Structural System in Architecture
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Module No # 02
Lecture No # 08
Shear Force Diagram

Hello students! Welcome to the NPTEL online certification course on Structural Systems In Architecture. We are in the module 2; the topic is Strength of Materials which is the 3^{rd} lecture under this module. And today we will discuss the shear force diagram.

Concepts Covered

In this particular lecture we will cover the following concepts:

- Introduction
- Shear Force
- Shear Force Diagram
- SFD for Cantilever Beams
- SFD for Simply Supported Beams

Learning Objectives

The learning objectives of this particular lecture are as follows:

- Overviewing the Shear Force.
- Understanding the Concept of Shear Force Diagram.
- Illustrating the Shear Force Diagram.

Introduction

Now in the introduction I want to say that any kind of a loading is integral part of a structural member because they are actually required to transmit the load of any super structure such as a bridge.

The load of the vehicles, pedestrians etc. act as an external load on a structural element such as a beam or a column. This load will create some kind of physical deformation in the body (as we have seen in the past 2 lectures) due to some axial force resulting in compression or tension in the members. But this physical transformation of the body is purely based on some of the typical parameters as mentioned below:

- Loading type and intensity

  As an illustration of this point, I'd like to mention that by loading type I mean to say it can either be a moment kind of a loading or maybe axial force or a force which is applied directly perpendicular to the axis etc.

- Loading geometry

  Secondly, by geometry of the loading I mean what are the distances or what is the length of a particular uniformly distributed load and what is the distance between the 2 consecutive loads and so forth.

- Types of support and the orientation of the structural element

  As you know that a structure behaves differently if the supports are different and the orientation of the structure is also different.

- Sectional property of the element

  The area, the moment of inertia, the section modulus etc. come under this category.

- Material property of element

  It is basically the Young's Modulus of Elasticity.

Orientation of various forces

The following figures show the orientation of different forces and their effect on a body.

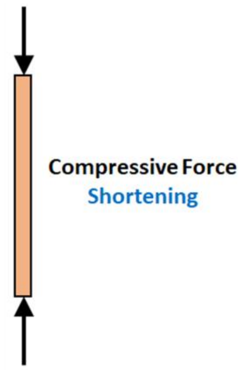


Figure 1 Compression

Compressive force

Compression force (or compressive force) occurs when a physical force presses inward on an object, causing it to become compacted. This change maybe either temporary or permanent depending upon the elasticity of the object.

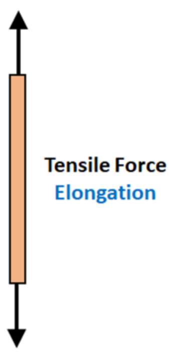


Figure 2 Tension

Tensile force

Tensile force is the stretching forces acting on the material and has two components namely, tensile stress and tensile strain. This means that the material experiencing the force is under tension and the forces are trying to stretch it. The elongation caused in the body due to this force also maybe either temporary or permanent depending upon the elasticity of the material.

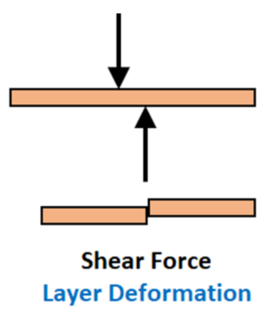


Figure 3 Shear force

Shear force

A shear force is a force applied perpendicular to a surface, in opposition to an offset force acting in the opposite direction. This results in a shear strain. In simple terms, one part of the surface is pushed in one direction, while another part of the surface is pushed in the opposite direction.

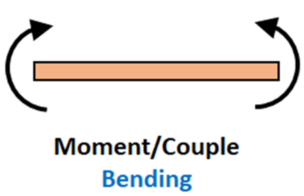


Figure 4 Moment

Moment

A moment is equivalent to a force multiplied by the length of the line passing through the point of reaction and that is perpendicular to the force.

Shear Force

The following figures show the effect of shear force on a horizontal member such as a beam supported over two walls having a gap between them.

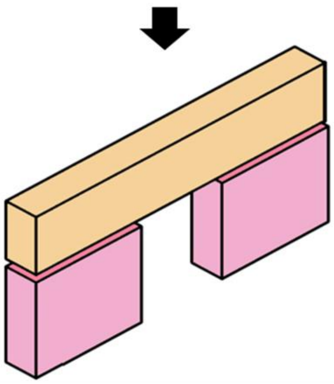


Figure 5 Shear failure step-1

A simply supported beam is subjected to a very heavy load at its center.

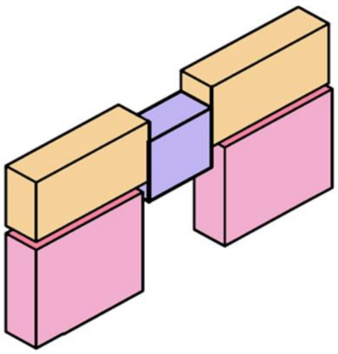


Figure 6 Shear failure step-2

Due to the excessive load the beam keeps bending until it can stand no longer and finally breaks from the center due to the resultant shear force.

The middle portion after breaking gets detached from the portions at the ends which are supported over the walls.

As has been noted that shearing forces are unaligned forces pushing one part of a body in one specific direction, and another part of the body in the opposite direction.

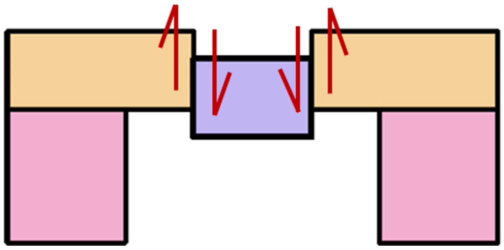


Figure 7 Shear failure step-3

Here In this case also there are two kinds of forces formed at the point of breakage. The downward forces are formed at the ends of the purple portion of the beam due to which it crashes to the ground while the upward forces are formed at the ends of the yellow colored portion of the beam which enables them to stay put. In other words, the yellow ends of the beam, towards the center, move upwards relative to the downward-moving purple portion. So we get a failure where the failure surfaces slide past each other causing a shear failure. Hence a shear force is rightly called as a cutting or slicing force.

Shear Force Diagram

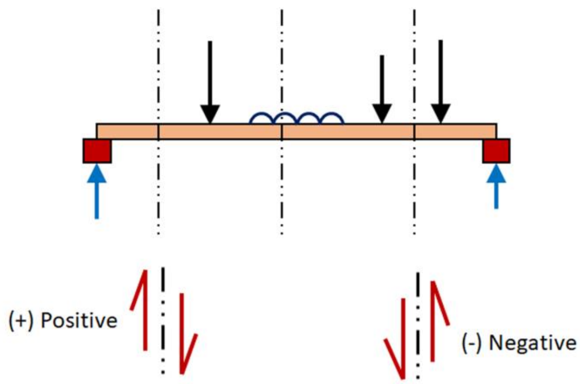


Figure 8 Simply supported beam having partly UDL and subjected to point load

A shear force diagram is one which shows variation in shear force along the length of the beam.

As you already know that a beam is subjected to various kinds of loads and also reactions at the supports due to these loads. Figure 8 shows a beam supported at both the ends represented by blue colored upward arrows. The beam is partly subjected to a uniformly distributed load (UDL) and three point loads represented with black colored downward arrows. The black colored dotted lines show the points of cross-sections where the resultant forces maybe calculated.

In order to draw the shear force diagram of a given beam, at first you need to find the reactions at each support due to the loads acting on it. Thereafter, you may proceed computing the forces by adding the positive ones and subtracting the negative ones at each point starting from left to right.

When you start computing from the left side, the forces shown with upward arrows (such as the supports) are positive whereas, that shown with downward arrows (such as the loads shown with black arrows in the Figure 8) are negative. Conversely, if you start computing from the right the vice-versa is true. These set of rules used to estimate the sign of particular force acting upon the given beam is known as sign convention. It is extremely important while drawing the shear force diagrams.

Example-1

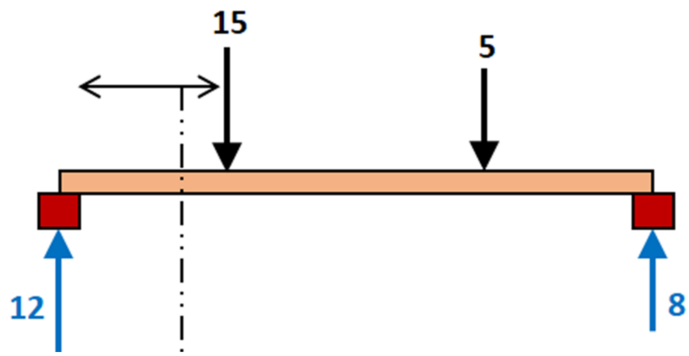


Figure 9 Example of SFD; Step-1

Now let us understand the process of making a shear force diagram with the following example. Given in Figure 9 is a simply supported beam subjected to two point loads of 15 units and 5 units respectively. The length of the span is not mentioned as this example is meant just for the basic understanding of how to approach a problem of shear force diagram.

Step-1: Find the reactions at each of the supports

Here we have,

Total load = 15 + 5 = 20 units

Then,

Sum of the total reactions = 20 units

i.e., reaction at left end + reaction at right end = 20 units

Now,

Since the span is not given, we cannot estimate the exact reaction forces.

So,

Let us assume that it is 12 units in the left hand side.

Then,

Reaction at the right hand side = 20 -12 = 8 units

Step-2: Calculate the forces at each point

The computation of forces is done here from left to right. In the present example the two point loads divide the span of the beam into three parts. So first, we have to consider the portion starting from the very left end and up to the point where the point load of 15 units is acting upon the beam, as shown in the Figure 10.

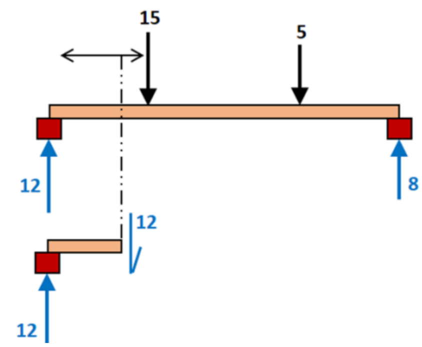


Figure 10 Example of SFD; Step-2(a)

Now in this particular section, the magnitude of reaction at the support is 12 units. In other words, an upward force of 12 units acts in the support at the left hand side of the beam. Hence this is a positive force. So from this point onwards and until the next point, where the point load of 15 units acts on the beam, the shear force will be constant, i.e. 12 units. It'll be constant as there is no other force acting on the beam between these two points. Figure 11 shows the shear force diagram at this point.

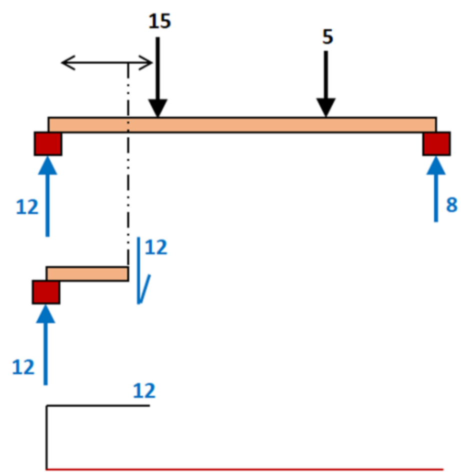


Figure 11 Example of SFD; Step-2(b)

Conversely, since the point load of 15 units is a downwards force, it is negative.

Thus,

The shear force at this point will be (12-15) units i.e., -3 units.

Therefore,

The shear force diagram at this point will go 3 units below the axis and will remain constant until the point where the point load of 5 units acts upon the beam, as shown in the following Figure 12.

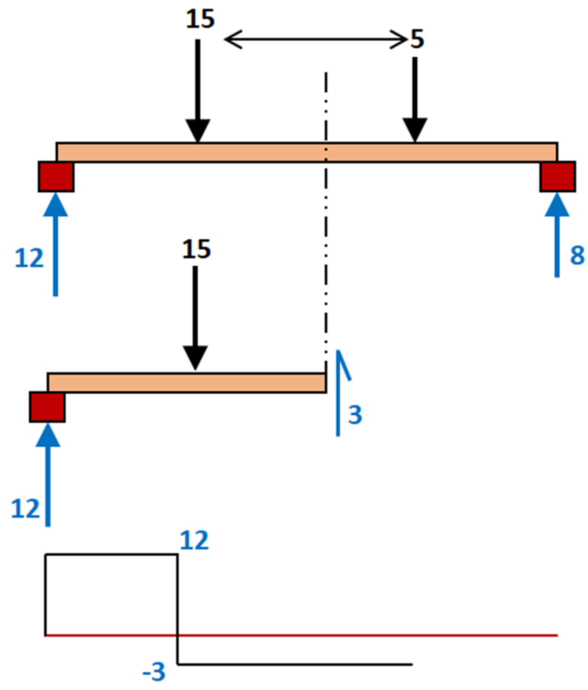


Figure 12 Example of SFD; Step- 2(c)

Again at this point, the point load of 5 units is a downward force. So it is negative.

Hence,

Shear force at this point is (-3-5) units, i.e., -8 units.

This value will also be constant till the right most end of the beam as shown in the Figure 13.

So,

At this point the value of the shear force will be (-8 + 8) units, i.e., 0 units.

Accordingly, the resultant shear force diagram can be seen in the Figure 13.

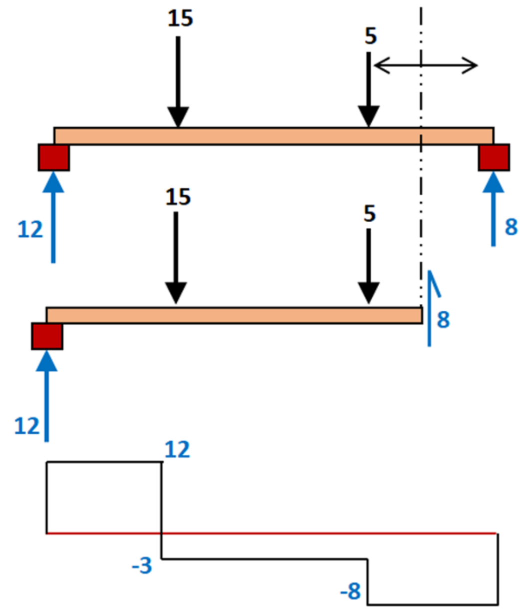


Figure 13 Example of SFD; Step-2(d)

Therefore, to sum up, we may say that,

A shear force diagram or an SFD is the representation of graphical variation of the shear force in different cross-sections over the length of the particular structural element.

Computation of Reactions for SFD

Case-1: Simply Supported Beam Subjected To a Concentrated or Point Load

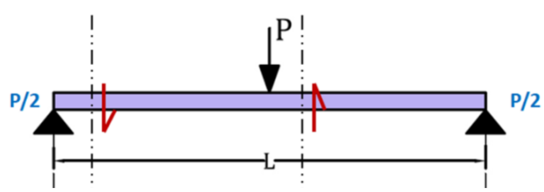


Figure 14 Simply supported beam due to concentrated or point load

Given in the Figure 14 is a simply supported beam spanning across length L and subjected to a point load P at its center.

Then,

Step-1: Compute the reactions

As we know that,

Sum of positive forces = sum of negative forces

i.e., sum of the reactions at both ends = P i.e., $\frac{P}{2} + \frac{P}{2}$ = P [Since, the load P divides the beam symmetrically]

Hence,

Reaction at each end of the given simply supported beam subjected to a point load at its center is P/2.

Step-2: Computation of shear force and the SFD

As mentioned earlier,

Starting the computation of the shear force from the left most end, we have shear force at the left support is +P/2. This value of shear force will remain constant until the center of the beam.

Then,

Shear force at the center = $\frac{P}{2} - P$ = - $\frac{P}{2}$

Hence at this point,

In the SFD, there is a visible drop in the graph from P/2 and –P/2, as shown in the Figure 14. From this point, the graph will again be constant till the far right end as there is no other load in between. At this end also there is positive force acting, having magnitude P/2.

So,

Shear force at the right end = - $\frac{P}{2} + \frac{P}{2}$ =0

Therefore, the SFD will end hear with a shear force value as 0.

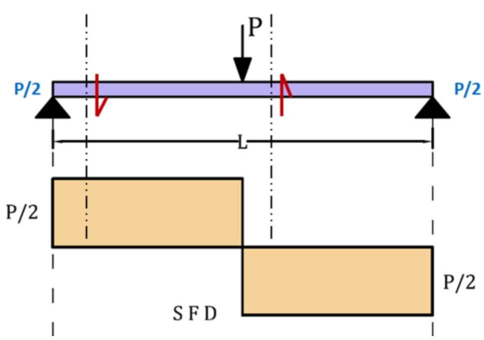


Figure 15 SFD of a simply supported beam subjected to point load

The final SFD of given beam can be seen in the above Figure 15.

Case-2: Simply Supported Beam Subjected to Two Point Loads

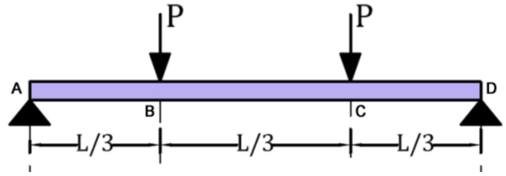


Figure 16 Simply supported beam subjected to two point loads spaced at equal intervals

Similarly, here we have another simply supported beam of length L, which is subjected to two point loads of equal magnitude P placed at equal intervals. Let these points be B and C respectively and the two end supports be A and D respectively. See Figure 16 for reference.

Then,

Step-1: Find the Reactions R_A and R_B

We have,

Total downward forces = P + P = 2P

So,

$R_A + R_D = 2P$

Here,

Though our common sense tells us that since loading is symmetrical so the reactions also must be equal, but let us go step-by-step so as to understand the process of calculating the reactions.

So as per the law of equilibrium of forces we also know that,

Total clockwise moment = total anticlockwise moment

So taking moment about A we have,

$$\frac{P \times L}{3} + \frac{P \times 2L}{3} = R_D \times L$$

i.e., $$\frac{P(L+2L)}{3} = R_D \times L$$

i.e., $P \times L = R_D \times L$

i.e., $R_D = P$

i.e., $R_A = P$

Step-2: Determine the Shear Force at A

Shear force at A = R_A = P

So we have,

The shear force at A as P and this value will remain constant till B as there is no other load in between.

Step-3: Compute the Shear Force at B

Shear force at B = R_A – Load at B = P – P = 0

Similarly,

The shear force at B is 0 and this will remain constant till C as there is again no other load between B and C.

Step-4: Calculate the Shear Force at C

Shear force at C = SF at B – Load at C = 0 – P = -P

Again,

The SF at C is –P which will also remain constant until D as there is no other load in between.

Step-5: Shear Force at D

Finally,

Shear force at D = SF at C + R_D [Since R_D is a positive force]

= -P + P = 0

Step-6: Plot the graph

Now plot the values of SF at each point A, B, C and D on a graph. The resultant drawing is the Shear Force Diagram. The SFD for this example can be seen in Figure 17.

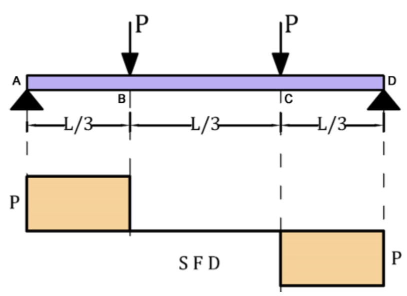


Figure 17 SFD of a simply supported beam subjected to two point loads.

Case-3: Cantilever Beam Subjected to Point Load at the End

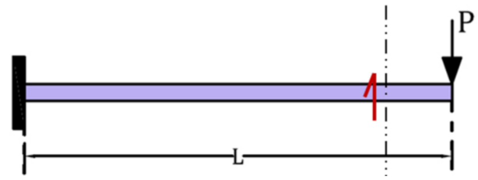


Figure 18 Cantilever beam subjected to point load at the end

Next, let us see how to compute the reaction for a cantilever beam subjected to a point load. Given in the Figure 18 is a cantilever beam with its supported at the left end and spanning across a length of L. Besides, a point load P is acting on the beam at the right most end of the beam as shown in the Figure 18.

Then,

Step-1: Find the Reaction

As per the law of equilibrium of forces,

Total upward force = total downward force

i.e., reaction at support = P

Hence,

The SF at the reaction is P and it will remain so till the other end where a point load P is acting and there's no other load in between.

Step-2: Find the SF at the right end

SF at the right end = SF at support – Load at the right end = P – P = 0

Therefore,

The SFD of the given cantilever beam can be drawn as shown in the Figure 19.

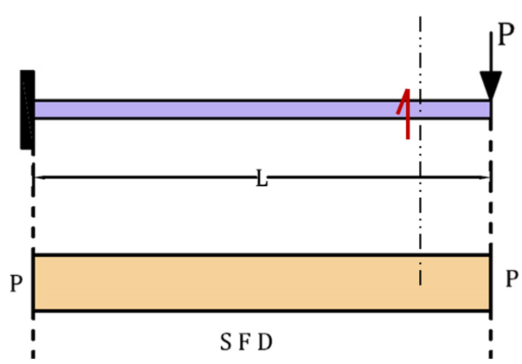


Figure 19 SFD of a cantilever beam subjected to a point load at the end

Example-2

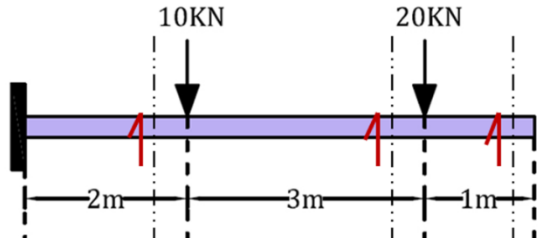


Figure 20 Cantilever beam subjected to two point loads

We can better understand the SFD of a cantilever beam subjected to point load with this example. Here in this Figure 20 we have a cantilever beam of span 6m supported at the left end. The beam is subjected to two point loads of 10 KN and 20 KN at a distance of 2m and 5m from the support respectively. For the ease of reference let us name each of these points as A, B, C and D respectively, starting from the support.

Now,

It is important to note that while computing the SFD of a cantilever beam you must proceed from the free end of the beam.

<u>Step-1: SF at D</u>

So at the point D,

We see that there are no loads acting on it.

Hence, SF at D = 0

<u>Step-2: SF at C</u>

As we can see,

There is 20KN of load acting on the beam at this point.

So, SF at C = 20 KN

<u>Step-3: SF at B</u>

We have,

Load acting at B = 10 KN

Thus,

SF at B = load at B + SF at C = (10 + 20) KN = 30 KN

Step-4: SF at A

Clearly,

There is no load acting between A and B. So the SF value at B will continue till the point A.

Also,

SF at A = total downward load

= (10 + 20) KN = 30 KN

Therefore, the final SFD can be drawn by plotting all these values in a graph as shown in the Figure 21 given below.

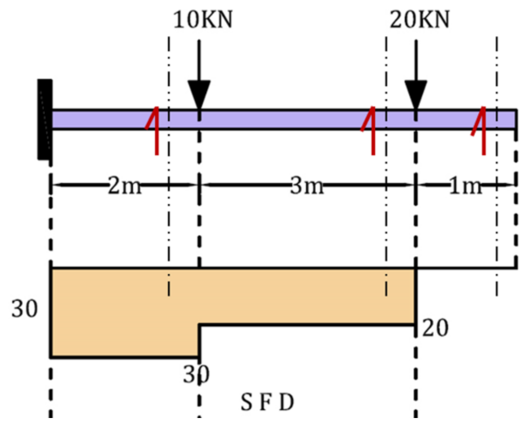


Figure 21 SFD of a cantilever beam subjected to two point loads

Case-4: Simply Supported Beam Subjected to Uniformly Distributed Load

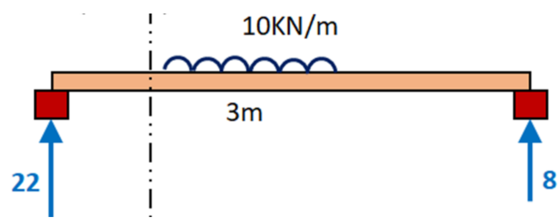


Figure 22 Simply supported beam when subjected to a UDL

In the earlier sections we've seen how to draw a shear force diagram in case of a point load. But now let us see what happens to the SFD when a beam is subjected to the UDL. So here we have a simply supported beam subjected to a UDL of 10 KN/m spread across a length of 3m. Again, the length of the beam is not provided as it is only meant for understanding purpose. However, the reactions at the supports are given to be 22 KN and 8 KN respectively. The diagram has been given in the Figure 22.

For our ease let us name the vital points as A, B, C, D and E where, A is the support on the left hand side, B is the point from where the UDL begins, C is the midpoint of the UDL, D is the point where the UDL ends and E is the support at the right hand side.

Then,

Step-1: SF at A

We have,

Reaction at A = R_A = 22 KN

So, SF at A is 22 KN.

Step-2: SF at B

As,

SF at A = 22 KN

This value of shear force will remain constant until the point B as there is no other load in between.

Also,

SF at B = 22 KN

Step-3: SF at C

Here,

C is the midpoint of the UDL.

So,

SF at C = SF at B – (load at C) [-ve sign because UDL is acting downwards]

= [22 – (1.5 × 10)] KN = 7 KN

Unlike the SFD of point loads, that of the UDLs comprise of sloping lines. So here, the line joining the SF value corresponding to the point B and that of point C is a sloping line starting from the value 22 KN and ending at 7 KN.

Step-4: SF at D

We have,

SF at C = 7 KN

So,

SF at D = SF at C – Load at D

= [7 – (10 × 1.5)] = -8 KN

Here also the line joining the SF value corresponding to the point C, i.e. 7KN, and that of point D, i.e. -8KN, is a sloping line.

Step-5: SF at E

Finally,

SF at E = SF at D + R_E [+ve sign because the reaction at E is an upward force]

= (-8 + 8) = 0 KN

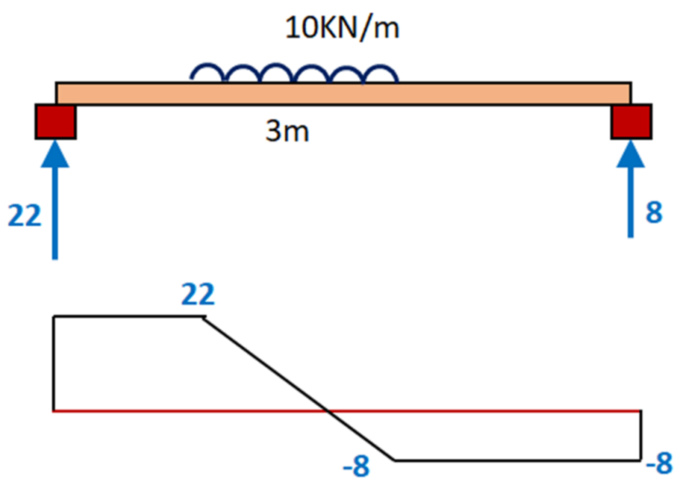


Figure 23 SFD of a simply supported beam partly loaded with a UDL

Therefore, the final SFD of the given beam will be as shown in the above Figure 23. Nevertheless, it is important to note that the shape of the SFD comprises of sloping lines only in the place of the UDL and not along the entire beam.

Case-5: Simply Supported Beam Subjected to UDL

Next, let us see what happens to the shape of the SFD of a simply supported beam when it is completely subjected to a UDL. So here we have a simply supported beam of length 5m subjected to a UDL of 12 KN/m.

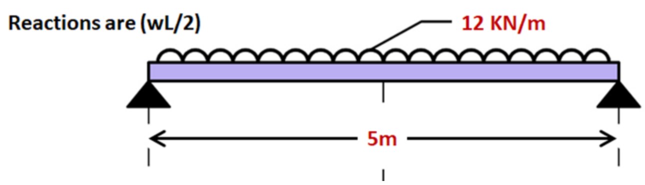


Figure 24 Simply supported beam subjected to UDL

Now,

Step-1: Find the reactions

Let us suppose,

that a simply supported beam of length L is subjected to a UDL of w units.

Then,

Total downward load on the beam = wL units

So,

Total upward load acting on the beam = wL units

i.e. sum of reactions at both supports = wL units

i.e., reaction at each support = $\frac{wL}{2}$ units [Since the reactions at both the supports are equal]

Therefore,

The formula for finding the reactions of a simply supported beam when subjected to a UDL = $\frac{wL}{2}$ units

Hence,

Reactions for the given beam = $\frac{wL}{2} = \frac{12 \times 5}{2}$ = 30 KN each

Step-2: SF at left end

We have,

Reaction at left end = 30 KN

i.e., SF at left end = 30 KN

So the SFD here will be a straight vertical line from the base axis and reaching up to the point 30 KN on the positive side i.e., above the base axis.

Step-3: SF at midpoint

As,

SF at left end = 30 KN

Then,

SF at midpoint = 30 - load at midpoint [-ve sign because UDL is acting downwards]

= 30 – (12 × 2.5) = 0

Therefore,

The SFD here will be sloping line joining the previous point and intersecting the base axis at the midpoint.

Step-4: SF at right end

We have,

SF at the midpoint = 0

So, SF at right left of the right end = 0 – UDL acting at this point

= 0 – (12 × 2.5) = -30 KN

But,

The support reaction at this end is a positive upward force of 30 KN.

Then,

SF at right end = (-30 + 30) KN = 0

Therefore,

Here again the SFD will be a straight inclined line joining the preceding point to the point representing -30 KN of SF and then it will go straight up with a vertical line joining this point to 0 on the base axis.

Hence,

The final SFD of the given beam is as shown in the Figure 25.

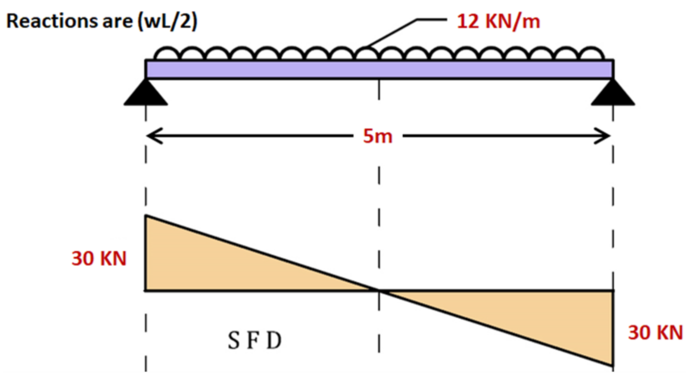


Figure 25 SFD of a simply supported beam subjected to UDL.

Case-6: Simply Supported Beam Partly Loaded with a UDL

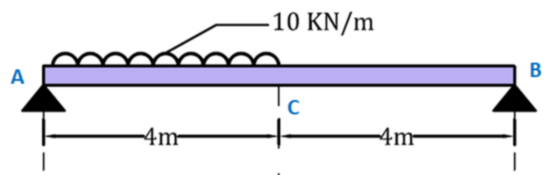


Figure 26 Simply supported beam partly loaded with a UDL

Further, let us see what happens to the SFD of a simply supported beam when it is partly loaded with a UDL. So given in the Figure 26 is a simply supported beam of span 8m which is loaded with a UDL of 10 KN/m till the geometric center of the beam, i.e., up to 4m.

For the ease of computation three points A, B and C has been marked in the Figure 26. Yet let us consider some more points P, Q and R where each of these points are placed at an interval of 1m along the UDL respectively. This is done so as to track the changes in the values of SF along the UDL at these points.

Next,

Step-1: Find the reactions at supports

We have,

Total downward load = 10 KN/m × 4 m = 40 KN

Then,

$R_A + R_B = 40$ KN

Taking moment about A,

$(10 \times 4) \times 2 = 8\ R_B$

i.e., $80 = 8R_B$

i.e., $R_B = 10$ KN

i.e., $R_A = 40 – 10 = 30$ KN

Step-2: SF at A

As,

$R_A = 30$ KN

So SF at A = 30 KN

Here the SFD will be a straight vertical line starting from the base axis and reaching up to the point corresponding 30 KN.

Step-3: SF at P

We have,

SF at the left of P = SF at A = 30 KN

So, SF at P = 30 – (10 × 1) = 20 KN

i.e., from the point A if you move towards right then after every 1m of distance we'll lose 10 KN of load. This happens because it is a UDL and its value is given as 10 KN/m.

For this reason,

The SFD here will be a straight inclined line starting from the point corresponding 30 KN and joining up to the point corresponding to 20 KN.

Step-4: SF at Q

Here,

SF at the left of Q = SF at P = 20 KN

So, SF at Q = 20 – (10 × 1) = 10 KN

Again,

There is a reduction in the load by 10 KN.

As a result, here also the SFD will be a straight inclined line joining the points corresponding to 20 KN and that of 10 KN.

Step-5: SF at R

Similarly,

SF at the left of R = SF at Q = 10 KN

Thus, SF at R = 10 – (10 × 1) = 0

Therefore,

The SFD at this point has to intersect the base axis.

Step-6: SF at C

Again,

SF at the left of C = SF at R = 0

Then, SF at C = 0 – (10 × 1) = -10 KN

Hence,

Here also the SFD will be a straight inclined line joining the previous point and the point corresponding to the SF value -10 KN.

Step-7: SF at B

Finally,

SF at the left of B = SF at C = -10 KN

So, SF at B = -10 + R_B = -10 + 10 KN = 0

As there is no UDL present so here the SFD will be a straight vertical line. The final SFD can be seen in the following Figure 27.

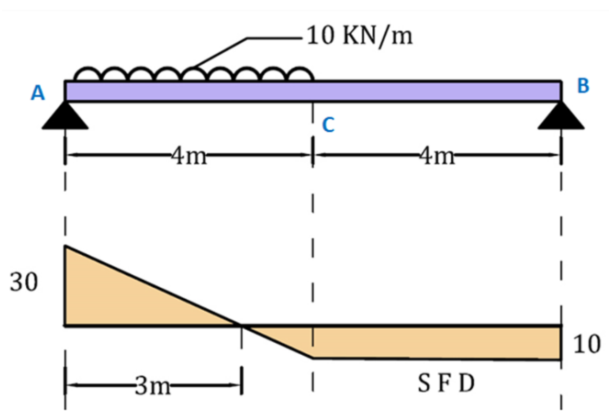


Figure 27 SFD of a simply supported beam partly loaded with a UDL.

Case-7: Cantilever Beam Completely Loaded With UDL

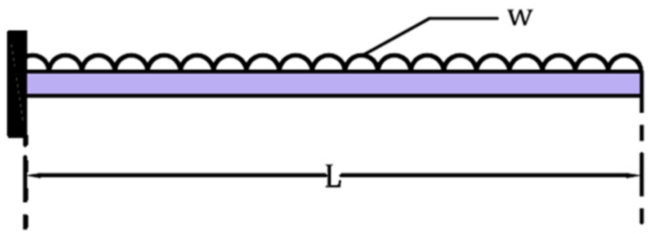


Figure 28 Cantilever beam completely loaded with UDL

Next, we'll discuss about the SFD of a cantilever beam subjected to UDL. Here we have a cantilever beam of length L subjected to a UDL of w units.

So,

Let X is a point on the beam at x distance from the free end.

Then,

SF at X = wx

Thus,

At x = 0, SF = 0

At x = L, SF = wL

Therefore,

The final SFD is given in the following Figure 29.

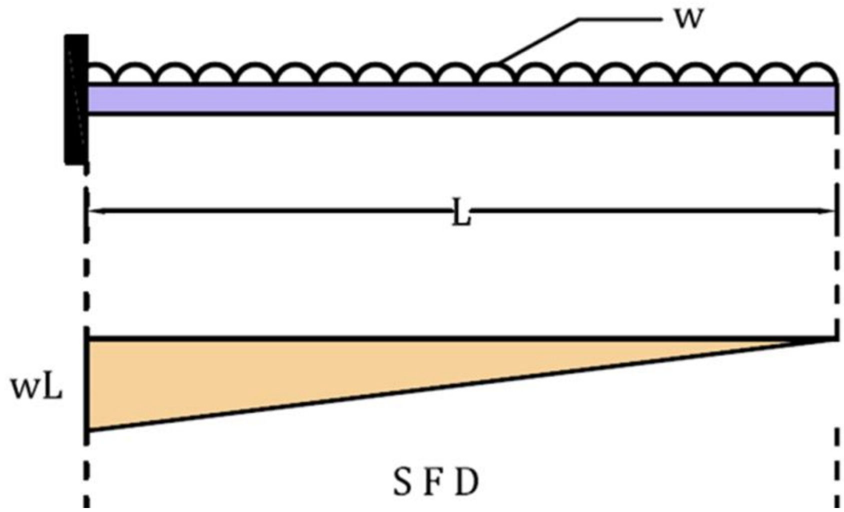


Figure 29 SFD of a cantilever beam subjected to UDL

Case-8: Cantilever Beam Partly Loaded With UDL and Point Loads

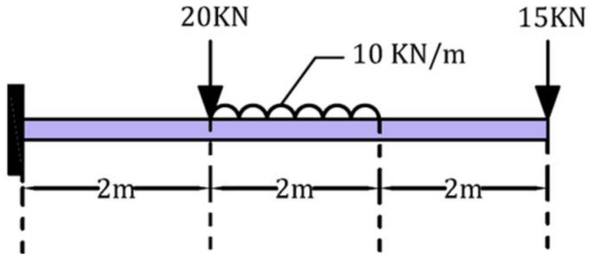


Figure 30 Cantilever beam partly loaded with UDL

Finally, in the Figure 30 we have a cantilever beam of 6m length. Now let us take four points A, B, C and D on the beam starting from the fixed end and then at intervals of 2m each.

Then, as evident from the figure, we have a point load of 20 KN at B, a UDL of 10 KN/m from B to C and another point load of 15 KN at D

Now starting the computation from the free end,

Step-1: SF at D

SF at D = 15 KN

Step-2: SF at C

As there is no load between C and D the value of SF at D will continue till C. But as evident from the Figure 30, we have a UDL from B to C and as you're aware from the previous sections that the SFD in presence of a UDL is an inclined line. So, it is important to find the SF at the left of C.

Thus,

Let us conside a point X anywhere between B and C, at a distance of x metres from C.

Then,

SF at X = SF at C + load at X = (15 + 10x) KN

So,

At x = 1m, SF at X = 25 KN

At x = 2m, SF at X = 35 KN

i.e., SF at the right of B = 35 KN

Step-3: SF at B

SF at B = SF at the right of B + load at B = (35 + 20) KN = 55 KN

Step-4: SF at A

As there's no load between A and B, so the SF at B will continue till A.

The final SFD can be seen in the following Figure 31.

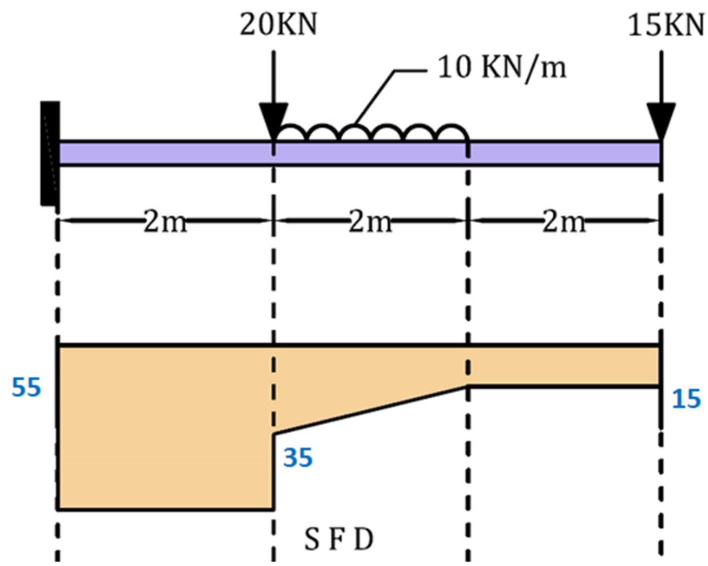


Figure 31 SFD of a cantilever beam subjected to UDL and point loads

References

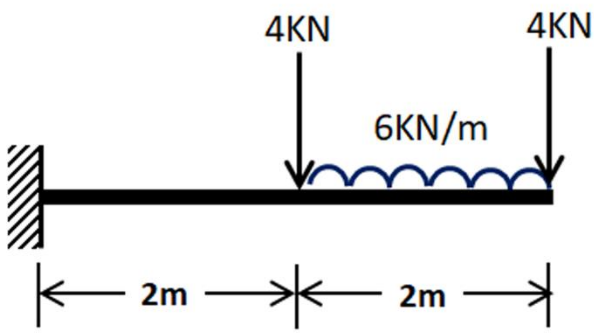
- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Basic Structures for Engineers and Architects** By Philip Garrison, Blackwell Publisher
- **Understanding Structures:** ***An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

Conclusion

- Shear is a slicing force that acts on a structural section due to external loading.
- The graphical representation of variation of shear force over the length of structural member is called Shear Force Diagram.

Homework

Q1. Draw the SFD of the following Cantilever Beam.



Q2. Draw the SFD of the Following Simply Supported Beam assuming a UDL of intensity 6KN/m as per following three locations:

(i) Entire Span, (ii) Only in the central 3 meter portion (iii) End 3 meter portions only, no UDL in the central portion