Network Analysis for Mines and Mineral Engineering
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Lecture - 09
Understanding the slack

Let me welcome you to the 9th lecture of NPTEL online certification course Network Analysis for Mines and Mineral Engineering. So, our topic of this lecture is Understanding the Slack.

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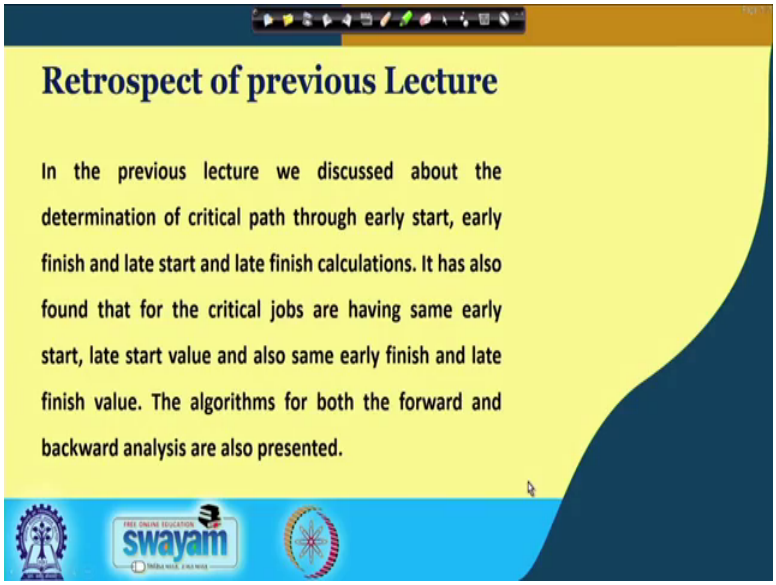

So, in let us retrospect our previous lecture; in previous lecture we have already discussed about the determination of critical path and we have found that for identifying the critical path we need to calculate early start, early finish and late start, later finish for each and every activities. After completion of these calculations, we have to find out for which jobs the early start, early finish is exactly equal to the late start, late finish time.

Then those jobs are considered as the critical job and those jobs in that critical jobs those path which is basically directing from the starting node to the end node is called critical path. And from by that way we can identify a critical path which has to be taken care of with most importance.

So, this is one way where we can identify the critical path and critical jobs; critical jobs are identified using early start, early finish, late start, late finish activities. And we have also discussed the algorithm for the forward and backward analysis. In forward analysis basically; in forward analysis we discuss about the early start, early finish calculation; early start, early finish this calculation is discussed in the forward analysis. And, in backward analysis we calculate the late start and late finish and those are already presented in the previous lectures.

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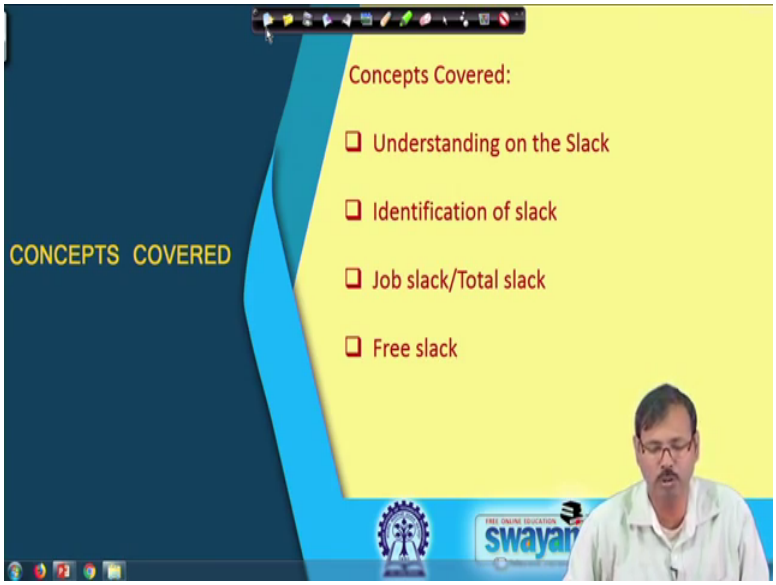

So, let us see what are the concept we will cover in this lecture. In this lecture we will understand what is slack, we will get the algorithm for identifying the slacks, then we will calculate for the slacks which is called job slack or total slack. And, we will also calculate another slack which is called free slack we will understand this as a definition of this slacks and how to identify them and calculate them their algorithm will be also discussed in this lecture.

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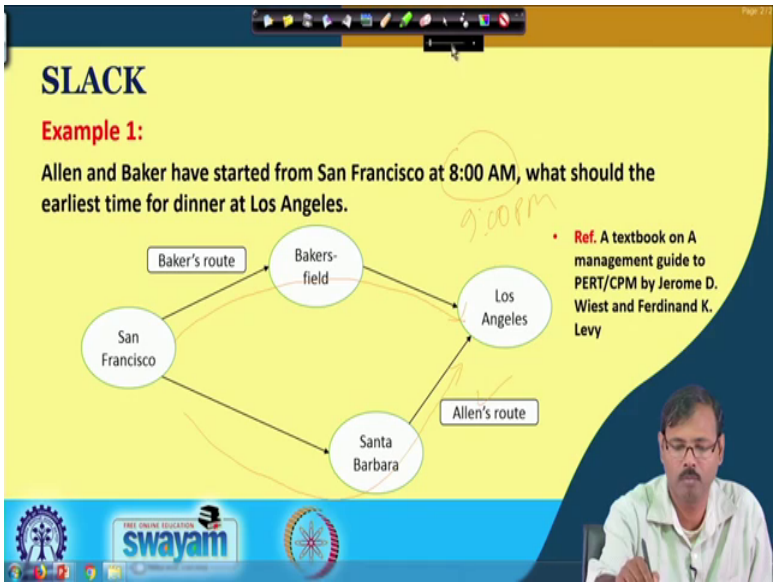

So, for understanding this let us have one example because, it is better to understand this mathematical problem with an example; solving one example then our understanding will be based about the algorithm of this. So, let us consider one example which is already discussed about the routes of Allen and Baker from San Francisco to Los Angeles.

Bakers route is route 1 which is route 1 which is from the Baker field and Allen's route is through the Santa Barbara. And in this problem we have seen that they both are starting from San Francisco at 8 AM and they are fixed a meeting, we have identified the earliest possible time for fixing the meeting is the 9. So, this is the 9 which is the earliest possible time to fix the meeting.

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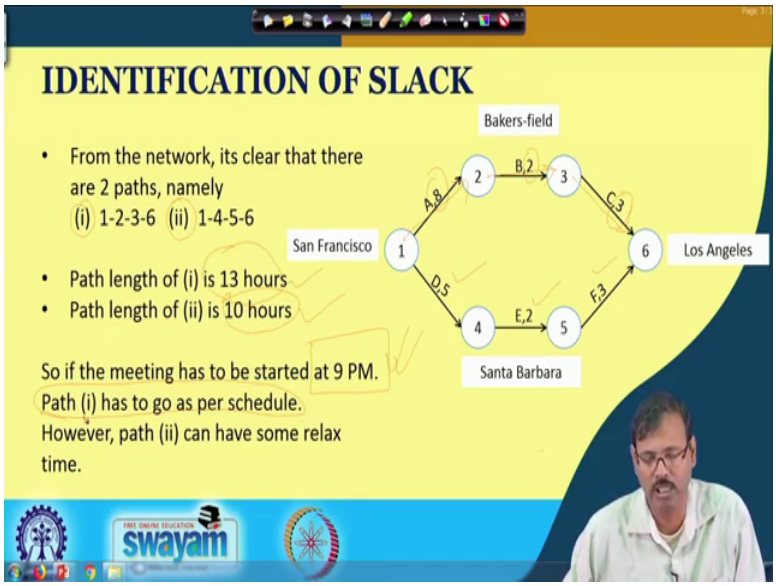

So, let us solve this problem. In this problem we have find out 2 paths: path i and path ii. Path i we have already discussed that is the route through Baker field which is taken by the Baker, the second one is through Santa Barbara which is taken by the Allen. And, both the paths are having three components that is 1 first job is A which is the travel of Baker from San Francisco to Baker field, then having a lunch in the Baker field and then third one is the Baker field to Los Angeles trouble the similar is therefore, the Allen's route

And it has been found that the path i is having a total duration of 13 hours because, the travel time of this one is 8 hours, this is lunch time is 2 hours and another next travel time is 3 hours. So, the total travel time total time required for complete this route is 13 hours and for the second one it is 10 hours that is 5 hours plus 2 hours plus 3 hours is 10 hours.

So, that is why we have found that the earliest possible time to start the meeting is at 9 AM and for achieving this target that the to strict be strict with the scheduled time of 9 PM meeting the Baker must have to follow the path i. And, he must drive his car with the desired speed, he has to take his lunch with the optimum time of 2 hours and again he has to drive his car with the desired speed to from Baker field to Los Angeles to strict be strict with the scheduled time for the meeting of 9 PM. But Allen has some relaxation in his part because he can complete his total travel including 2 hours lunch at San Francisco

even if that it can be completed in 10 hours; that means, there are some relaxation available for the Allen in the path ii.

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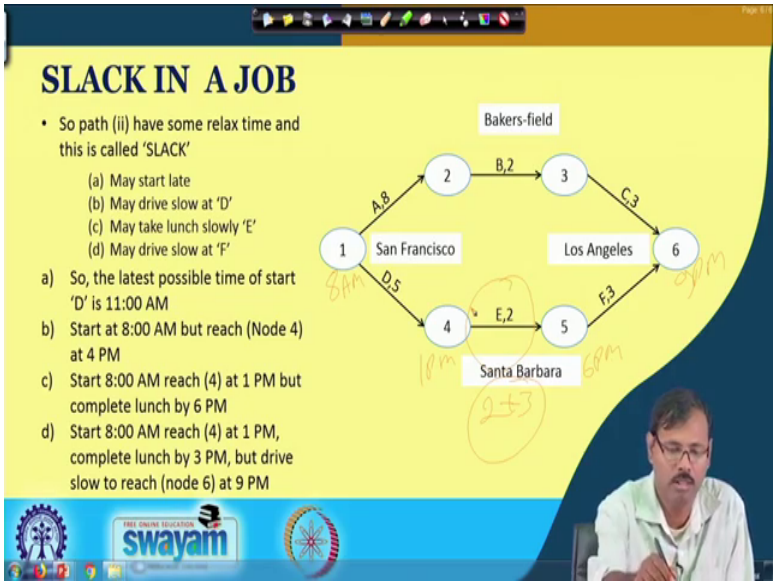

In fact, these relaxations are termed as the slack; that means, Allen is having some slack hours in his jobs despite those slack hours he can address the meeting attend the meeting at 9 PM without selling it. So; that means, he is having some options some slacks in his decision making and it can be seen that this relaxation it can be seen that relax time is called slack and this is the slack for the jobs.

And this slacks allow this slacks allow Allen the with this options that either he may start late or if you see. So, he can start instead of 8 AM he can start latest by 11 AM, then also if you start here at 11 AM he will reach at this point at 4 PM, he can have to 2 hour lunch and he can start from Santa Barbara at 6 PM and then also he can reach Los Angeles at 9 PM and can attend the meeting successfully.

So, he can start late is the first option, second option that he can drive slow; that means, in this path from San Francisco to Santa Barbara instead of cover it in 5 hours, he can cover it in 8 hours also. So that means, he can drive slow between San Francisco to Santa Barbara up to 3 hours drive slow then also he can able to attend the meeting in case of that he will start here; he will start here at 8 AM.

But instead of reaching here at 1 PM he may reach here at 4 PM at 4 PM then also he will reach a take the lunch at 6 PM and he can reach here at 9 PM. So; that means, he may drive slow in the option two in between San Francisco to Santa Barbara then also he can successfully attend the meeting third option is that he can start here at 8 reach here at 1. And, instead of having 2 hours lunch he can spend more time and start here at 6 PM; that means, 2 plus 3 hour time will be spend at this position may be which is relative he can spend.

And just by starting here at 6 then also he can reach here at 5 PM; that means, here he is having some slack hours at this point he can enjoy that one or in the last point he can start here at 8 reach here at 1 starting here from 3, but drive this portion slowly and he reached here by 9. So, these are the possible slacks available with all the activities associated with Allen and Allen is basically enjoying a 3 hour slack in its total path. So that means, path i does not have any slack, but path ii is having some slack hours and he can enjoy those slack hours may be in job D may be in job E may be in job F as for his own wish.

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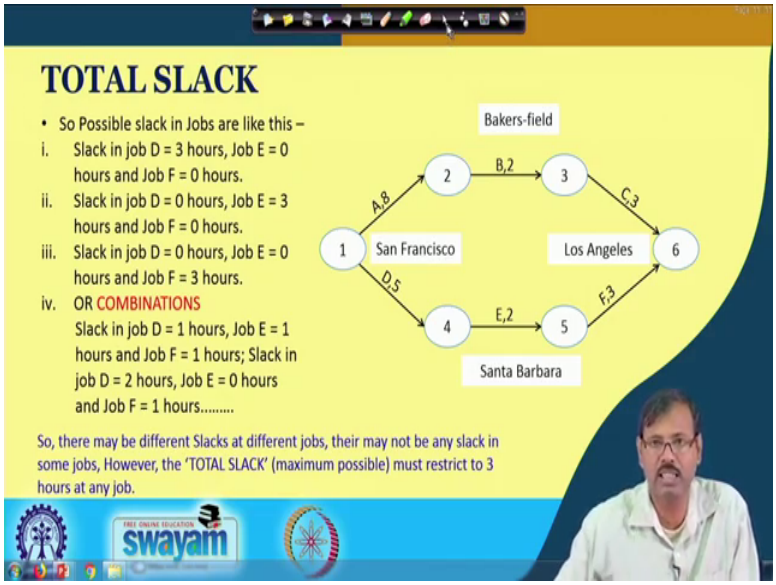

So, now, we can see this options can be slack in job D for 3 hours slack a in job E for 3 hours slack in job F for 3 hours or there may be combinations we can have 1 hour slack here, 1 hour slack here, 1 hour slack here, or may be 2 hour slack here, 0 hour slack here,

1 hour slack here. So, these combinations are also possible and he can enjoy those possible combinations of slack hours in his path.

So, basically there may be different slacks at different jobs there my not be any slack in some jobs. So, this jobs are does not have any slack the total slack is the maximum possible, but restricted to 3 hours at any job for this path ii. So, maximum possible slack for this path ii is 3 hours that may be assigned to any job. So, that is the restriction without failing the completion time of this we may think of enjoying the slack hours in some activities.

So, basically slack is the relaxation hours available to some activities which does not differ the completion date which does not differ the completion date, but the job activities can be differed a little bit to completion without affecting the completion date of the network.

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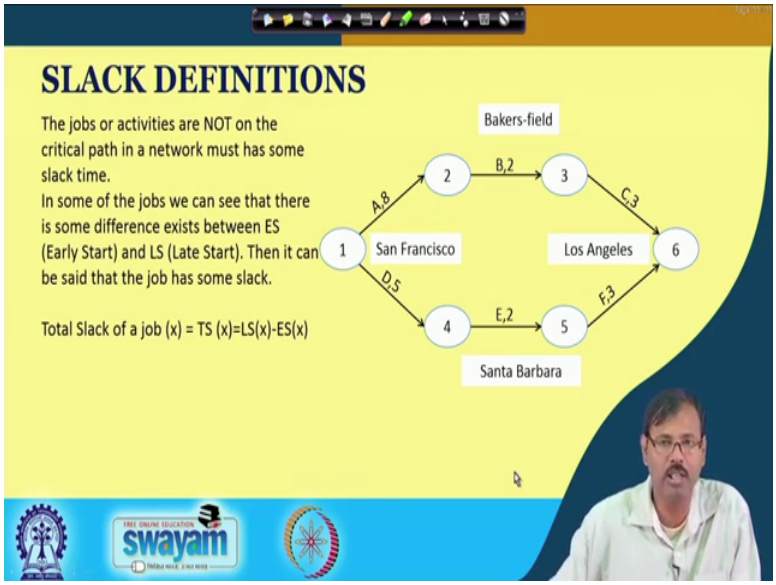

So, basically jobs are some jobs are having some slack hours that can be enjoyed. And the moment we talk about slack it is called total slack total slack of a job is basically calculated by the term late start of activity x minus early start of the activity x gives us the total slack of possible in that activity x.

So, jobs or activities are not on the critical path in a network must has some slack time. Some of the jobs we can see that there is some difference exist between the early start

and late start time and the difference is basically gives us the idea about the slack hour possible in that particular job.

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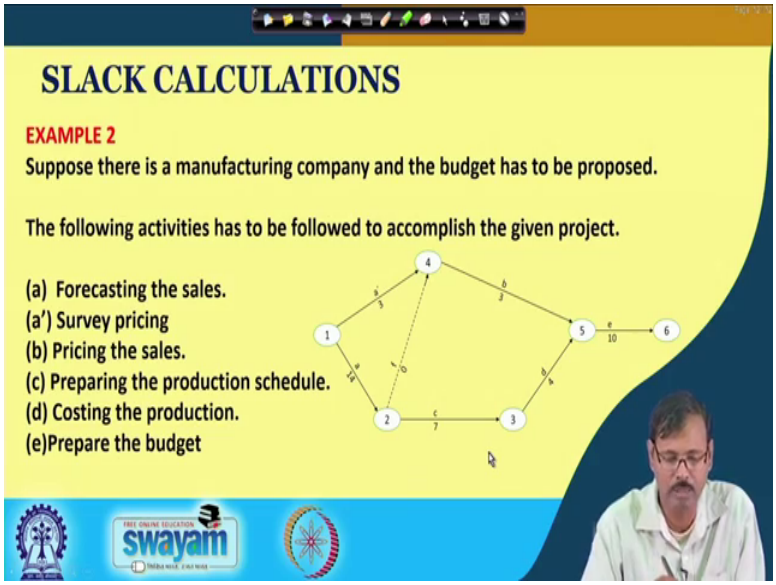

So, let us solve one big problem. So, that this can be easier for us this problem is also known to us we have already discussed in our previous lecture. This is the problem of a one budget preparation for a new product to be launched which is already discussed and these are the components forecasting of sales, survey pricing, pricing the sales, preparing the production schedule, costing the production and prepare the budget for that particular product.

So, the network is already discussed in our previous class and it can be found that while we are developing a activity on arrow diagram in that case for this we need a dummy job to complete the network that is F is presented here. So, this is the first activity a forecasting of sales, this is the survey of pricing, this is the preparing of production schedule, this is the pricing of sales and this is the costing and this is the preparation of the budget the final activity. So, let us calculate the early start early finish time late start late finish time for this in that work hence from that we can easily identify the slack.

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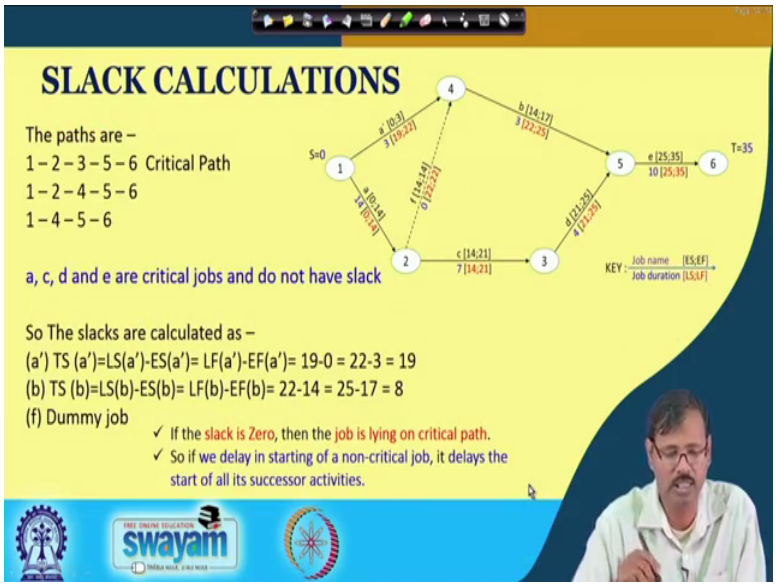

So, this is already computed in the last class I have taken this example. So, you can see I have used different color code. So, job activities names are given in black color here and early start early finish time is given under third bracket in black color in the top. And job duration is given in the blue color in the bottom part and in the red color late start late finish times are given in under third bracket in the lower part of a arrow of an arrow.

So, this is the way we have presented you may have your own criteria, but this is easier to understand for. So, you have used different color code for presenting this one for yourself. So, you can see there are 3 path possible here; the 1st path is this one, the 2nd path is this one and the 3rd path is this one and it has been found that the 3rd path here it is mentioned in the first the is the critical path.

Because, the early start early finish time is equal to the late start late finish time of this particular job. So, if you see the early start early finish time for this job is same, early start early sorry early start early finish time and late start late finish time for this job is same, early start early finish late start late finish time is same here also, here also, here also.

So, these 4 jobs are basically critical or the critical jobs where no slack can be available and we have we should be must be we must be very very careful for completing this job on time because there is no relaxations. So, the path comprising by this jobs are showing

us the critical path. So, this is the critical path and in critical path as per our formula if you deduct the late start minus early start time it will be 0 for all the cases.

So, 25 minus 25, 21 minus 21, 14 minus 14, 0 minus 0, gives us slack is equal to 0. Similarly, if we go for late finish minus early finish late finish minus early finish late finish minus early finish is also 0 for all the cases. So, this is the critical path and the slack available to the critical path is always found 0. Now, let us determine the slack for the other paths. So, let us calculate it for the job a dot for job a dot total slack a dot is LS minus LS a dot minus ES a dot means late start a dot minus early start a dot that is equal to late finish a dot minus early finish a dot and that is coming to 19 here.

Similarly, we can determine the total slack in job b also and it has been found the total slack of job b is 8 hours or 8 days whichever it is. And f is dummy job though f is also having some slack hours, but as it is not existing job we are not calculating slack for the dummy jobs that is not required unless and until we need to calculate it for other types of slack.

So, the slack hours available with job a and job b has been found nineteen and 8 8 respectively. So, let us find out slack is zero if the job is lying on the critical path and if we delay in staring of a noncritical job that delays the start of the all successor jobs. Let us see how it is possible? Say as we can see from the late start time if we start job a dot if we start job a dot at 19th hour 19th day then also there will not be any delay in the total schedule.

But, if we are starting it an 19th day then the starting of job b is not possible on 14th day it is it will be differ to twenty second days which is the latest possible start time for the job b. So that means, it is slack of a dot job is basically effecting the start of the job b, but if you look into this in little bit other way suppose instead of 19 hour 19th day starting job a dot suppose you are carrying out its on 5th day.

So, you have started your a dot job and 5th day then the age of job a dot job will be finish by 8ghth say. But see the earliest possible time for starting job b is 14; that means, if we are utilizing the highest portion of the total slack available with the a dot job that is differing the earliest possible time earliest possible starting time of the job b which is in successor.

But, if we start the a dot job on the 5th day it is not effecting the earliest possible start time of job b; that means, there is a portion of the total slack of 19 hour which is available with a dot job if we utilize that slack then also the earliest possible slack time of the subsequent jobs or successor jobs will not be effected.

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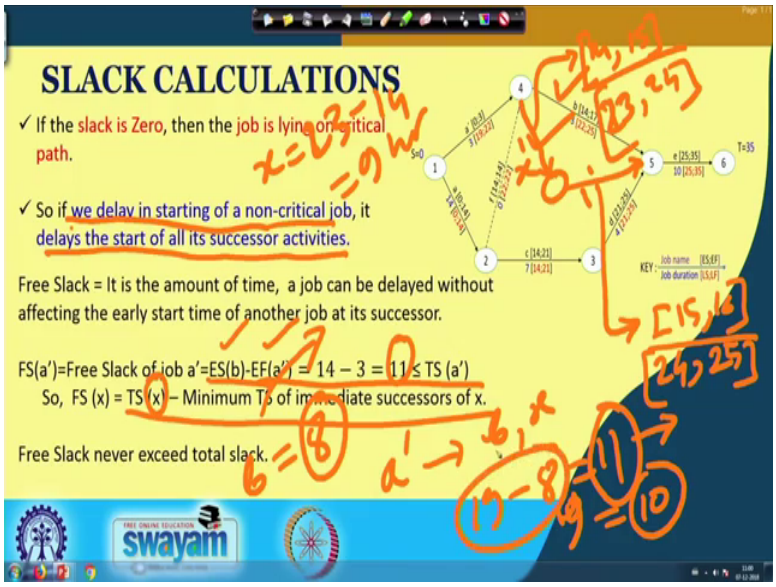

So, you can understand that we if we have delay in starting of a noncritical job it delays the start of all its successor activities, but in some cases where free slacks are existing then that free slack can be enjoyed which is not effecting the starting of the activities of is successor jobs.

So, suppose the formula is already given to you understand that this is the formula where early start of the successor job and minus the earliest finished of the previous job gives us the idea about the free slack. It can be also calculated by using the formula total slack of that particular job x minus the minimum of the total slack of all its immediate successors is basically gives us the idea at about the free slack.

So, suppose let us consider we are having another job here we are having another job here which is having a time duration requirement of 1 hour or 1 day. And if we calculate its early start early finish time then it can be calculated us for this job let me write here, it can be calculated as 14 hours and 15 early start and early finish.

For this job it is 15 and 16 early start early finish time and that that is not effecting the job e though the these two jobs are added and if we consider the late start late finish time. So, the late finish time of this one is 25 so, it you can consider the 24 is the late start time and for this case the late start time is 24 and the late finish time is 23.

So; that means, if we are considering two successor jobs of a dot is one is b another is new one say let us consider x. So, this a b job and x job are the immediate successors of a dot job and the slack available with b is already calculated b job is having a total slack of 8 hours 22 minus 14 and x job is having total slack of 23 minus 14 is equal to 9 hour. So; that means, if the minimum of the total slack of the immediate successors is the 9 hour and 8 hour minimum is 8 hour. So, if we subtract total slack of x 19 hour minus 8 hour that is coming 11 hour or 11 days. So, the free slack enjoyable is 11 days or 11 hour.

Now, let us see what will happen if we deduct 9 hour from the total slack of this then it will become 10 9 19 minus 9 will become 10. That means, we it is considered that a 10th hour if we start that is not effecting the early start, but that is not true because, if we start at 11th hour then also it is not effecting the early start time of b job and x job which are being started at 14th hour or 14th days.

So; that means, the formula will be total slack of job; total slack of job x minus the minimum time minimum total slack of the all its immediate successors are giving us the free slack or in otherwise early start time of its successive activities. You see the early start time of all the successive activities will remain same it is 14 for all and the early finish time of that job a dot that is 3. So, this 14 minus 3 is giving us the free slack time.

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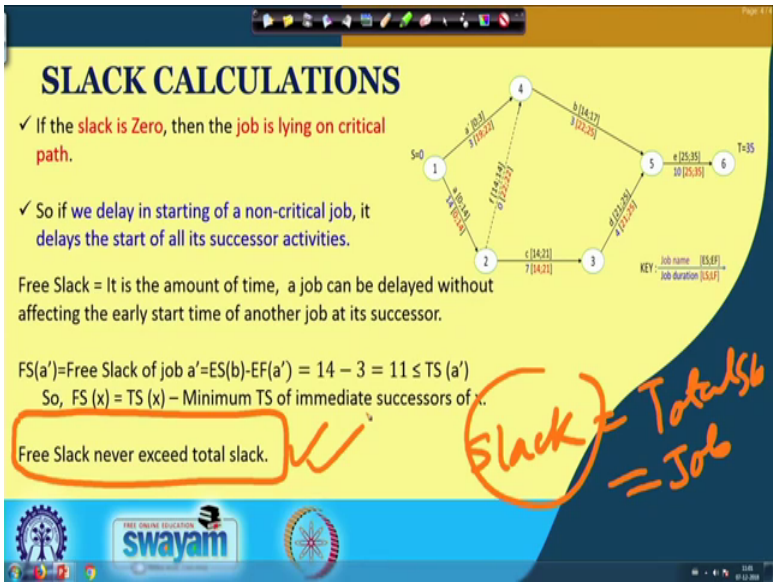

So, this is now easily understood and free slack one condition is very very important free slack never exceeds the total slack it is a particular portion of the slack; it is a particular portion of the slack or total slack. You remember one thing if it is only called slack means it is total slack or it is also called job slack also. So, this job slack total slack is equal to slack; if it is called free slack then it must be termed as free slack free slack is always lesser than the total slack there may be free. Free slack there may not be free slack, but this is a part of total slack only if it is existing at all.

So, free slack is important in one aspect that a free slack gives a some idea about the time which can be carried out independently in that particular machine. That means, it is not effecting the production of the overall network, but it that on that particular job on that particular machine some maintenance some manuring can be carried out that is the benefit of the free slack.

Basically that is why we carry out the calculation of total slack and free slack. Free slack gives us the time independent time where that particular job or activities can be differently used. That means, the production in the total network is already going on, but still in a dot job we are having 11 hour and in that 11 hour or 11 days we can carry out something with that machine may be over rolling or may be some maintenance work can be carried out, but the production work is going on that soft floor or on that area. So, free slack is giving us some benefit and that is why this calculation is very very important.

Thank you.