Course on Momentum Transfer in Process Engineering
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Module 3
Lecture No 11
Application on Navier Stoke's Equation for finding out viscosity (Part 2)

So in the previous problem we have seen that this is nothing but a jugglery of the boundary conditions, the original equation remaining similar. Now in the 2 cases we have seen that outer cylinder was rotating and inner cylinder was fixed, right? And that can give you way of finding out the velocity profile also the way of finding out the viscosity in many instruments they had we had earlier said that Brookfield viscometer uses the similar situation. Now if the condition is little different that if instead of outer cylinder rotating and inner one fixed and we played with the radii, right?

We made the outer cylinder 1 radii, inner cylinder another radii, of course the velocity expression that becomes different right, so if we do the other one then the outer one is fixed and the inner one is rotating, right. In that case you take the former equation that is the first one like that and then let us see how it happens? Okay.

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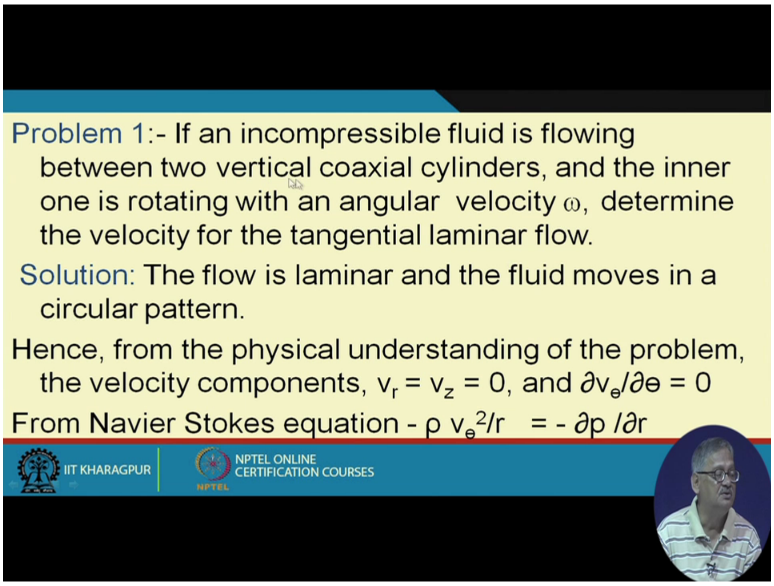

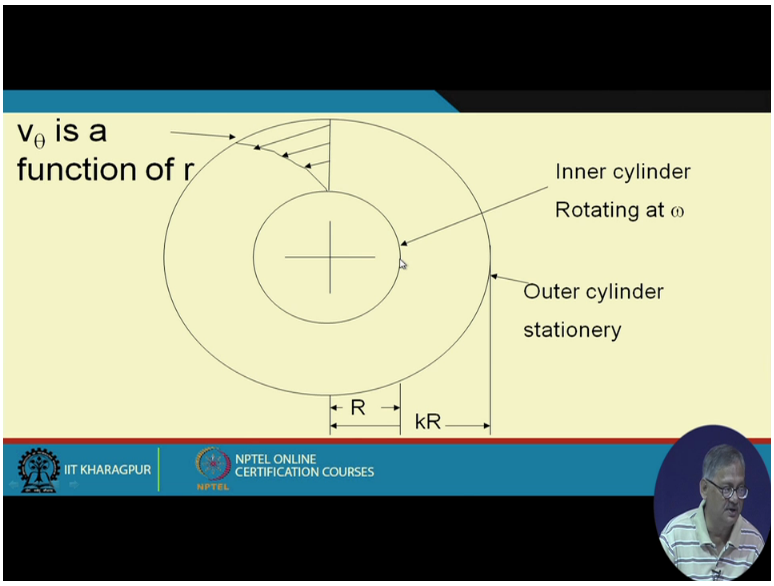

So here the same problem if an incompressible fluid is flowing between 2 vertical coaxial cylinders, and the inner one is rotating with than angular velocity omega, determine the velocity for the tangential laminar flow, right? Now the inner one is rotating and the outer one is fixed, so this outer cylinder is stationary and inner one is fixed rotating rather with an angular velocity of omega, right?

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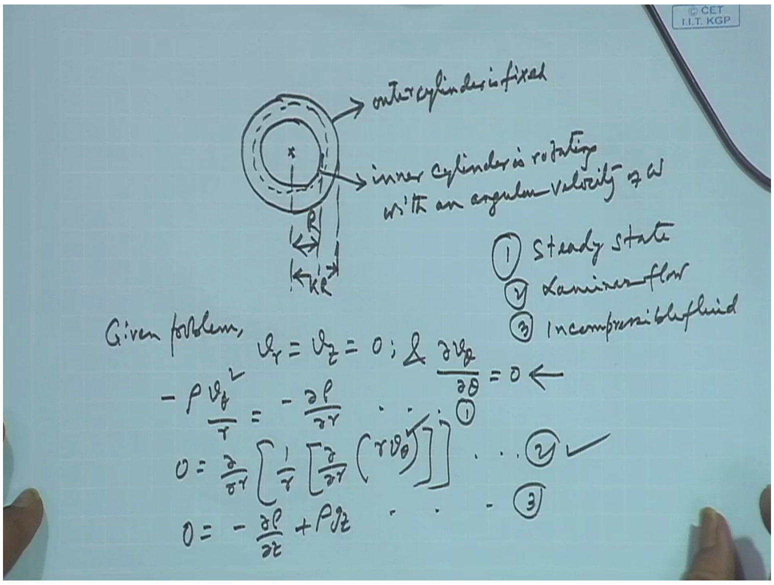

And kept that R and KR as the 1st case that this is R and this is kR, right. If you keep them like that then we can say that the problem can be defined like this, we have one inner cylinder, we have one outer cylinder this is the center, right. This outer cylinder is fixed and the inner cylinder is rotating with an angular velocity of omega, right. And we get as earlier the same, this is R and this is KR that is K is multiple more than one, right. So that KR is greater than R right? So if we take this problem and solve so like the previous one on earlier cases if we use the Navier Stokes Equation, right.

So we can write and here also we have the number 1 steady state number 2 number 2 your laminar flow, number 3 is incompressible fluid and of course others like flow is fully developed there is no in defect all these are also there, right. So we can write from the Navier Stokes Equation right the same as earlier right from the given problem by physical understanding of the problem we can write Vr is equal to Vz is equal to 0 and del V theta del theta is also 0 that means this is not changing with the theta, right.

So if that be true then this is the understanding of the problem we can write and from the Navier Stokes Equation we can write rho V theta square by r this equals to minus del p del r, right? This was our 1st equation, 2nd equation we can write that 0 is equal to del del r of one by r del del r of r V theta right, so this was our 2nd equation and the 3rd equation we can write 0 is equal to minus del p del z plus rho g z, right. Now from these 3 questions since we need the solution of V theta.

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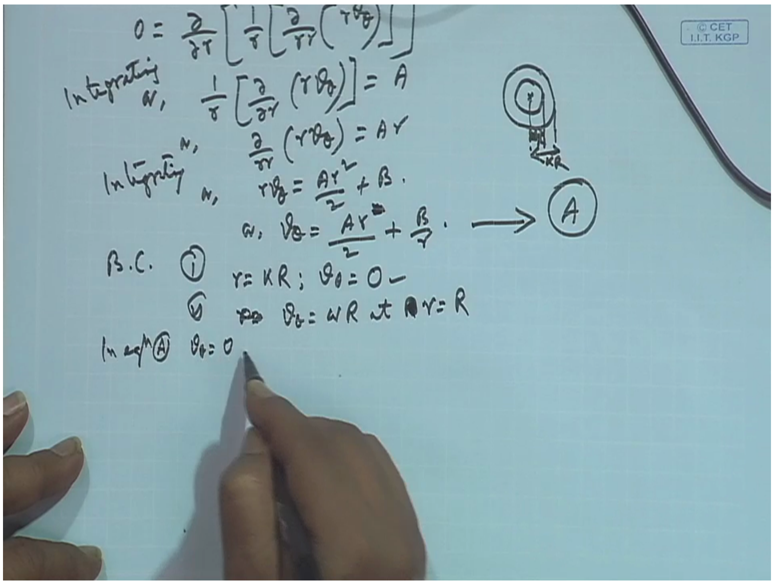

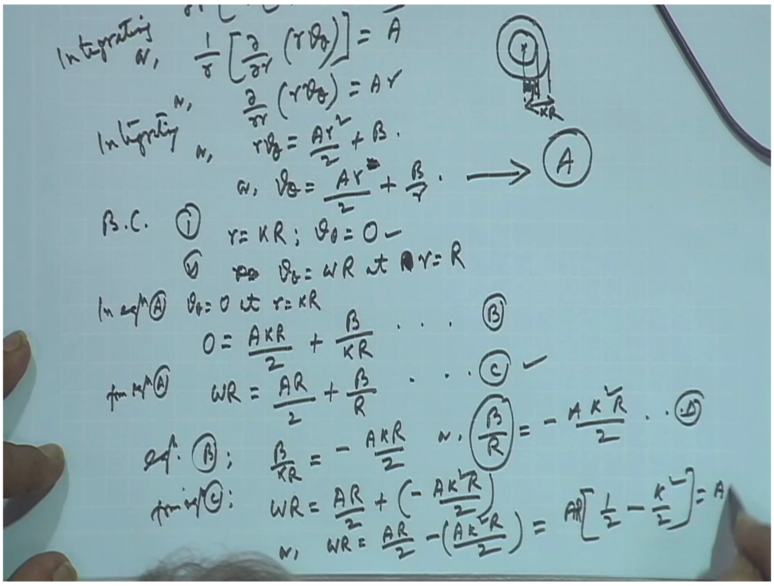

So this equation we take then we can lead to the solution and that equation if we rewrite we can write del del r of one by r del del r of r V theta, this is equals to 0, right and we also wrote that on first integration we write that on first integration we write 1 by r del del r of r V theta, right this equals to A. On simplification we write del del r of r V theta is equal to A r, right. On 2nd integration we will write, integrating again integrating. On 2nd integration we can write that r V theta is equal to A r square by 2 plus b or V theta is equal to A r square or r square goes out A by 2 r plus B by r.

So again up to this that is the equation A the solution remains identical. Where we are seeing the change? The changes are in the boundary conditions because our boundaries are changing, right? Either with the diameter or radii or with the condition of physical condition of the problem, they are changing so boundary conditions are changing. So this is again nothing but a change of the boundary conditions, right.

So we write the boundary condition 1 what is given we have been given said that the inner cylinder is rotating and the outer cylinder is fixed right. So we have that R is equal to R and that R is equal to KR, two different conditions, so what are those 2 different conditions? (()) (9:50) one if we write that r is equal to KR V theta is equal to 0 because this outer cylinder is fixed, right outer cylinder is fixed, so V theta is equal to 0 and r is equal to KR, right. So r is equal to KR V theta is equal 0 this is boundary one and boundary 2 is r is equal to sorry that is that is V theta is omega r, V theta is omega r at r is equal to sorry at r is equal capital R, right so these are the 2 boundaries we have to substitute, right?

These other 2 boundaries we have to substitute and when we are substituting them then we see that the first one at r is equal to KR V theta is equal to 0, we put this in equation 1 or A, if we put V theta is equal to 0 at r is equal to kr we get what so V theta is 0, r is KR so it is AKR by 2 plus B by KR right, so this is say equation B and second equation is V theta is omega R at r is equal to KR so we right here again from equation A we get that V theta is equal to omega R right at r is equal to capital R so it is AR divided by 2 plus B by R, right. So again by substituting one this is C, so by substituting one that is that is B is from B we can write we can write B by KR is equal to minus AKR by 2 or we can write B by R is equal to minus A K square R by 2.

So if we substitute this B by R right if we substitute this B by R into C we can write from equation C we can write that omega R is equal to AR by 2 plus B by R has become minus plus minus A K square R by 2, right or we can write omega R is equal to AR by 2 minus AK square R by 2, right so this on simplification we can write A if we take out or AR also if we take out then we can write this is 1 by 2 minus this is K square by 2 is equal to AR into 1 minus K square by 2, right.

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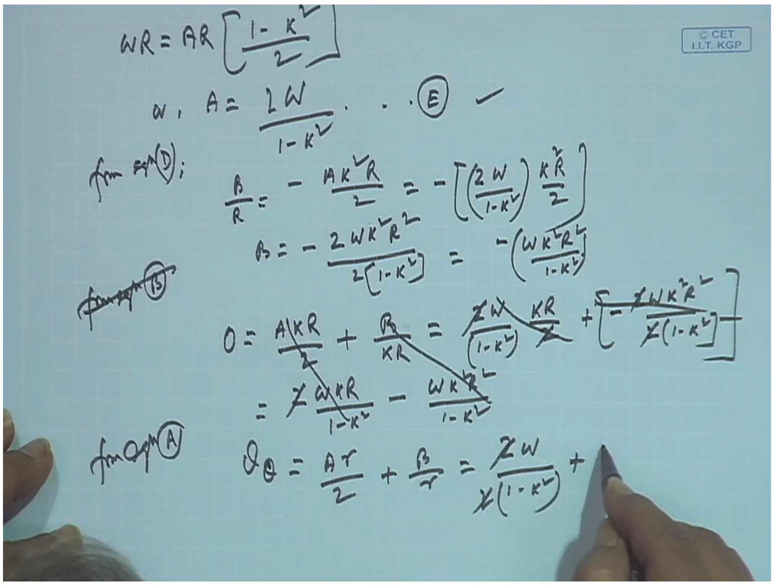

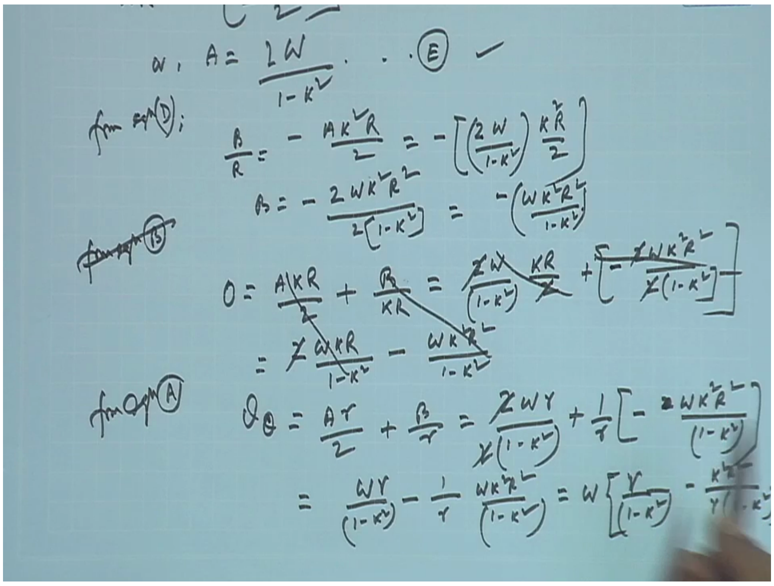

So we can write from there that we can write from there that omega R is equal to AR into 1 minus K square by 2 or A is equal to omega or 2 omega by 1 minus K square. So if A is equal 2 omega 1 minus K square so this is relation E we can write from the other one relation B from equation D we can write B by R is equal to minus A K square R by 2, right is equal to if we write K value so it is B by R is equal to A is minus then 2 omega by 1 minus K square, this is one into K square R by 2, right. So V (())(16:23) equals to minus 2 omega K square this R into R goes R square by 2 into 1 minus K square right, so if you substitute A and B in the original equation that is that is either B or C right.

So if we substitute them in B we have 0 is equal to AKR by 2 plus B by KR right, so this is equal to value of A we know, this is 2 omega by 1 minus K square right and KR by 2 plus right this is minus we are substituting B by KR right so this is 1 by KR we will write afterwards so B is equal to minus 2 omega K square R square divided by 2 into 1 minus K square so this of course 2 2 goes out so we could write it minus omega K square R square by 1 minus K square, this is for B so we substitute this 2 and we can re-write this is equal to 2 omega KR omega KR so this 2 and this 2 goes out so this 2 also was not there so 1 minus K square right, omega KR by 1 minus K square this is minus so this is omega K square R square by 1 minus K square right.

So if that is true we can write this substitution we should have done not here at we should have done at this equation A from equation A sorry this should not have been otherwise V theta is not coming. V theta is Ar by 2 plus B by r, right. So here we are substituting A as 2 omega by 1 minus K square into 2 so this 2 this 2 goes out this is the Ar by 2 plus plus b or 1 by r into minus 2 omega K square R square divided by 2 this 2 of course goes out so one minus K square right.

So we can write now that omega by 1 minus k square minus 1 by r right. Ar by 2 so it was 2 omega into r right, so omega r by 1 minus K square 1 by r minus and this is omega K square R square by 1 minus K square right. So this is the V theta which of course we can further simplify as omega if we take common then 1 r by 1 minus K square minus minus omega is out so K square R square divided by r into 1 minus K square, right. So this is the solution right, so if we look at then this should be the solution.

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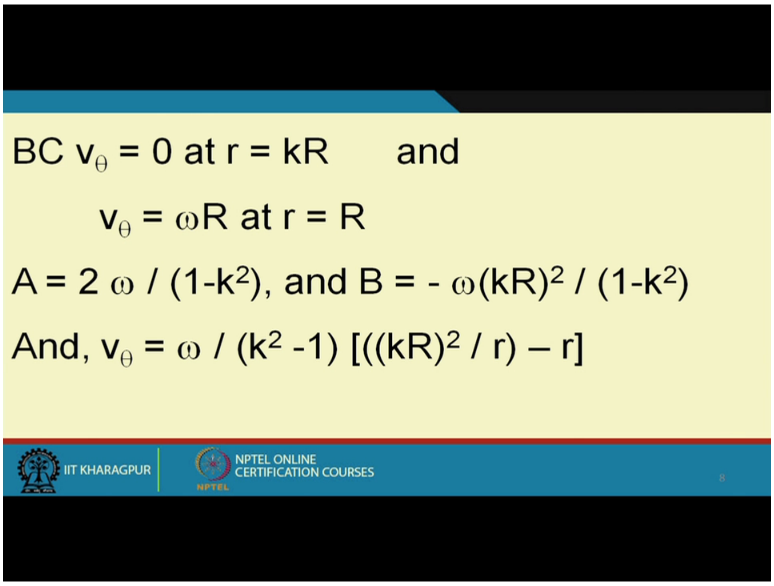

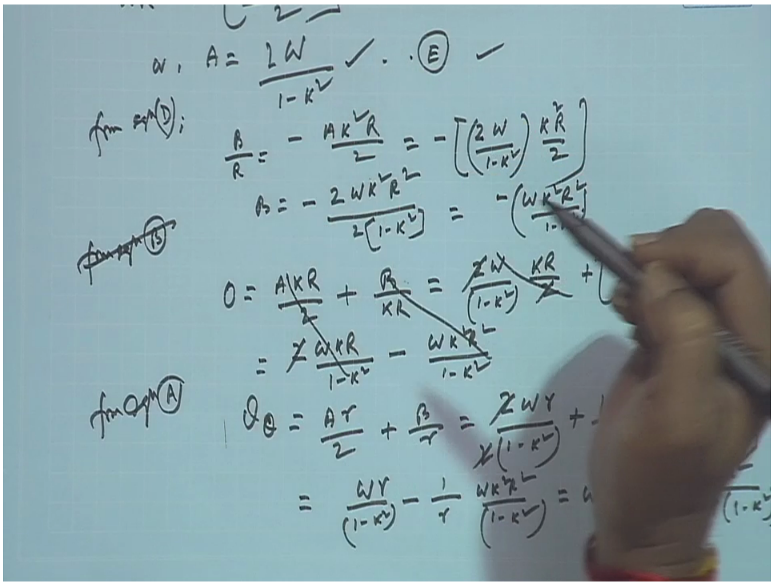

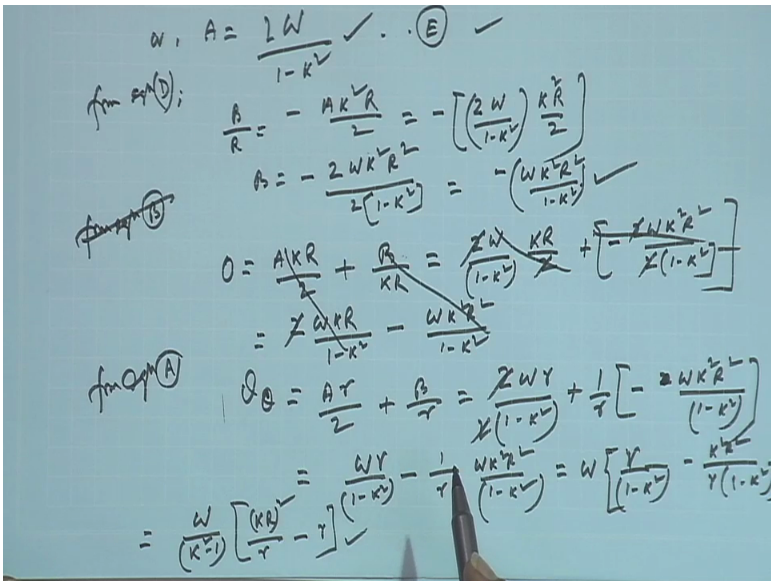

Let us look into in the final solution which we have gotten on the same way on the same thing we have done, you see boundary condition was V theta is equal to 0 with V theta is equal to 0 at r is equal kR right and V theta is omega R at r is equal to R right V theta is omega R at r is equal to R, this was our V theta is 0 at r is equal to kR and V theta is equal to omega r at r is equal to R, right. So if we do that we got A is equal to C we got A is equal to here we got A is equal to 2 omega by 1 minus K square right and B you we got is equal to minus omega kR square omega K square R square by 1 minus K square.

That is true and we got V theta is equal to ultimately this omega divided by omega okay omega we can also write in this form this is equal to omega by K square minus 1 if we take common from here then K square minus 1 so 1 negative we have taken inside right, so that becomes equal to this into then first term is this one so kR square kR square divided by this r remains there right and this r then goes there minus r so omega by K square minus 1 into K square kR whole square by r minus r.

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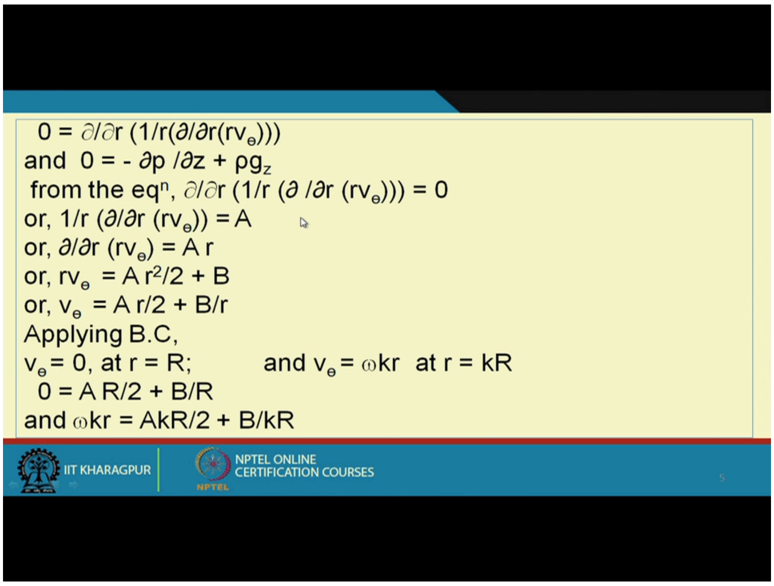

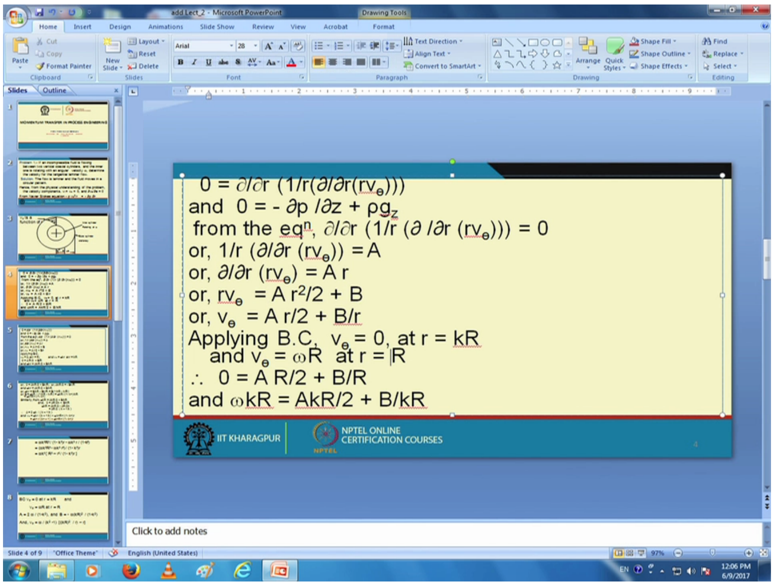

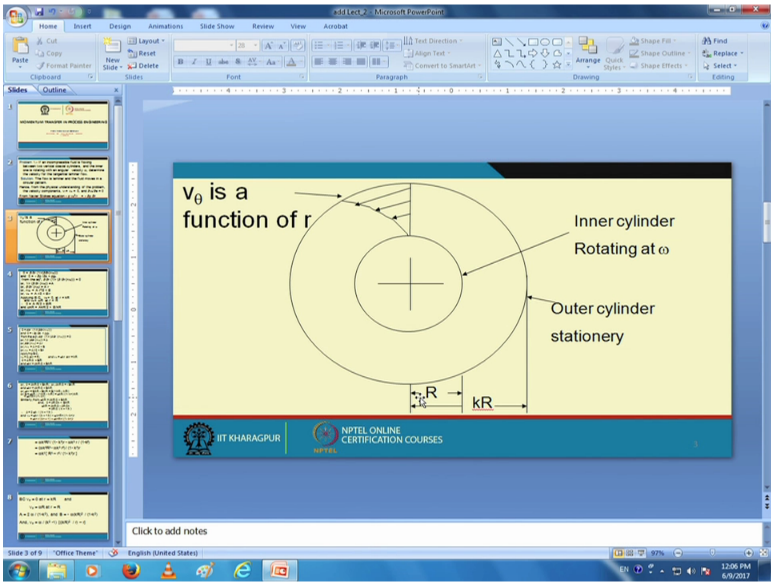

This from and this form are same so we can say we have done the right thing. Only the thing which is required which you need to do here in between if you look at the solutions, the same solution original which we have started with, we have done up to this the same right, then we have changed the boundary, right so boundary was originally your V theta is 0 at r is equal to R and V theta is equal to kR omega kR at r is equal to kR. Here it is getting changed that V theta is zero at r is equal to kR and V theta is equal to omega r at r is equal to capital R so this change we have to do and then as we have shown the solutions here, so in that way if you go and solve I hope you will get the same.

Please do the required changes please do the required changes in the in the in the in the slides okay. Let us also since we can do it let us look into this and do that boundary condition was V theta was equal to 0 at r is equal to kR at r is equal to kR and we can write V theta is equal to omega R at r is equal to R right, in our new system here as we have seen this is omega R and kR, right so at r is equal to R this V theta is equal to this is fixed.

So at r is equal to R this is V theta is omega omega R and at r is equal to kR V theta is 0 at r is equal to kR V theta is equal to 0 and this now if we substitute them, then we can write that in the 1st one 0 is equal to here right 0 is equal to A into r by 2 plus B by r. This V theta goes to 0 at r is equal to kR then this should be 0 is equal to A kR by 2 and this is b by kR right and in the second case omega R is equal to AR by 2 and B by R, right so from there from there the same this of course repeat so hopefully we can delete this.

Okay so from there we on simplification we do not write on these because we have shown you there we can write simply that what is A, so A has come 2 omega by 1 minus k square 2 omega by 1 minus k square. This is 1 minus minus k square, right. So we put here a square 1 minus k square right and B is equal to but it is also required and B is equal to we have we have a obtained here as you see that V was this minus omega k square minus omega into k into k into R whole square that is much easier and better.

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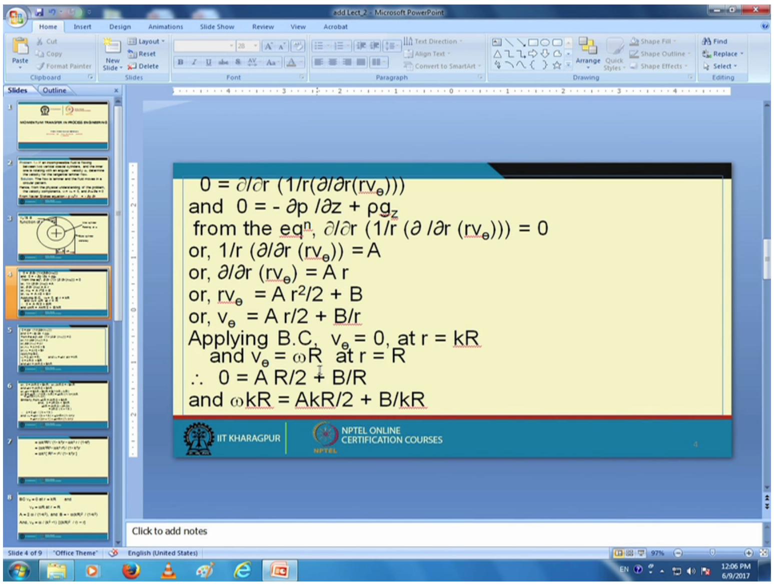

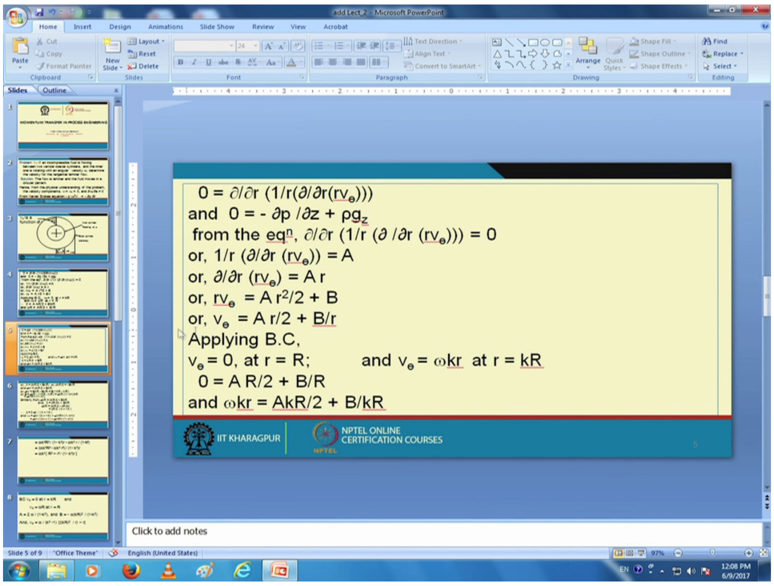

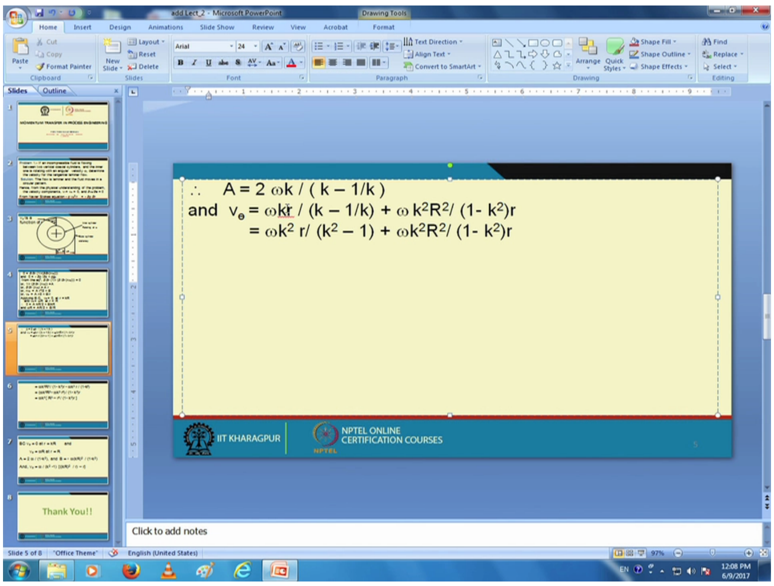

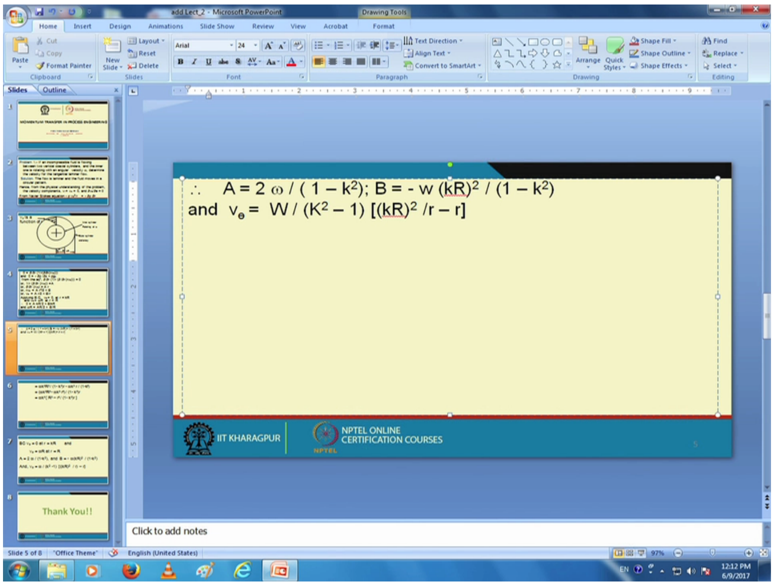

K into omega square right and this divided by divided by it was 1 minus k square, right 1 minus k square right, so that k square is this 1 minus k square yeah. So if that be true and then the final equation which we got for V theta was this V theta was omega by k square minus 1 omega by K square minus 1 K square minus 1 right times if it take this times kR square by r, kR square square by r minus r, right.

So this was our solution, so we got it so we have corrected also, so as we as we as we requested you in the previous class in the previous problem that we have done on the paper but corresponding slides were not properly corrected and we requested you that if you go it to that slide get that thing connected and because we have done on the paper. Here we have done both on the paper as well as on the slides, so this is the final V theta expression that V theta is equal omega by K square minus 1 into kR square by 2 by R square by r minus r. So hopefully we have seen it and that is the final. Thank you.