Course on Momentum Transfer in Process Engineering
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Lecture 17
Module 4
Laminar and turbulent flow in a pipe

Okay, we had then seen what is the frictional factor how it is defined and how we can find out from Moody's chart, right? Now, let us look subsequent things that after the Moody's friction factor once that is done, we can see that okay next we we handle it with a problem for given one this is of course nothing but a mathematical problem,

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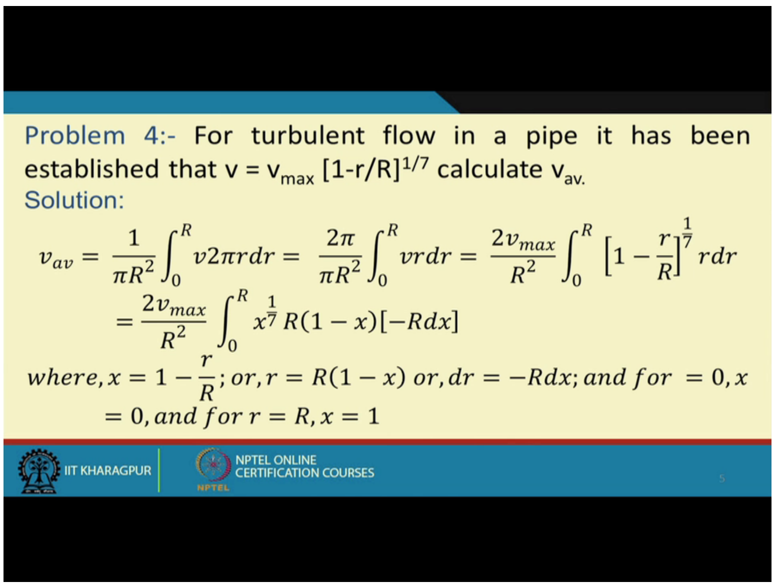

for a turbulent flow in a pipe it has been established that v is equal to v max into 1 minus r by R to the power 1 by 7, so then you have to calculate the average velocity.

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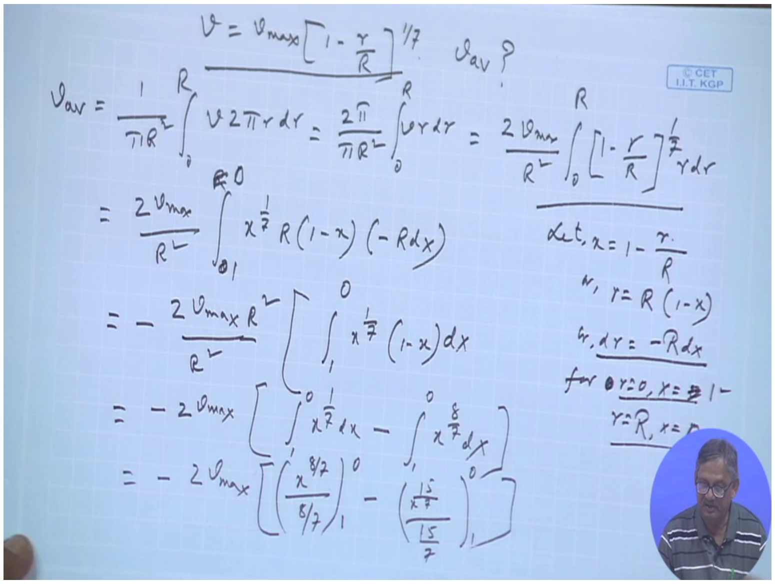

So the problem is for turbulent flow in a pipe it has been seen that v is equal to v max, right? Times 1 minus small r to capital R to the power 1 by 7, so if this is there then what is the value of v average? Right? Now we have shown earlier for pipe flow also we have shown that when you are doing it for average flow that average flow is defined by this way that v average is equals to 1 by pi R square 0 to R, right? v 2 pi r dr, right?

And this can be written as 2 pi divided by pi R square, right? Into vr dr, right? This is this is between 0 to R, right? Now this can be written as 2 v max over R square, right? And 0 to R and 1 minus r over R 1 minus r over R to the power 1 by 7 r dr, right? So this we can also write is equals to 2 v max, right? Over R square between 0 to R, right? Now let us define this as x is equals to 1 minus r over R let x is equals to 1 minus r over R, right? Therefore we can write small r is equals to small r is equals to capital R into 1 minus x, right?

Or we can also say dr is equals to minus r dx, right? So if that is true we can say for 0 or r is equals to 0 x becomes equals to 0 and for r is equals to R x becomes equals to 1, so this which we had earlier done, okay now this we can replace for R is equals to 0 to R is equals to r. So R is equals to r it is 1 and r is equals to 0 it is 0, right? So this is x to the power 1 by 7 R 1 minus x into minus R dx becomes there is minus R dx, right? So between 0 to R R is equals to 0, x is equals to 0 r is equals to 0 x becomes equals to 0, no for r is equal to 0 you see r is equal to R so this goes out x becomes equals to 1,right? And for r is equals to 0 x also becomes equals to 1, okay.

For r is equals to 0 x becomes equals to 0 for r is equals to R x becomes equals to 1, so it is 0 to 1 x to the power to 1 by 7 R 1 minus x into minus R dx, okay. So if this is true, then we can write this is equals to minus 2 v max, right? Over R square, right? And here also R square this R and this R makes R square, right? And this we can write between between 1 to 0 x to the power 1 by 7 1 minus x dx, right? 1 to 7 this minus we have taken out, right? This minus we have taken out minus 2 v max this R this R has made R square already there was one R, okay and this was 0 to 1, right?

x to the power 1 by 7, right? 1 minus x dx dx, right? and only one thing which is x is equals to 1 minus r by R, so when x is (equa) r is equals to 0 so x becomes equals to 1 when r is equals to R, x becomes equals to 0 that is what was the problem, r is equals to 0, x is equals to 1 and when r is equals to R x becomes equals to 0 that is what it is when r is equals to 0, x becomes equals to 1, r is equals to 0, x becomes equals to 1 and when r is equals to R, x becomes equals to 0, when r is equals to capital R then this is 1 x becomes 0.

So it should have been 0 to R so from there it is x is (equal) r is equals to 0, that means x becomes 1 and for this 0, so this was our original, right? 1 to 0, so that means this we can write to be equal to minus 2 v max R square by R square 1 to 0 x to the power 1 by 7 1 minus x dx, right? On simplification we can write 2 v max, right? R square goes out and now with a negative then we can write it to be 1 to 0 x to the power 1 by 7 dx, right? Minus 1 to 0 x to the power 8 by 7 dx this is 1 by 7, right? Minus 1, so it is 8 by x to the power 1 by 7 into 1 that is 7 plus 1 8 by 7 so x to the power 8 by 7 dx, okay 1 to 0. Now this we can re-write to be minus 2 v max, right? and between between x to the power 1 by 7 means x to the power 8 by 7 divided by 8 by 7 this is between 1 to 0 minus this one x to the power 8 by 7, right? 8 by 7 plus 1 so 15 x to the power 15 by 7 over 15 by 7, right? So this is also between 1 to 0, right?

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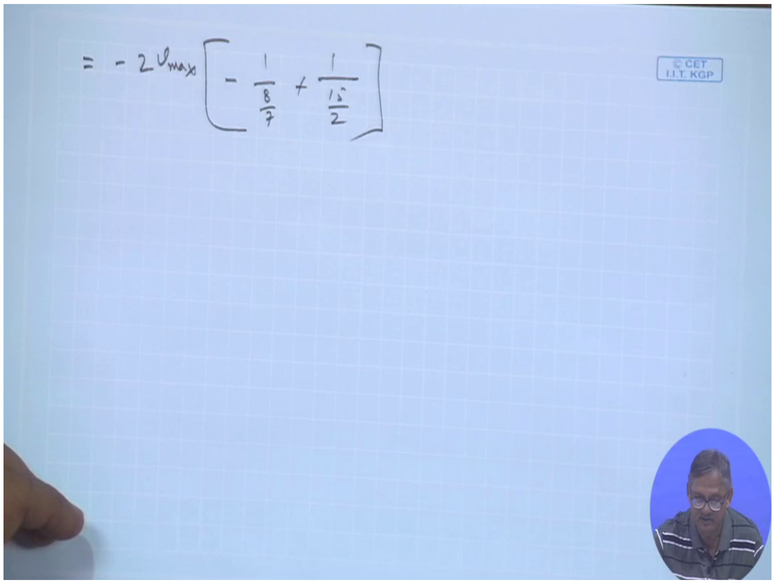

So if this is true, then we can write that this is equals to minus 2 v max, right? Times now this we can write x to the power 8 by 7 and 0 and 1 when we put this limit then we write 1 by 8 by 7 plus 1 by 15 by 2, right?

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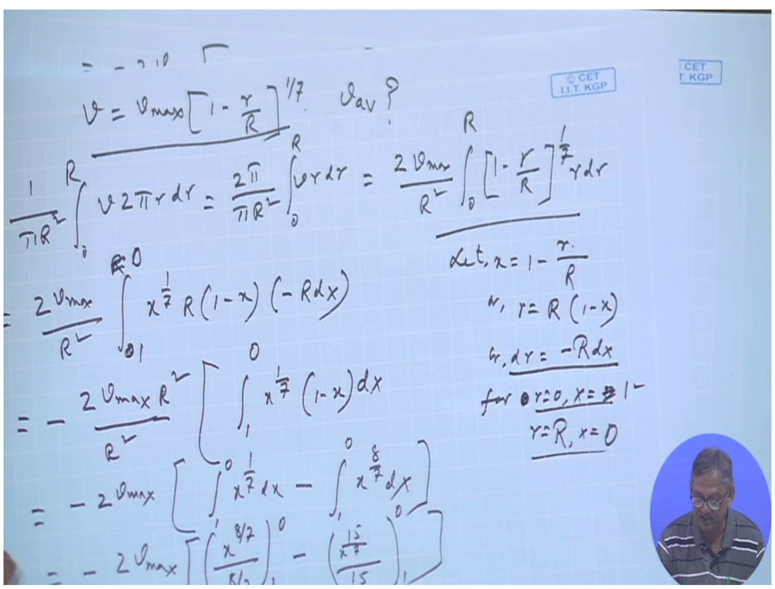

Because this when we had seen this was between the two minus 2 v max we maintained minus 2 v max, right? And this was x to the power 8 by 7 by 8 by 7 when x is 1, right? Then automatically 1 minus comes in when x is 1 automatically because this is 0 minus 1 so when x is

0 this is 0 then then 1 minus 1 minus this that 1 by 8 to the power 8 by 7 and this is minus 1 by 15 by 7.

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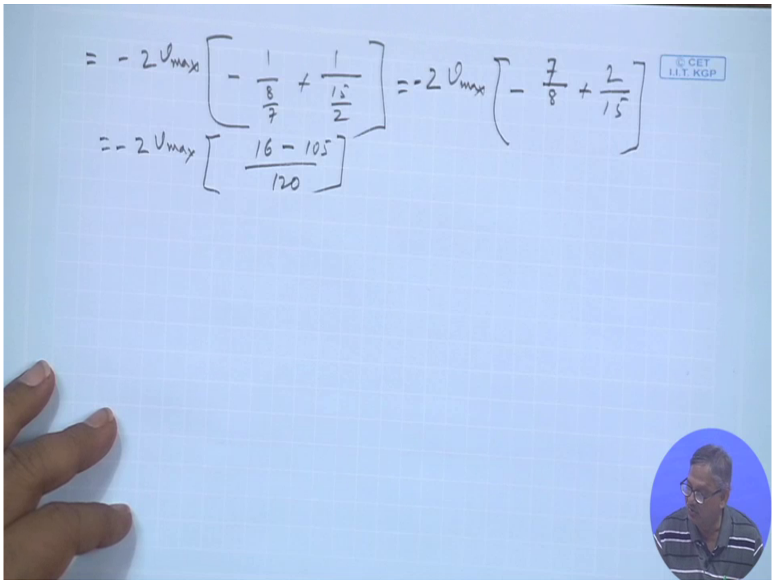

So that is what we got and this 1 minus and that minus made us this is plus, right?

So this on simplification we can write v max, right? v max times (1) minus 1 by 8 by 7 which we have written, so it is 7 by 8 minus plus 2 by 15, right? So this is nothing but 2 v max so (ou) our our denominator 2 v max of course here we had a negative which we forgot minus 2 negative we have now 15 8 120 and this becomes 15 this becomes 8 two’s are 15 8 two’s 16 and 56, right? This is 8 two’s are 16 minus 15 7’s are 105 15 7's are 105, right? It is it is okay okay if we take if we take 7 or 7 by 15 to where and 7 by 8, no this 15 by 15 by 7 how come it is 2,

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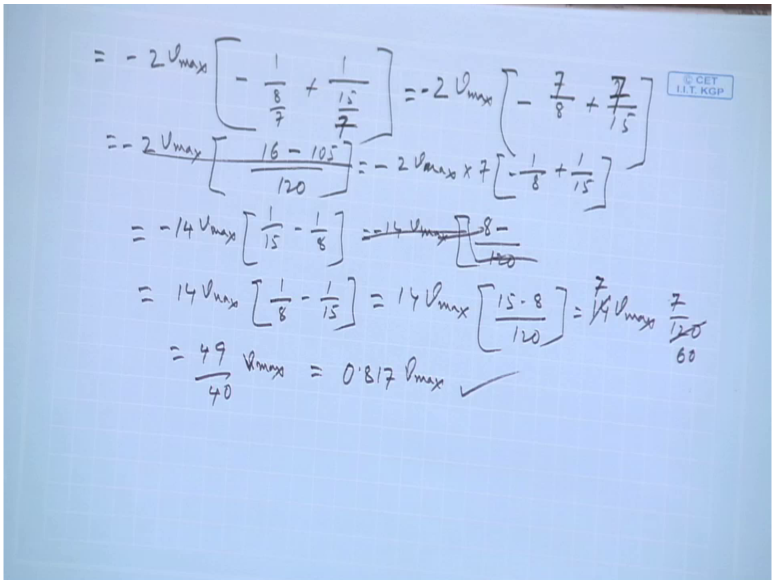

How come it is 2, this is x to the 1 by 15 by 7 and 8 by 7, okay. So this is 1 by 8 by 7 this is 15 by 7 or that is what the mistake was, so it is 15 by 7, so 2 v max 16 not 16 we can re-write minus 2 v max, right? Into this 7 if we take common then it becomes 1 by 8 minus plus 1 by 15, right? Or is equals to 7 two’s are 14 v max, right? 7 two's are 14 v max and this becomes 1 by 15 minus 1 by 8, right? So this is 14 v max, right? 14 v max 120 so this becomes 8 and this becomes this becomes minus with this negative was there this negative is still there so if the negative is still then why do we carry out.

So this we can take that negative inside 14 v max, right? 1 by 8 1 by 15, right? That is 1 by 15 over 1 by 8, right? 1 by 15 1 by 8 minus 1 by 15 this is 14 v max, right? And 120 now, so this is 15 minus 8, right? So this is 14 v max 15 minus 8 is 7 by 120, right? And if we if we divide this becomes 2 or 7 and this becomes 60, right? So it becomes 49 by 60 v max, right? This is nothing but 0.817 v max, right? So we could find out what is the what is the average velocity, so average velocity is 0.817 v max this is a typical typical your problem this kind of relations you can also generate and then find out what is the average value with the actual value with the maximum value, right?

But you must also know that the relation which you have put here you if you remember we started with v max v (avera) having a v is equals to v max so much that relation which we get and so you must have one such relation where it will have a meaningful relation and some value like here we have found out this is to be equals to 0.817, right?

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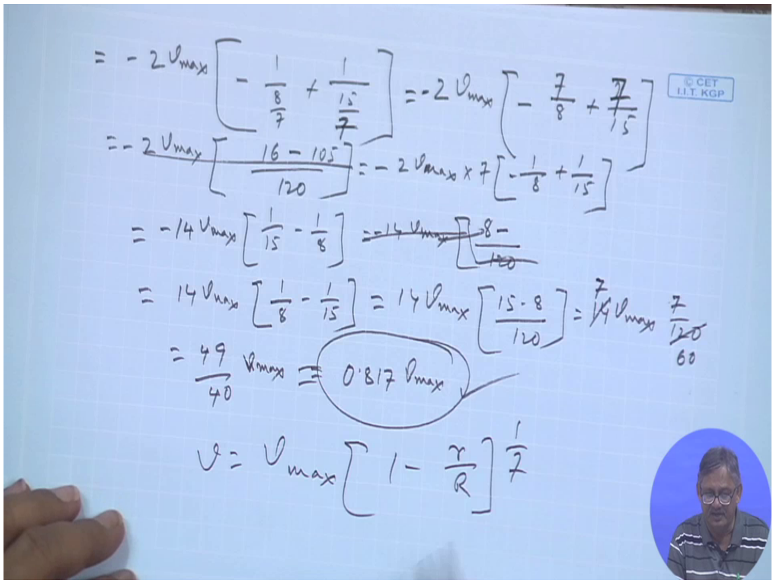

Whereas we started with that relation was v is equals to v max into 1 minus r over R to the power 1 by 7, right? So any such relation if we have, then you can get the average velocity in terms of maximum velocity.

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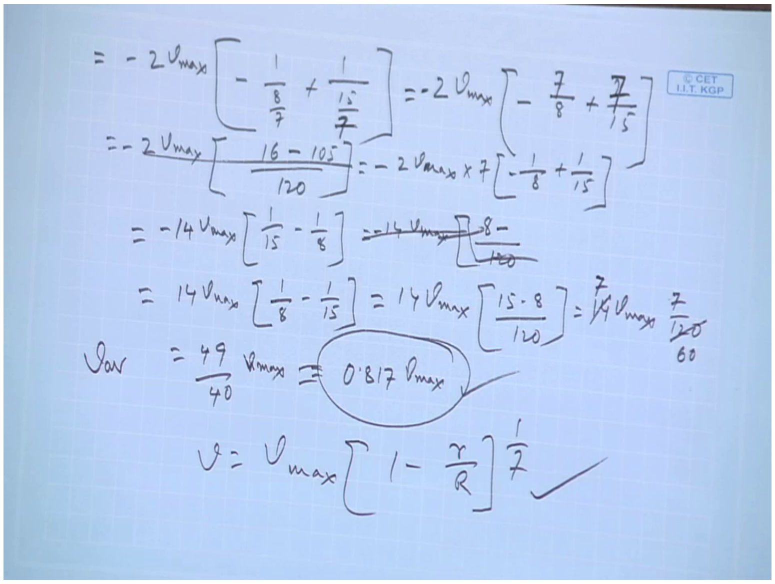

So we got the average velocity this is equals to maximum velocity of so much 0.817 times of maximum velocity. Keep in mind that this relation is a just for 1 exercise to find out the average velocity, right? Because average velocity we if you go back to your previous and previous that 1 by area times the area total so where that integrated between the between the limit 0 to 1 or 0 to 2 pi then r to capital R like that depending on your regime which you are handling with and then find out and then get into it, right? So this way we can determine a relation, okay.

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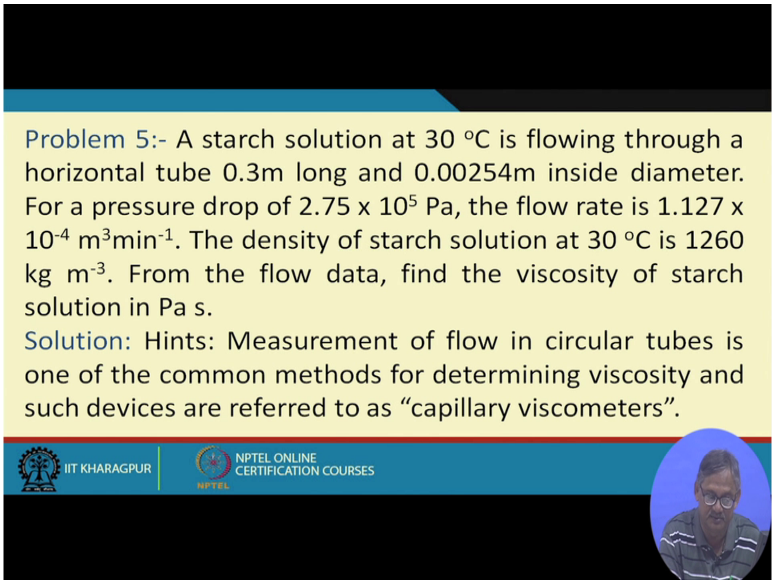

Then let us also solve another problem quickly so that we are accustomed with this that A starch solution of a starch solution at 30 degree centigrade is flowing through a horizontal tube 0.3 meter long and 0.00254 meter inside diameter. For a pressure drop of 2.75 into 10 to the power 5 Pascal, the flow rate is 1.127 into 10 to the power minus 4 meter cube per minute. The density of starch solution at 30 degree centigrade is 1260 kg per meter cube. From the flow delta, find the viscosity of starch solution, right?

I repeat, A starch solution at 30 degree centigrade is flowing through a horizontal tube 0.3 meter long and 0.00254 meter inside diameter. For a pressure drop of 2.75 into 10 to the power 5 Pascal, the flow rate is (126) the flow rate 1.127 into 10 to the power minus 4 meter cube per minute. The density of starch solution at 30 degree centigrade is 1260 kg per meter cube. From the flow data and the viscosity of starch solution in from the flow data, find the viscosity of starch solution in Pascal second, right? One thing given us hint that measurement of flow in circular tubes is one the common methods of determining viscosity and such devices are referred to as "capillary viscometers"

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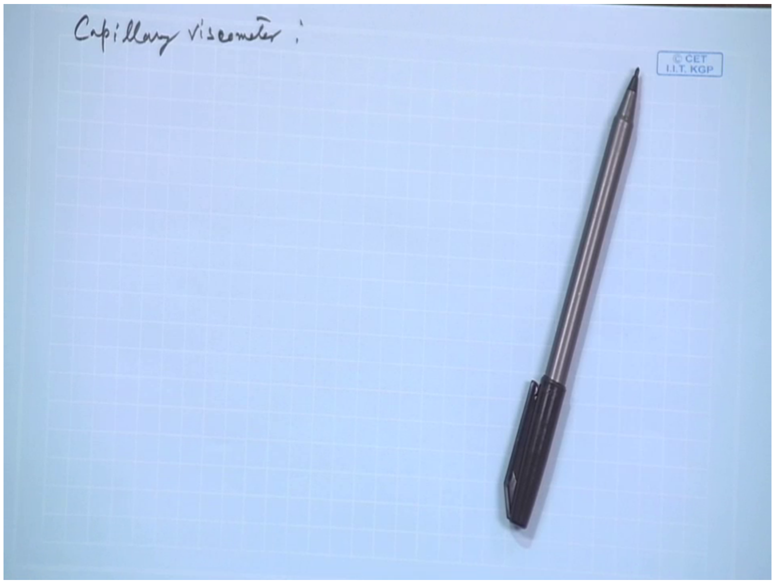

So this is nothing but a problem of capillary viscometer, right? So if we solve if we find the viscosity we will tell that this is the viscosity as measured by the capillary viscometer. So capillary viscometer if it is with your institutes you can find out and check knowing the other values and check whether your predicted value and the actual value are matching or how far they are close that you can determine, right?

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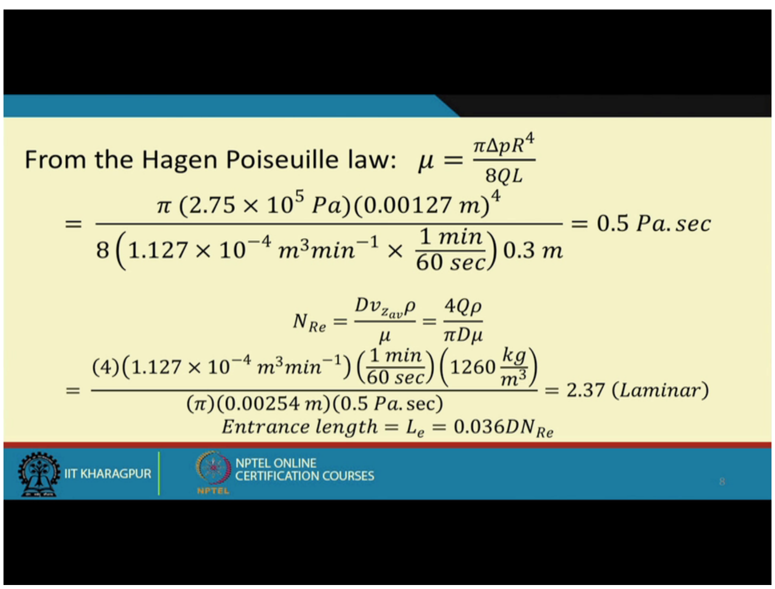

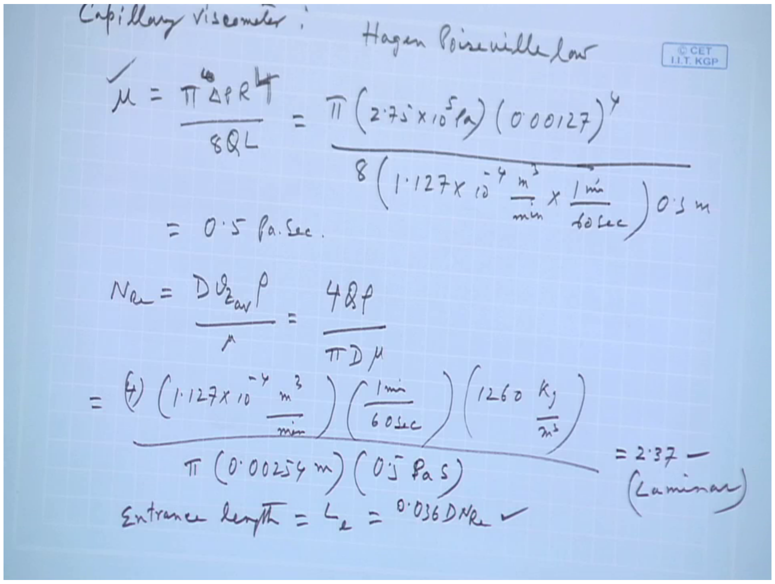

So let us look into the solution from the from the Hagen–Poiseuille's law we know that mu is equals to mu is equals to pi into delta P to R square divided by 80 or 8 Q L, right?

Mu is equals to pi delta P R to the power 4 not R square R to the power 4 by 8 Q L, right? So delta P is rather given pi into delta P is given 2.75 into 10 to the power 5 Pascal, right? And R is also given as 0.00127, right? To the power 4 and this divided by 8 Q is also given as 1.127 into 10 to the power minus 4 meter cube per minute, right? Times 1 minute by 60 seconds into 0.3 meter, that is L, right? So this becomes equals to 0.5 Pascal second, right?

So viscosity we found out so we can say Hagen–Poiseuille so Hagen–Poiseuille law we found out the viscosity what it is what the value is, right? Now, we find out Reynolds number NRe is equals to D vz average into rho by mu that is equals to 4 Q rho by pi D mu, right? So if we substitute this values, then this we can write this is equals to 4 times 1.127 into 10 to the power minus 4 so much meter cube per minute 1 minute per 60 seconds 1260 kg per meter cube divided by pi 0.00254 meter times 0.5 second or 0.5 Pascal second given, right?

So this becomes equals to 2.37 which is nothing but the flow is laminar, right? So this becomes the the flow is laminar. Now there is a term that is also called entrance length so and this entrance length Le is related with NRe as 0.36D 0.036 D NRe, right? So this problem we can we have handled and okay we found out this to be 2.37, right?

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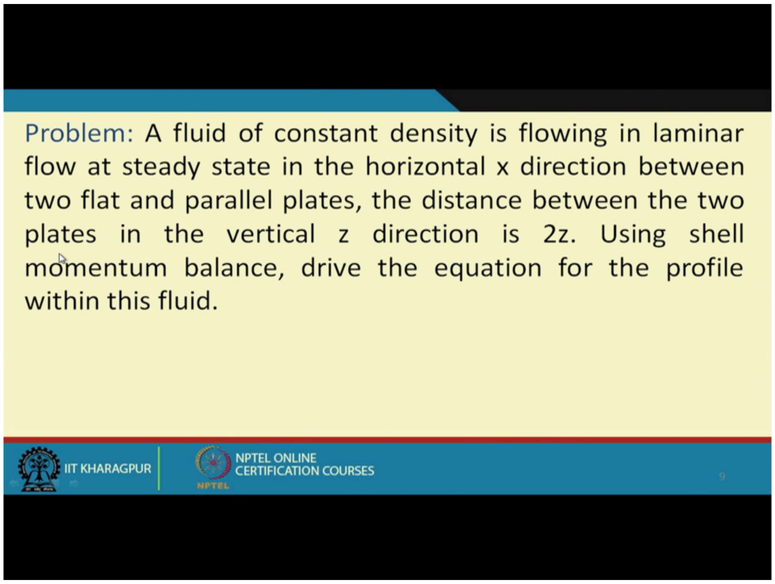

Now I give you another problem which I I do not tell you that this is assignment, but okay some other day if you are not able to answer we will solve it and you first try that is the best thing you first try if you are not able to find out the solution I will definitely tell this do not take it as assignment, but okay giving a problem which you can try with, right?

So for example, A liquid of constant density is flowing in laminar flow at steady state in the horizontal x direction between two flat and parallel plates, the distance between the two plates in the vertical z direction is 2z. Using a shell momentum balance, derive this is not drive derive the equation for the profile within this fluid, so I repeat, right? And hope you can do this that is my expectation that you can do this till now whatever we have done from that basis you can do this if again you are not able to do obviously I will give you the solution, but first you try, okay at least till now whatever has been covered by that it can be easily done.

So A fluid of constant density is flowing in laminar flow at steady state in the horizontal x direction between two flat and parallel plates, the distance between the two plates in the vertical z direction is 2 zero using shell momentum balance, derive the equation for the profile within this fluid, right? So I hope you can do this because we have already done similar thing and that similar thing the pipe flow we have done so we can do it subsequently, right? So in this problem with this given problem let us complete this class today and next time if you are not able to solve we will come across and solve it, okay thank you.