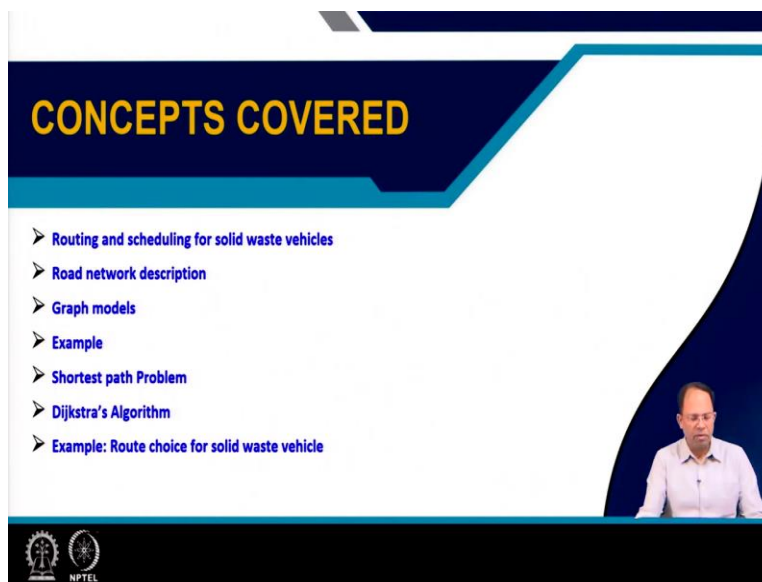


Urban Services Planning
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Lecture 24
Routing and scheduling for solid waste vehicles Part I

Welcome back. In lecture 24, we will be talking about routing as scheduling for solid waste vehicles and this is part 1, of the lecture.

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The slide features a dark blue header with the title 'CONCEPTS COVERED' in yellow. Below the header, a list of topics is presented in blue text, each preceded by a right-pointing arrowhead. The topics are: 'Routing and scheduling for solid waste vehicles', 'Road network description', 'Graph models', 'Example', 'Shortest path Problem', 'Dijkstra's Algorithm', and 'Example: Route choice for solid waste vehicle'. In the bottom right corner of the slide, there is a small video inset showing a man in a light-colored shirt. At the bottom left, there are two circular logos, one of which is the NPTEL logo.

- Routing and scheduling for solid waste vehicles
- Road network description
- Graph models
- Example
- Shortest path Problem
- Dijkstra's Algorithm
- Example: Route choice for solid waste vehicle

So, the different concepts that we will cover on Routing and scheduling for solid waste vehicles, Road network description, Graph models, we will then talk give an example, then Shortest path problem, Dijkstra's algorithm and finally, an example of Route choice for solid waste vehicle.

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Routing and scheduling for solid waste vehicles

Primary collection: From solid waste from generator to a storage depot/transfer station/disposal site/waste processing center.

Secondary collection: From community bins/storage depots to a transfer station/waste processing center/disposal site.

Tertiary collection system: From transfer station to disposal sites/waste processing and treatment facilities.

Collection and transfer of waste is the most costliest part of MSWM and thus requires detail planning and optimization.

- Each vehicle has a fixed capacity.
- A vehicle has to travel between two points along a road network such that travel time and/or cost is minimized (e.g., Between transfer station and landfill site, between collection points)
- A vehicle has to travel to many collection points along a road network such that the overall route travel time and/or cost is minimized (e.g., collection from community bins)
- Not all sites are accessible at all times of the day.
- Not all roads are accessible (e.g., road width restrictions, one-way, road maintenance)

The slide also features a small video inset of a man in a light blue shirt speaking, and logos for NPTEL and IIT Madras at the bottom.

So, as we have learned earlier that there are three types of collection systems that we are that we usually deal with during solid waste collection and transfer, that is primary collection, secondary collection and tertiary collection.

Now, in primary collection we are collecting waste from the different generators directly and taking it either to a storage depot for example, residential household level waste that is generated is usually collected using door to door operation and then transferred to a storage depot and from storage depot usually use we use a larger vehicle which is a secondary collection and we transfer that waste to a transfer station or directly to a disposal site or a processing center.

But sometimes it may happen that even in primary collection like for example yard waste or maybe even construction and demolition waste, it may be directly collected by some vehicle and taken to the final waste processing center of the disposal site as well. So, there are different varieties of it, but primary collection usually means that the collection system that starts with the generator and it may end either at the storage depot or at the transfer station or disposal site or even at the waste processing center.

The secondary collection is more about collecting from stores either community bins, if there is along with door-to-door collection system there are many cases we have also got community bins like in very congested areas, in slum areas and so on or sometimes even large apartment

complexes do collect their waste internally and then they store it and the quantity of waste is also large. So, we collect this kind of waste directly from those kinds of bulk generators or you can say that from largely community bins as well in that area or for from storage depot.

Storage depots and committee bins is sometimes synonymous sometimes community bins are also used as storage depot. So, storage depot is somewhere where the primary collection from door-to-door waste is being brought at this secondary collection involves taking garbage either from storage depot or from community bin to a transfer station waste processing center or disposal site.

Finally, tertiary is collection system is transfer station, transfer of waste from transfer station to disposal sites or waste processing and treatment facilities. So, these are the three kinds of collection system involved and what it means is there is a vehicle involved and the vehicle travels between two points or it may travel in along a sequence of points. So, this is what we are planning for that means we have to generate a route for the vehicle and also a scheduled for that particular vehicle that at which time this vehicle will reach which point and so on and so on.

So, collection and transfer of waste is it is the most costliest part of municipal solid waste management and of course, it requires detailed planning and optimization. So, that means if I can reduce some amount of cost in this part of the process in this management process, then we will have significant savings in terms of money.

Now to before we even start about talking about how do we do routing and scheduling of solid waste vehicles we have to understand some basic concepts. One is each vehicle has got a fixed capacity, that means there are two ways we can consider that, that can see per vehicle starts from a particular community bin and then keeps on collecting waste from multiple community bins, it can only take till its capacity is, it is filled.

So, it is filled, then it cannot take further waste and it has to take it to that disposal site. And once it takes it to the disposal site then again it has to come back to the point from where it would start collection again. So, in case the time taken by the vehicle, is over that is the overall working hours of the vehicle is over in that case the vehicle another vehicle has to be utilized because this vehicle alone is not able to take care of all the community bins.

So, we will give up detailed example later on. So away, the second point is a vehicle has to travel between two points along a road network such that travel time and or cost is minimized. So, that means, whenever we are traveling along our road network, we in most cases, we are traveling between two points, what are these two points.

It could be from the depot of the vehicle to the first community bin, or it could be a depot of the vehicle, or from one community bin to another community bin, or from the point where its capacity actually exceeds that means when the vehicle is filled, from that point, we can actually take the vehicle to the landfill site. So, any any point of time the vehicle is traveling between two points.

And it we end in case of a large city because it involves a lot of link, a lot of roads, a lot of intersections and all, the vehicle has to travel along those links and intersections. So, we have to make sure that the vehicle has to when it travels between two points in a road network, we have to make sure that the total travel time or the cost, cost is it could be like the fuel cost or the distance as per the distance the vehicle will consume fuel.

So we have to make sure that it does not travels extra. So, we have to find the shortest path or the smallest route from the first point to the second point. So, for example, it could be between transfer station and landfill site or even between the collection points. Now when a vehicle has to start from a depot, go to multiple community bins and then go to a disposal site and then even returns back again to start or continue with the collection process.

In that case, a vehicle has to travel to many collection points along the road network, such that the overall route travel time or cost is minimized. So, that means it is not only traveling between two points. So, we are minimizing the travel time or the distance between two points, but it also in what sequence we should select these travel points so that the overall journey of this particular vehicle is minimized.

So that is the second part. So, that is a vehicle has to travel to many collection points along the road network says that the overall route travel time or cost is minimized. So, for example collection from different community bins in an area and then taking it to a landfill site. So, the

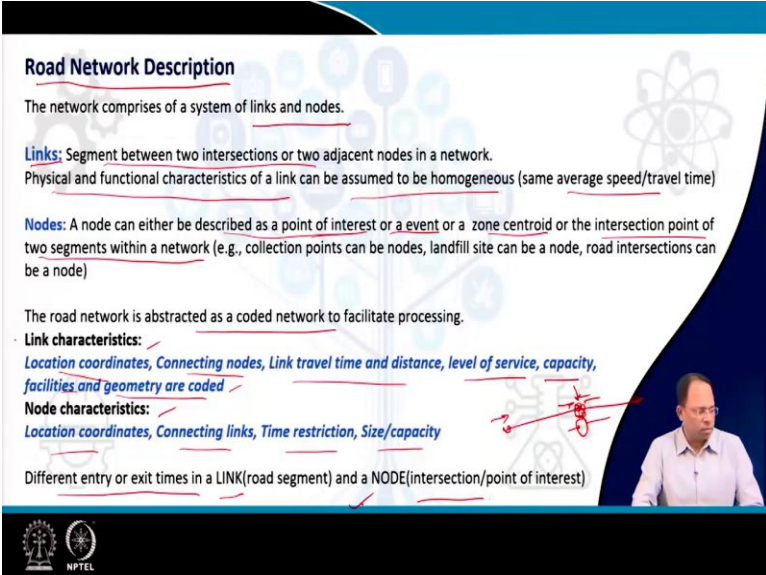
overall route time, it has to be designed in such a way so that the overall route, the time taken to travel this overall route is minimized. Not all sites are accessible at all times of the day.

So, there may be a lot of restrictions with some sites, for example, in certain areas, garbage vehicles or even, larger, heavy vehicles like lorries and all are not allowed at certain times of the day. So, that is the restriction so we have to only reach there after a certain time period is over or maybe in the evening or so on. So that means that also needs to be considered if I am planning a route for the vehicle, I have to make sure that this vehicle will reach that point after a certain time. So, maybe that also needs to be considered when we plan for the route.

So, there could be time restrictions as well. And sometimes what happens is some links are one way so we cannot take the vehicle along this route. So, accordingly we have to also change that. So, not all sites similarly not all roads are also accessible throughout the day. So, that is what is said over here.

That is road sometimes roads are not accessible because it is designed as a one way but sometimes because of road width some vehicles cannot go into a certain road or sometimes because of road maintenance, some roads are close. So, these are the things which we have to keep in mind when we design a routing route and scheduled for solid waste collection vehicles.

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Road Network Description

The network comprises of a system of links and nodes.

Links: Segment between two intersections or two adjacent nodes in a network.
Physical and functional characteristics of a link can be assumed to be homogeneous (same average speed/travel time)

Nodes: A node can either be described as a point of interest or a event or a zone centroid or the intersection point of two segments within a network (e.g., collection points can be nodes, landfill site can be a node, road intersections can be a node)

The road network is abstracted as a coded network to facilitate processing.

Link characteristics:
Location coordinates, Connecting nodes, Link travel time and distance, level of service, capacity, facilities and geometry are coded

Node characteristics:
Location coordinates, Connecting links, Time restriction, Size/capacity

Different entry or exit times in a LINK(road segment) and a NODE(intersection/point of interest)

The slide features a background graphic of a network with nodes and links. A small video inset in the bottom right corner shows a man speaking. The NPTEL logo is visible in the bottom left corner.

Now, where do we start, the first starting point is to first determine the road network or first to describe the road network. So, as we can say that if I have a GIS map of a road network, our first job is to convert it into a system of links and nodes. Why? Because at the end of the day, the computer does not understand what is the road, what is its effect on the width and so on.

So, we have to actually convert it into certain in a format where the computer understands and we can do mathematical calculations with those values. For example, every link has got a start and an end point and also there is a distance or a time attached for this link. So, we are giving some properties to the link. Similarly every node is connected by certain links or it has got a certain location in a space, we can determine that using giving it some properties.

So, these are the things that we have to first decide on. So, what are links? It is the segment between two intersections or two adjacent nodes in that network. So, it is any segment between any two nodes. But what are these nodes in that case? So, a node can either be described as a point of interest, for example, the location of a community bin or a event or a zone centroid, for example, a landfill site maybe taking up a huge area.

So, when I say the road to the landfill site, it would be to the centroid of that landfill site that is what we assume. So, it could be a zone centroid or it could be the intersection point of two segments within a network that means, when two roads intersect that point is also a node, why because from here you can make a decision should I go on the left side or the right side, we will continue in the front straight through that particular intersection or in that through that particular node.

So, a node is a decision point we can say, that is from that point, we can again change direction or we can stop there. So, that means we will, we can either stop there or we can change direction. So, it is a point where we can, we have to take a decision so that is why it is different. So, coming back to links physical and functional characteristics of the link can be assume to be homogeneous, that is if you have a very long road one part of the road is two lanes, whereas, the other part of the load road is six lanes. In that case, I will consider them as two separate links.

So, usually a link is designed in such a way so that we can assume that to be homogeneous in terms of the average speed of the travel time along that particular link. Similarly, in case of

nodes as we have discussed that this could be point of interest or could be intersection. So, we have to design which is a node and which part would be the links passed along and we have to convert a road network into a link node diagram.

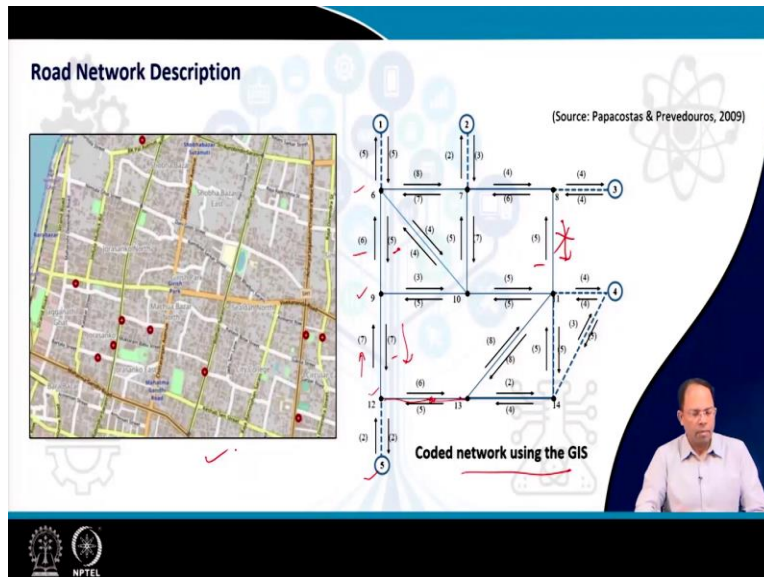
So, the road network is abstracted as a coded network to facilitate processing via computers and all this link characteristics and node characteristics are entered for example, location character coordinates, connecting nodes, link travel time and distance, level of service, capacity, facilities along that particular link, geometry of the link, these are all coded for this particular link.

For node characteristics, location coordinates, connecting links, time restriction, size and capacity restrictions, all these things also are considered. So, these are the different characteristics that we consider and once these are all coded, we have to make sure that once we are processing this information using a computer there is different entry or exit times in a link, that means, if I have got a link, and then the link continues, there is an entry time of the vehicle in that link and then there is exit time on the vehicle to the link.

Similarly, for a node also there is a entry time and exit time. For example, when if I considered this node as a intersection that means it will take some time in the intersection for the vehicle to travel, it maybe we can also consider it a 0. Similarly, if it is a solid waste your community bin, in that case also there would be a time taken for the vehicle to park, load or unload into that community bin and then again move on from that community bin.

So, that also will take some time. So, this actually we have to enter these values or we have to make sure that there is an entry and exit time for each node or each particular link, during the calculation process.

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So, this is how it looks, as you can see that we have a road network on this side, you can see there are different points in the node network and now we have coded it using GIS or we are we have abstracted that entire road network into a link node diagram. And in this case, you can see that these are the different nodes, this is one we can call this as a centroid node, these are road intersections and we can if we want to that there is a community bin, somewhere inside suppose over here in that case, I can break this into even into two links as well.

So, overall, you can see that there are two directions to each link, because this is a directed graph you can say that means the travel time, while you travel towards the north and or you travel towards the south this to travel times are different. So, that is why we have got two values, in this particular link it is same but over here you can see these are different. So, that means sometimes the link may take same time in both directions, but sometimes not.

For example, during peak hour you will see the in one direction it takes more time compared to the other direction. So, these are the things then this link as you can see has got only one direction, what it means is, it is a one-way link and vehicles can travel from this side to this side, but from 8 to 1, 8 to 11 you cannot travel. So, in this direction, travel is not allowed. So, when I consider a route I cannot come along this particular way. So, this is the first what we have to, convert our road network into a link node diagram.

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(NPTEL: Trip Assignment: <https://www.youtube.com/watch?v=Ryqip8aF0pw>)

Graph Models

The coded network, is represented through graph models.

A graph is an ordered pair of sets, represented by: $G = (N, L)$

Where, N (nodes or vertices) and links or arcs, i.e., $L \subseteq N \times N$, a set of pairs of nodes belonging to N

Graphs are oriented, i.e. the links have directions and the nodes are ordered pairs

$G \equiv (N, L)$
 $N \equiv \{1, 2, 3, 4\}$
 $L \equiv \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)\}$

Network flow models

O-D Pairs	1 to 4	2 to 4			
Paths	1	2	3	4	5
Links					
1	1	1	0	0	0
2	0	0	1	0	0
3	1	0	0	1	0
4	0	1	0	0	1
5	1	0	1	1	0

A path, k , is a sequence of consecutive links connecting an initial node (origin) and a final node (destination)

Different paths can be extracted out of the graph.

A binary matrix called the link-path incidence matrix, Δ , can represent the relationship between links and paths.

Each path k is associated with a path cost.

The total path cost for a network depends on the link-path incidence matrix and the costs associated with the links.

Link-path incidence matrix

Now, once we have created a link node diagram, then we usually can do a lot of calculations using that link node diagram. And for a computed to we can do manual calculations we can track how a vehicle will travel along a route, we can keep on adding the distances of the link and we can find the length of a particular route, then we can evaluate another route as well, in this way we can do calculations but as you can understand in a big network these kinds of calculations cannot be done using directly, we cannot do it one by one.

So, we have to use computers for that and we also have to use many mathematical procedures or certain techniques to do this process faster. And to make all these things happen, we have to first convert our link node diagram into a graph model or we have to consider it as a graph model and the coded network what we have just did earlier is represented through graph models, our graph is an ordered pair of sets represented like this, where N is node or vertices no N is the number of nodes and vertices and links or arcs, L is the links or arcs and L is always as a set of pair of nodes belonging to N , as you can see over here.

So, in this particular image, you can see that there are N links, N nodes and N links, 1 2 3 4 are the number of nodes and these are ordered, it is we give them some value and it is in sequence and then link is connecting each of these particular nodes, these are the links 1 2, 1 3, 2 3, 2 4 and 3 4, and over here if you can see like for example, I have start node as 1 and suppose I want to go to node 4, that is this particular point.

So, in that case, what are the ways I can travel, I can travel like this. So, first I will go along link 1 2, and then from 2 to 4 or I can travel like this 1 to 3 and 3 to 4 or I can travel like 1 to 2, then I can travel to 3 and then I can also travel. So, there are three ways to travel from, from 1 to 4. Now, in case this network size increases there could be millions of ways in which a person can travel from between two points.

So, how do I evaluate that so that that is where this operate mathematical operations comes in? So, before we even go into any kind of mathematical operations, we usually, we have to determine a path, like we just determined that path from 1 to 4 and for that we build our network flow models.

So, this is the basic based on which we can apply many algorithms using which we can determine either shortest path or many other things could be determined like the shortest route or you can say the route which connects all the points using the shortest network or using the least amount of travel time different things can be determined.

So, first we have to develop network flow models. So, in network flow models, we have to determine paths. So, a path K is a sequence of consecutive links connecting an initial node which is origin and a final node destination. So, we are finding a path between two points, and different paths can be extracted out of this particular graph as we were doing in this particular graph, we can find different paths.

A binary matrix called the link path incidence matrix can represent the relationship between links and paths, what it means is within this particular path is certain links present or not, if it is present, we say it is 1, if it is not present, it is 0. And then if I multiply this incidence matrix with the actual distance of each of these links or the distance matrix, then we will get the size of each path or the time taken to travel each path and then the computer can decide which path is the shortest one.

For example, in this particular figure, you can see that if I want to travel from 1 to 4, that is starting from here to here, there are three paths that we have discussed this is path number 1, path number 2, path number 3, in path number 1, you see link number 1 is present, in path, in link

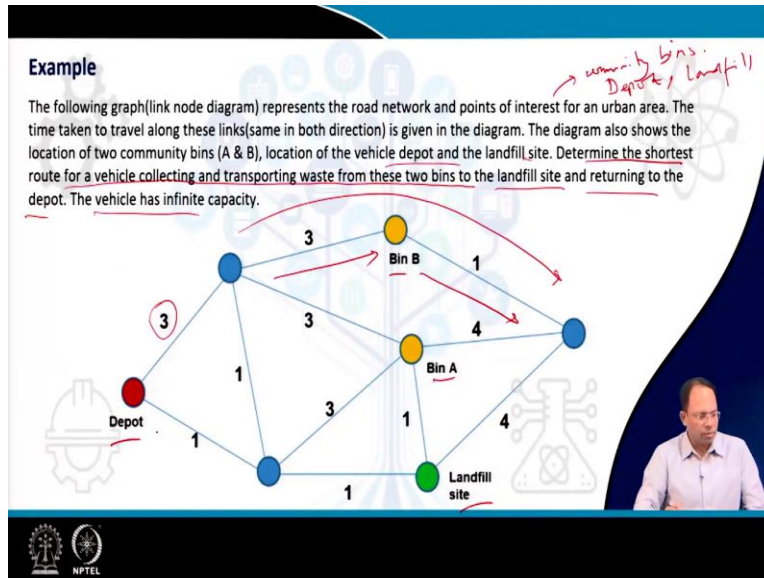
number 2 is not present, link number 3 is not, is present. And then link number 4 is not present again link number 5 is present.

So, this is this path. In path number 2, link number 1 is present and link number 4 is present, the other links are not present. So, in this way, it is easy for the computer to understand how to compute which paths will include which links and accordingly and it can take decisions that when it reaches the end of a particular link, it can take a call if it is connected or not, if it is not then it will find the path which is connected and then make it into one.

So, in that way it will keep on generating the options and the computer does it in a repetitive way. So, that is why in this way if I code it I know that which are the links considered and this is one option, and we also can track until which part the computer has, has taken a decision and then it will take another decision after that or it will branch from the decision. So, in this way we can actually decide and network flow models helps in coding the entire network in such a way so that we can easily determine the overall path cost.

So, each path, k , is associated with a path cost and the total path costs for a network depends on the link path incidence matrix and the cost associated with the links and the one with the minimum path cost the shortest link. So, this is one example, but this is how a network, a link node diagram is converted into a graph model and then we can use network flow models within that to do a lot of computers computations using this kind of, networks.

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Now, coming to a practical example of what we are concerned with, so we are concerned with solid waste collection. So, we will deal with example, based on that. Let us go through this example, this particular graph or linked or diagram if you want to call it, represents the road network and point of interest for an urban area, point of interest over here is nothing but community bins, then that depot, the starting point of the vehicle, this is the where all the vehicles stay and then the landfill site.

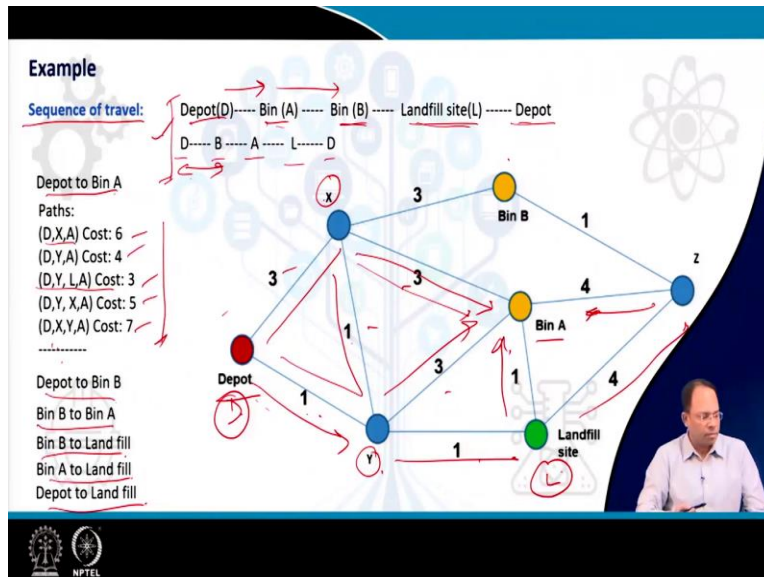
So, this is my depot, this is my landfill site, you can see this is the location and this is the road network or and within suppose the within this particular road, we have got the bin over here. So, I have divided this particular road into two links. One is starting over here and goes to this and the other is from here to here. And at the along with that we have also given this as the cost of using that particular link, it could be time it could be costs it could be anything. So, it is just a relative measure which we have given for all the links.

The diagram shows the location of two community bins, bin number A and bin number B, and location of vehicle depot and the landfill site. So, what we have to do is to determine the shortest route for a vehicle collecting and transporting waste from these two bins to the landfill site and returning to the depot, the vehicle has infinite capacity.

So, we are not putting any restrictions on the vehicle size. So, that means if bin A and bin B, even though they are they contain like 20 tons of waste, we will say the way our vehicle has

capacity of more than 20 tons. So, we are assuming infinite capacity. So, that means we would not be able to say that the vehicle will get filled at that once it collects from bin A or either bin B, so that is being ignored.

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So, what it means, that means from depot. So, so what would be the sequence of travel or in what sequence or what routes would be generated. So, I can start from the depot then I can go to bin A, then I from bin A to I can go to bin B, then I can go to the landfill site and from there I can return back to the depot. So, this is one path that is or one route that is possible. I could have also gone from depot D to bin A then to landfill site then back to bin B then to landfill site and then back to depot could be there.

But as you can understand that would obviously be much larger than depot A to depot, from first bin A to bin B, or the alternative which is depot to bin B then to bin A, and then to the landfill site and then back to the depot. So obviously the other alternatives are much more larger because you have to travel certain distances multiple times, like if I go from bin B, to landfill site and again back to bin A and then again back to bin B obviously it would be larger than these two options. So, these are the two sequences that we are considering.

Now, let us take the first sequence depot D to bin A then to bin B, then landfill site and then to depot. So, to do this, first we have to determine how do I travel between depot A to bin A. So, this is depot and I have to travel to bin A. So, as we learned in the previous lecture that we have to first determine the different paths through which we can travel. So, let us look at the paths which are possible. The first path is DXA. So, this is D, this is L, this is X, so DXA.

So, the path cost is 3 plus 3 6, then DYA, D this is Y, so the path is like this and then over here to A, so this path pass cost is 1 plus 3 which is 4, then DYLA, then from D I can go to Y then from here I can go to L and then to A so I take the same path I go like this and then I will go over here. So, here the path cost is 1 plus 1 plus 1 which is 3, then DYXA, this is the part DYXA, D then Y then X then A.

So, I will come like this, then I will go like this, then I will go like this. So, this path cost is 1 plus 1 plus 3, so that is 5. Then DXYA, D X Y and then A, directly so DXYA is 3 plus 1 plus 3, so that is 7. So, you can see we can keep on constructing paths we can even go from L to Z and then from Z back to A, so that means 1 plus 1 plus 4 plus 4, so 8, 9, 10.

So, there in that way we can keep on building paths or we can go from Z to this bin B, then X, and then again come back to it. So, we can go keep on generating these kinds of paths. So, you have to determine which one is the shortest one. So, based on whatever we have found obviously, if I go in this direction, it will keep on adding values and all, because we are repeating certain paths and all and so in that case, we can say that this is the shortest path DYLA.

Now, we are able to say that because we have only limited options, but if it is a very large network, we cannot do this manually. So, that is where there is some way to determine this using some other techniques or procedures. Similar to depot to bin A, this is the first segment bin A to bin B also need should be find out so what is the bin B to bin A or bin A to bin B, what is the shortest path, then depot to bin B also needs to be found out because this is one option which is here bin B, to landfill, bin A to landfill, depot to landfill.

So, all the shortest path passage is to be found out. Then combining the shortest path, we have to find the right sequence and then we have to find the smallest sequence or the smallest route sequence and then that would be the route for this particular weight. So, that means, we are doing this in two stages.

First, we are determining the shortest path between the points and in the next stage what we are doing is we are combining this shortest path as per the sequence that we are proposing or as per the order in which we will visit the sites and then we will determine which one to choose from.

So, this order of sites is also very very important sometimes I can give it on my own that this is some sequence which we have to follow or sometimes if I say that no the order should be as per the optimal value then the algorithm or the there has to be a mathematical procedure. So, that we can find which order will give us the best possible result or which sequence will be the best for the maker to travel.

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Shortest path problem

Dijkstra's Algorithm Solves single-pair, single source and destination shortest path problems for positive edge weight graphs

Floyd-Warshall Algorithm Dynamic programming algorithm to determine shortest path in a negative or positive edge weighted graph. Shortest path is determined for all pairs.

Bellman Ford Algorithm Solves single-pair shortest path in a weighed graph where some edge weights are negative. Can also detect negative cycles (where edges sum to a negative value).

Dijkstra's Algorithm

Step 1 : Consider any vertex, i , as start vertex in the given network. Set i to zero, and distances to all other vertices from i as ∞ (infinity).

Step 2: Visit the unvisited vertex with the smallest known distance from the start vertex.

Step 3: For the current vertex, examine its unvisited neighbours and calculate the distance of each neighbour from the start vertex. If the calculated distance of a vertex is less than the known distance, update the shortest distance. Update the previous vertex for each of the updated distances.

Step 4: Return to step 2 until the list of unvisited neighbours is empty.

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So, now, first the shortest path problem, that is we just found out that the shortest path in case of a large network could be many many options for shortest path and we have to find the, in a very easy way. So, there are several mathematical procedures or algorithms for determining the shortest path in an easier way compared to, evaluating all the alternatives. So, some of these are Dijkstra Algorithm, Floyd Warshall and Algorithm, Bellman Ford algorithm. So, these are some of the common popular algorithms that are there.

So, Dijkstra's algorithm is the most common one that we use and it solves single pair that means single source and destination shortest path for positive edge weight graphs. So, that means in the previous slide, we have seen that there is a time or a cost for each of these particular links or each of these arcs. So, in that case these are all positive weights, that means we are giving them a positive value, but sometimes they can have a negative value.

In transportation travel time or cost it is sometimes difficult to say that some values will be positive some are negative, but in other cases there will be some positive and some negative these values. So, we are only kept, this algorithm can only deal with positive edge weights and only between two pairs one pair that is one between single pair which is source and destination, what is shortest path.

Floyd Warshall algorithm it is a dynamic programming algorithm to determine shortest path in a negative or positive edge weighted graph. So, it can deal with both negative and positive weights, and shortest path is determined for all pairs it can say between any two nodes, which is the shortest path. Finally, Bellman Ford algorithm, it solves single pair shortest path in a weighted graph, where some edge weights are negative same as the earlier one.

And it can also detect negative cycles where edge sums to a negative value, where there is a mix of positive and negative weights, sometimes it can create a loop inside our path. So, because if it makes a loop then I cannot get out of that particular loop. So, this algorithm can avoid that. Anyway, so we are mostly concerned with the Dijkstra algorithm, because there are not too many negative weights to deal with in our case.

So, in each case, we can consider all of them as same weights, the total cost increases along if I keep on adding links to a route the cost of that route would be increasing. So, let us see the steps of Dijkstra algorithm. So, the first step is to consider any node and as the start vertex or you can also consider the start point in a given network, we first set it as 0 value, add distance to all the other vertices are considered as infinite, that is as most as possible.

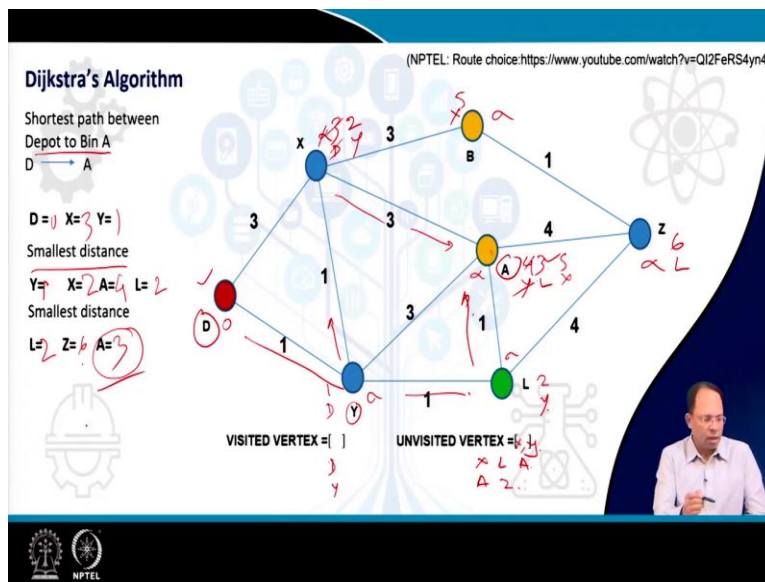
Then from this particular node, we visit the unvisited vertices, that means from any point we will go to the other nodes and we will find which one is the smallest distance from this start vertex. So, we will find the link which has got the shortest distance. So, from this new node, at the end of that particular shortest link, we will examine which of the unvisited nodes or neighbors and we will calculate the distance of each neighbor from the start vertex and if the calculated distance of the vertices is less than the known distance, we have to update the shortest distance.

So, that means if we have already calculated the shortest distance and we as soon as we write, we determine that which are the unvisited vertices, we also have to write the distances to that

vertices from the previous vertices. So, once this is done, but we come back to a particular node is from another direction and if we find that the distance is less, we will update that with the lesser distance.

And I will also keep a track of what was the route choice for to reach that particular distance that particular node as well, what was the previous node visited, update the previous vertex for each of the updated distance and return to step 2, until the list of unvisited neighbors is empty or we have reached the destination. So, that is how the algorithm works. So, I will take you through an example then it will be absolutely clear.

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So, in this particular, we will go back to our problem that we have given so we have depot to bin A, we are trying to find that this is depot and this is bin A. we are trying to find the shortest path. So, let us start with this D point or the depot point, this is the start point. So, we give a value of 0 and we give a value of infinity to all the other points, infinity in terms of the cost of reaching that particular point or the cost of that particular route is infinite.

Now, from D I can go in two directions the unvisited vertices. So, this from D there is there is no visited vertices everything is unvisited. So, both X and Y are unvisited vertices from D, so I can go to either one of them. So, when I go to X, I can update this infinity value by 3, because that is the distance from where it is coming from and the previous node is D, over here I can go to this

particular node and the unvisited and the value is one and the previous node is D. So, out of both these particular things, sorry this D is 0 then X equal to 3 and Y equal to 1.

Now out of this we will choose the smallest distance then this one is obviously the smallest distance the value is 1. So, from 1 we will now find out which are the unvisited vertices. So, in case sorry from Y we will find which are the unvisited vertices. So, for phi visited vertices is D, unvisited is X, and also L and also A. So, these are all the unvisited vertices, so what is the distance from Y to X? Now we will find that already the distance to Y is 1, from the start point.

So, we will add the total distance that means we will of course we will update this value with the distance from the start point. So, when I travel via this particular route or this particular link to X, then I add 1 plus 1 that is 2, which is lesser than this particular value. So, I will replace this by 2, and I list the previous vertices as Y, so I will list that. Similarly, for this one, the value would be 3 plus 1, 4 and the previous vertices Y and for this one the value would be 1 plus 1, 2 and the value and the previous vertices Y.

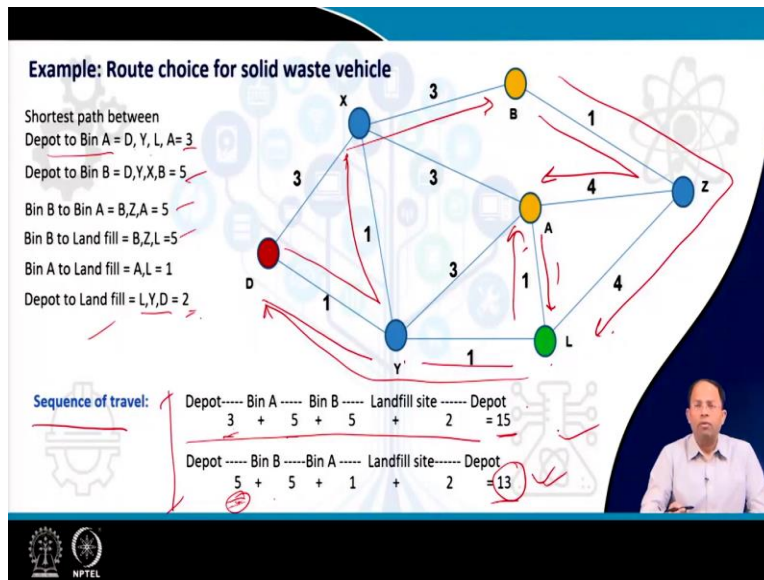
So, Y equal to 1, then X equal to 2, then A equal to 4 and L equal to 2. Now, out of all this the list value could be either taken as this one because this is 2 and this is 2. So, let us assume that we will go with this one. So, in that case in that case we say that L value is 2, then from L unvisited visited what is already Y, A is unvisited vertices, Z is unvisited as well.

So, distance from L to A is another 1, so 1 plus 1, that is 3, so this is already 2, so 2 plus 1 that is 3 and the previous node is L. For this one, the value will become 2 plus 4, 6 and the previous node is L. So, this is 6 and this A is becomes, 3. So, now, out of all these you can see of course, this is the least value and because this is the least value we can actually come and we can say that we have already reached the destination.

And there is no way we can actually increase improve upon this particular solution and the shortest path between D to A is via this particular route, which is 3, that is the cost of this path is actually 3. Now, if we had gone from this point then what happens I could have gone instead of choosing this, if I choose this then this value would have been 3. So, this is not possible you can see that 3 plus 2 this is it this would have been become 5, the previous value should be X and then this value this side the value would be 3 plus 2, this is 5 and the previous value is X.

And now, from out of this both are we have reached this. So, this is one solution. So, this is obviously larger, and we can take this particular value. So, in this way we can find that, A is the shortest path and out of all the because both of them are there. So, we can evaluate both and we can see obviously, this one is better.

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So, now, shortest path as we have found for depot to bin, the is along DYLA, which is 3, so D Y L A, so this is 3, for depot to bin B, this is DYXB, D Y X B, so this path is of size 5, then bin B to A, these BZA, B Z A, so that is like this, so obviously, the bin A to B is also the same then bin B to landfill is BZL, B Z L which is 5. So, B to landfill is this particular way. A to landfill is AL directly which is only 1 and depot to landfill is LYD which is 2, L Y D, which is 2.

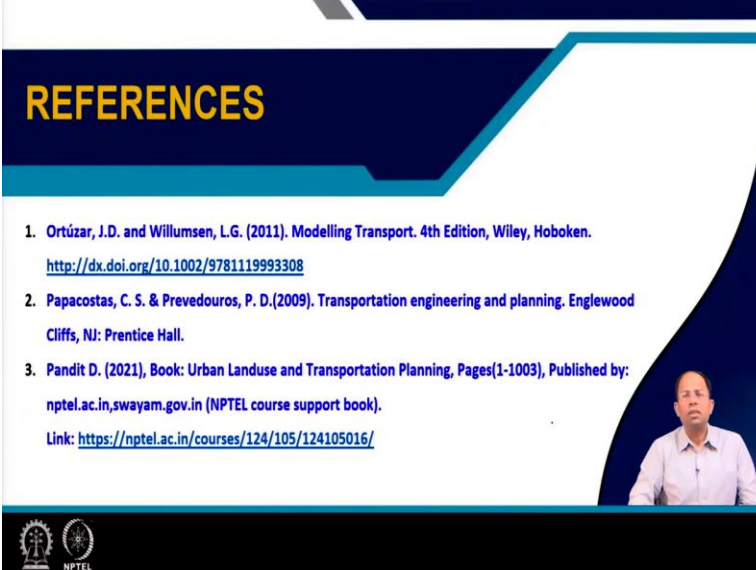
So, this is the sequence of shortest, this is the values of shortest path for each of this particular for your distance between any two points. Now, when I build the sequence or that particular route, if I now, I have to add this particular shortest path values, depot to bin A is 3, bin B to bin A to bin B is 5, and so on I get along this particular sequence the value comes around 15.

Whereas, if I go to depot to bin B and then to bin a then to landfill site the value comes around 30. So, which is actually lower. So, out of these two path this is the more optimal path or the most shortest path. Now, interesting thing is initially depot to bin A is, like over here it is 3 and over here it is 5. So, even though the starting this is larger, but eventually it becomes lower the

overall sequence leads to a lower value. But if I take a call that, should I go to bin A or bin B immediately I say that okay bin A is faster, I will go to bin A, it would have been a wrong decision.

So, this is why both shortest path between two points is important as well as the shortest route also needs to be evaluated to determine the overall shortest path for the entire, for the vehicle that it can adopt. So, this is how we can determine the shortest distance between two points along a particular road network.

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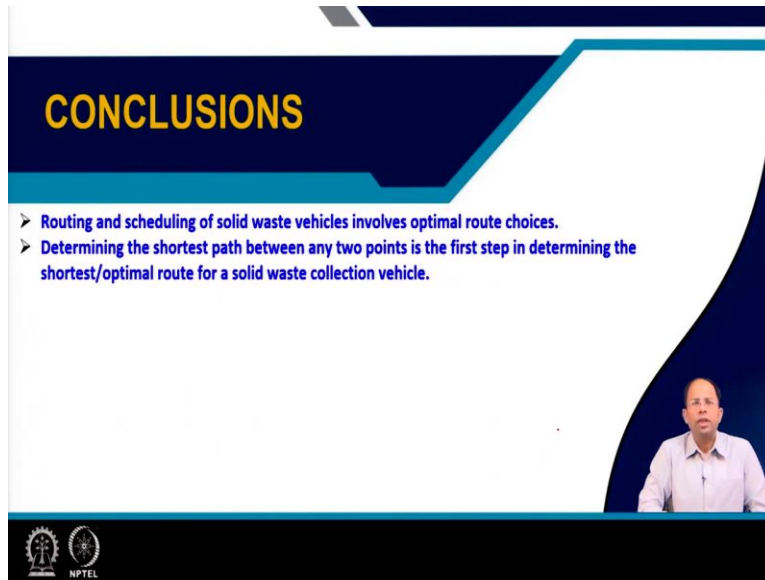
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Link: <https://nptel.ac.in/courses/124/105/124105016/>

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So, these are some of the references you can study.

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CONCLUSIONS

- Routing and scheduling of solid waste vehicles involves optimal route choices.
- Determining the shortest path between any two points is the first step in determining the shortest/optimal route for a solid waste collection vehicle.

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And to conclude, routing and scheduling of solid waste vehicles involve optimal route choices, and determining the shortest path between any two points is the first step in determining the shortest or optimal route for a solid waste collection vehicle. Thank you.