**Course on Momentum Transfer in Process Engineering**
**By Professor Tridib Kumar Goswami**
**Department of Agricultural & Food Engineering**
**Indian Institute of Technology, Kharagpur**
**Lecture 12**
**Module 3**
**Application of Navier Stoke's equation for finding out viscosity (Part 3)**

You remember that we have been doing this combination of boundary conditions and also the implication of the different conditions of the cylinders we have two cylinders one is fixed and other is rotating, right? So under that situation what will happen that we are observing, right?

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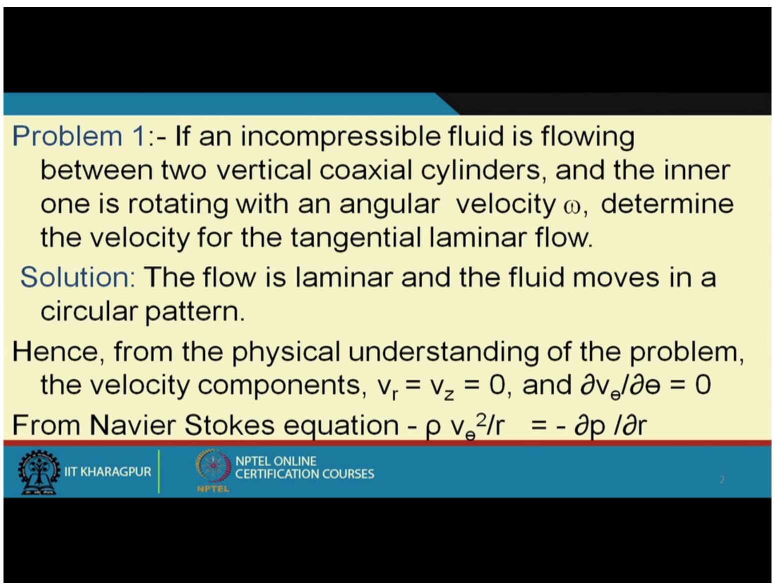


And you can also try with this that let us see the problem was like this that in an incompressible fluid is flowing between two vertical coaxial cylinders and the inner one is rotating with an angular velocity omega, determine the velocity for the tangential laminar flow.

So repeat, if an incompressible fluid is flowing between two vertical coaxial cylinders and the inner one is rotating with an angular velocity omega, determine the velocity for the tangential laminar flow, right?

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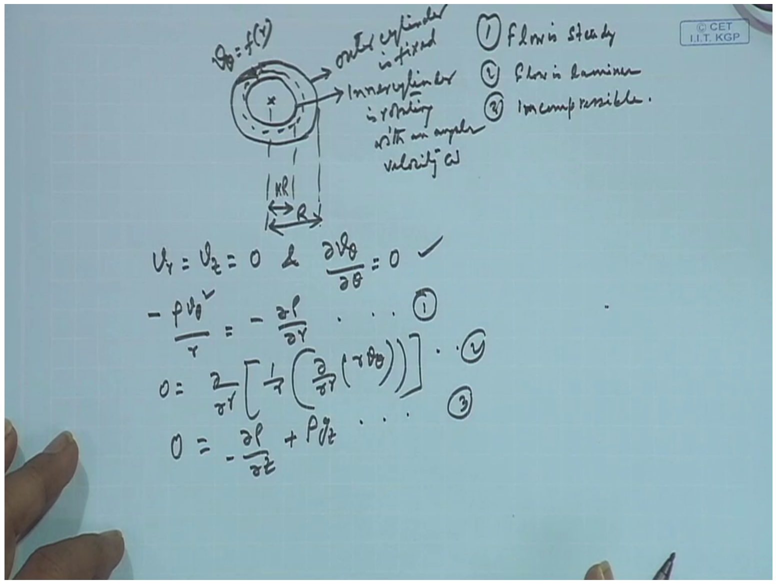


So as usual we have seen that the flow is steady, then the flow is laminar and the fluid is incompressible these are the three given conditions of course the others like that flow is fully developed there is no end effect all these are also there, right? Then, we said that we have two coaxial cylinders like this, right? And the inner cylinder is rotating with an angular velocity omega, right? Of course since doing it on the computer online omega writing is little difficult in many cases earlier also we have replaced this omega with w, right?

So that you change or you keep in your mind and the outer one is fixed or not rotating, right?

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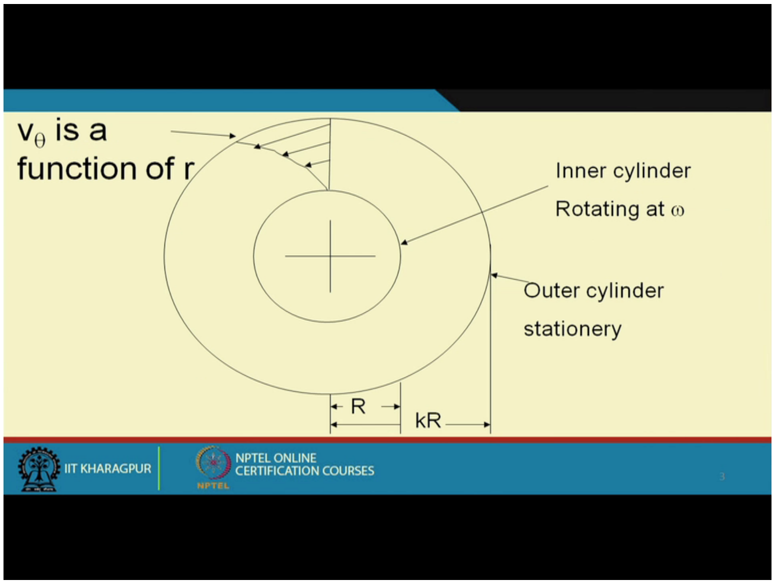


So outer cylinder is fixed or we can say that this is fixed, okay so like this that outer cylinder is stationary and inner cylinder is rotating and there is a velocity profile within as it is v theta is a function of r, right? And in earlier case if you remember we have done this problem with inner one is r and outer one is kR. Now if we do the reverse that the inner one is kR and the outer one is r, so what will be the change in the expression, right? So that is our fundamental thing which we are repeatedly showing you so there you have all sought of problem solved.

So it can have four combinations, one inner cylinder fixed, outer cylinder rotating in two conditions inner cylinder fixed outer cylinder rotating there inner cylinder is r and outer cylinder is kR or inner cylinder is kR and outer cylinder is r or inner cylinder is stationary and outer cylinder is rotating again there with inner cylinder r outer cylinder kR or inner cylinder kR and outer cylinder r. in all the four cases you can see the expression for the velocity is all to be the different, right? That we have to do.

So of course this thing I am doing it online because earlier I have also not checked it what will be the final expression, hopefully we will not do any anything wrong and if there be then you please bring to my notice when you will go through and we will also have it counter checked, but hopefully we will proceed in the right way as we are proceeding and this we will be doing online. So when we are doing it, then from the Navier-Stokes equation from the given problem again we can write that vr is equals to vz is equals to 0 this is 1 and del v theta del theta is equals to 0 that is there is no velocity component in the theta direction there is no change in velocity v theta in the theta direction that is why del v theta del theta 0, why we kept it like that? Because we said that the velocity is laminar, right?

So flow is laminar, so if the flow is laminar so there will be no rippling, there will be no mixing all these will not be there so there will be no velocity component in the theta direction in the z direction also, right? So b theta is a function of r only like this, right? b theta is a function of r. So if that be true if this is be true, then as usual as we have done in earlier cases, so from the first equation of the equation of (momen) equation of that Navier-Stokes equation first we can write minus rho v theta square by r this is equals to minus del p del r, right?

This is equation number 1, then similarly we can write on the other that is 0 is equals to del del r of 1 by r del del r of r v theta, right? So this we have done earlier also, second and the third one is 0 is equals to del p del z with negative plus rho gz, right? And as usual we have done in the earlier cases also that we need one unknown that is v theta, so one equation is good enough and obviously we also said that the equation out of the three which one you will pick up.

So depending on which variable you were solving, in this case it is the tangential velocity, so the velocity component will be taking had it been that pressure then we would had taken been pressure had been some other we would had taken that some other variable. So in this case it is said that we have to find out the tangential laminar velocity what it is, right? So one unknown so one equation is good enough to solve this out of these three, so which what we pick up? The one which will obviously lead to the solution that in this case as you see out of all these three the second one has the velocity component.

So v theta in the first one is also there but there pressure terms are also there, so the equation is little complicated and obviously with that we until some other conditions are same we are not able to find out because del p del r until we know we cannot say what is the v theta and del p del r is not spelt out here, right? So in cases when del p del r will be spelt out that this is that then you can also try with that or if you have to find out given the velocity profile what is the del p del r you can also find out from there, so depending on the problem which you are solving or you are which you are handling, right?

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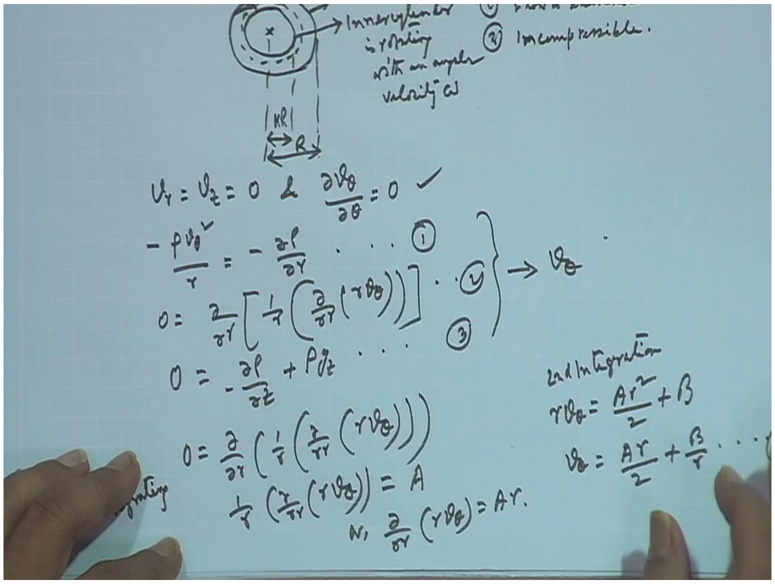

So here we then again take this second sub equation and that second equation is 0 is equals to del del r of 1 by r del del r of r v theta and this if we solve we get on first integration we get 1 by r del del r of r v theta, right? And this is equals to integral constant A which on rearrangement we can write del del r of r v theta is equals to A r and on second integration we can write this as r v theta is equals to A r square by 2 plus new integration constant B, so which on simplification we can write v theta is equals to A r by 2 plus B by r, right?

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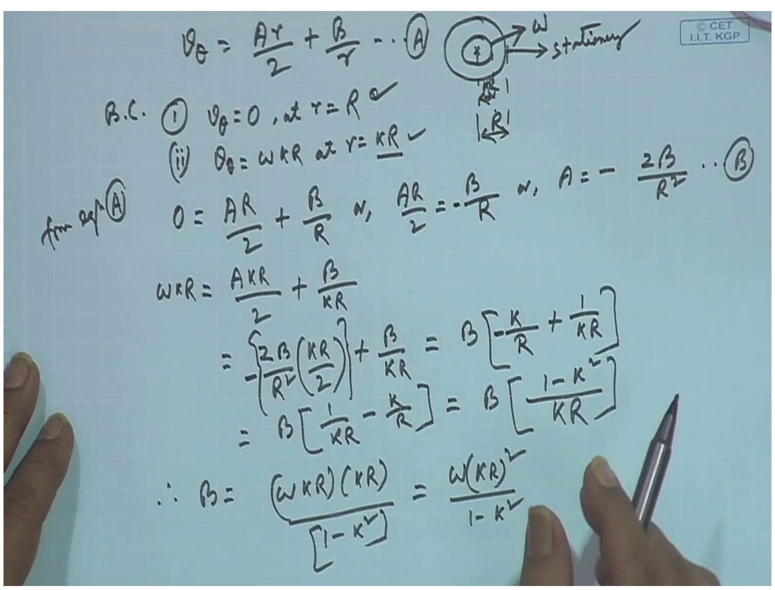

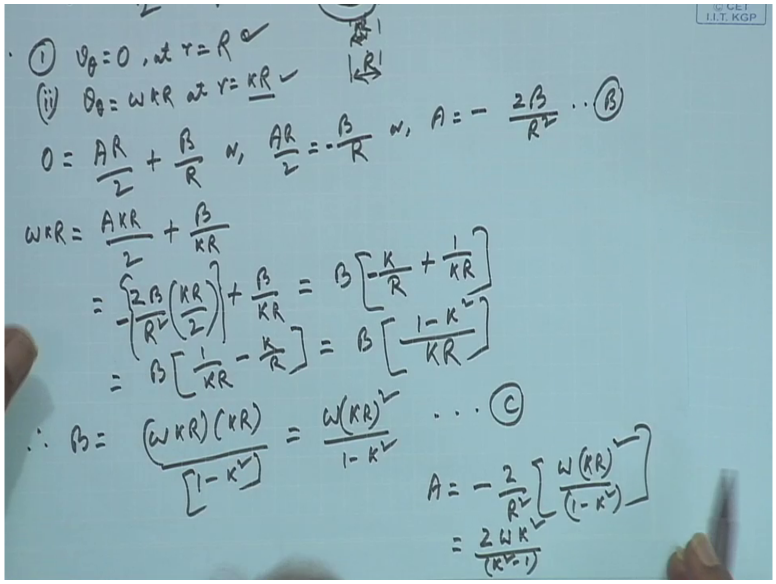

So this we take as equation a, right? So we are then taking as equation a as v theta is equals to A r by 2 plus B by r, right? Now the boundary condition, so this is the outer this is the inner this is the center so we said that the inner radius is kR and the outer radius is r, right? If inner radius is kR and outer radius is r and we said that this inner one is rotating with an angular velocity of omega and outer one is stationary, right? Then, sorry, then the boundary condition we can write that boundary condition 1 is the outer one is fixed, right?

So that means v theta is equals to 0, right? That you remember we said that the fluid which is clinging to the surface if the surface is moving then the fluid is also moving but if the surface is at rest or in stationary condition, then the velocity at that surface is 0, right? So this is what we said during clinging, right? It is called clinging clinging on the surface surface is stationary so the velocity component there also is stationary.

So that is why v theta is 0 and if v theta is 0 when at r is equals to capital R, right? This is boundary 1 and boundary 2 we can write v theta is equals to omega k R, right? Omega kR at r is equals to kR, right? So if this or the two boundaries then, we can substitute them in this equation which we wrote as to be equal to say a then from equation a we can write that v theta is equals to 0, so 0 is equals to at r is equals to r, so AR by 2 plus B by R, right? From there we can simplify AR by 2 is equals to B by R, right? With a negative or we can write A is equals to minus 2 B divided by R square, right? A is equals to minus 2 B by r square.

So this is A we write it to be equation b, right? Then, from the second boundary this was probably first boundary, so from the second boundary we write v theta is omega kR at r is equals to kR. So v theta is omega kR at r is equals to kR so A kR by 2 plus B by kR now A we have already found out is to be minus 2 B by r square. So we can write this to be A equals to minus 2 B by R square so 2 B by r square into kR by 2 plus B by kR is equals to omega kR.

So this we can simplify by taking B as common so B if we take common, then we get this r square this r that goes out, so this 2 and this 2 goes out. So we get k by R, right? Now, is there any negative problem? Yes, A is equals to minus of that so it was a minus, right? So minus there was a A this A is minus 2 B by R square so when you substitute A so it is minus 2 B by R square into kR so this we write with the second bracket then it becomes confusion less or no confusion rather.

So B if we take common then R square R goes out 2, 2 goes out then it remains B out k by R k by R minus plus B has been taken out 1 by kR, right? So this on simplification we can write B into 1 by kR minus k by R, right? So this on again further simplification we can write this kR so 1 minus this is k so k square, right? Therefore, B becomes equals to omega kR into kR by 1 minus k square, right? So that we can write this is equals to omega kR whole square over 1 minus k square, right?

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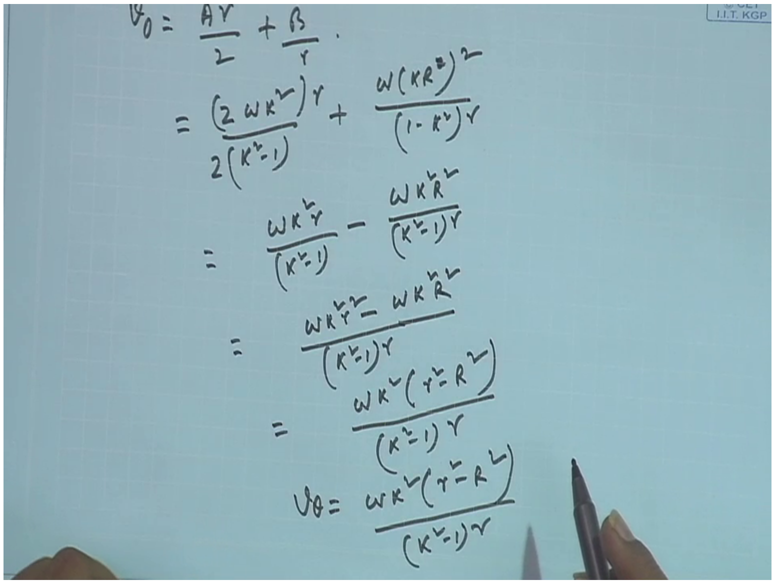

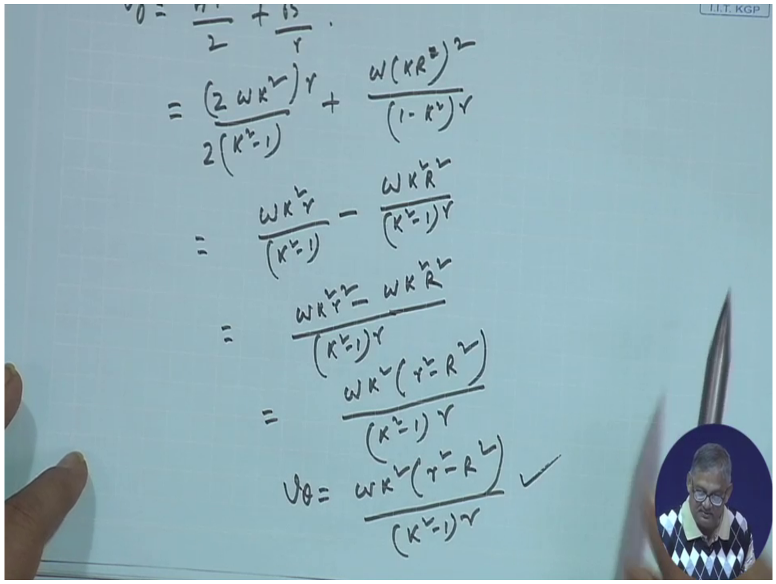


So this on substitution we can write that v theta which was here v theta is equals to A r by 2 plus B by r, now substituting the values of A and B. Now of course B has becomes this, then so this is c, then we should also find out what is the value of A, A becomes equals to minus 2 by R square into this omega kR whole square divided by 1 minus k square, right? So this again we can write that this R square this R square goes out, right? These 2 remains so 2 omega k is square k square divided by instead of 1 minus k square we can write k square minus 1.

So A is 2 omega k square by k square minus 1, so here if we substitute we can write this to be 2 omega k square divided by k square minus 1 here it was 2 into r plus B we found out to be omega kR square kR whole square divided by 1 minus k square, right? And this divided by r, right? So this is what we got, right? Now on further simplification we can write this 2 this 2 goes out as omega k square r divided by k square minus 1, right? Minus this we can write omega k square R square, right? Divided by k square minus 1 into r, right?

Which on further simplification if we want to simplify we can write in the denominator k square minus 1 into r, right? So this becomes omega k square r square minus this becomes omega k square R square, right? So this on simplification can be written omega k square common, then r square minus r square divided by k square minus 1 into r. So v theta is equals to omega k square r square minus R square divided by k square minus 1 into r, right?

So this we can do, right? So v theta, we have done. Now let us look into that what changes we need on the this, now here if we look at that boundary condition is a little different because here we have done the boundary conditions that v theta is equals to 0 at r is equals to R, right?

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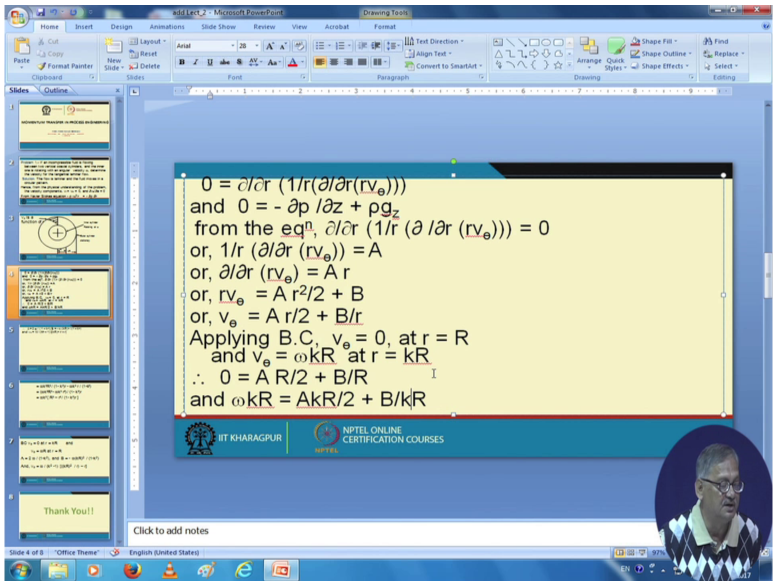


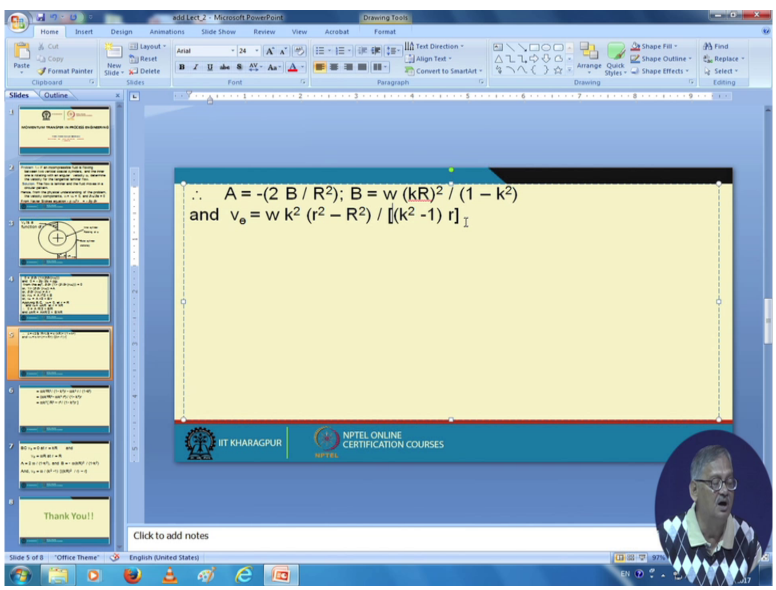


So instead of that we write here v theta is equals to 0 at r is equals to R and v theta is omega kR. So here it is omega kR, right? And at r is equals to there it is kR, right? Then we can write that in the first case 0 is equals to AR by 2 so AR by 2 is equals to minus B by R that is AR by 2, right? Minus AR by 2 sorry this is (le) (le) let it remain plus so undo B by R, right?

This is for first case, second case we have written omega kR omega kR is equals to omega kR is equals to AkR by 2 plus B by kR, right? So we have done this one correct. Then it becomes how much? I think this we should remove and say okay since A is there A is there so we can say that A is equals to minus 2B by r square. So let us re-write minus 2 capital B by R square so instead of this let us write it to be R square, right? So minus 2B so let us make one bracket here and hopefully, no this is not taken so let us make it normal so this so (mi) A is equals to minus 2B by R square, right?

And also we got B is equals to omega kR whole square by omega kR whole square by 1 minus k square 1 minus k square but there is no negative in this, right? So if that be true, then ultimately we got v theta is equals to let us re-write this this became equal to v theta then became equal to hopefully yeah, here it is omega k square so omega we are writing as we said that small w if for easy doing it omega k square so that we have done omega k square into r square minus R into r square not r square minus capital R square so we write it like this capital R square divided by so this bracket closed divided by k square minus r square.

So to write it we write it this like this k k square, right? k square minus 1 1 into r. So this we write in third bracket here also in third bracket, so v theta is omega k square into r square minus capital R square by k square minus 1 into r. So we have done all the four cases that all r's are changing or inner and outer radei are changing and also inner and outer radie inner and outer cylinder conditions are also changing either outer cylinder is fixed, inner is moving or rotating or inner is fixed and outer is rotating or moving, right?

So this way you try to solve as many problems you come across and then find out the solution, okay so this way we then complete rest of the things of course we are not required so we close it here today, thank you.