Course on Momentum Transfer in Process Engineering
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Lecture 2
Module 1
Equation of continuity expression and their interpretations

So as we finished in the last class that we will be dealing now with the equation of continuity, right?

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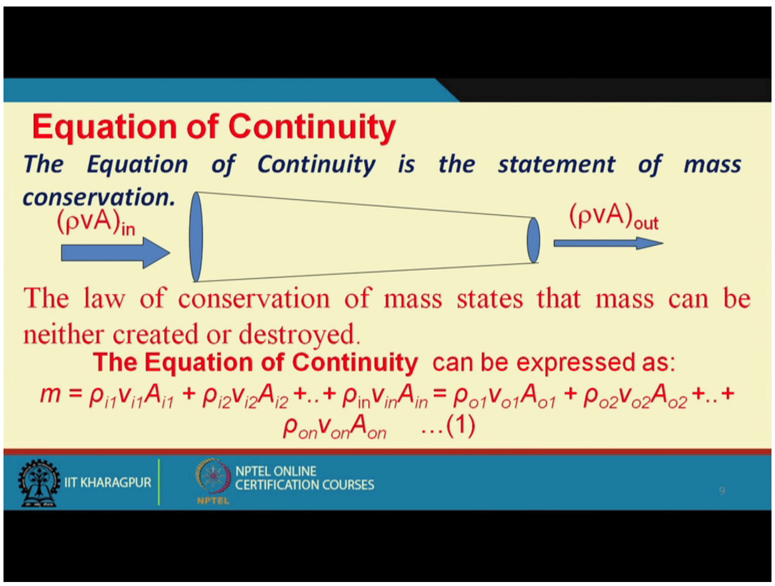

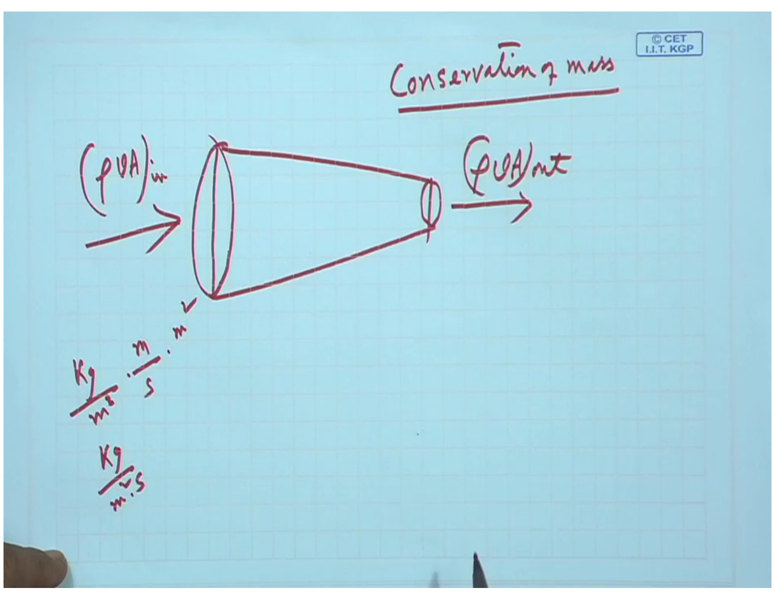

Continuity equation that will you so we will start with a as we said in the previous class that finite volume element we will take and there will be inlet and there will be outlet, right? So the inlet and outlet depending on that we will say this is say we have taken a volume element like this, right? Say like this and, right? We have here the (consa) this we will do equation of continuity based on the conservation of mass, right. So this is equation of continuity totally on the conservation of mass, right?

And we say that rho into v into A, right? This rho into v into A, rho is what? Kg per meter cube, v is meter per second and A is meter square, then what it means? Kg per meter square per second, right? So kg per meter square per second that is any anything which is per unit area per unit time that is called with a flux that is called to be the flux or we called to be mass flux, right? Per unit area per unit time of any parameter that is known as to be flux.

So in this case we can call it to be mass flux, right? Rho into v into A what is coming in and that is going out as rho v A out, right? And from the law of conservation of mass we can say that nether we can create mass nor we can destroy mass this is note, so conservation of mass says that we neither we can create nor we can destroy. So whatever will come total mass whatever will go that total mass if there be any accumulation this all put together will remain same, otherwise we will be generating some mass not possible from the conservation of mass.

If there is some mass going out no accounting, then that is also not possible because we cannot destroy the mass, so by from the conservation of mass we can say that whatever is coming in that is to be going out if there is no accumulation, right?

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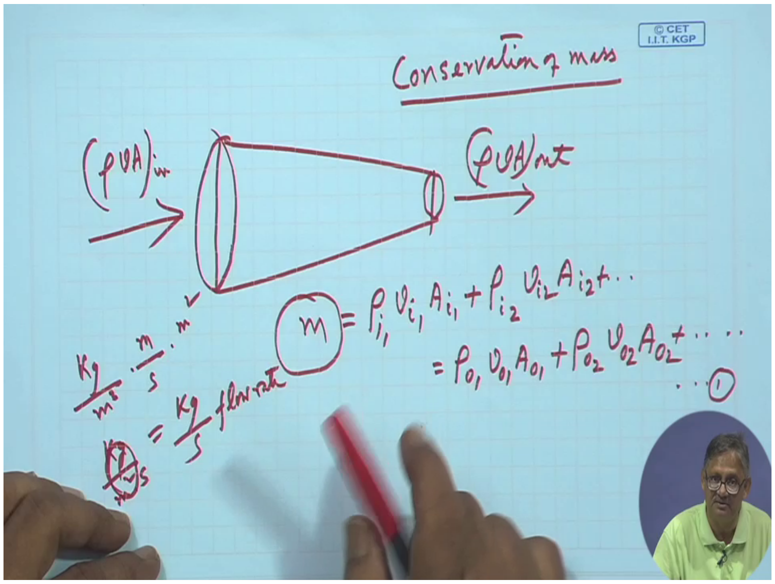

So here that is what we are saying, now if we tell to be total mass aim so that is equals to say at different points rho 1 into velocity at that point in at the inlet v1 into area of the inlet at that point Ai1 plus similarly rho i this is at the point 2 v at the point 2 and area at the point 2.

So these all these plus all so many this must be equals to the rho at the point outlet at point 1 times v velocity at the point outlet at point 1 times area at point outlet at point 1 plus rho outlet 2 into velocity outlet at the point 2 and area at the point 2 plus as many points we consider. So this must be equals to the total mass in, right? So this if we tell then we can say this to be say equation number 1 where all the points are added in terms of rho times v times A this is the mass flux kg per meter square per second, right?

How it came? Rho is kg per meter cube, v is meter per second, A is meter square so this makes this meter meter square and this meter cube rather sorry this is meter meter square into meter that is meter cube and this meter cube it comes kg per second or sorry mass flux mass only that is that is the mass flow rate, right? This is the mass flow rate, right? Yes, as we said that if the meter square is in the denominator then it becomes that you need per unit area per unit time than that becomes the flux then it becomes the mass flux we will come again.

So this is only mass flow rate that is kg per second meter square meter so meter cube over meter cube so it goes out so it is kg per second that is the mass flow rate with time so much kg is flowing and if that is constant this much of mass m is going, so that is equivalent to that inlet as many point we consider rho 1, 2, 3, 4, 5 as many. Similarly outlet rho 1, 2, 3, 4, 5 so many points addition of these three so we will give you that multiplication of these three will give you that mass flow rate of that system, okay.

Now let us look into, okay as we said it is kg per meter my aim is kg per meter second kg per second rho is the density kg per meter cube gives us velocity or speed, of course velocity and speed we know the difference velocity we say to having a definite direction, but when the same thing is said without any direction then it is called to be speed, right? So the train has this much of speed 60 kilometer per hour, so we never say that it is going from one place to the other place. So the (mement) the moment we say that it is going from this place to the other place then it becomes velocity because then it is directed from this place to that place it is going, right?

So that why we are also we have taken to be speed because we have not said how we know which direction it is going so meter per second then area as meter square, right? Now if we assume that in all this inlet and outlet in all this inlet and outlet the density is not changing, right? And we remember in the first class we said that if the density is not changing that means it is having constant density and if that is a constant density then we also say it is to be incompressible fluid, right?

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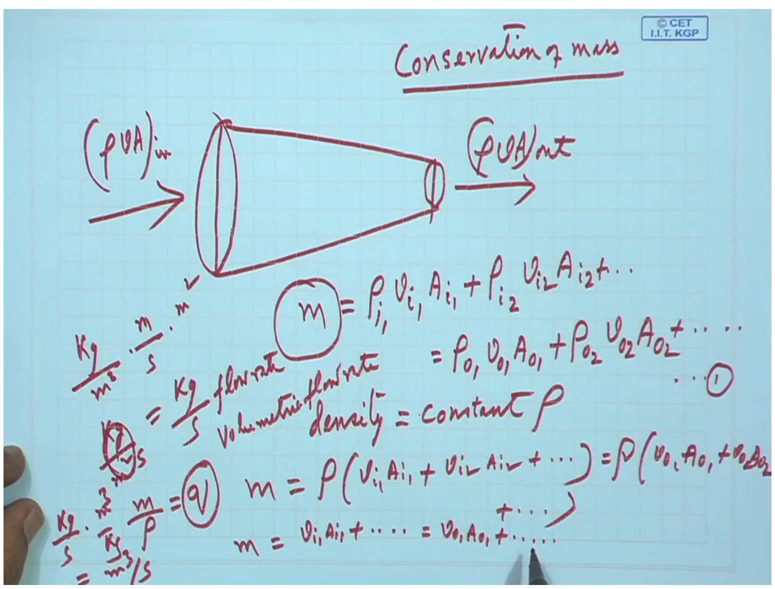

So here it is applying that if it is to be incompressible fluid that is if the density is not changing that is density is constant if density is constant, then we say it to be all row, right? So then, this row and this row then we say this row is equals to rho into vi 1 Ai 1 plus vi 2 Ai 2 plus as many is equal to v this rho into vo 1 (pla) Ao 1 plus vo 2 Ao 2 plus as many, right? So this is true, so that means again we can write m is equals to vi 1 Ai 1 plus dot dot dot dot is equals to vo 1 Ao 1 plus dot dot dot dot, right?

So where now we can say that if this row we divide by m, right? So we had m by rho is equals to q if we say this q to be the volumetric flow rate that was mass flow rate, right? How? This was kg per second and this is kg per meter cube so it becomes meter cube per second. So meter cube per second means the volumetric flow if you take that to be volumetric flow, then m this row we divide it here, so m by rho is equals to this v1 A1 v2 A2 (v) plus all is equal to vo A0 1 vo A0 2 all these points, right?

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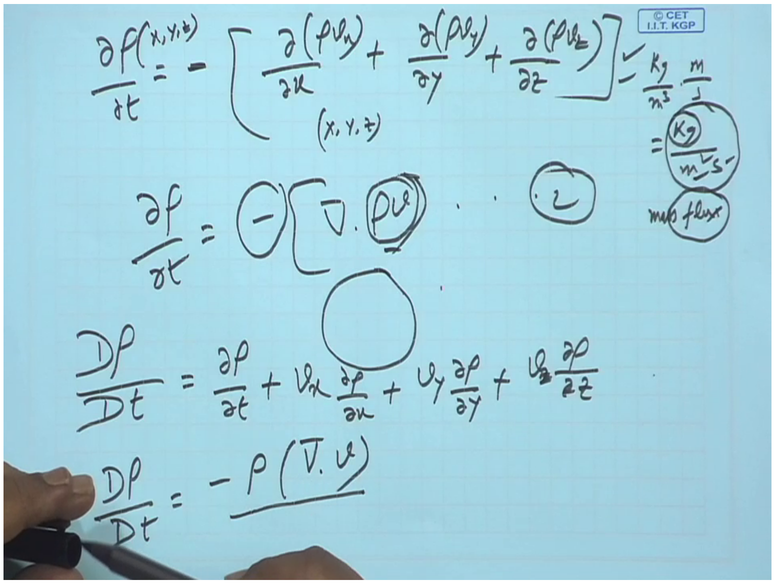

Then we can say that by adding this we can come to this level of equation of continuity in general we can write that del rho del t we can write del rho del t this is to be equals to minus del del x of rho vx plus del del y of rho vy plus del del z of rho vz, right? So this is one form of equation of continuity del rho del t change of density with time that is equals to minus del del x of rho vx plus del del y of rho vy plus del del z of rho vz, right? You know here you say rho is kg per meter cube and v is meter per second here it becomes mass flux. So earlier by mistake we said that without noticing it to be that it was kg per meter cube meter per second meter square, so we said by mistake that it was kg per meter square per second that is a flux mass flux but it is not so, it is so here now as kg per meter square per second that is the mass flux.

So in this case kg is the mass so area is the meter square, second is the time so anything any parameter per unit area per unit time is called to be flux of that particular variable, this variable here is the mass so we call to be mass flux, right? So this is one form of writing equation of continuity that del rho del t del is minus del del x of rho vx plus del del y of rho vy plus del del z of rho vz, do not worry we will derive this subsequently that how this is true why how it this is true in what way we are saying it to be there this of course in Cartesian coordinate how we are saying that this is the equation of continuity that we will come and derive subsequently, right?

Now, if the significance of this is that the rate of change of density at a fix point results from the changes in the mass velocity vector rho v or in other words results from the changes in mass flux in all directions because this is the mass flux of the z direction, this is the mass flux of the y direction, this is the mass flux of the x direction. So changes so what is the change with respect to that position del x that mass flux, sum of all these put together is coming to the rate of change of the density at a fixed point, so that fixed point in the Cartesian coordinate is x, y and z, right?

So with that x, y and z del rho del t this obviously we do not normally write x, y, z, right? But it is so that del rho del t this rho is at a point x, y, z is nothing but the changes in the mass flux or mass velocity that whatever it called this is mass velocity is normally said. So mass velocity that mass times velocity, right? At that fixed point this summation of all these results of all these changes in that direction, okay and it is coming from all the directions of x, y and z.

This is one form of saying equation of continuity, right? So in the vector term also this can also be said in that way that del rho del t is equals to minus this vector rho v vector, right? So this also has a significance this is the say in second form of equation of continuity this normally we call to be second form of equation of continuity and in that case that is said the significance of this can be re-written can be said that the (vec) vector rho v this rho v vector is the mass flux and its divergence has a simple significance it is the net rate this the net rate of mass flux per unit volume, right?

So like here also it was the net because this was the net of all that whatever in all directions x, y, z changes are occurring mass flux here also that rho v vector the net flux per unit volume is the rate mass flux, right? It can also be said in the other way that the fact remains like this the rate of increase of density the with a small volume element which is fixed in a particular space which is fixed in a particular space that the rate of increase of density or this can be increased of density or decrease either way depending on the case, increase or decrease of the density with a small volume element fixed in a particular space that is equal to the net rate of mass in a flux or in flux to the element that is divided by its volume, right?

So this is another way of stating the equation of continuity, right? So this (eq) way we can say equation of continuity in different ways the we have said the two ways express in two ways, similarly we can also express it in another way, right? That is called that equation of continuity in terms of partial this substantial time derivative no, equation of continuity in terms of substantial time derivative if we look into that, then we can say that that is D rho capital then substantial time derivative the operand is different that is the capital D, so D rho Dt is equals to del rho del t plus vx that is the velocity components in the x direction vx del rho del x plus vy del rho del y plus vz del z del rho del z, right? del rho del z.

So we said if you remember in the substantial time derivative when we defined it there we said that when you are moving in the canoe, right? On a canoe you are moving and that is in the direction of the stream only, right? So in that case that substantial time derivative can be written in this way capital D operand, so D rho Dt capital D rho Dt that is equals to the change in density with time del rho del t plus the velocity component in the x direction vx into change of the density in that position del rho del x plus velocity component in the y direction vy into the change of density in that y direction del rho del y plus velocity component in the z direction that is vz del rho del z, right?

(obi) why it is coming? Because when you are moving in the canoe if you are flowing say in the x direction, right? You have some more velocity effect on that, whereas the other two direction if it is stream line if it is not turbulence there is no storm and other things happening if it is normal so other two directions they will have not that significant effect on that but you have to consider. So how we are considering? We are talking the velocity component of the velocity v, right?

And in that case this is vx vy and vz which are taken to account of this equation, right? So this we can significantly say the significance of this equation we can say that del rho capital D rho

Dt, right? Which is nothing but the entire substantial time derivative of the density. So it says that this is for a path following the stream of the flow, flow of the stream. So this is following the flow of the stream which we are repeatedly saying, right?

So I am repeatedly saying that this is nothing but the stream of the flow through which we are calculating and that is the substantial time derivative and this can in another way also be said that the rate of change of density that can be seen by an observer when floating along the stream of the fluid, right? So this substantial time derivative we can say this is nothing but that the rate of change of density when you are flowing along the stream of the flow, right? Okay.

Then this is of course in the (ti) in terms of vector in that can also be said that D rho Dt the vector operand that minus. So this has also a meaning that this can be said in the rate of change of density as seen by the observer flowing with the direction of the fluid, right? So if we now we have said the different forms of the equation of continuity both in the normal (cond) coordinate form and also in the vector form. So in that case we will next class we will be definitely deriving that based on the Cartesian coordinate or maybe in cases in cylindrical coordinate also in cases we can derive and then bring to this equation that how the equation was developed, right?

We just wanted to tell the significance because we have said the definition of derivatives and now how those derivatives also getting impact on the different term that we have said, right?

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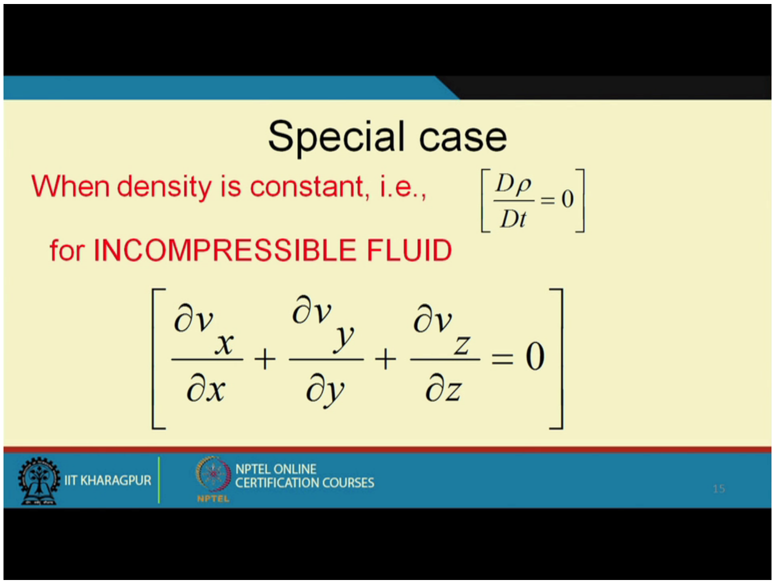

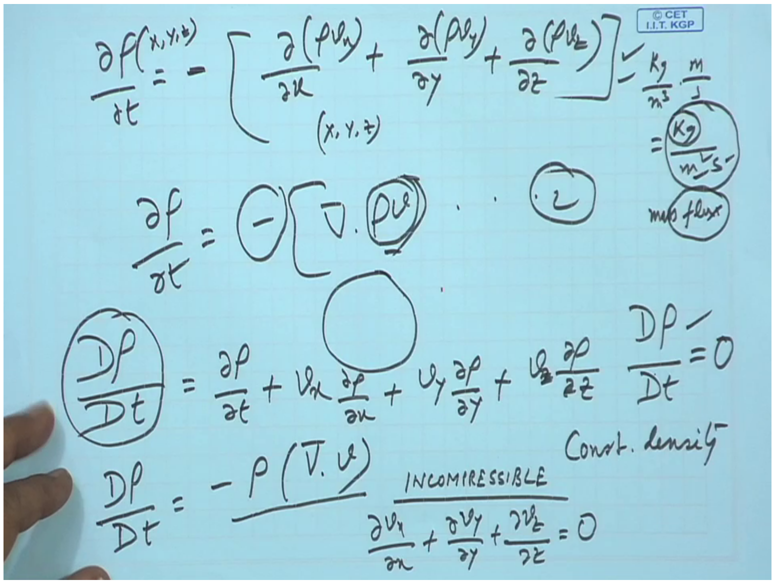

Now if we take this to be a special case that we have said the different forms of (substa) equation of continuity, right? Now if we take a special case, that is if the density is not changing that is if the density is constant if density is constant, then we can say that capital D (ro) rho capital Dt this is nothing but equals to 0, if we take this capital D rho capital Dt this is nothing but equals to 0, right?

So if that be the situation, that is under constant density condition under constant density means density is not changing in that case we can say that the fluid is incompressible and if the fluid is incompressible then capital D rho Dt is 0 and the equation which wrote earlier that can be written as del vx del x plus del vy del y plus del vz del z equals to 0 because this D rho Dt has become 0. So if this has become 0, right? Then, del rho del t also has become 0 then this individual components because this is incompressible so in that case we can write that del rho del vx del x that first equation del vx del x plus del vy del y plus del vz del z this is equals to 0, right?

So that if we see that, yeah so here we have right? del del vx del x del vy del y and del vz del z this becomes equals to 0, okay. So this is a special situation where the fluid is incompressible and D rho Dt is equals to 0, right? That means if the fluid is incompressible that is the density is not changing it is constant then we can write that D rho Dt is 0 and that implies the equations of (cont) equation of continuity to be equal to del vx del x plus del vy del y plus del vz del z equals to 0 mind it this relations will further be utilized subsequently maybe for developing the equation of motion, right?

The entire flow regime is primarily depending on the sets of equations known as Navier-Stokes equation which we will subsequently develop obviously and there this relations of equation of continuity will be really helpful and you will that is why giving so much emphasis on this that you must also keep in mind that this under special condition that D rho Dt is 0 and then del vx del x del vy plus del vy del y plus del vz del z is 0 at when the density is constant this you keep in mind and subsequently we will do it when we are developing the Navier-Stokes equation or that is called equation of motion, right?

So from the equation of continuity the equation of with the help of equation of continuity equation of motion is also developed. So utilizing these relations so keep in mind we will keep these relations utilized developing the equation of motion, right? And so next class we will do the derivation of equation of continuity in both Cartesian coordinate and in cylindrical coordinate. So obviously in Cartesian coordinate it will be x, y, z and in the cylindrical coordinate it will be r, theta, z, right? But basic thing will remain in both the cases same that the basic equation based on which as we have done here in conservation of mass.

So that will remain same, so in that case it will not be changing so whatever is coming on that must be equals to whatever is going and if there is any accumulation so that we will keep in mind and do in the next class, right? So we now finish it here so thank you for this class.