Course on Momentum Transfer in Process Engineering
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Lecture 19
Module 4
Flow of film or film flow

Yeah, so continuity to the last class or previous class where we were discussing about the shell momentum balance of the of the film flowing over an incline surface and this application also we had said say like concentration of concentration of say food material in from (lite) low concentration to high concentration over the heating surface and this heating surface also we said can be totally vertical or also can be horizontal, but we have started we have taken an angle beta, right? With the with the reason that what is the angle beta and then we have developed the basic we assumed to something and then on that assumption we have developed the equations for momentum in and momentum out both by convective transfer as well as the molecular transport mode, right?

So let us start then from there and then since we also said it is the steady state so the convective momentum that comes out because it is a steady state so at the surface we both the in and out will become same and in that case that cancels out so only the molecular transport will remain.

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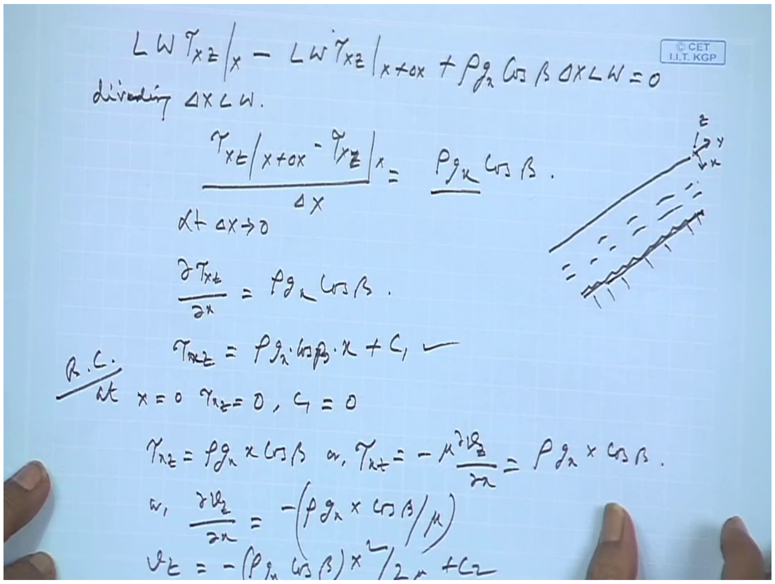

So let us look into that so then it becomes L w from the previous equation L w tau xz at the phase x minus L w tau xz at the phase x plus delta x plus rho gx cos beta into del x L w this is equals to 0 that becomes by putting the individual into the into the governing equation, right?

That governing equation we said the rate of momentum in minus rate of momentum out plus sum of the forces acting is equals to rate of momentum accumulation, right? So we have taken that steady state so accumulation is not there and also we have stated that steady state so momentum in by convection momentum out by convection becomes identical, so that also goes out. So now if we divide dividing with delta x L w both the sides, right? And we make that tau xz at x plus delta x minus tau xz xz at x over delta x this is equals to rho g x because this minus this divided by delta x will make it a differential, so that is why with there is a negative and this negative on this side will make that both thing on the other side.

So this negative will go to that this remains so this rho g x becomes positive and this is also positive. Now if we put limit delta x tends to 0 from the definition of the derivative we can tell that tau xz del del x of tau xz is equals to rho g x, right? Into cos beta, right? rho del del x of tau x is z is equals to rho g x cos beta, right? So on integrating this we can say that tau xz that becomes equals to rho g x cos beta into x plus C1, right? rho g x into cos beta into x plus C1 integration constant.

Now to find out the integration constant we can now say at boundary condition is at x is equals to 0 tau xz is equals to 0, so that C1 becomes equals to 0 at x is equals to 0 tau xz is 0. Now you remember we had taken this to horizontal solid board or solid material on which the film was there we have taken the volume element like that and we said this was the surface this was the liquid which is on the solid surface and this was the open surface which is the liquid gas interface, right?

This was the liquid solid interface this is a liquid gas interface and we took our our coordinate to be here that this is x this is y or this is z whatever, right? So we are concerned with this, then tau xz is (li) like that. So so we can write that in the at x is equals to 0 so here that tau xz is equals to 0 and that is why it is C1 equals to 0, right? So we can write then tau xz is equals to rho g x into x cos beta, right? Or or similarly tau or xz is equals to minus mu d del vz del x is equals to rho g x x cos beta, right? Or we can write this term xz is del del del x of vz so so we can say that del del x of vz so we can say that del vz del x is equals to minus rho g x x cos beta by mu, right?

So this on integration we can write vz is equals to minus rho g x cos beta into x square by 2 mu plus C2, right?

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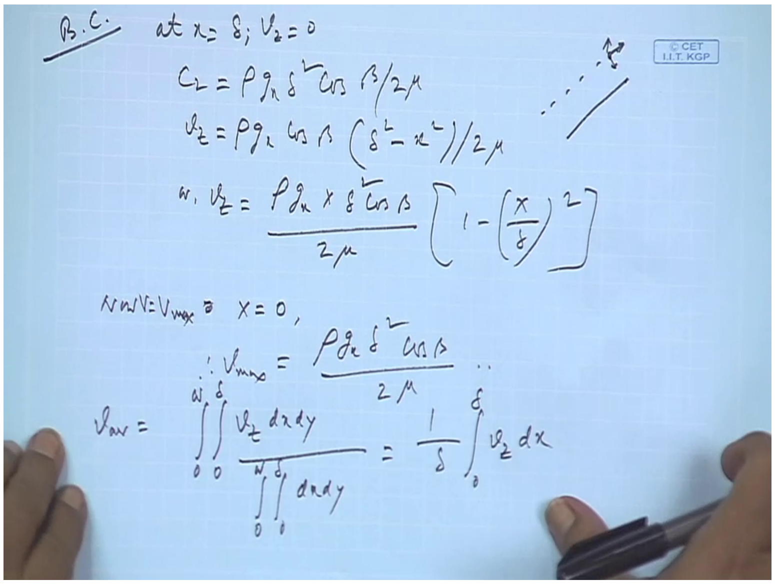

So if that be true, then we can if that be true then we can say that by putting the boundary conditions that at x is equals to delta, right? vz is equals to 0, right?

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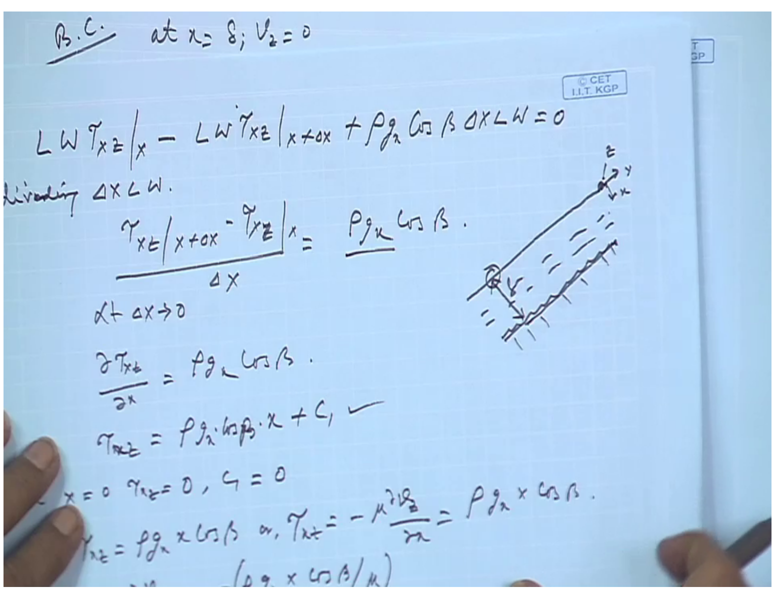

Now at x is equals to delta means if you remember we had said that the thickness of this film is delta, so we started from here this is the coordinate point so at x is equals to delta this is the open liquid gas interface, right? So at x is equals to delta, vz is equals to vz is equals to 0, why? Why?

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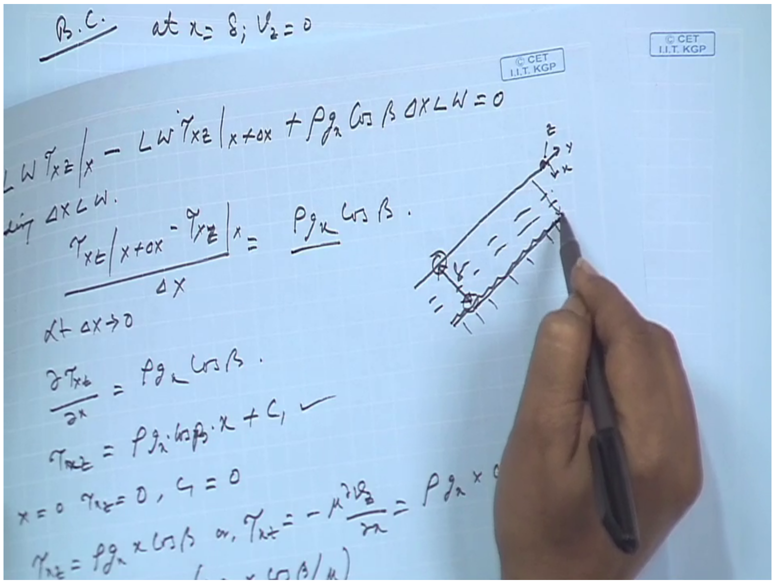

That was liquid solid interface and this was liquid solid interface and this is liquid liquid gas interface this is liquid solid interface, so our point was liquid gas interface it is the 0th, so delta is this so del means we have liquid solid interface. So at the liquid solid interface solid liquid is clinging to the surface so vz becomes equals to 0, so if that be true we can write C2 is equals to rho g x del square cos beta over 2 mu, right? Or vz is equals to (rho) vz is equals to rho g x cos beta into del square minus x square by 2 mu right? Or vz we can write is equals to rho g x into x into rho g x into into del square into x cos beta by 2 mu into 1 minus x by del whole square, right?

Now v max or v (is) v is equals to v max that can be when x is equals to 0, right? When x is equals to 0 for again for understanding this was the open surface this is a solid surface we started our x this way so this is 0 x is equals to 0 this is another this is third direction so in that case that that at x is equals to 0 means this is the open that is the gas liquid interface. So v becomes v max at x is equals to 0, therefore v max can be written as rho g x del square cos beta by 2 mu this is the maximum this is the maximum velocity rho vx.

So average velocity or v average this can be written as again we write that between 0 to w between 0 to del, right? vz vz dx dy over integral of 0 to w, 0 to del dx dy, right? This means this is equals to 1 by del, right? 0 to del 0 to del vz dx, right?

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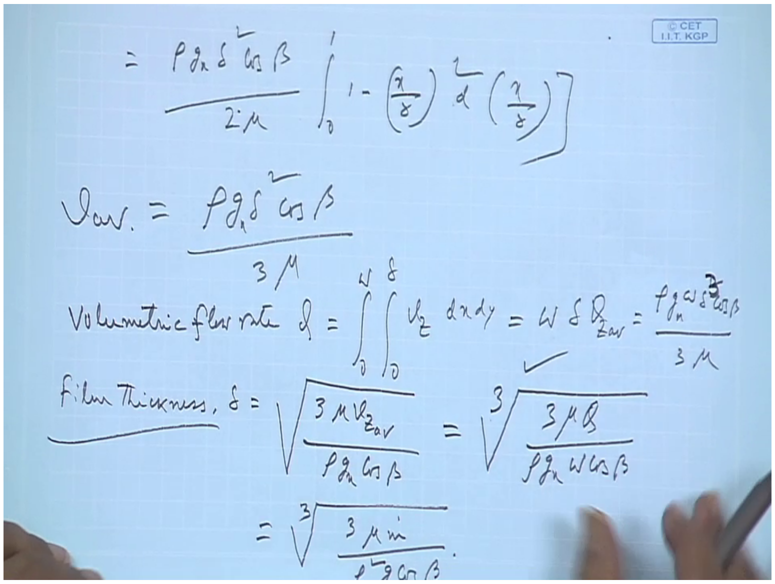

vz already we have found out and this can be written as this can be written as equal to rho g x del square cos beta by 2 mu into 0 to 1, 1 minus x by del whole square d of x by del, right? So that is equals to rho g del x del square cos beta by 3 mu.

So average velocity is rho g x del square into cos beta by 3 mu, right? So this is v average, so v max we have seen v average we have seen, now volumetric flow rate, so volumetric flow rate or Q this can be written as 0 to w 0 to del vz into area dx dy this is equals to w del vz average that is equals to rho g w g x rather w del square cos beta over 3 mu, right? Over 3 mu this is the volumetric flow rate, that is either w del vz average or rho gx w del square cos beta by 3 mu, right?

So v average was rho g x del square cos beta by 3 mu, volumetric flow rate is rho g x w del square cos beta by 3 mu, then the film thickness film thickness delta this can be found out from the volumetric flow rate or average whatever with the way we want. So this can be written as 3 3 mu vz average over rho g x cos beta, right? And this also can be written as this also can be written as as this was del, right? w del vz average and vz average we had 1 del so it will becomes del cube not square, right?

This becomes del cube this was 1 del square and there is 1 del from here 0 to del, right? So in that case 0 to del x so in that case becomes like that del 1 and del del square so del cube. So that is why under cube root under cube root of 3 mu Q, is the volumetric flow rate by rho g x w cos beta, right? This also can be written in terms of cube root 3 mu m dot over rho square g cos beta, right? So this is the film thickness, right?

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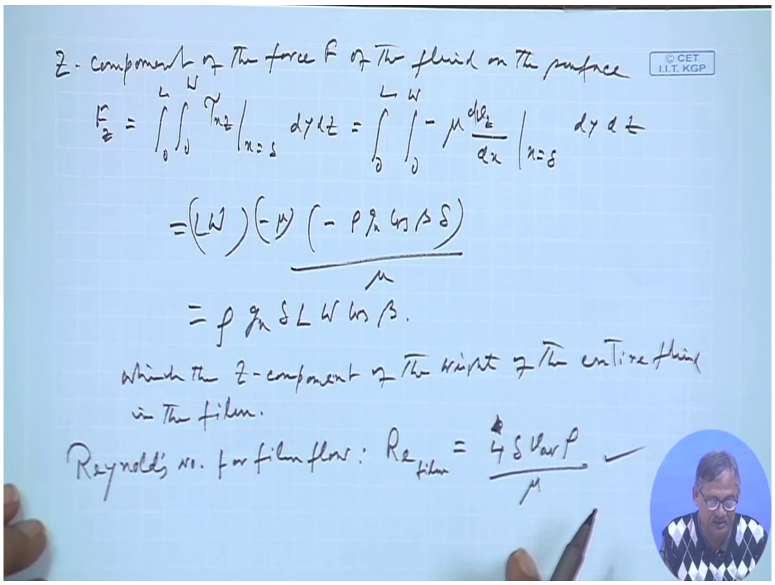

Then we also look into what is the force that is z component of the force z component of the force f of the fluid on the surface or it can be written as fz is equals to 0 to L 0 to w tau xz at x is equals to del dy dz this is 0 to L 0 to w minus mu dvz dx at x is equals to del dy dz this can be written is equals to L w L w into minus mu into minus rho g x cos beta into del over mu, so this on simplification can be written rho g x del L w cos beta, right? Which is this is known as the z component of the weight z component of the weight of the entire fluid in the film, right? So this is the z component of the entire film, right? So some more things which we need to know are like this Reynolds number.

So Reynolds number for the film film flow we can write Re film that is equals to 4 that is equals to 4 del v average into rho by mu 4 del v average rho by mu, right? Instead of d v average rho by mu that is the pipe flow d v average by mu here we can say it is 4 del v average rho by mu that is the film Reynolds number. Now for films Reynolds number there is one good thing that unlike the pipe flow Reynolds number where it was 2100 and 2100 and 10 to the power 4 that is at 2100 and 4000 above.

So this this can be this can be very smaller in terms of film because film you have different thing which is having low low Reynolds number for both laminar and the turbulent.

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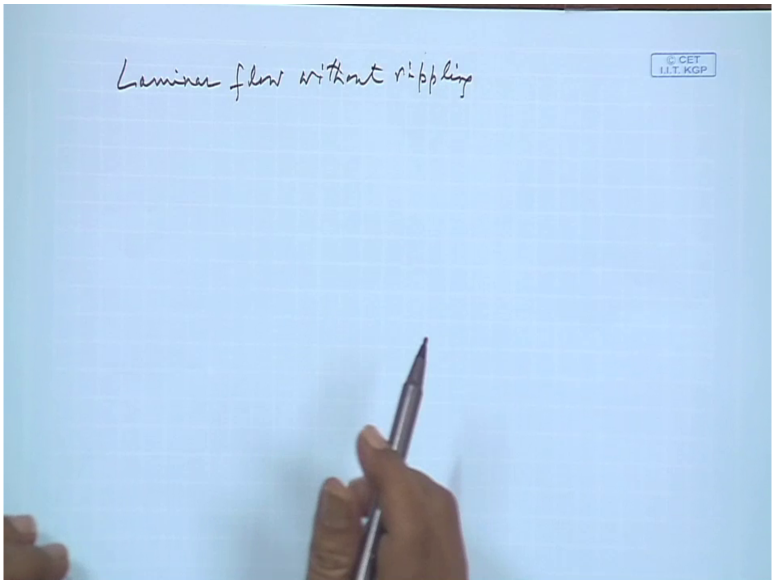

For laminar flow for laminar flow without rippling now for this rippling means, what we understand by rippling? I give you a simple example that in your child during your childhood you must have played with this that you stayed on the on the bank of maybe river or pound or whatever, right?

So say on the bank of the pound on the on the side of the pound you had threw a stone, right? The stone when inside the fluid there is water, right? After that there is a there is a wave kind of thing which was generated, right? This kind of situation is known as the rippling, right? This kinds of situations are known as rippling, there is no turbulence there is no turbulence we have thrown over a simple stone and the stone created a wavy pattern in the in the fluid and this you known as the rippling and this is very very useful when it is used in the in the falling film or similar to that.

So this you have to keep in mind that rippling without rippling and with rippling means there is a wavy form in the (ri) in in the in the flow and then that can be said as to be equal to the rippling.

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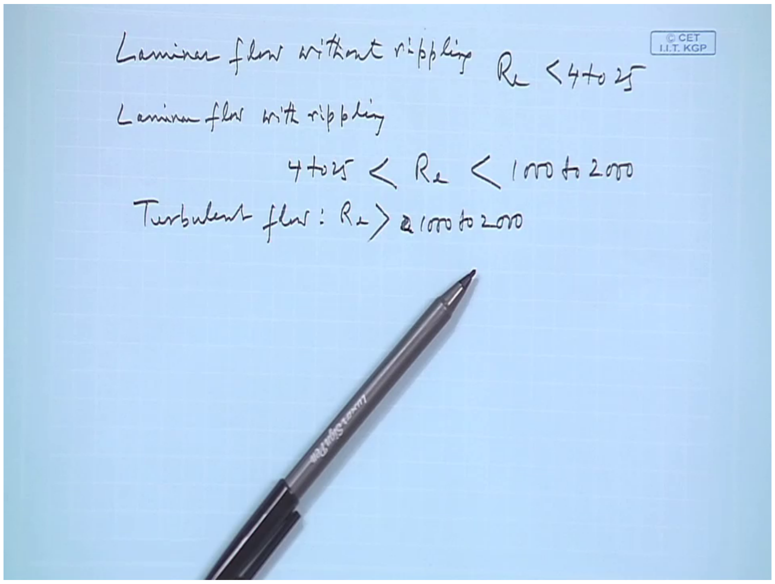

So laminar flow without rippling in that case Reynolds number becomes less than 4 to 25, so laminar flow without rippling is Reynolds number without rippling between 4 to 25 then laminar flow with rippling that can be between 4 to 25 plus then Reynolds number plus then 1000 to 2000 it is it is quite big number 1000 to 2000 and turbulent flow turbulent flow can be between or greater than (2000) 1000 to 2000, right?

So you see that film Reynolds number film Reynolds number you are definition or your expression is different than that of the that of the flow to through pipe, right? So subsequently you will also see this is also different when it is flowing through fluid is flowing through a very small sneak, right? The one which we had shown as the two two plates to that, similar to that if you have a sneak small small or if it is small hole small opening through that when it is being flowed then it is called that flow through the sneak, right?

And there you will see that this is not so easy this is not not what I say not so easy means this is difficult different than that of the falling film and the flow through the pipe. So Reynolds number expressions are quite different for three types of this kind of flow one is for the pipe flow and another is for the film flow and third is for the flow though the sneak of course flow through the sneak we have not done, but Reynolds number through the pipe flow we have done that is dv rho by mu, right?

Whereas the Reynolds number through this film we have shown that to be equals to 4 that to be equals to 4 del v average rho by mu, right?

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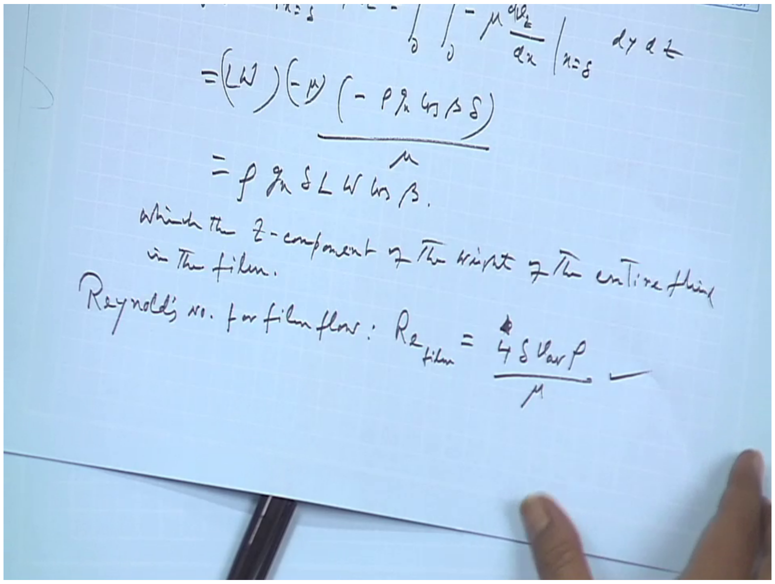

For del v average rho by mu and the values are also quite quite different and our nature is also different in this case we have seen that it has rippling and we have also said the meaning of the rippling rippling is the similar one which you have experience during your childhood, throwing a stone on to the wave on to the pound that creates a wave this kind of wave formation is known as the ripple, right?

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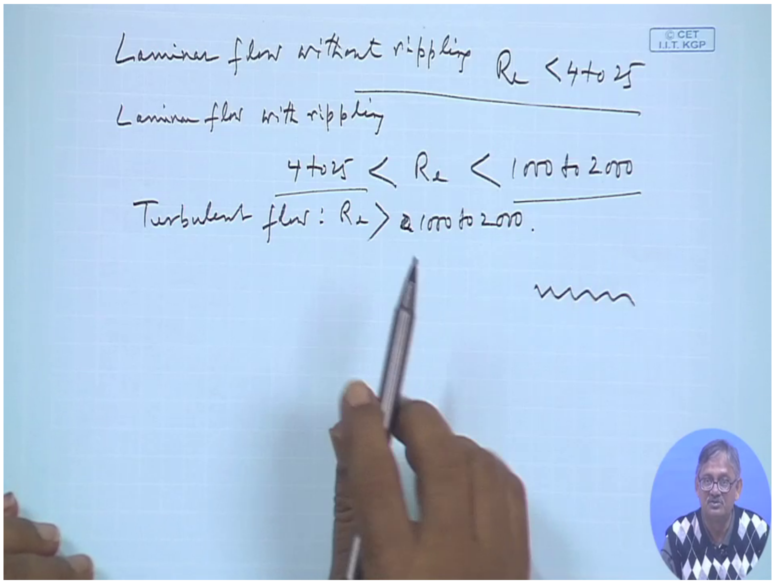

So rippling, like this, right? So this is like that and when it is like this then we call it to be rippling so without rippling between 4 to 25 with rippling it is less than 4 to 25 so with rippling it is between 4 to 25 and 1000 to 2000 and if it is without rippling with rippling other than that that turbulent flow then it is between 4 to 25 and 1000 to 2000. So film flow that is also very very important in term of processing in terms of heat transfer, in terms of in terms of fluid flow because this is a really a commercial aspect which it has wide application.

So falling film we have to also develop very accurately or this this expressions you have to develop and here we have assumed that this fluid was clinging to the surface and the free flow or rather free end that is the interface between the between the gas and liquid or gas and fluid that was also where the where where the tau was 0 considered to be 0 to (momen) momentum transfer is then 0 and and we found out the expressions for them, right? So keep in mind and do this practice with derivation of the film flow, thank you.