Engineering/Architectural Graphics - Part 1
Orthographic Projection
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Lecture - 35
Orthographic Projections
Sections of Cones

Good morning, welcome to the last lecture of this week for the ongoing online course on engineering or architectural graphics. And in this week, we have been discussing about the orthographic projections for sections of solids. So, so far in the last 4 lectures of this week, we have already covered how to draw orthographic projections for sections of prisms, sections of pyramids, sections of cylinders. And today in the same series, we are going to cover the sections of cones, circular right angled cones.

So, the fundamentals the basics remain the same as that of pyramids, the only thing being that we do not have edges here, because the base is a circle and there are infinite number of generators. So, what we have to do? Similar to what we did in cylinders or what how we learned to draw the circular planes which are inclined? We will be dividing the circle of the base into multiple equal numbers of parts and then we will be drawing generators and taking the projections here.

Another thing, the fundamentals of pyramids section of pyramids apply here also. So, if the base, if the section plane is parallel to the base, we will be seeing the same shape as that of the base just that the scale will be different depending upon where in height the cone is cut. If it is perpendicular to the base and parallel to the axis, in that case, since there are infinite edges here in case of a cone, we will actually be seeing a curve.

So, if you remember the conic sections, which we did earlier in this course, we would either be seeing depending upon how the plane is placed, we would either be seeing the parabola or a hyperbola. And if the cone is cut by a section plane, which is inclined to the base in such a manner that the entire cone is cut and the base is not being cut, in that case, we will be seeing an ellipse.

So, let us see how to draw these 4 or 5 different conditions and then we will arrive at the solutions following the same process. The process essentially remains the same.

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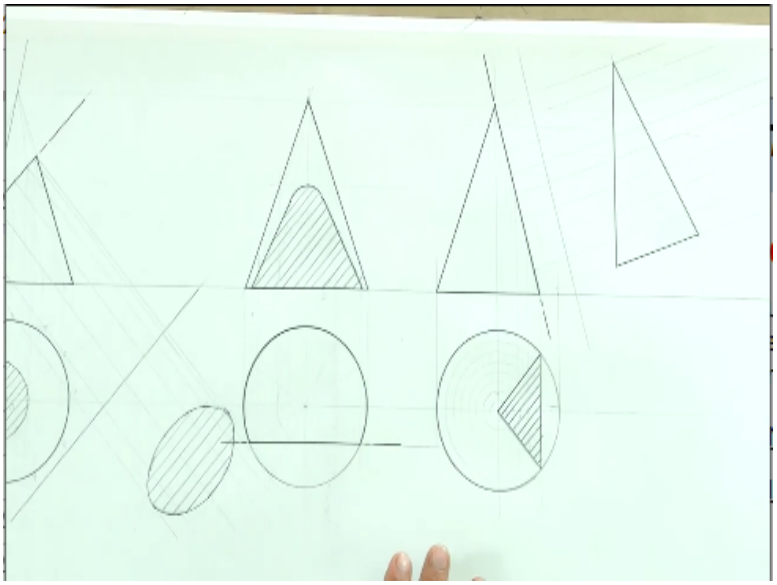

So, for this particular class and assuming that we have a cone with a base of 4 centimetre circle. So, we will draw a simple cone and it is an upright cone. It is a right angled cone. So, the radius of this base is assumed to be 4 centimetres. This is the base and these are the points in the base. Assuming the height of the cone to be 10 centimetres. This is the cone that we get. So, this is the original position of the cone which we are assuming and we will take section planes as different conditions in this cone.

Now, in the first condition, let us assume that this cone is being cut by a section plane, which is parallel to the base. So, I drawn a section plane here, which is parallel to the base. Now, in this case, it is very simple. Simply if I connect this, so, what we have? I have a generator which is connecting these points. And since it is parallel, we will be getting to see a circle here at this distance from the apex.

So, this is what we will get in case the section plane is parallel to the base. This is what you see from top. The base remains intact because, it is not getting cut. It is only at the top portion that this is cut. In the front elevation, we just have the remaining portion of the cone which will be visible. So, this is one of the simplest possibilities in which a cone will be cut.

So, if you remember our lecture on conic sections, we had clearly stated that we get a circle. Circle is one of the conic sections that we always get that is when, the section plane is parallel to the base. And which is what we can see here that this is what we have got. So, this is how the section of the cone is going to look like, in case, the section plane is going to be parallel. Now, let us assume another condition and draw another cone.

So, here we are drawing again the same cone. We are assuming the same condition here. So, this is the cone in the same position, it is kept on ground. It is an upright cone perpendicular. We just retain the dimensions of the cone as the same. So, this is the original cone. Now, let us assume that this particular cone is cut by a plane, which is inclined to HP and perpendicular to VP. And let us assume that it is passing somewhere through this through the axis.

So, I am assuming that this plane here is making an angle of 45 degrees with the HP and it is perpendicular to VP. Now in this case, we will have to divide our circles into 12 equal parts. So, that is what we will do here. So, these are the 12 parts in which we have divided our circle. And now, we will have to take where the generators are cut onto back onto this top view.

So, I will just project these generators where they are touching the base and each of these generators will now be connected joined to the apex because, all these generators are connected to the apex. So, now, we have joined all these generators, generator points to the apex. And wherever this plane cuts, each of these generators is the point which we get here. So, what we have? This generator which is joining this, I will just number these, so, it is easier for us to locate the points.

So, we have again divided the entire circle into 12 parts and wherever this is the apex O. So, wherever this apex is this generator joining apex and one of the points is cutting, is the point which we will get. So, this O1 is getting cut here, so, we will get back the projector. So, this is the point which we will get. Now, the next is O2 or O12 which is this generator. So, this generator is getting cut here. So, what we will do? We will get it back onto these 2 generators.

Now, this is 3 and 11. So, it is getting cut here, you get it back like this. So, these are the 2 points. Now, our only problem is what do we do here, where do we cut? So, we will come to it later, let us finish the rest of the ones 5 and 9. So, we get it here 5 and 9. This is the point 4, 6 and 8 and this is the point 4, 7. So, this is what we will get. Now, for this one again, if you remember the same concept, if we were to have a section plane parallel. So, what we will get? A circle, same distance at these 2 generators.

So, we will take the same distance at this generator here, which is this point. And now, the same distance as this point has from the apex will be the distance that we will have on these 2 generators. So, these are the remaining 2 points. So, if we see it from the top, this is going to be; this will be appearing as a distorted ellipse. So, that is the shape that we are getting here and the apex is chopped off. We still have the rest of the cone intact, but these generators are all imaginary. So, we will not have them here.

So, we will not be seeing them in the final drawing. The final drawing will actually be having only this. So that is what we will darken. So, what I have done here is, I have just tried to join these points together to give it the shape which this ellipse has. So, this is the shape of the ellipse that we are getting. The base of the cone remains intact and the central portion which you have here is actually the section.

So, we will just hatch this portion which represents the section of the cone and it is the distorted ellipse. It is not the true shape of this section. Because we are not seeing parallel to the section and in the elevation, what do we see? We only see the remaining portion of the cone which is like this. And then we see the section plane appearing as a line and the base of the cone. So, what we get here is this ellipse. Now, what we have to know is the true shape of the section here.

So, we can draw the an auxiliary plane which is parallel to the section plane, which is what I have taken this side, because there is no space. However, you could take it there wherever it is convenient, you could be taking. The only thing is it has to be parallel. And all we are

going to do is we are going to bring back the projectors for the generators. So, all I am doing is, I am bringing back the projectors of the generators here.

And now, what we have to do? So, what we have is this is for 1, this is for 2 and 12, this is for 3 and 11. This is for 4 and 10. This one is for 5 and 9. This is for 6 and 8 and this is for 7. And now, what we have to do is we have to mark, the same distance as these points have from the reference line. So, that is what we are going to do now. Same process, I just keep repeating the process and demonstrating it for different conditions. If you have fairly grasped how to do it, probably you will not even need to look at this.

So, I am just marking these points which we are getting and each of these generators. This is where the central generator is going to come. So, all these 12 points on the generators, they will be marked here. So, orthographic projection might appear to be a tedious thing to do. But it is quite scientific. And once you have got a grasp of it, it becomes fairly simple to arrive at the solution. So these are the points that we have. And if you can make out a shape, you can clearly see that we have arrived at a perfect ellipse here.

So, let us again try to fit the French curves here. So, our attempt is to at least fix a curve at any given point of time on 3 of the points. Only then will you be able to get a smoother curve. So, this is what I have just tried to fit. However, we could have improved it. We could have done it better. Anyways try to do it even better or smoother curve as smooth as you can get. So, this is the ellipse which is the true shape of this here.

So, we will just touch it and it becomes, it represents the true shape of this section for us. So, that is what we get to when we have a section plane, which is inclined to the base of the cone and it is not cutting the base or the apex. So, if it is cut in between, we will always see an ellipse which is what we had explained while we were discussing the conic sections as well as when we were introducing the sections of cones, sections of different solids.

So, that is another condition. Now, what happens if this cone is cut by a section plane which is perpendicular to the base and parallel to the apex? So, let us quickly draw the projections for such a case as well. So, we have the same cone for the reference. This is our cone and this

is where the apex is going to lie. Now, let us assume that this cone is cut by section plane which is parallel to VP and perpendicular to HP like this.

Now, how do we take the projections? In the top view, of course, it is going to look like this as it appears, but where are the points going to come if we are going to see the front view? So, for this again, we will divide into, let us see whether we can get the solution by dividing it into 12 equal parts or not. Because, all the times, we do not get the solution to the conic sections by drawing generators.

Sometimes, we have to use some other methods, but let us see if we can get the solutions here. Now, what happens here? If we look at it from top, so, we will first draw the generators in the elevation as well and we will connect the generators. So, these are the generators. Now, let us see where each of these generator is going to be intersected. So, what we see in the base? This is the point where the section plane is going to cut. So, this is where the section plane in the base is going to be. This generator is getting intersected in this point.

So, if I make it like this, these are the 2 points I get. Now, the next generator is this. So, we get 2 points here and again the same for this one, we will take it. This is at the same. This is going to remain at the same distance. So, assuming that there is a plane, which is parallel, so, we just rotate it, take it on this generator, which is the generator here and take a plane which is parallel.

So, what we have? We actually have these points which give us a parabolic shape or a hyperbolic shape depending upon how the section plane is going to cut. So, what we have here is, we have these points like this. So, this is the shape of section that we are going to see and this is also the true shape of the section. If you see because this plane is parallel to VP, so, we will see this as the true shape of the section and the apex in this case remains intact.

So, we will see the outer part of the cone except that there will be a cut section here and in the plan, we will see the remaining part of the cone the base. So, we just have this much visible. This is the section line which you have and this is the section surface which is also the true

section of the surface, true shape of the section surface. So, this is what we will see when the cone is getting cut by a plane which is perpendicular to the base and parallel to VP.

Assume that it was being perpendicular to both the planes. In that case, the same thing would be seen here in side elevation and in the front elevation, we would just be seeing the straight line of a section plane and the plan would again be a similar thing like this. You can also try a couple of conditions where the conditions will remain probably the same. In this case, just imagine that this plane is now parallel to one of its generators.

What happens if this inclined plane becomes parallel to its generators? If you know theoretically, of course, we will be getting a parabolic shape. So, just try drawing this section plane parallel to one of its generators. You can also see if the cone is cut by a plane which is perpendicular to the base and passing through the apex or there could also be a possibility where we have an inclined plane which is passing through the apex like this.

So, what if we have a cone which is being cut by a plane like this? So, try following the same method and you will be able to arrive at the solution following exactly the same method. Sometimes, we may not be able to get the solution using the generator method. So, I will just show you one last method where the plane is going to pass through the apex and it is cutting. It is incline and it is cutting the base.

So, let us look at this condition where the cone is again kept in an upright position in HP. So, we will draw the cone in its original position. Now, we are assuming that we have the section plane, which is parallel to its generators. And we are assuming that we have a section plane which is passing through the apex and it is cutting through the base. So, I am just drawing the section plane like this.

So, what we get in the base is that it is cut in the base like this. Straight line will come here. And if you look at this, this is a generator. If you join it, what you see in the top view is just a triangle like this. So, this is going to be the top view in this case but, what is the true shape of this section for this particular case? For that, what we will have to do? We will have to follow the method of concentric circles.

Now, what we do in concentric circles is, we draw certain number of concentric circles. So, I am just drawing concentric circles here. So, these concentric circles also represents the horizontal planes which are parallel to the base, but at different heights. That is what we have seen in earlier cases also. So, I am just making this very thin concentric circles here. I have kept it equidistant. It is not necessary to keep them equidistant, but it makes our job easier if you make them equidistant. So, we will just draw these concentric circles.

Now, as we remember the concept that each of these concentric circles represents a circle parallel to the base. So, what we are doing? We are going to take these projections up. And it is on this generator that each the circle is going to pass and that is where a parallel plane, parallel to the base plane will be passing. So, I will now, draw these horizontal planes represented by these horizontal lines. So, these are the planes horizontal planes which are now intersecting in this.

So, if I want to now draw the true shape of this section which is going to be parallel to this. So, because in top view, we will only be getting to see this triangle. In the front, we will only be getting to see the straight line. So, let us try to draw the true shape of the section here. So, I am fixing it parallel to this section plane again, which is what it is and I will now take the projections.

So, what we have is, we can take each of this point being projected on the representing circle. So, what we have? If we look at this generator, maybe this generator, so, this concentric circle which is the first circle here is where you will get this point. So, this is the point which we are, these are the corresponding points which we are seeing on this horizontal circles. So, now, if I take the lines, projection lines perpendicular to each of this, this is where ideally it should be.

So, this is where our points should ideally come and this is the apex right. So, what we now have is the distance that each of these points has from the reference line is what we will see here. So, this apex is at this distance. The next point, we have 2 points and we are going to mark this distance. Similarly, we will keep marking all the points. We just have to be very

careful cautious about where the projection, the distances are taken and just follow that the same projectors are being taken to mark the points.

If you take the distance somewhere and you mark the point somewhere, it will go wrong. We have to be careful about marking of these points. The process is simple; the process is same. It is just the meticulous marking which will make the difference in your drawings. So, we will just mark all these points wherever they come. And finally, what you have is the final shape. Now, if I try, these are all the points that we would have got.

So, if I try to connect this as a straight line, I am drawing a very thin line. We will see that there is a slight difference in all these points which is coming, but it is more or less the straight, same straight line which is what we are going to get here. So, if it is passing through the apex, this is the kind of shape, we are going to get. This is true shape of the section which we were not able to see either in the front elevation or the plan.

So, this is the true shape which we will get. In the plan, we will only be seeing this triangle and rest of the base. So, I will just darken the rest of the conic portion and this portion is going to be hatched. This is the section which we are seeing and in the front view, we will just be seeing a straight line to the section. So, this is what the section line would be. In this manner, you could also derive.

So, these are the 2 sections, which I have told. One is that of a concentric circle, which is this method and the other one of that of generators. So, sometimes, you will be getting the solution using these concentric circle method. And the other times, you will be getting the solutions using the generator method. Both methods will yield equal results. Sometimes, both the methods can be used. Sometimes, one of the methods could be used.

So, we will shade this one also and this is how we will derive the sections of right angled upright cones. So, I hope with this, you have fairly grasped the method of drawing sections of cones and in various different conditions. So, thank you very much for being with me in this week and for the last lecture of this week. We still are left with sections of spheres which we will cover in the next week, which is the last week of our course.

So, thank you very much once again for being with me here today. See you again next week with sections of spheres and few more lectures. Thank you. Bye, bye.