Underground Mining of Metalliferous Deposits
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Lecture 37
Tributary Area Method

ROOM AND PILLAR METHOD

PILLAR DESIGN

Pillar support is intended to control rock mass displacements throughout the zone of influence of mining, while mining proceeds. Pillar design considerations that need to be taken into account include :

- Pillar load
- Strength of pillars (failure criteria)
- Orebody geometry
- Geological characteristics of a deposit
- Load-deformation characteristics of the pillar and stiffness of the loading system

TRIBUTARY AREA:

Lunder and Pakalnis (1997) proposed that it is possible to reconcile the analysis of pillar stress using the tributary area method.

- Tributary area method is used to determine primarily, the load on an individual pillar.
- This method is based on a force balance between the load carried by the pillar and the tributary area conveying load to the pillar.
- Each pillar supports the volume of the rock over an area(pillar cross sectional area and the portion of the room area).

For rectangular shaped pillar,

Tributary area for pillar 'A' = Room area – pillar area

$$= (a + \frac{c}{2} + \frac{c}{2}) \frac{c}{2} + \frac{c}{2}) \times (\times (b + \frac{c}{2} + \frac{c}{2}) \frac{c}{2} + \frac{c}{2}) - ab$$

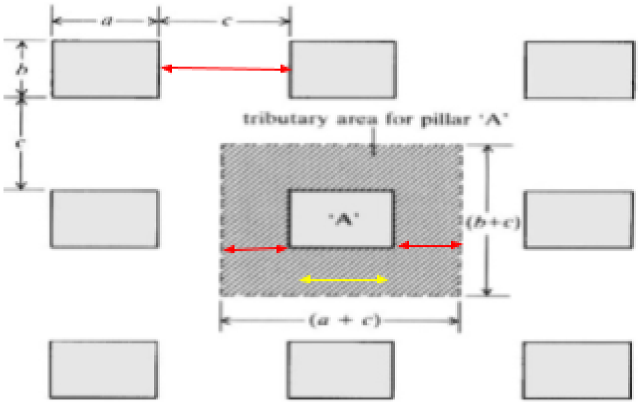
=(a+c)(b+c)- ab **[**length=a, width=b, c=span of the room**]**

For Square shaped pillar,

Tributary area $= (a+c)^2 - a^2$ $(a+c)^2 - a^2$ [length= width=a; c= span of the room]

For irregular shaped pillar,

Tributary area $=$ **Rock column area − pillar area**
Rock column area − pillar area



PILLAR STRESS:

Traditional strength-based pillar design requires estimates of pillar stress and pillar strength.

We know that, Pillar stress is expressed as

$$\boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}(w_o + w_p)}}{\boldsymbol{w_p}}$$

where, w_o w_o = span of the room

w_p w_p = span of the pillar

$(w_o + w_p)$ $(w_o + w_p)$ = area which is tributary to the representative pillar

σ_p σ_p = average axial pillar stress

p_{zz} p_{zz} = vertical normal component of the pre-mining stress field

In-situ vertical normal stress (P_{zz}) at a sub-surface point is given by:

$p_{zz} = \boldsymbol{\gamma z}$

where,

γ γ = unit weight of rock (kN/m^3)

z = depth of mining horizon (m)

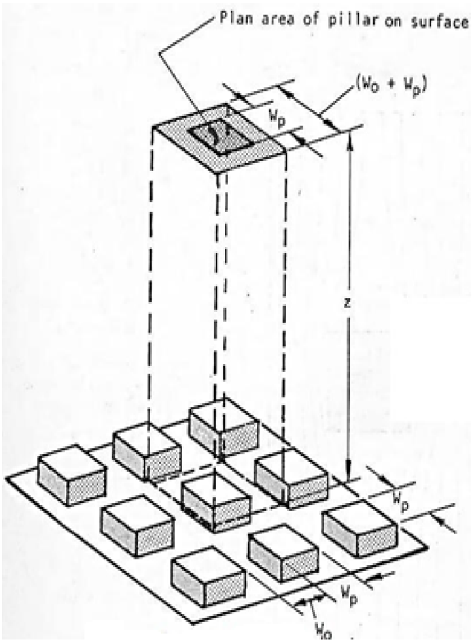


Figure2. Plan area of pillar on the surface

Considering the requirement for equilibrium of any component of the structure under the internal forces and unit thickness in the anti-plane direction, the free body diagram is shown in Figure 3.

The force balance on the pillar section is as follows:

$$\sigma_p w_p = p_{zz}(w_o + w_p) \ \sigma_p w_p = p_{zz}(w_o + w_p)$$

$$\boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}(w_o+w_p)}}{\boldsymbol{w_p}} \ \boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}(w_o+w_p)}}{\boldsymbol{w_p}} \qquad \text{.......}(i)$$

or, $\boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}[(a+c)(b+c)]}}{\boldsymbol{ab}} \ \boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}[(a+c)(b+c)]}}{\boldsymbol{ab}}$ [i.e., for rectangular or irregular shaped pillar length=a, width=b]

Also, $\boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}(a+c)^2}}{\boldsymbol{a^2}} \ \boldsymbol{\sigma_p} = \frac{\boldsymbol{p_{zz}(a+c)^2}}{\boldsymbol{a^2}}$ [i.e., for square shaped pillar; pillar length=width=a]

where,

$w_o \ w_o$ = span of the room

$w_p \ w_p$ = span of the pillar

$(w_o + w_p) \ (w_o + w_p)$ = area which is tributary to the representative pillar

σ_p σ_p = average axial pillar stress

p_{zz} p_{zz} = vertical normal component of the pre-mining stress field

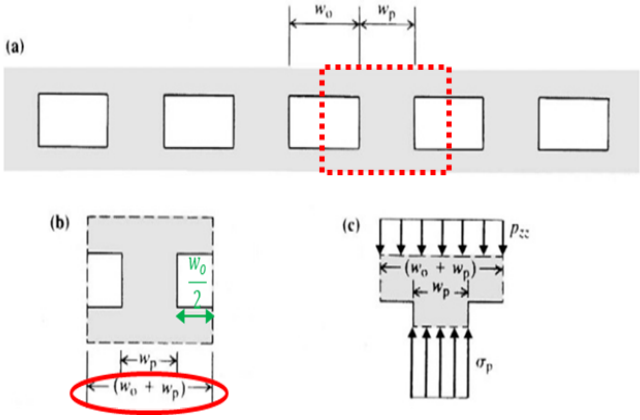


Figure 3. Tributary area method: Free body diagram of the pillar.

EXTRACTION RATIO

Extraction ratio (r) is the ratio of the area mined to the total area of the orebody.

Extraction ratio (r) $= \frac{\textbf{Area mined}}{\textbf{Total area of orebody}} \frac{\textbf{Area mined}}{\textbf{Total area of orebody}}$

Now, again considering the representative element of the ore body illustrated in figure 7(a) and 7(b), for which w_o and $(w_o + w_p)$ w_o and $(w_o + w_p)$ are the area mined and total area of illustrated orebody respectively, the extraction ratio (r) is expressed as

$$\text{Extraction ratio (r)} = \frac{w_o}{(w_o + w_p)} \text{(r)} = \frac{w_o}{(w_o + w_p)}$$

Or,

$$r - 1 = \frac{w_o}{(w_o + w_p)} - 1 \; r - 1 = \frac{w_o}{(w_o + w_p)} - 1$$

$$r - 1 = \frac{w_o - w_o - w_p}{(w_o + w_p)} \; r - 1 = \frac{w_o - w_o - w_p}{(w_o + w_p)}$$

$$r - 1 = \frac{-w_p}{(w_o + w_p)} \; r - 1 = \frac{-w_p}{(w_o + w_p)}$$

such that,

$$1 - \text{r} = \frac{w_p}{(w_o + w_p)} \; 1 - \text{r} = \frac{w_p}{(w_o + w_p)} \qquad \ldots\ldots(ii)$$

Also,

$$\text{r} = 1 - \frac{w_p}{(w_o + w_p)} = 1 - \frac{w_p}{(w_o + w_p)} \qquad \ldots\ldots(iii)$$

Putting the dimensions of the pillar (a and b for rectangular or irregular shaped pillar) and span of the room (c) in eqn (iii)

$$r = 1 - \frac{ab}{(a+c)(b+c)} \quad r = 1 - \frac{ab}{(a+c)(b+c)}$$

***Therefore,* Extraction ratio** $$(r) = \frac{(a+c)(b+c) - ab}{(a+c)(b+c)} \qquad (r) = \frac{(a+c)(b+c) - ab}{(a+c)(b+c)}$$

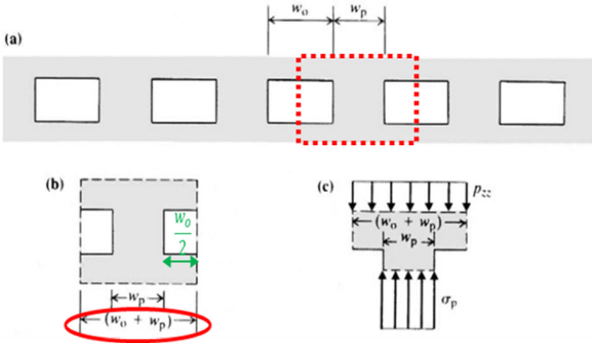


Figure 4. Representative element of the ore body

In case of square pillars,

$$r = 1 - \frac{w_p}{(w_o + w_p)} = 1 - \frac{w_p}{(w_o + w_p)}$$

$$r = 1 - \frac{a \times a}{(a+c) \times (a+c)} = 1 - \frac{a \times a}{(a+c) \times (a+c)}$$

Extraction ratio $$(r) = 1 - \frac{a^2}{(a+c)^2} \quad (r) = 1 - \frac{a^2}{(a+c)^2}$$

where, a= pillar length= pillar width

c= span of the room

RELATION BETWEEN EXTRACTION RATIO AND PILLAR STRESS:

Substituting the equation(ii) in eqn (i), we get the relationship between extraction ratio and pillar stress

$$\sigma_p = p_{zz} \times \frac{1}{(1-r)} \quad \sigma_p = p_{zz} \times \frac{1}{(1-r)} \qquad \ldots\ldots(iv)$$

where,

σ_p σ_p = average axial pillar stress

p_{zz} p_{zz} = vertical normal component of the pre-mining stress field

r = extraction ratio

For rectangular or irregular shaped pillar, $\sigma_p = p_{zz} \times \frac{1}{(1-r)} = p_{zz} \times \frac{(a+c)(b+c)}{(ab)}$

$$\sigma_p = p_{zz} \times \frac{1}{(1-r)} = p_{zz} \times \frac{(a+c)(b+c)}{(ab)}$$

For square shaped pillar, $\sigma_p = p_{zz} \times \frac{1}{(1-r)} = p_{zz} \times \frac{(a+c)^2}{(a^2)}$

$$\sigma_p = p_{zz} \times \frac{1}{(1-r)} = p_{zz} \times \frac{(a+c)^2}{(a^2)}$$

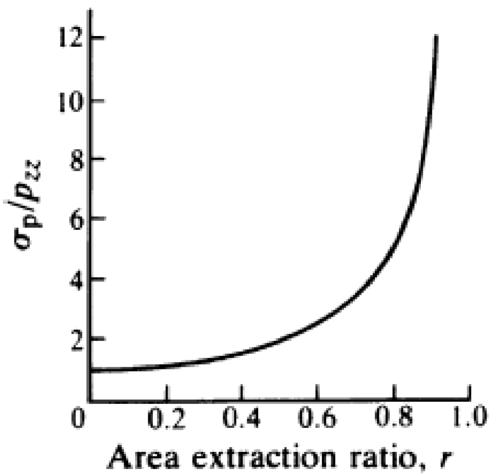


Figure5. Variation of pillar stress concentration factor with area extraction ratio

PILLAR STRENGTH:

Hardy and Agapito (1977), noted that the effects of pillar volume and geometric shape on strength S are usually expressed by an empirical power relation of the form

$$S = S_o v^a \left(\frac{w_p}{h}\right)^b$$

$$= S_o v^a R^b = S_o v^a R^b \qquad \ldots\ldots(v)$$

S = strength of a pillar (Mpa)

S_o S_o = strength parameter representative of both the orebody rock mass and its geo-mechanical setting

v v = pillar volume

w_p w_p = pillar width

h h =pillar height

R = pillar width/height ratio

a and **b** = reflect geo-structural and geo-mechanical conditions in the orebody rock.

An alternative expression of size and shape effects on pillar strength is obtained by recasting equation (v) in the form

$$\mathbf{S} = \boldsymbol{S_o\, h^{\alpha}\, w_p{}^{\beta}}\ \mathbf{S} = \boldsymbol{S_o\, h^{\alpha}\, w_p{}^{\beta}} \qquad \text{……. } (vi)$$

For pillars which are square in plan, the exponents α, β , a, b in equations (v) and (vi) are linearly related, through the expressions

$$a = \frac{1}{3}(\alpha+\beta)\ a = \frac{1}{3}(\alpha+\beta), \quad b = \frac{1}{3}(\beta-2\alpha)\ b = \frac{1}{3}(\beta-2\alpha)$$

PILLAR STRENGTH: Various pillar strength equations:-

Overt-Duvall/Wang Formula : $\boldsymbol{S_p = S_1(0.788 + 0.222\frac{w_p}{h})}$ $\boldsymbol{S_p = S_1(0.788 + 0.222\frac{w_p}{h})}$

Holland-Gaddy formula : $\boldsymbol{S_p = \frac{K\sqrt{w_p}}{h}}$ $\boldsymbol{S_p = \frac{K\sqrt{w_p}}{h}}$

Holland Formula: : $\boldsymbol{S_p = S_1\sqrt{\frac{w_p}{h}}}$ $\boldsymbol{S_p = S_1\sqrt{\frac{w_p}{h}}}$

Bieniawski Formula : $\boldsymbol{S_p = S_1(0.64 + 0.36\frac{w_p}{h})}$ $\boldsymbol{S_p = S_1(0.64 + 0.36\frac{w_p}{h})}$

Salamon-munro formula: : $\boldsymbol{S_p = k \times h^{\alpha} \times w_p{}^{\beta})}$ $\boldsymbol{S_p = k \times h^{\alpha} \times w_p{}^{\beta})}$

where,

k = 7.176 7.176

α α = -0.66

β β = 0.46

h = height of the pillar

w_p w_p =width of the pilar

FACTOR OF SAFETY:

The **Factor of safety** for the pillar is then calculated by dividing pillar strength by pillar stress.

$$FS = \frac{Strength}{Stress}$$

What constitutes an acceptable safety factor depends on the tolerable risk of failure.

Safety factor value:

2 - typical for pillars in main development headings or panels during advance mining.

1.1 to 1.3 - typical for panel pillars after retreat mining.

<1.0 - Possible for panels where pillar failure is the eventual intent.

CONVERGENCE MONITORING:

Sometimes when pillars are partially removed without proper planning, or when pillars that are too small are left after first-pass mining, the pillars begin to break up and serious convergence accelerates uncontrollably. One of two things must be done to save the area:

(1) Massive pillar reinforcement, if there is time before collapse;

(2) Massive backfill in the entire area

Complete network of convergence stations should be installed throughout the affected area.

- Convergence of 0.0254 mm (0.001 in.) per month is not significant.
- Convergence of 0.0762 mm (0.003 in.) per month indicates a serious problem, but is controllable with immediate action.
- Convergence of 0.1778 mm (0.007 in.) per month indicates that acceleration is getting out of control and the area may be lost.

PILLAR DESIGN:

- Interaction between the pillar ends and the country rock results in heterogeneous, triaxial states of stress in the body of the pillar, even though it is uniaxially loaded by the abutting rock.
- The adjacent stopes are separated by a barrier pillar, similar to the division of panels in a coal mine
- The maximum extent of any collapse is then restricted to that stope pillars itself

Problem: Consider the following example.

Q: A 2.5 m thick horizontal orebody is located at a depth of 80 m, with the rock cover having a unit weight of 25 kN m^{-3} . An initial mining layout is based on 6.0 m room spans and 5.0 m square pillars, with the full orebody thickness of 2.5 m being mined.

Find the factor of safety for the pillar.

Solution: The tributary area analysis of this prospective layout is as follows:

(a) **Pre-mining stress**

$$P_{zz} = 80 \times 25 \text{ kPa} = 2.0 \text{ MPa}$$

(b) **Average axial pillar stress**

$$\boldsymbol{\sigma}_p = 2.0 \times [(6.0 + 5.0)/5.0]^2 \text{ MPa} = 9.68 \text{ MPa}$$

(c) **Pillar strength**

$$S = 7.18 \times 2.5^{-0.66} \times 5.00^{.46} \text{ MPa} = 8.22 \text{ MPa}$$

(d)Factor of safety

$$F = \frac{8.22}{9.68}\frac{8.22}{9.68} = 0.85$$

The low factor of safety provided by this prospective layout indicates that redesign is necessary to achieve the required factor of 1.6.

The options are:

(i) to reduce the room span, thereby reducing the pillar stress level

(ii) to increase the pillar width

(iii) to reduce the pillar (and mining) height.