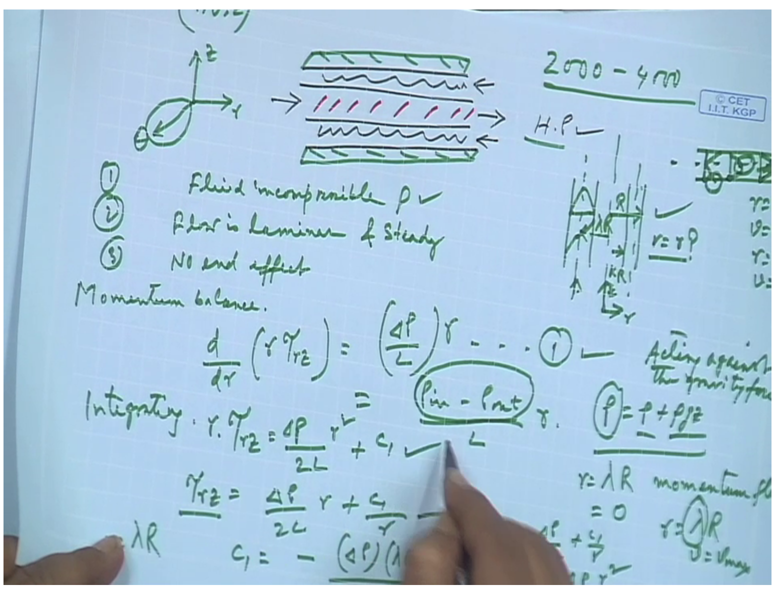

So tau rz we can write because substituting the values value of C1 into this and then taking common delta P by 2 L into R, then we can write this delta P by 2 L R times r by R minus lambda square capital R by small r, right? So if this is tau rz, then we can also say this tau rz is defined as tau rz is equals to this minus mu d v z dr, right?

So d vz dr if that is there, then we can substitute this d vz dr is equals to this negative goes there, right? And we can write that this is equals to minus delta P by 2 mu L into R into r by R minus lambda square into R by r this, right? So this is what we can write in terms of division. Now integrating this division we can say that vz this is equal to delta we had minus delta P R by 2 mu L so this was there so on first this integration will come, so it is that r if we take out and then rearrange, then we can say this is delta P by 4 mu L, right? Times R square r by R whole square minus 2 lambda square ln r by R plus C2, right?

This is the second, right? Okay this C2 we here we can write that is equals to C2, right? So then if we substitute the boundary conditions boundary condition 1 is at r is equals to at r is equals to kR at r is equals to kR we have vz or at r is equals to kR we have vz is equals to 0, right? And also at r is equals to R R we have vz is equals to 0, right?

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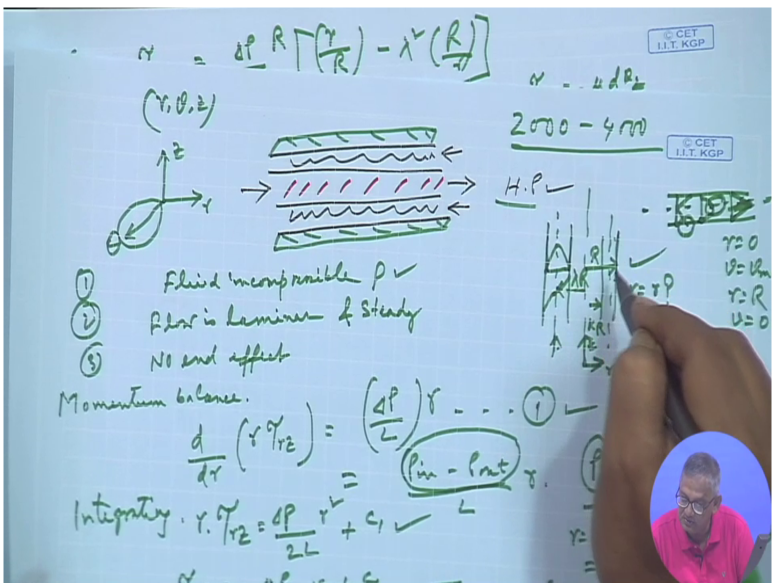

These two because there if you remember, so we had this at r is equals to R and r is equals to kR because because that is the wall.

So this is the solid wall, so where where our our fluid is getting filled and this vz becomes equals to 0. So that boundary condition if we put and substitute these boundary condition into C2 we can write that 0 is equals to 0 is equals to delta P over 4 mu L R square with negative into 1 plus C2 and and from there we can find out from there we can find out C2 is equals to minus 1 and 2 lambda square is equals to 1 minus k square divided by ln of 1by k.

So we find out the value of lambda lambda is equals to 1 minus k square divided by ln of 1 by k, right? This half, right? Into rather this is half and this to the power of again half, right? That is the value of lambda.

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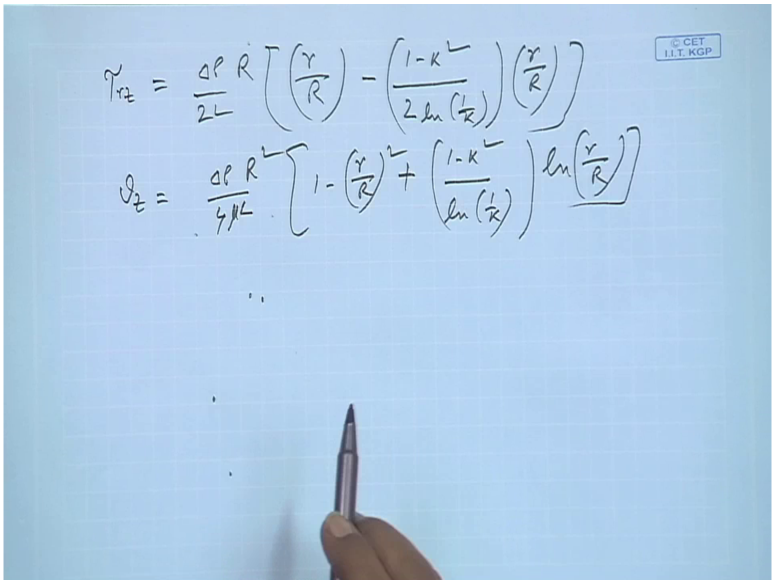

So if we know C2 and then lambda, then we can say that tau rz now tau rz is now becoming equal to delta P over 2 L into R is r by R, right? Minus 1 minus k square divided by 2 ln of 1 by k, right? Into 1 by or r by capital R this is tau rz and vz is equals to delta P divided by 2 L or 4 mu L rather delta tp divided by 4 mu L to R square into 1 minus r by R whole square, right? Plus 1 minus k square over ln of 1 by k, right? Times ln of r by R.

So this is the velocity distribution and this is the (mo) momentum flux distribution, right? Now here we have to find out, okay in many cases in earlier cases also we have seen that we can we can determine these velocity profile and shear stress profile and based on the relations we obtain then we can substitute we we can substantiate whether our development is correct or not by putting some conditions that we will do in the next class because in this class time is over, thank you.