Course on Momentum Transfer in Process Engineering
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Lecture 14
Module 3
Hagen–poiseuille equation from Navier stokes equation

Yeah, so we have seen that Hagen–Poiseuille equation how we have developed in a pipe in for a fluid which is incompressible and under steady state and laminar flow this there is no end effect, there is no velocity development in along the flow of the fluid that we have seen, right? Velocity profile was normal so developed in the flow of the fluid uhh along the pipe, so this we have already found out. Now one more thing we had said earlier that uhh you can utilize the equation of motion or this uhh equations which we developed known as the Navier-Stokes equations. From the Navier-Stokes equation also this you can find out and develop so we will do this today also again, right? So for that let us let us go to that situation that we have taken

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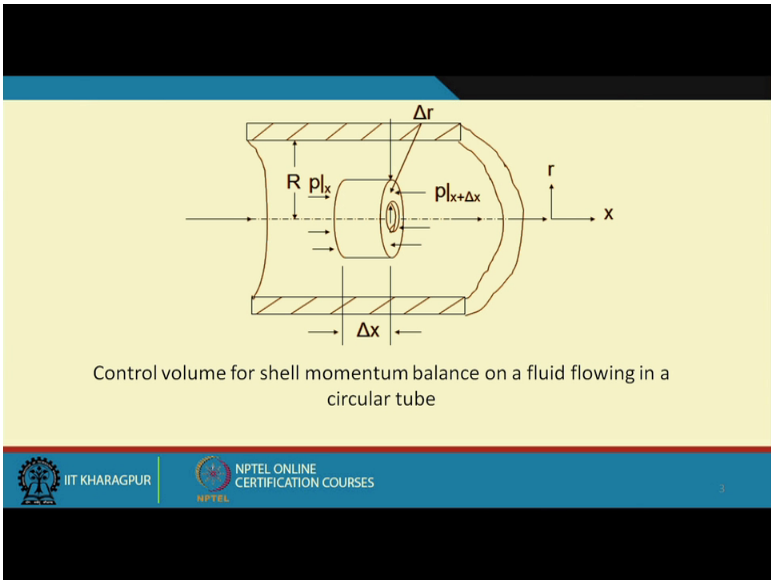

this fluid we have taken the shell momentum balance we have done, right? So here we will do with the Navier-Stokes equation if you remember the Navier-Stokes equation with the r theta and z components r theta z component equations we had used, so similar to that we will use also here r theta z component, right?

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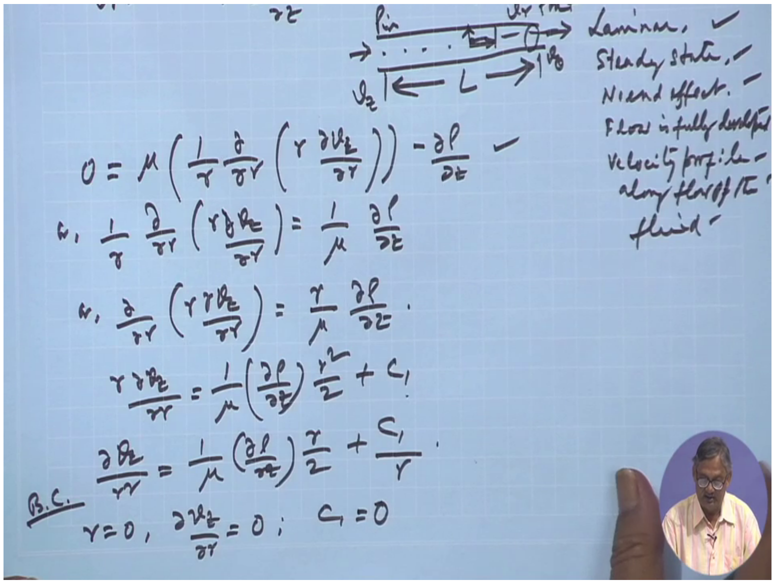

So let us take in that case we know that vr is equals to v theta is equals to 0 and del vz del z is equals to 0, right? vr v theta v since that was r theta z, right? So here also we take r theta instead of x and z, right? Instead of x as z, so r theta z if you take so vr is equals to v theta is equals to 0, because what we are supposed to do? We are supposed to find out that v is flowing like this, right? v is flowing like that so it is vz which is flowing like that and vr is this.

So this vr and v theta is that which is like that, so v theta will be taking since the flow we assume assumed is laminar, we assume steady state, we assume no end effect, we assume flow is fully developed and we also we assume there is no velocity profile along the flow of the fluid. So we assumed all these, so in this case knowing these conditions the uhh conditions of the flow this is called the problem description. So from the problem description we can tell vr is equal to v theta is equals to 0, that is vr is 0 and v theta is 0 there is no velocity component, right? In the r direction there is no theta component v theta in the theta direction this is 0 and v del vz del z there is no uhh flow in the in the direction of the flow there is no velocity profile that del vz del z is also 0.

So in this case if we assume all these three, then we can write from the Navier-Stokes equation if you remember that the first equation, second equation and third equation if we take the uhh **uhh** if we take this equation that mu into 1 by r del del r into r del vz del r, right? This is minus del p del z this is equals to 0, right? Since it is horizontal so rho gz we can tell to be equals to 0, right?

So we can write 1 by r this is the from the Navier-Stokes equation, right? And uhh in the z that r theta z this is the z one del del r of r del vz del r, right?

So 1 by r then 1 by uhh 1 by r del del r of r r del vz del r, right? This is equals to 1 by mu del p del z so this we have taken to the other side, this del p del z this is equals to we can write del del r of r del vz del r, right? This is equals to r by mu into del p del z, right? So on first integration we can write r del vz del r is equals to 1 by mu del p del z into r square by 2 plus C1, why you have not taken in del p del z in the in the (in) integral part because del p and del z they are they can be said to be given, right?

So del p means p in minus p out that is del p and del z is nothing but the length, so once we know the length that is for a given length uhh, right? For a given flow we know del p we know del z so del p del z is (())(8:12) so that is why it is not going into the integration. So we can write r del vz del r is equals to on the first integration 1 by mu del p del z r square by 2, right? plus C1, right? So this on uhh first integration and we can uhh also say on the uhh rearrangement of this as del vz del r is equals to del vz del r is equals to 1 by mu del p del z, right?

And this r goes there so it is r by 2 plus C1 by C1 by r, right? At r is equals to now the boundary condition which we can apply at r is equals to 0 del vz del r is also equals to 0, right? At r is equals to 0, that is this point, right? at r is equals to 0 del vz del r that is also equals to 0, therefore we can write C1 is nothing but is equals to 0 at r is equals to 0 del vz del r is 0 so C1 is also equals to 0 r is equals to 0 this goes out, right?

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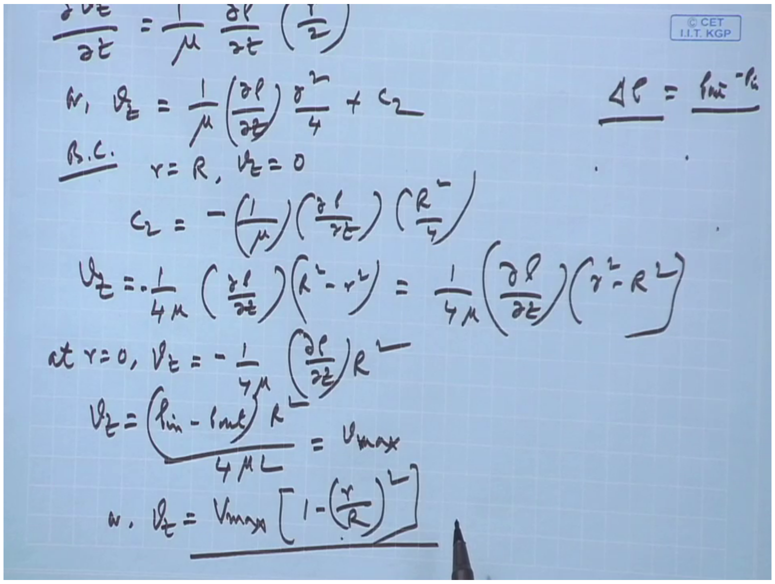

So that also goes there so we can write that uhh C1 is equals to 0 and then we can also say that del vz del z is equals to 1 by mu del p del z into r by 2 since C1 was equals to 0. On second integration we can write vz is equals to 1 by mu del p 1 by mu del p del z that is constant into r square by 4 plus C2. Now, we can put this second boundary condition (boundar), second boundary condition is what? At r is equals to capital R vz is equals to 0, right? If that be true at r is equals to capital R vz becomes equals to 0, then C2 becomes equals to minus 1 by mu into del p del z, right? into R square by 4, right?

Therefore, we can write vz is equals to 1 by 4 mu, right? into del p del z, right? into R square minus small r square capital R square minus small r square, right? C2 is this, right? 1 by 4 is uhh this is with a negative, right? Because that is negative so this can be written 1 by 4 mu del p del z, right? into r square minus capital R square, right? So at r is equals to 0, we can write vz is equals to minus 1 by 4 mu del p del z into R square, at r is equals to 0, right? This minus remains vz is equals to minus 1 by 4 mu del p del z R square, right?

Now we can say that vz is equals to nothing but p in minus p out, right? into R square by 4 mu into L, right? Which is nothing but v max. So vz is p in, how we have taken this minus in? minus in has become this delta p again any delta p is generally out minus in, right? Anything delta is higher minus lower but that is the (out) exit minus inlet. So in this case since we are taking p in minus p out as the delta p so this negative goes in there and this becomes p in minus p out by 4 mu L into r square that is nothing but v max, right?

Or we can write vz is equals to v max, right? terms times rather 1 minus r by R whole square, right? vz is nothing but v max 1 by 1 minus r by R square this is what we have done in the in the uhh flow of fluid in pipe, right? This we have done earlier and with the help of Navier-Stokes equation we have seen that you can also do the same thing for the uhh pipe flow and you can find out the velocity distribution in the pipe, right?

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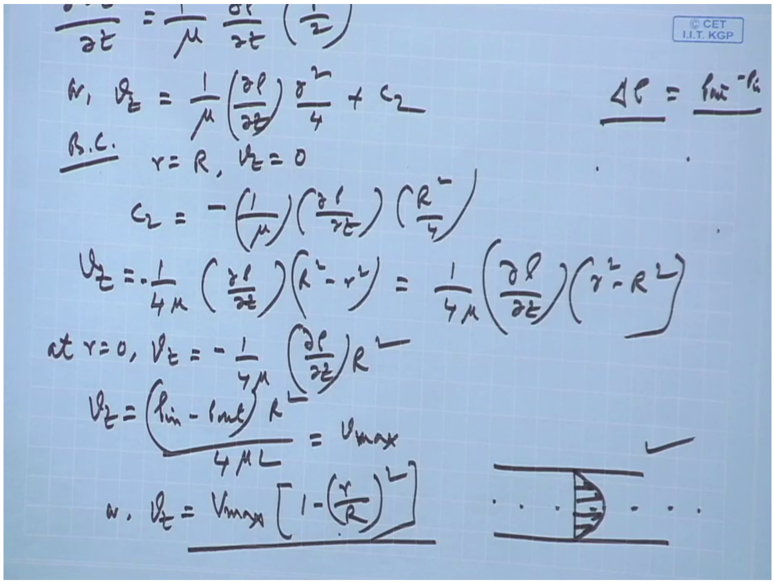

Then, vz is v max so how the velocity distribution comes in? This is the pipe, right? This is r, so velocity is 0 at the pipe and vz is maximum at r is equals to 0 so if this is the maximum so then this will have a velocity profile like this, right? So this will be the velocity profile in the pipe, right? So now here you see in the previous class we had done with the shell momentum balance, but in this present class we have also done the same thing with the help of the Navier-Stokes equation keep in mind Navier-Stokes equation if properly the terms can be identified, if properly you can substitute the values of the terms, then you can easily come to the solution and for the flow of pipe we have done it carefully, right?

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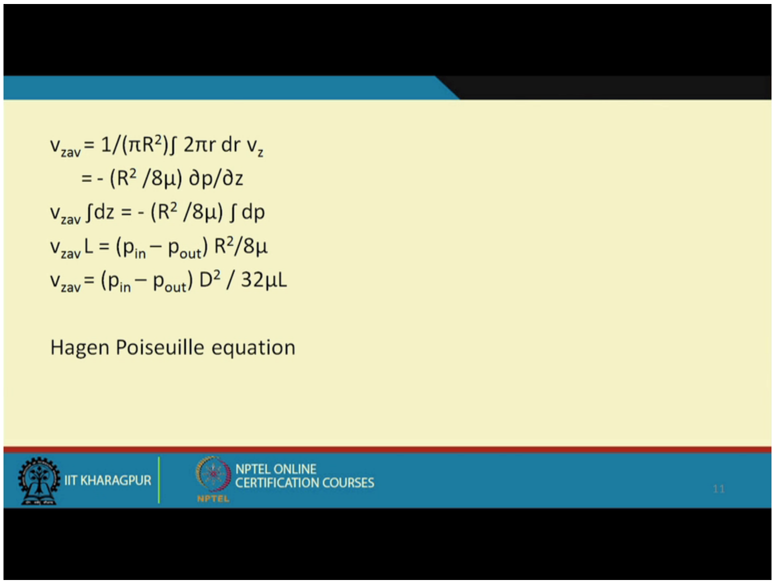

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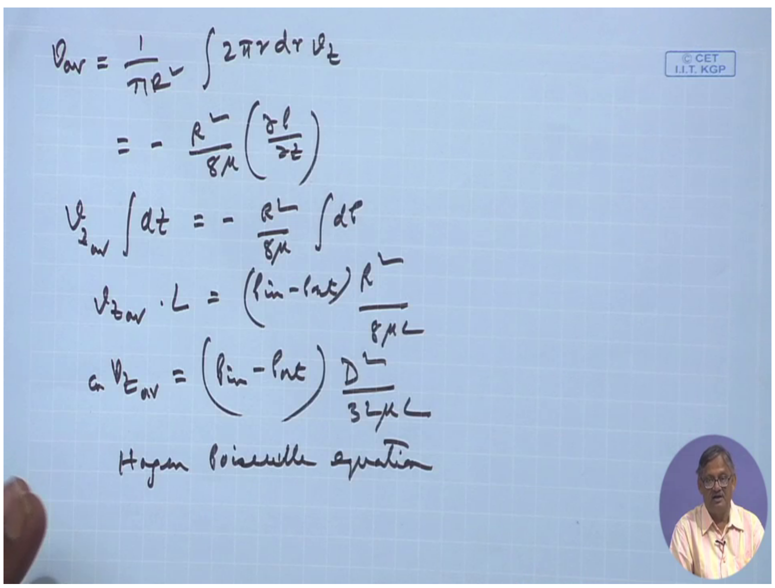

So we can say that like this so v average, then can also be written as this is 1 by pi r square that is the area into into 2 pi r into dr vz, right? That is equals to minus R square by 8 mu delta p over delta z, right? That is R square by 8 mu delta p by delta z, right? So v then then vz average into dz is equals to minus r square by 8 mu into dp, right? or vz average into L is equals to p in minus p out that is delta p R square by 8 mu or z vz average vz average is equals to p in minus p out, right? into D square by 32 mu L and this is Hagen–Poiseuille equation, right? So this is Hagen–Poiseuille equation so Hagen–Poiseuille equation we have found out with both flow through pipe and flow in the uhh flow through pipe with the shell momentum balance as well with the with the uhh Navier-Stokes equation, right? Then, Navier-Stokes equation again and again I am uhh I am I am forcing on that that Navier-Stokes equation you do not have to remember the entire equation is not possible to remember the entire equations all the components r component, theta component and z component.

When you are solving you must take that this Navier-Stokes equation and all the r component, theta component and z component they are with you in the right form and then you understand them in the proper way and the moment you understand them in the proper way you can figure out in most of the cases you will see that from the given problem you will figure out what are the velocity components over there all three of them are uhh active or some of them are nullified or it can be taken as 0.

So if you can identify them, if you can put them, then you do accordingly and then find out whether this Navier-Stokes equation is applicable there or not, how it can be applied, what are the terms which are like in this case we have seen that vr and v theta is 0 and uhh we also have found out that v del vz del z that is also 0 but these these verse or these terms we have said in setting the problem in you remember while setting the problem we have said that uhh the velocity component in the in the flow of the direction of the flow of the fluid does not have any velocity profile.

That means it is not having any velocity profile in the direction of the flow, that means del vz del z is 0, right? We also said and and when you are integrating finding out the integral constants, there what you were putting you are putting the boundary conditions, now here we have put the boundary condition at r is equals to 0 that is the central point at r is equals to 0 you have vz which you have found out, right? vz is equals to max v max, right? And at r is (())(21:01) capital R that is at the at the surface or the at the at the inner inner surface of the cylinder, right?

Where the fluid is clinging if this is the surface and if this is the layer of the fluid, so it is clinging to that as if if the surface is moving it is also moving but if the surface is fixed it is not also moving, right? So that is called clinging it is clinging to the surface and that means the surface surface as the velocity 0. So there r at capital R we have taken to be constant uhh at capital R to be velocity to be 0 and we have shown how the velocity profile has changed, right? In the uhh (al) (al) along the radius, right?

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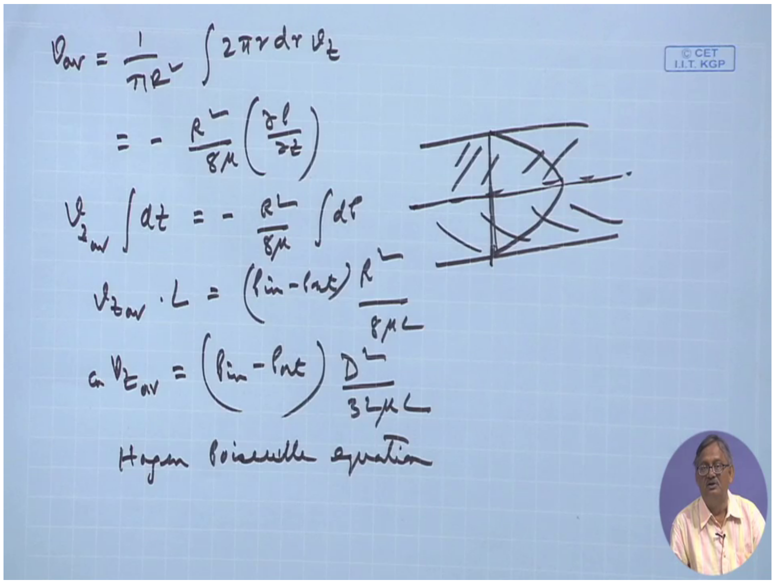

So that we have shown that so that we have shown that if this is the pipe and if this was the axis, then velocity here it was 0 it became maximum and then the this is nothing but a mirror image of this half this half is the mirror image of this half so whatever was there the same thing came out to be here, right? In many cases that is why subsequently you will see once if for a symmetrical thing if you do half of it the rest of the things can be assumed to be the same, right?

So let us then the next we will do when we will have other things, right? Most probably next we will be going for finding out the another thing as if from here it is appearing that this is clinging, right?

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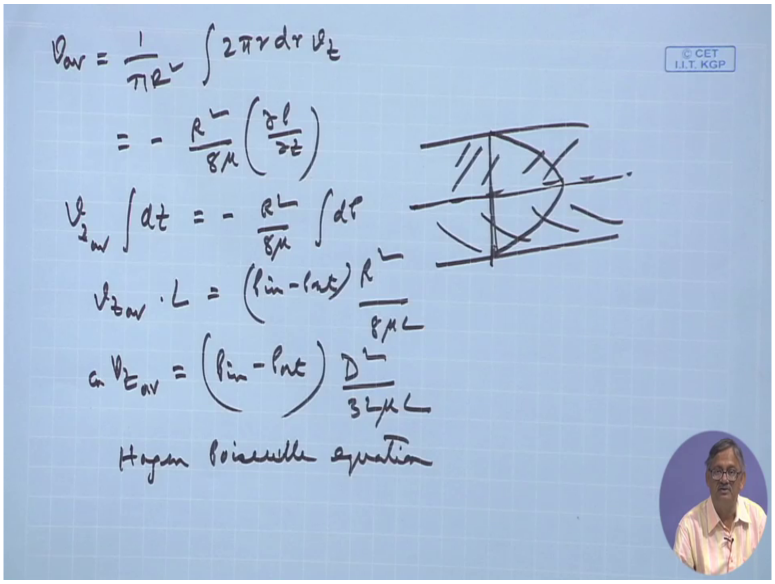

The surface which is clinging to the wall is not moving, right? So this is known as friction, right? So one layer with the other layer with the other layer there having some friction, other is why p out and p in there will be some difference, why p in and p out there will be some difference, who is taking in this is taken by that frictional force which is dragging or drag force one other, this we will do in the subsequent class, okay thank you.