Structural System in Architecture
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Lecture – 10
SFD & BMD: Special Cases

Welcome to the NPTEL online certification course on Structural Systems in Architecture. We are in the module 2 i.e., the Strength of Material is our topic of concern of the second week. And this is the last lecture of this module, which is based on the SFD and BMD special cases.

Concepts Covered

The following concepts are covered in this lecture:

- Relation Between Shear Force and Bending Moment
- Point of Contra-flexure
- Two-sided Overhang Beams
- Loading Diagram from SFD & BMD

Learning Objectives

The learning objectives of this lecture are given below:

- To differentiate and relate between BMD & SFD.
- Illustrating the overhang beam.
- Relating the Loading diagram from SFD & BMD.

Relation Between Shear Force and Bending Moment

Case-1: Cantilever beam subjected to UDL

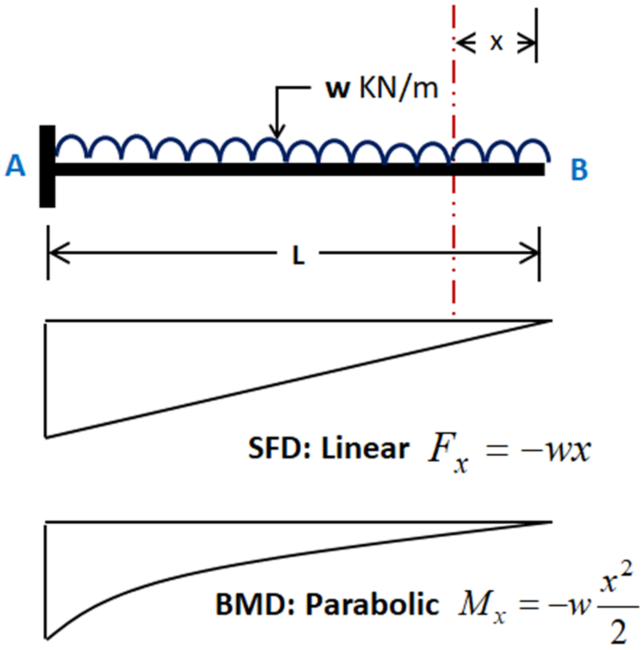


Figure 1 Cantilever beam subjected to UDL

Here we have a cantilever beam AB of span L. the beam is loaded with a UDL of w KN/m as shown in the Figure 1. The figure also shows the SFD and the BMD of the given beam.

We know that,

The SFD being linear, can be expressed as

$F_x = -wx$

Differentiating the above expression we get,

$$\frac{d F_x}{dx} = -w = load\ intensity$$

Also, we know that

The BMD being parabolic, can be expressed as

$$M_x = -w\frac{x^2}{2}$$

Differentiating the above expression we get,

$$\frac{d M_x}{dx} = -2 \times w\frac{x}{2} = -wx = F_x$$

Differentiating the above expression again, we get

$$\frac{d^2 M_x}{dx^2} = \frac{d\,F_x}{dx} = Loading\ intensity$$

So finally we may conclude that the differentiation of bending moment is the shear force and the differentiation of the shear force is nothing but the loading intensity. In other words, the second order of the differentiation of the bending moment is the differentiation of the shear force i.e., the loading intensity. The motto behind discussing this is not to make your life complex but to explain this concept to you so that it'll be helpful to you in the future while discussing some of the principles of the deflections and also when you do need some amount of this differential calculus. But those will be very simple.

Case-2: Simply Supported Beam Subjected to UDL

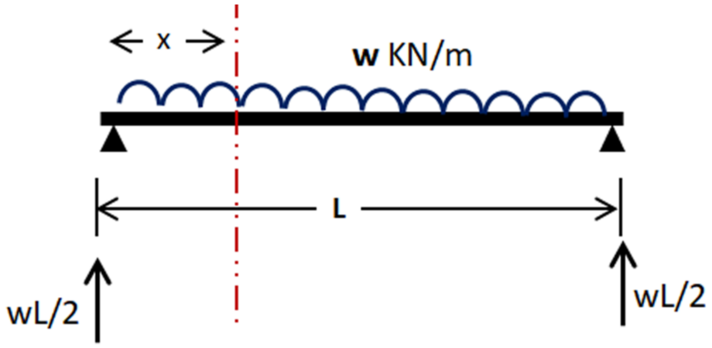


Figure 2 Simply supported beam subjected to UDL

The Figure 2 above shows a simply supported beam of length L subjected to a UDL of w KN/m.

Let X be a cross-section anywhere on the beam at a distance x meters from the left end.

Now,

The reactions on either side = $\frac{wL}{2}$

Then,

Shear force at X = $F_x = \frac{wL}{2} - wx$

Differentiating the above equation we get,

$$\frac{d\,F_x}{dx} = 0 - w = -w = loading\ intensity$$

Also,

Bending moment at X = $M_x = \frac{wL}{2}x - \frac{wx^2}{2}$

Differentiating the above equation we get,

$$\frac{d\,M_x}{dx} = \frac{wL}{2} - 2 \times \frac{wx}{2} = \frac{wL}{2} - wx = F_x$$

Clearly, the results of the Case-1 and Case-2 are same which is worth noting.

Point of Contra flexure

So next let us discuss another very interesting thing called the Point of Contra flexure.

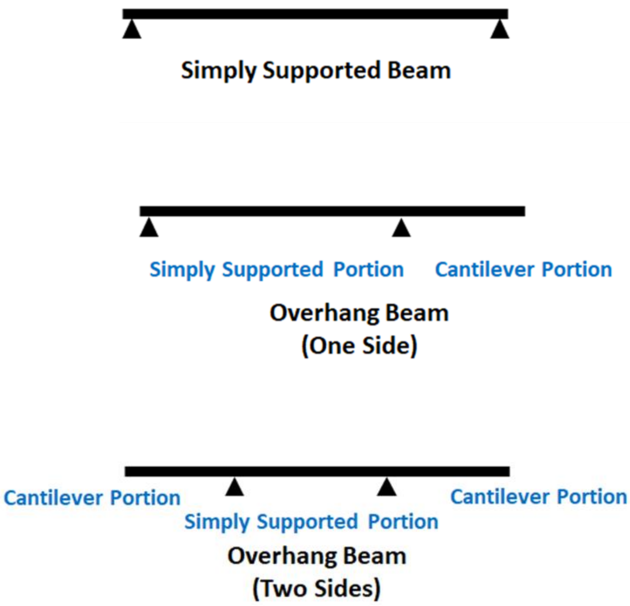
In a beam that is flexing (or bending), the point where there is zero bending moment is called the point of contra flexure. At that point, the direction of bending changes its sign from positive to negative or from negative to positive. (It may also be thought of as a change from compression to tension or vice versa).

For this change to happen, it must pass through zero – the point of contra flexure. On a bending moment diagram, it is the point at which the bending moment curve intersects with the zero line.

An analogy may be made with a speeding train travelling west on a single track. In order to reverse direction and travel east, it must decelerate, stop then accelerate in the opposite direction. The point it stops, even if momentary, is zero – the neutral point, where it is not travelling.

Theoretically, when considering a structural member under load, such as a reinforced concrete beam, the point of zero bending moment would seem to suggest no reinforcement would be required. However, omitting reinforcement at that point is considered inadvisable as, in a real-life situation; it may be difficult to locate the exact point of contra flexure.

Moreover, you're already aware from the previous lectures that where does the tension and compression zones lie in different types of beams, how is their BMDs and what is the sign convention in each of the cases. So now let us see what happens to simply supported beam if we alter the position of its supports. See the Figure 3 for reference.



Here we have a simply supported beam.

Moving one support towards the centre of the beam results in an overhang beam from one side. It consists of a simply supported portion and a cantilever portion.

Moving both the supports towards the centre gives an overhang beam from two sides. It consists of a simply supported portion in the middle and two cantilever portions on both the ends.

Figure 3 Point of contra flexure

Therefore, as the various portions of the beam changes from being simply supported to being cantilevered, so do their corresponding zones of tension and compression and thus, so do their sign of bending moment (explained in Lecture 9). So, the point(s) where the curve of BMD will change its sign will be the point(s) of contra flexure.

Case-1: One-sided Overhang Beam

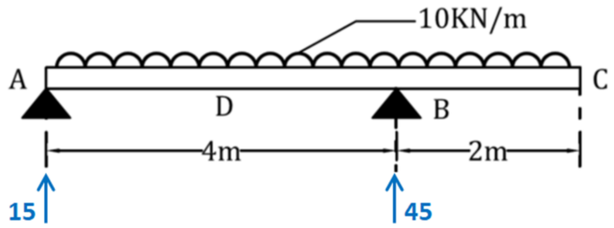


Figure 4 One-sided overhang beam subjected to UDL.

Here we have a one-sided overhang beam subjected to UDL of 10 KN/m as shown in the Figure 4.

Step-1: Compute the support reactions

We know that,

$$R_A + R_B = (10 \times 6)\ KN = 60\ KN$$

So taking moment about A,

$$4R_B = (10 \times 6) \times (3) = 180$$

i.e., $R_B = 45KN$

i.e., $R_A = (60 - 45) = 15KN$

<u>Step-2: Find the shear force</u>

SF at A = R_A = 15 KN

SF at D = $R_A - (10 \times 2) = -5$ KN

SF at the left of B = $R_A - (10 \times 4) = -25$ KN

SF at B = $R_A + R_B - (10 \times 4) = 20$ KN

SF at C = 0 (as there are no supports)

<u>Step-3: Draw the SFD</u>

Plot the above values of shear force in a graph to get the SFD of the given beam, as shown in the Figure 5.

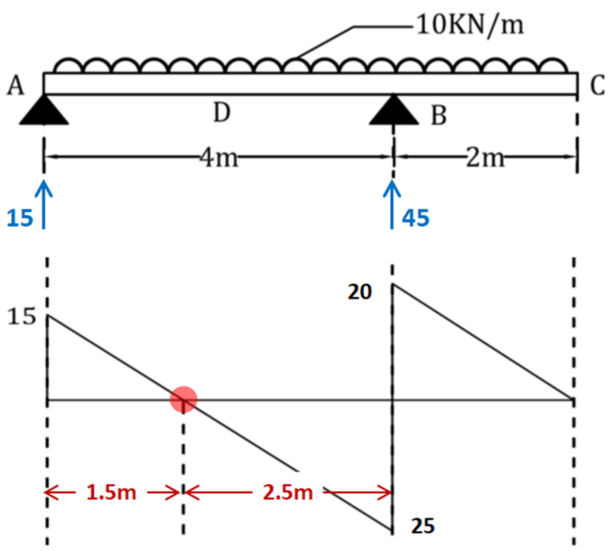


Figure 5 SFD of the given one-sided overhang beam

Here it is important to note the shear force is 0 at a distance of around 1.5 m from A. So let us cross-check this.

Let us take a cross-section Y over here.

Then,

SF at Y = $R_A - (10 \times 1.5) = 15 - 15 = 0$

Step-4: Compute the bending moment

In C-B, Taking a section at 'x' from C

$$M_x = -10 \times \frac{x^2}{2} = -5x^2$$

Then,

$At\ C,\ M_C = 0,\ as\ x = 0$

At B, $M_B = -5 \times 2^2 = -20,\ as\ x = 2$

Again,

In B-A, Taking a section at 'x' from C

$$M_x = 45(x-2) - 10 \times \frac{x^2}{2} = 45x - 90 - 5x^2$$

So,

$At\ B,\ M_C = 45X2 - 90 - 5X4,\ = -20,\ as\ x = 2$

At D, $M_D = 45 \times 4 - 90 - 5X16 = 10,\ as\ x = 4$

Next,

In B-A, Taking a section at 'x' from C

$$M_x = 45x - 90 - 5x^2$$

Differentiating the above equation we get,

$$\frac{dM_x}{dx} = 45 - 10x$$

Equating the above equation to 0 (i.e., when SF = 0) we get,

$45 - 10x = 0$

i.e., x = 4.5

In other words, the shear force of the given beam is 0 at a distance of 4.5 m from C i.e., 1.5 m from A as also known from the SFD. Hence, the bending moment of the beam has to be the maximum at this point because whenever the derivative of any mathematical equation is 0, the function is either the local maximum or the local minimum at that particular point.

So, $M_{max} = 45 \times 4.5 - 90 - 5 \times 4.5^2$ = 11.25 KN-m

Further,

In B-A, Taking a section at 'x' from C we have,

$$M_x = 45x - 90 - 5x^2$$

So, when $M_x = 0$

$45x - 90 - 5x^2 = 0$

i.e., $x^2 - 9x + 18 = 0$

i.e., $x^2 - 6x - 3x + 18 = 0$

i.e., $x(x - 6) - 3(x - 6) = 0$

i.e., $(x - 6)(x - 3) = 0$

i.e., x = 6, 3

In other words, the bending moment of the given beam is 0 at distances of 3m and 6m from C. Let us verify that from the BMD given in the Figure 6.

<u>Step-5: Draw the BMD</u>

The BMD of the given beam can be obtained by plotting the above values of bending moment in a graph as shown in the Figure 6.

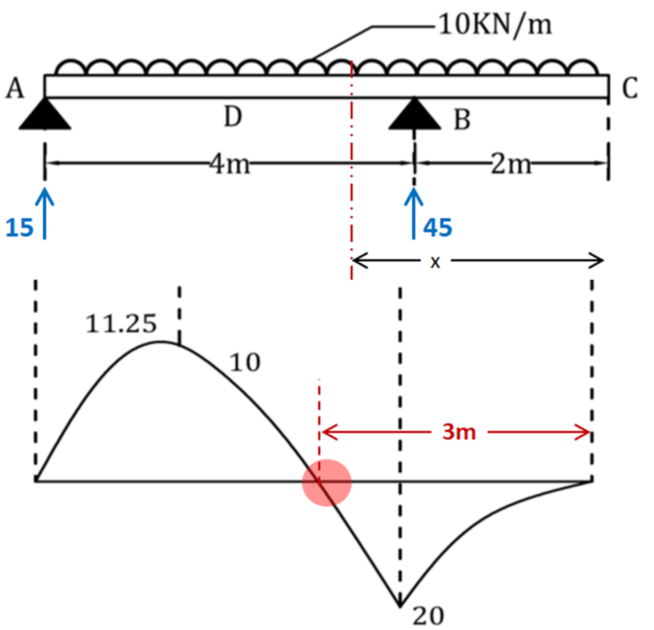


Figure 6 BMD of the given one-sided overhang beam

In the Figure 6, the point at a distance of 3m from C, which is also marked in red, is the point of contra flexure. It is so because as we know that the point of contra flexure is when the bending moment is 0 and also, the sign convention either changes from +ve to –ve or vice versa. Moreover, the change of sign occurs depending on whether the type of moment is sagging or hogging which you've already learnt in the Lecture 9. So here, for the given beam the portion of affected by sagging and that by hogging is shown in the Figure 7.

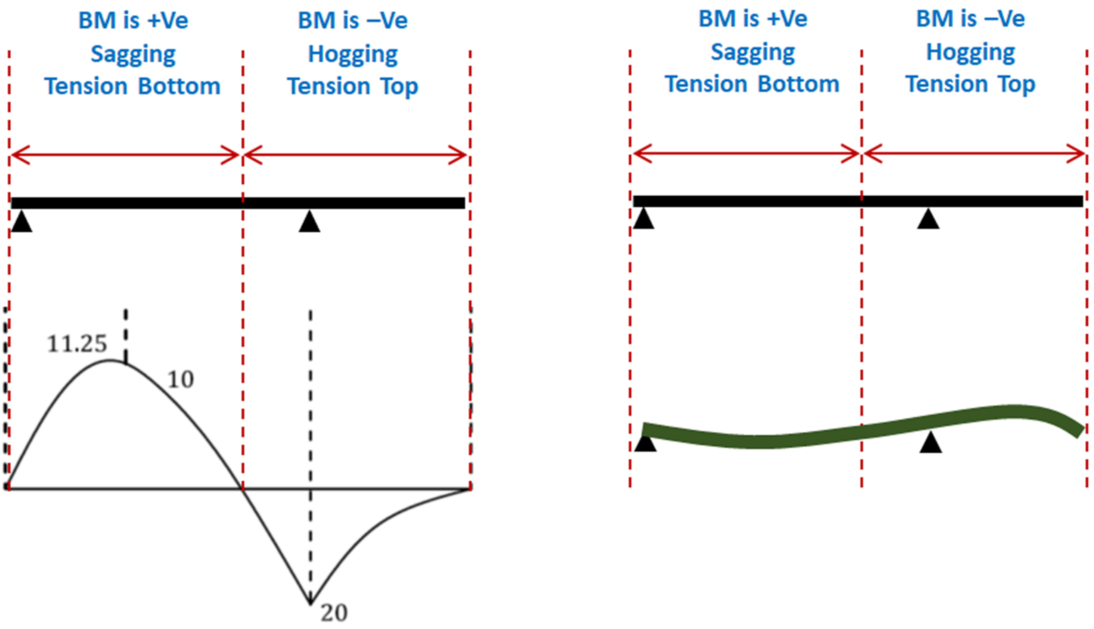


Figure 7 BMD of the beam with respect to sagging and hogging moment

Points to remember:

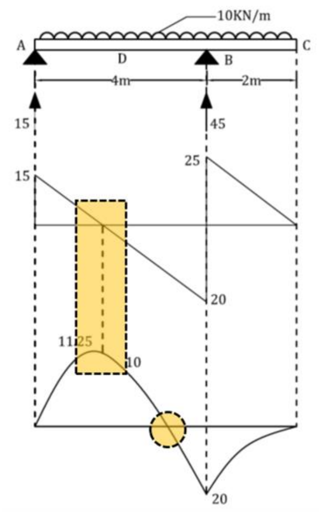


Figure 8 Points to remember

Bending Moment is Maximum at the point where the Shear Force is Zero.

Point of contra-flexure is a point on a bending beam, where bending moment is zero (changes its sign).

Case-2: Two-sided Overhang Beam

Here we have a double sided overhang beam whose supports are repeatedly moved so as to show the variation in their corresponding SFDs and BMDs.

Case - 2 (a)

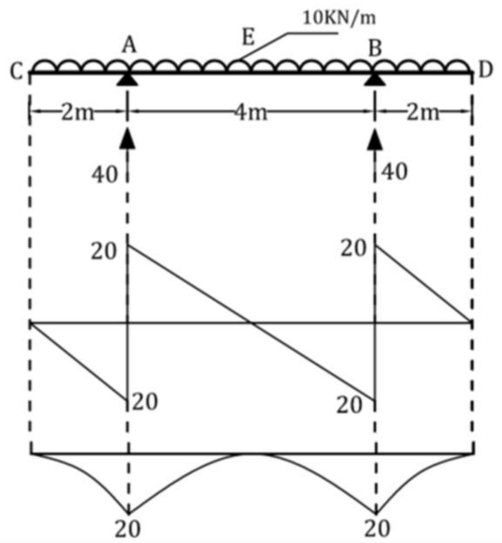


Figure 9 Case-2(a)

Overhang Length = a = 2m

Total Length = L = 8m

a/L = 2/8 = 0.25

$$M_A = M_B = -10 \times 2 \times \frac{2}{2} = -20$$

$$M_E = (40 \times 2) - (10 \times 4 \times 2) = 0$$

Case – 2 (b)

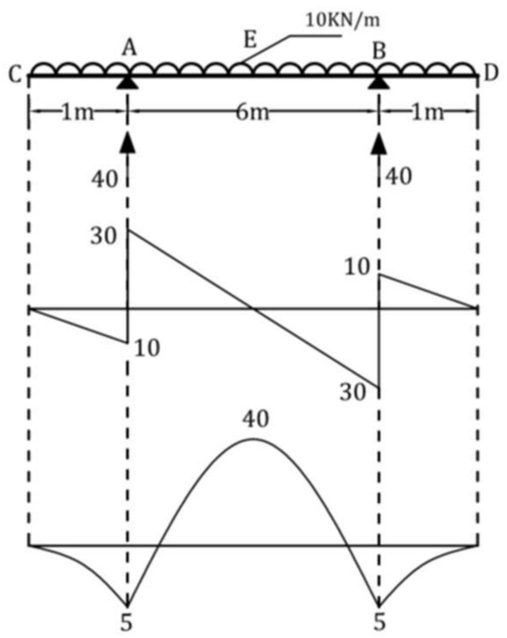


Figure 10 Case-2(b)

Overhang Length = a = 1m

Total Length = L = 8m

a/L = 1/8 = 0.125, (Less than 0.25)

$$M_A = M_B = -10 \times 1 \times \frac{1}{2} = -5$$

$$M_E = (40 \times 3) - (10 \times 4 \times 2) = 40$$

Case – 2 (c)

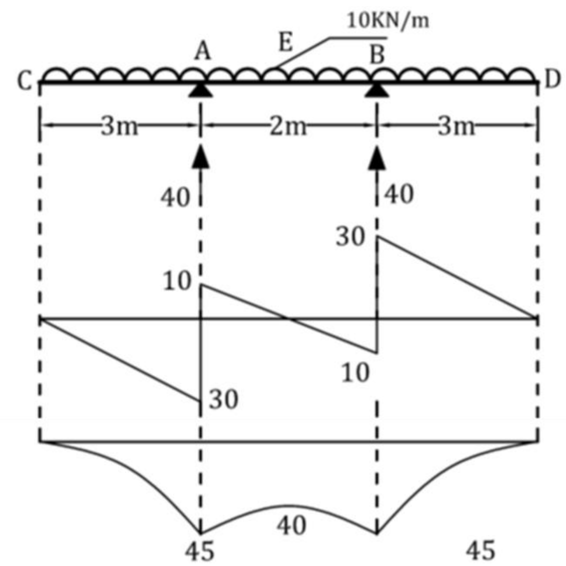


Figure 11 Case-2(c)

Overhang Length = a = 3m

Total Length = L = 8m

a/L = 3/8 = 0.375, (Greater than 0.25)

$$M_A = M_B = -10 \times 3 \times \frac{3}{2} = -45$$

$$M_E = (40 \times 1) - (10 \times 4 \times 2) = -40$$

Case – 2 (d)

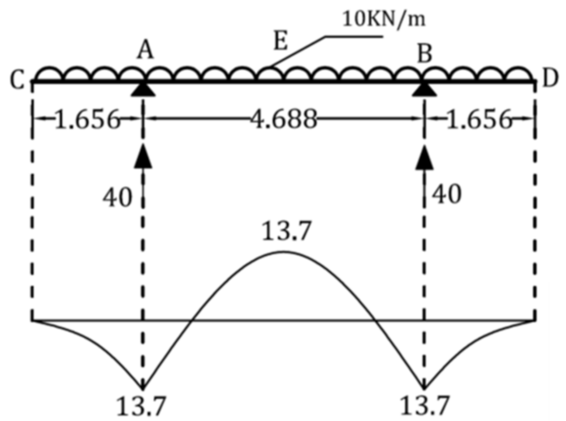


Figure 12 Case-2(d)

Overhang Length = a = 1.656m

Total Length = L = 8m

a/L = 1.656/8 = 0.207

$$M_A = M_B = -10 \times 1.656 \times \frac{1.656}{2} = -13.7$$

$$M_E = \left(40 \times \frac{4.688}{2}\right) - (10 \times 4 \times 2) = 13.7$$

Clearly, cases 2(b) and 2(d) consist of two points of contra flexures whereas the other two cases i.e., 2(a) and 2(c) consist of none.

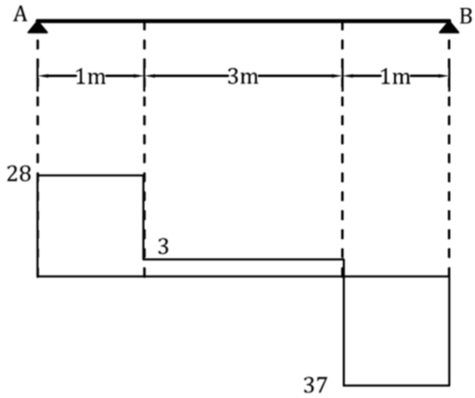
In case of the reinforcement of the beams it should be noted that the reinforcement should be provided in the tensile zones. Hence, for sagging moments the reinforcement has to be provided at the bottom whereas, for the hogging moments it should be provided at the top.

Further, in the fourth case, the maximum positive value of bending moment is equal to the maximum negative value of the same. Hence this kind of a structure is symmetrical and similar reinforcement can be used at the top as well as the bottom. Therefore, it may be concluded that when the overhang is 20% of the length of the beam the structure is beneficial.

Loading Diagram from SFD and BMD

We're discussing this particular topic purely because of academic interest as this kind of questions often comes in the competitive exams.

Q1. Given below are a beam and its SFD. Draw the loading diagram for the given beam.



Solution:

Clearly from the given SFD we know that,

$R_A = 28$ KN

Now,

Let C and D are two points at the distances of 1m and 4m from A respectively. Also, let the loads at C and D be P_1 and P_2 respectively.

Again,

From the SFD we have,

SF at C = 3 KN

i.e., $R_A - P_1 = 3$ KN

i.e., $P_1 = 25$ KN

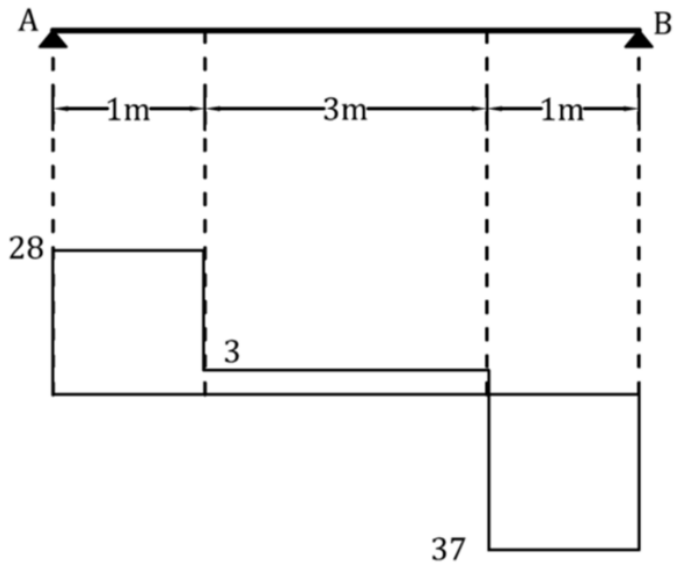
Similarly, we know that,

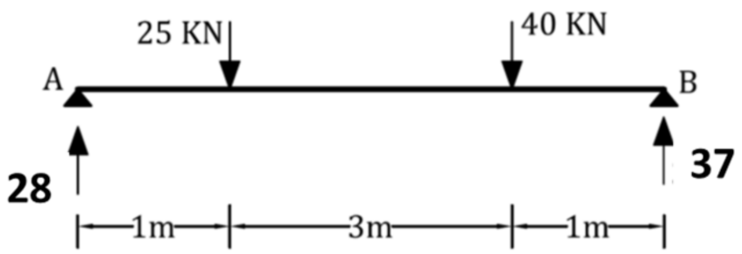
SF at D = -37 KN

i.e., 3- P_2 = -37 KN

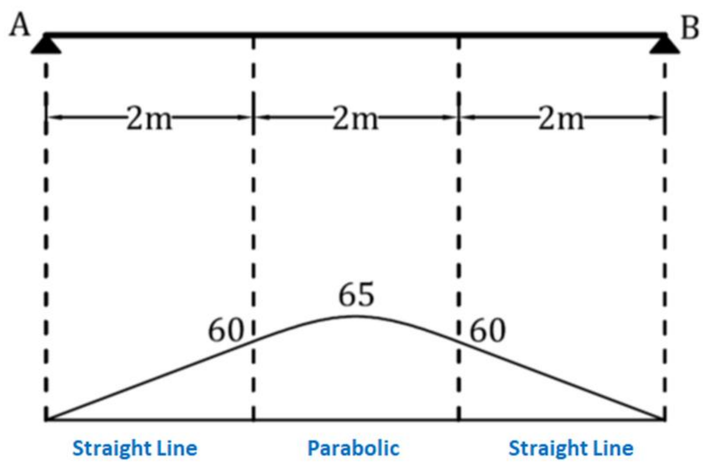
i.e., P_2 = 40 KN

Therefore, the final loading diagram can be drawn as shown in the figure below.





Q2. Given below are a beam and its BMD. Draw the loading diagram for the given beam.



Solution:

Let C and D be two points at the distances of 2m and 4m from A respectively. Let the loads acting on C and D be of P KN each. Also, since the curve of the BMD between C and D is parabolic so there must be a UDL acting here. So, let this UDL be of w KN/m.

Then,

$M_c = R_A \text{ x } 2 = 60$ KN-m

i.e., $R_A = 30$ KN

i.e., $R_B = 30$ KN (as per symmetry)

Thus from symmetry,

$(P + w.1) = 30$

i.e., $P = (30 – w)$

Now, let X be another point at the midpoint of the beam.

Then,

$M_X = (R_A \times 3) – (P \times 1) – (w \times 1 \times 0.5) = 65$

i.e., $90 - (30\text{-}w) – (0.5w) = 65$

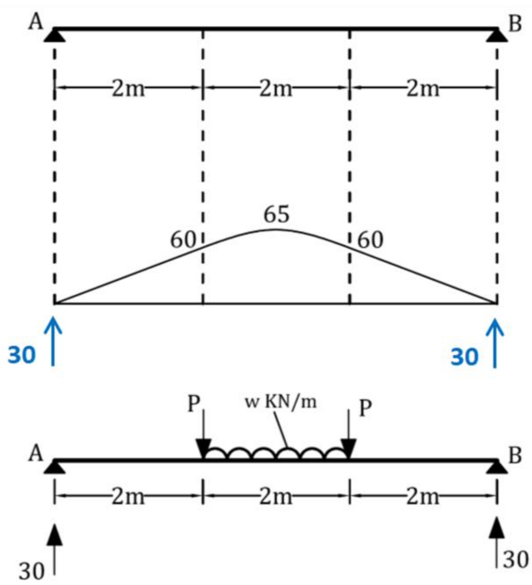
i.e., $60 + w – 0.5w = 65$

i.e., $60 + 0.5w = 65$

i.e., $0.5w = 5$

i.e., $w = 10$ KN/m

Therefore, the final loading diagram can be drawn as shown in the figure below.



References

- Engineering Mechanics by Timishenko and Young McGraw-Hill Publication
- Strength of Materials By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- Basic Structures for Engineers and Architects By Philip Garrison, Blackwell Publisher
- Understanding Structures: *An Introduction to Structural Analysis* By Meta A. Sozen & T. Ichinose, CRC Press

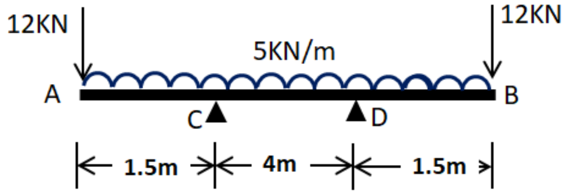
Conclusions

To sum up the lecture we can note the following:

- The differentiation of bending moment expression gives the expression of shear force.
- Point of contra-flexure is a point where bending moment is zero (changes its sign).
- The loading diagram can be established from BMD or SFD.

Homework

Q1. Draw the SFD & BMD for the overhang beam shown in the figure below.



Q2. A simply supported beam AB and its BMD is shown in the figure below. Draw the Loading Diagram and SFD. Beam is supported at A & B