Course on Momentum Transfer in Process Engineering
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Lecture 13
Module 3
Flow through pipes

Again you remember that we said how the understanding of the flow behavior how the velocity components they are acting this we have found out with the help of equation of continuity and equation of motions. We also have done on the application of these equations how we can solve with the help of these equations some of the problems we have done, right? And where Navier-Stokes equations understanding of the individual components of the equation there we have shown, right?

Obviously as an when your problems will be complicated, your solutions also will be complicated, but depending on the cases we assume something which are valid not only valid which can be really assumed and all in all cases where this is more feasible those things we are dealt with, right? Now, let us go into some other which are really further required for your not only understanding but also applications, for example if you are in industries you are dealing with flow through pipes, right?

You have seen if you are in the hostels that your pipe is coming the fluid is coming through water is coming to some pipes and those pipes are maybe somewhere some horizontal somewhere vertical depending on the cases from where it is coming etcetera. And one more very good example is that when you look at this is the real picture of the hostels wherever it would be that some or other day if you fortunate enough or rather unfortunate enough to come across such situations that you (pi) you started taking bath and suddenly there was no flow in the in the toilet or in the wash room.

It might may happen it might have happened to many of you who are students in our students life in the hostel life this is a common situation everywhere, right? And then when after shouting and after doing this and that some people went to the supply and again supply came up and when this supply is coming, then initially at the time past when it came it was not coming fully it was coming a little sprinkle like thing rest for little flow and then stop a little flow and stop with the sound like “phish phish phish phish phish” like that this kind of sound they used to come and then we would come to know, yes now the water is again coming and after sometime the fully developed flow used to come.

And this situation is also applicable in many cases, right? Now, what we will look into is that pipe flow or flow of fluid through the pipes or we call flow through pipes or pipe flow or flow flow through circular tubes, right? Or conduits anything it can be. So that we are that is we have a circular conduit through which flow is fluid is flowing like that if it is a pipe in this the liquid is flowing from one end to the other end, right?

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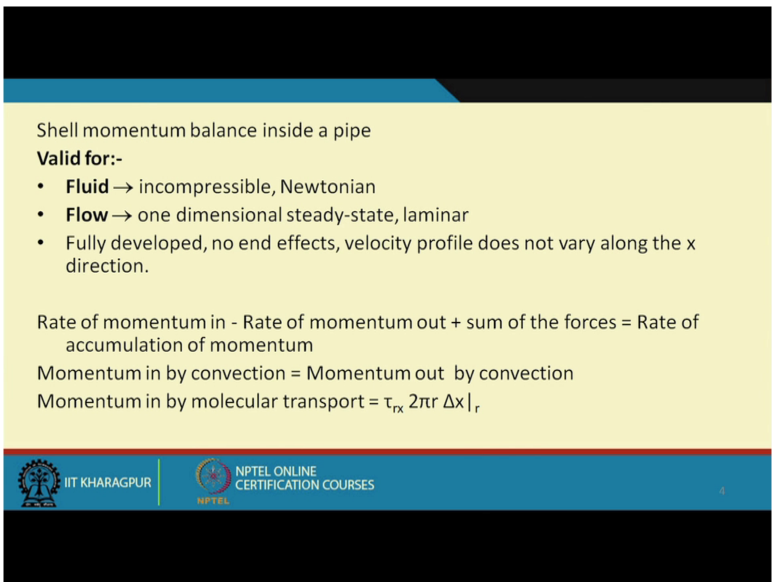

In this certain assumptions we make those assumptions we have to follow and these assumptions are like we will do the shell momentum balance inside a pipe there is (())(4:49) so we will make a we shall make a shell momentum balance that is we will take a shell in the pipe and we will do the momentum balance in that shell and then integrate over the entire region of the pipe, right?

Some conditions which we are imposing that the fluid incompressible, right? The fluid is incompressible it can be Newtonian so if it Newtonian, then that all of which will come afterwards (())(5:28) equations they are becoming very simple so it is Newtonian equation or Newtonian fluid rather incompressible fluid that is again incompressible fluid means the fluid is having constant density, right? Flow is one dimensional steady state as well laminar.

So these conditions we are imposing in the beginning that what is the situation flow is laminar, flow is steady, (flow) fluid is incompressible and the fluid is Newtonian fluid. So these things along with the flow is fully developed, right?

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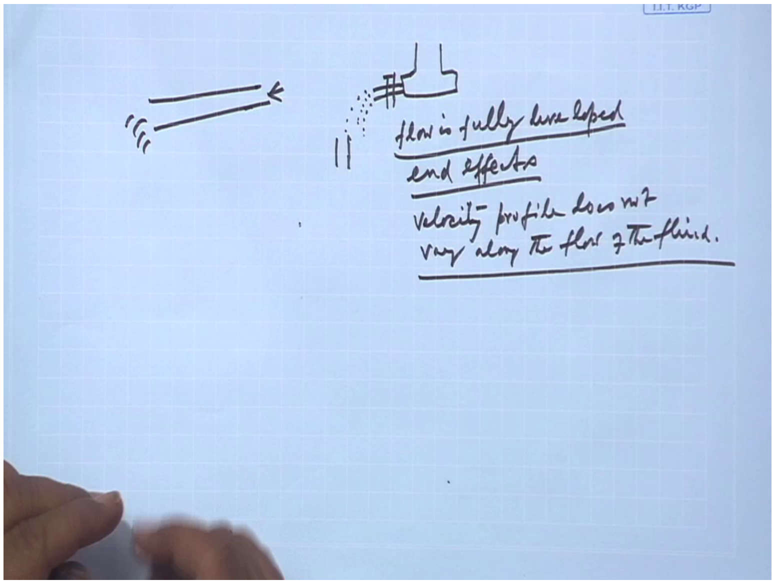

As we said the flow is fully developed if you think that here a reservoir here, right? Like this and with that some pipe there is a say valve here, right? In this valve you open then there will be some point when it will be like this, right? Till the flow becomes a continuous flow as we give the example of this hostel, right? So that flow is fully developed this flow is fully developed and there is no end effect this is what we are saying end effect means in the pipe with that whenever it is coming in there will not be such this kind of this kind of maldistribution of the flow, alright?

This in either side of the pipe is not there so it is no end effect is also there this flow is fully developed there is no end effect and velocity profile does not vary along the flow of the fluid, right? This also we are imposing velocity profile does not vary along the flow of the fluid, right? So these conditions we impose, right?

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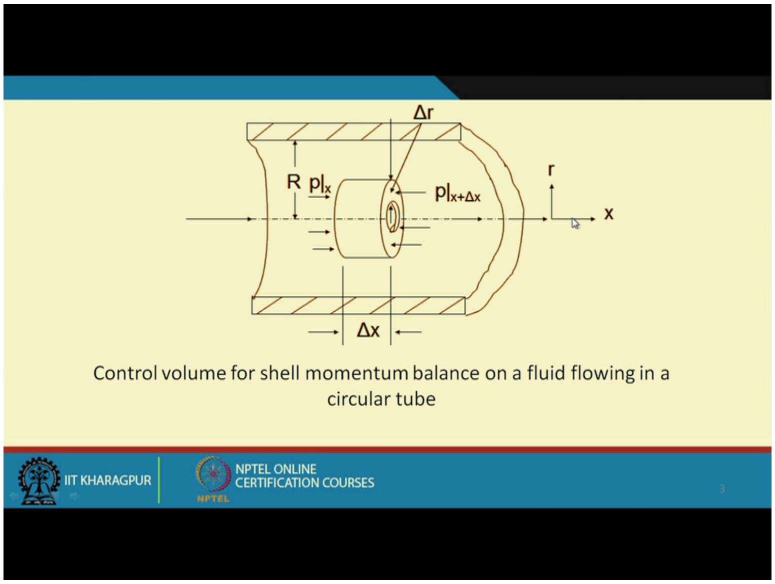

And then by doing a shell momentum balance we can tell now shell momentum balance means here we are taking that shell this is the section of the pipe we assume that in this pipe this pipe is in the r and x direction, right? If we assume it to be horizontal pipe so z direction is not there.

So it is horizontal so in that case z direction is the there so r is this in the pipe radius and x is the flow of the in the direction of the flow, right? This we are taking and in that we have taken a shell like this this shell having a thickness of delta x, right? Having a length rather of delta x and say thickness of delta r, right? This shell we have considered this shell of delta r we have considered, right? And in we will do the shell this shell in this momentum balance we will do and this called shell momentum balance or control volume for shell momentum balance on a fluid flowing in a circular tube conduit in that we are dealing with.

So this we will do today that is the flow through the pipe, right? And we assumed or to be horizontal pipe, we assume the flow to be fully developed, we assume it is a steady state, we assume that the fluid is incompressible and the fluid is also Newtonian and also we assume that there is no end effect in the flow and there is no velocity component or there is no velocity profile changing along the flow of the the velocity profile is not changing along the flow of the fluid.

These assumptions we have made in the beginning and we had doing axial momentum as we have shown in the diagram in the slide that we are taking a small elemental volume in that pipe section and this we will do the momentum balance on the shell and then integrate over the entire pipe, right?

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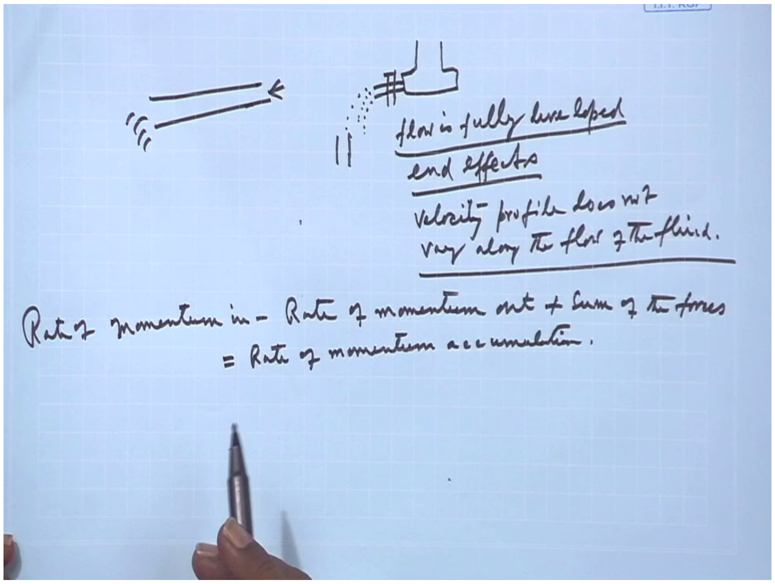

So to do that now let us do that the given equation is rate of momentum in minus rate of momentum out plus sum of the forces acting on the volume element is equals to rate of momentum accumulation, right?

Now, momentum can be transferred as we said earlier also it can be transferred by two ways one by the bulk momentum that is the bulk flow of the fluid when the fluid is fully flowing the bulk flow of the fluid is one way transferring the momentum and the other one is by the sheer force that is, though it is laminar by we (sa) we said that the though laminar but the fluid we can we can assume that the layers to be so close so small that the molecules they vibrate and due to the vibration of the molecules there will be the transfer of energy and that is we said earlier by the molecular momentum transfer, this we said earlier.

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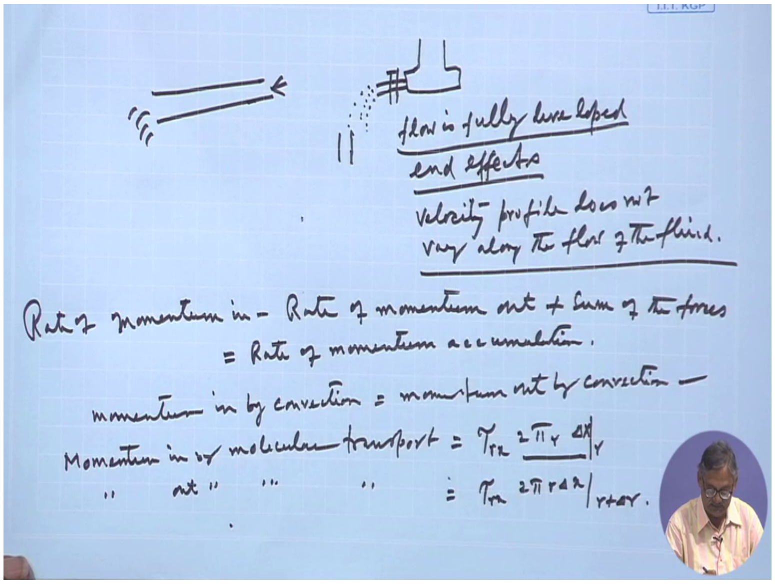

So in this case since it is a steady state, then we can say momentum in by by the flow or convection is equals to momentum out by convection, right? So momentum in and momentum out both are same because it is a steady state, so by bulk flow there is no momentum getting transferred, right? It is a steady flow. Then the momentum in by molecular transfer momentum in by molecular transfer is tau rx 2 into area 2 pi r at the phase delta x r, right? 2 pi r delta x r the phase r 2 pi r delta x is the area and at the phase r.

Similarly this is momentum in by molecular transport so momentum out by molecular transport at the phase r plus delta r is tau rx, right? 2 pi r delta x at the phase r plus delta r, right?

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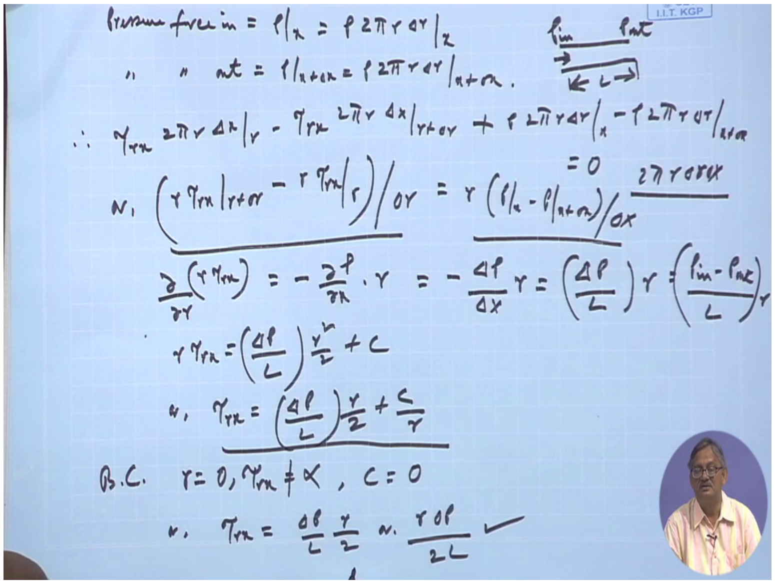

And then out of some of the forces acting on the on the shell we can say that pressure force in is equals to p at the phase x that is equals to p into area 2 pi r delta r at the phase x and (pr) pressure force out that is equals to p at the phase x plus delta x that is equals to p into 2 pi r delta r at the phase x plus delta x, right?

So if we know apply that the control control equation which we had started with, right? if we apply that now we can say that tau rx 2 pi r delta x at r minus tau rx 2 pi r delta x at r plus delta r plus p 2 pi r delta r at x minus p 2 pi r delta r at x plus delta x, right? This is equals to 0 or we can now divide with delta x and delta r all the cases, then this becomes and we can divide the this 2 pi r delta r into delta x that is the volume, right? 2 pi r delta r to delta x so if we simplify divide with this, then we can write r tau rx at r plus delta r minus r tau rx at r over delta r this is equals to r times p at x minus p at x plus delta x, right? Over delta x, right?

So from the definition of the derivative we can write from this two that this can be written as del del r of r tau rx, right? This is equals to since this is px minus p at x plus delta x from the definition of derivative it is p at x plus x minus p at x over delta x is the derivative. So it becomes one negative so del del x of p this into r is this side, right? So we can write that this is del del r into r tau rx is equals to minus r into del p del x, right?

Now del p del x is nothing but minus del p del x into r is equals to this is del p delta p means you have this pipe this is p inlet this is p outlet, right? So p inlet is higher than p outlet otherwise there will be no flow. So p inlet is higher than p outlet but from this definition it is p outlet minus p inlet by delta x that is negative, right? So from there we can write this is delta p over if this length is L, right? If the total length of the pipe is L delta p by L into r, right? So why this negative have gone out? You hopefully have understood that delta p is from inlet high to outlet low, right? By definition of this derivative this is p outlet minus p inlet by del x that becomes the derivative.

So this negative has been taken care of by delta p where this is nothing but is equals to p in minus p out over L into r, right? So if this be true, then we can write del del r of r tau rx is equals to delta p by L into r, right? So on integration we can write that r tau rx is equals to delta p which is constant L which is constant this r is r square by 2 plus C plus C. So we can write tau rx is equals to delta p over L into r by 2 plus C by r, right?

So this we can write, so therefore we can say that tau rx is delta p over L into r by 2 plus C by r. Now if we put the boundary conditions, the boundary conditions are what? At r is equals to 0 since tau rx is not equals to infinity at r is equals to 0 since tau rx is not infinitely we can write C is equals to 0 at r is equals to 0 since tau rx is not infinity, then if at r is equals to C as to have some value, then tau rx has to have (infini) infinite value since it is not infinite it is having a finite value.

So C is equals to 0, right? Then we can write tau rx is equals to delta p by L into r by 2 or r delta p by 2 L as the tau rx, right? so this is in general we will use these subsequently in many cases you will see that we will tell that this when we are doing the either shell momentum balance or similar we will tell that we will starting from here if there is no no bulk flow or effect of bulk flow is negligible or not negligible it is not there it is steady by any chance and then we will start in many cases when the fluid will be Newtonian you will non Newtonian you will see that we will tell that we start from here that tau rx is equals to r delta p by 2 L or delta p into r by 2 L this we will start with, right?

You remember this that this is the starting point in many cases we will see, okay.

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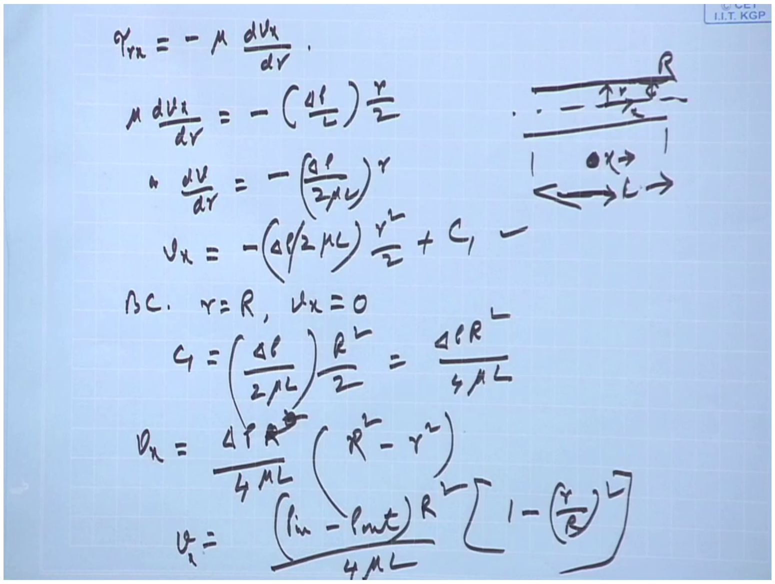

Now, since by definition tau rx is nothing but is equals minus mu dvx dr so all substitution we can write mu dvx dr is equals to minus delta p over L into r by 2, right? Or dvx dr is equals to minus minus delta p over 2 mu L, right? Into r, right?

So on integration we can write vx is equals to vx is equals to minus delta p 2 by 2 mu L, right? Into r square by 2 plus the integration constant C1, right? So if we write like this, then we can also write that the boundary condition that boundary condition is what at r is equals to capital R at r is equals to capital R vx is equals to 0 that was this, right? So this is the axis and this was r, right? This was delta L or x axis this is delta L or rather way, right? So this is delta x or in the x axis this is r axis r and x in that case at vx is r is equals to r this r is equals to r is capital R, right?

So at that is at the at the wall at the wall r is equals to capital R vx is 0 that there is no velocity on the wall, right? The layer which is clinging to the wall there is no velocity that we know. So if this is true, we can write in terms of maths that r is equals to capital R, vx is equals to 0, therefore we can we can write this equation at C1 is equals to vx is 0 so C1 is equals to delta p by 2 mu L, right? Into this 2 so that r is equals to r so r square by 2, right? So that is equals to delta p r square by 4 mu L is C1.

So therefore, vx we can write vx we can write that this is nothing but delta p R square by by by 4 mu L or this is 4 mu L, right? If we take that to be common then it becomes r square and first this R square will come this is plus so capital R square minus small r square, right? If we substitute here value of C1 then it becomes delta p R square by 4 mu L R square minus r square, right? So this we can we can re-write as vx is equals to delta p means p in minus p out, right? Into R square if we take this R square to be there by then then then then this R square does not come, right?

This now if we take R square inside then this does not come, right? By 4 mu L, now we can write this 1 minus r by R whole square, right? So if this is true then vx is p in minus p out R square by 4 mu L 1 minus r by R square, right? So this is the velocity at any point this is the velocity at any point x.

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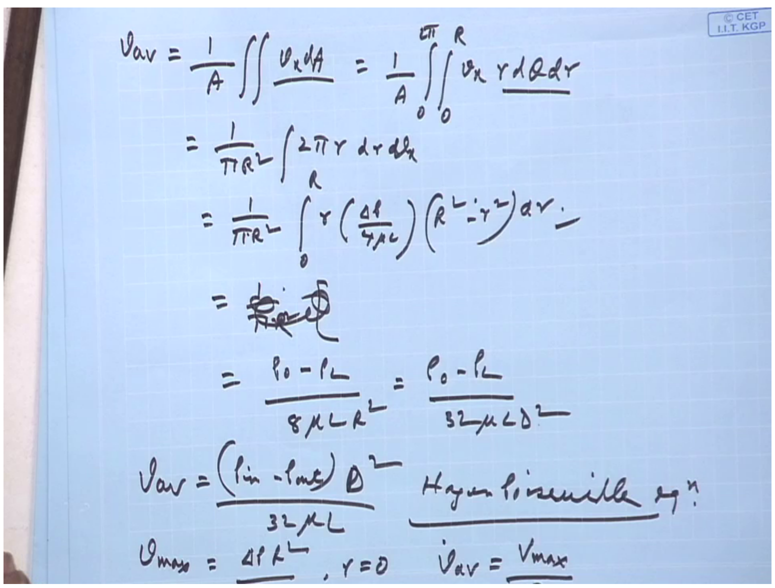

Now if we have the average velocity v average that can be written as 1 by area integration of vx into d area, right? vx d this area, right?

Now this velocity we can substitute with the vx value this is 1 by A into integration of this area is vx r d theta d r this is the elemental volume, right? r d theta into d r that is area and this (val) varies between 0 to R and this theta is varying between 0 to 2 pi so that we can write and this area we can write 1 by pi r square to be the area, right? And we can say this is 2 pi r d r rather vx, right? So on substitution of the value of vx we can write 1 by pi R square, right? Integral of r into delta p by 4 mu L, right? Into R square minus r square, right? Into d r this all simplification we can write this is again between 0 to capital R.

So 1 by pi r square 1 by pi capital R square and this on simplification can be written this on simplification can be written as 2 pi R, okay this pi r 2 pi r that comes out, so pi R square so that means this pi R that goes out, right? From here. So we can then write as this is equals to p0 minus pL by 8 mu L into R square that is equals to p0 minus pL by 32 mu L D square, right? So v average then is equals to p in minus p out, right? Over 32 mu L into D square if it is r, then (()) (32:22) or there is this D, right? If it is 8 then r then if it is 32 then D.

So p in minus p out by 32 mu L is (())(32:37) v average that is p in minus p out into D square by this, this is called Hagen Poiseuille equation, right? This is a famous equation which is equation for pipe flow if you are asked then that under steady state if the flow is laminar for a Newtonian fluid, how can you find out the pressure drop? So you can say you can use the Hagen Poiseuille equation and find out the delta p over that and with this equation that the v average is delta p into D square by 32 mu L that is the Hagen Poiseuille's equation.

Obviously one more thing here we can average we can find out v max, right? v max is equals to delta p that is p0 minus pL r square by 4 mu L this be true at r is equals to 0. So from this equation which we had gone through that from this equation that this on integral relation in terms of r if we put r is equals to 0 here, then we can say delta p R square by 4 mu L is the v max. Therefore that can be said that v average is nothing but v max by 2, right? Average velocity for a pipe flow is the maximum velocity by 2 or v max by 2 when the fluid is fluid is incompressible fluid is under laminar steady, there is no end effect, there is no no no velocity profile along the flow of the fluid in the pipe and in that case we can say that average velocity is half of the maximum velocity, right? So here we stop today and thank you (())(35:24) that you can show, okay so thank you.