Network Analysis for Mines and Mineral Engineering
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Lecture - 07
Algorithm for critical path early start and early finish times

Let me welcome you, to the 7th lecture of NPTEL online certification course Network Analysis for Mines and Mineral Engineering. In this lecture the title is Algorithm for critical path early start and early finish times.

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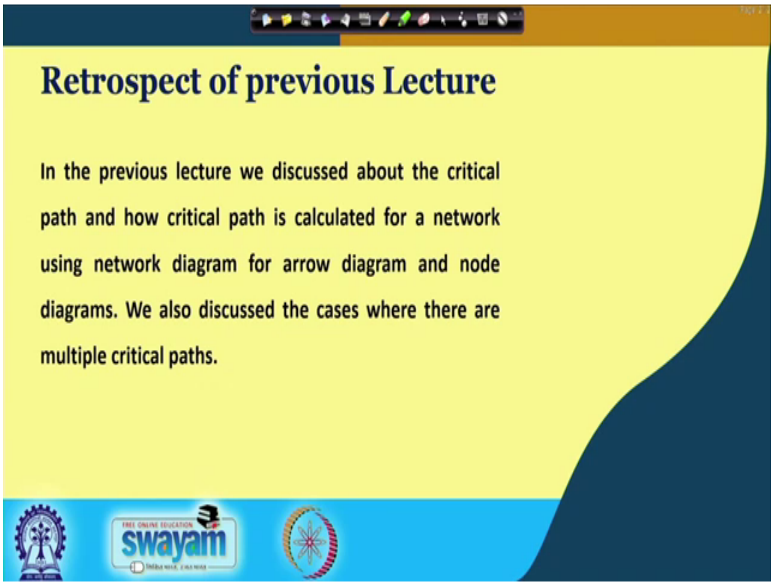

So, let us retrospect what we have a discussed in our previous lectures. In previous lecture we understood what are the different terminologies, jobs, activities, we understood also what is the network. And in last class we have discussed about the critical path, how the critical path is calculated for a network using network diagram.

And we have already understood that the network diagram may be up two type’s; arrow diagram and activity on node diagram. And we are now able to determine the critical path analysis for both arrow diagram and activity on node diagram. So, manually we are able to identified the critical path, but let us understand how we can developed an algorithm for determining the critical paths.

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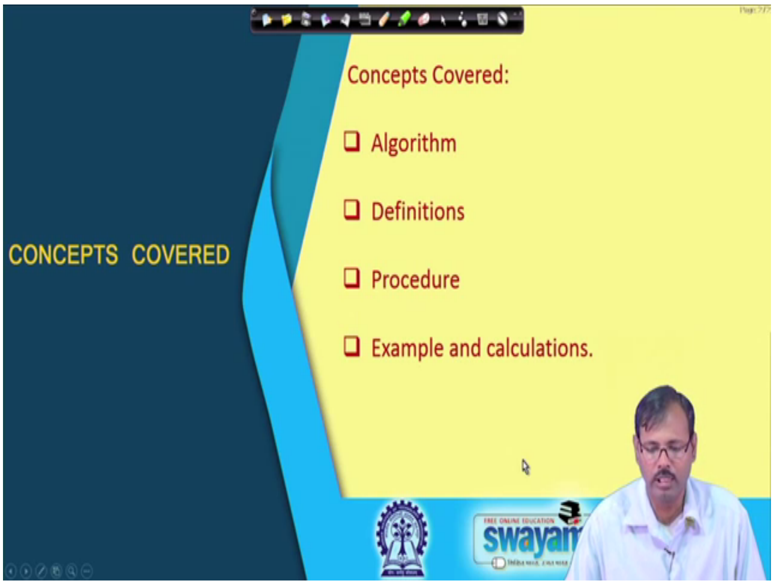

So, in this lecture we will try one or we will start the develop of algorithm using early start early times early finish times calculation. And based on that the critical path will be analyzed will come to that later on. But in this lecture we will learn, how we can calculate early start and early finish time of any job or activities in a particular network

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So, what we will do let us first understand what is algorithm. Basically algorithm may be defined like this algorithm is a set of procedures or calculation of rules specifying calculation which lead to a desired result.

So, that means it is basically the development of logic to calculate something for arriving at a decision. And algorithms are used to find the critical path of a project so that for the big networks we can computerize the algorithm. And the critical path may be calculated by the computer, not manually it has to be carried out.

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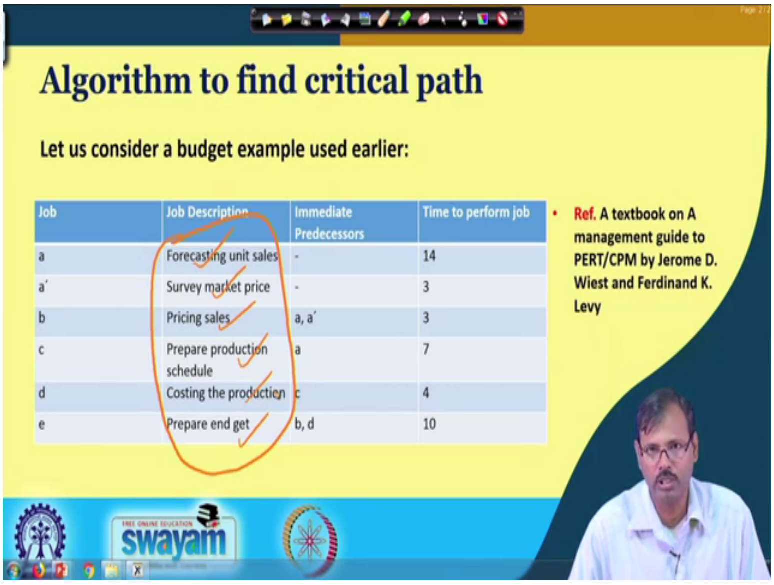

So, basically this logic has to be developed so, that the critical path may be calculated using the computers automatically and human may not be required for that. So, we will learn how to do that one for this analysis let us consider one example which we have already discussed in the earlier classes; that is the forecasting example.

This forecasting example is already discussed in the earlier class where we have found that there are 6 activities. One is the forecasting of the unit sales, then the market survey, then the pricing of the sales, then production scheduling, costing of the production and then the final is the end budget has to be prepared based on the everything. So, this is the particular example has to be we will solve this example for the critical path analysis in this case.

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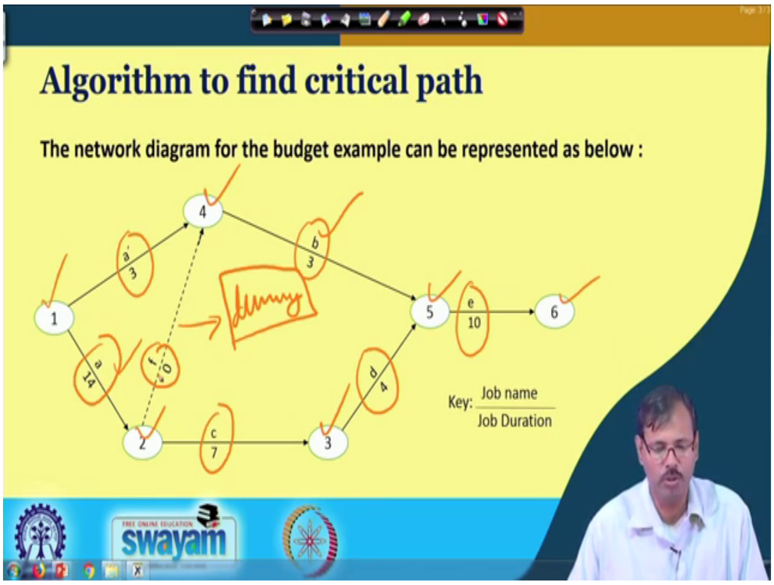

So, what was our network when we have developed this network for this for this network we when we have placed it in a arrow diagram mode then we have found this is the first job. This is the second job, this is the third and fourth jobs and this is fifth and this is the sixth job. The nodes are termed as 1 2 3 4 5 6 and to complete this network we had to introduce one dummy activities dummy activities.

This dummy activities f has to be introduced. So, that the dependency of b activity on the; a activity can be presented in the network. So, this essentially this dummy activity doesn't need any cost or the time requirement. So, this is the network may be presented and our presentation node was that we presented the activity name in the top. And the duration in the bottom of the arrow it is by this way you have presented it.

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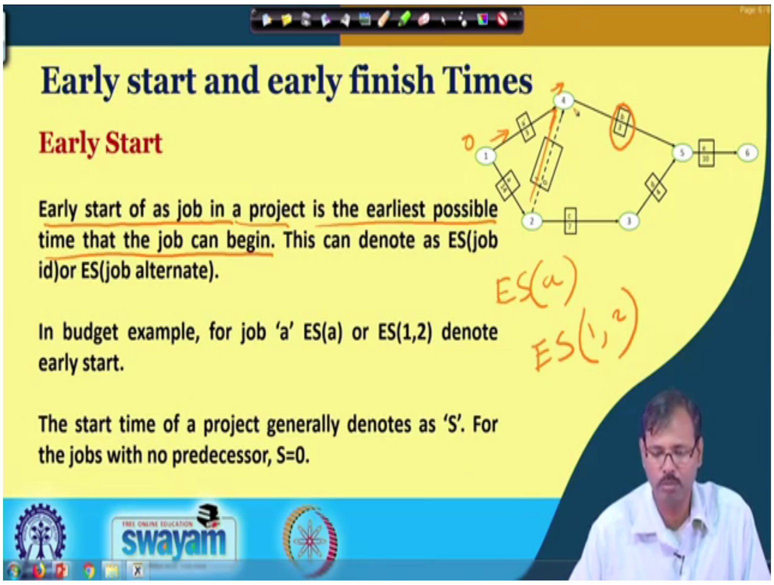

So, let us understand what is called early start. Early start of a job in a project is the earliest possible time that the job can begin. And this is denoted as the early start and the job name may be a sometimes you can use the alternate also that is the early start. This is the alternate name also for denoting the early start time. Now, let us understand, what is the earliest possible time to start that is supposed to job start; the first job the earliest possible time is 0.

That means we can start the job immediately we do not need any preparation for that. But if you consider this job this job cannot be started unless and until this job is finished or this job is finished. So that means, its early start is dependent on the ending of the other activities. But for the first activities the earliest possible time to start that job is 0. So, this is basically earliest possible time which is denote as the earliest early start time.

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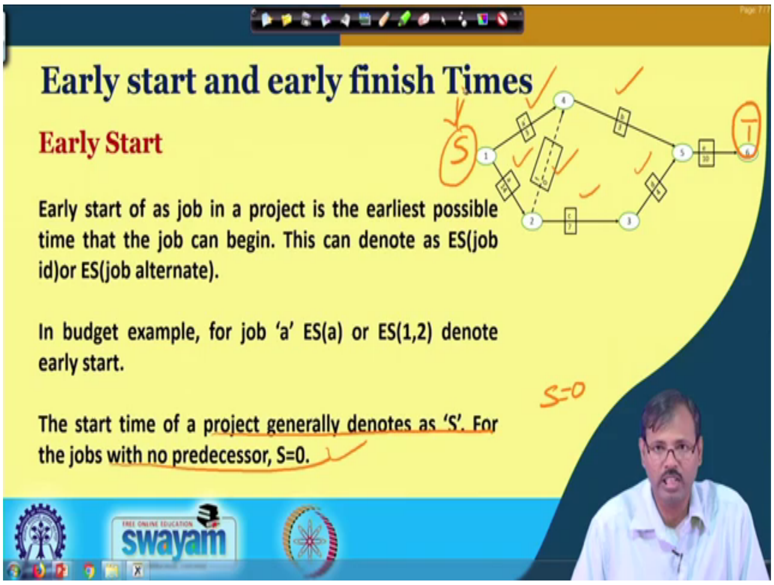

And obviously for the first job the early start time is 0. What there are two more terminology we must understand. First is the, this S is basically the starting time of the project. That means, S is the starting time of the complete project and generally S is equal to 0 or it may be some particular hours minute or something like that death hour minute if we launch something in that case.

Otherwise S is considered as to be 0 because that is the start time for the first activity or the network is being started on that particular time. T is basically the ending time the ending time of the project is considered as the T or terminating time of the project also of an it is also called target time. But this ending time target time and earliest possible ending time these are basically different and depend on the completion of this activities.

But basically our starting time is more or less fixed it always start with the 0 without the delay. But we are having the possibilities we can start it late for few of the activities. So, let us consider the staring time of the project and it is 0 and in this zeroth time we can start few of the activities like this one and this one. So, these activities can be started at the zeroth time because those are the initial activities and does not have any predecessors on them.

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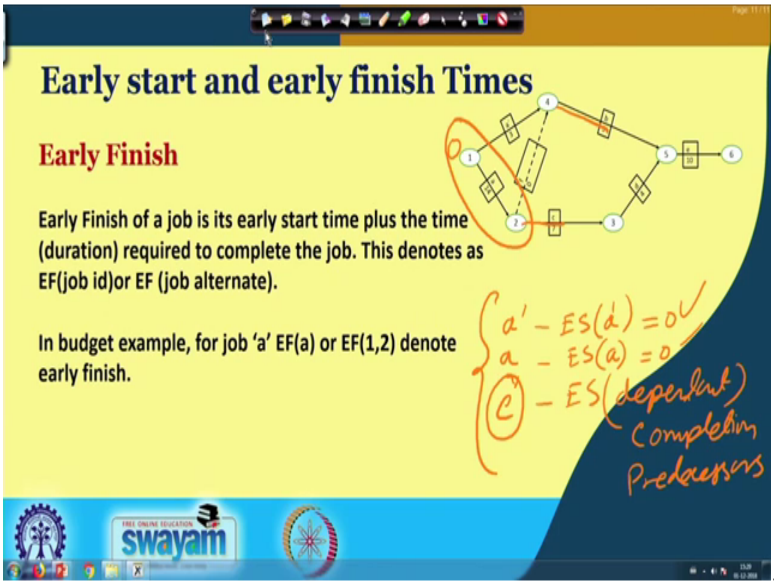

So, let us consider and calculate one by one say consider in this case the activity a dot which starts at 0 at 0th time which starts at 0th time. So that means, the a dot is having the early start time of a dot is equal to 0. So, that means we can start a dot activity at the 0 time. Similarly for a also early start time of a is also 0 at 0th time we can start activity a dot we can start activity a also.

And this two activities can be started at 0th time because these are they do not have any predecessors before that. But, if you look into the next job this one or this one. So, first let us consider the c job let us consider this c job which has to be started only after this completion of these activities.

So that means, the some of the activities are having the early start time dependent on the completion of the predecessors activities completion of the predecessors activities. So, that is why these are essentially required this calculations are essentially required the completion of the predecessors jobs for this a particular activity of c.

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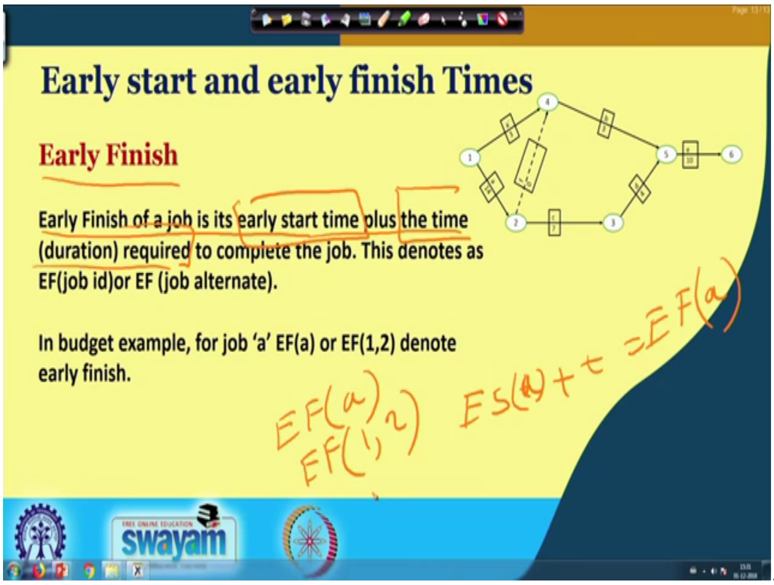

So, we will calculate it in our following slide, but before that let us understand what is called early finish. Early finish of a job early finish of a job is defined that the earliest possible time to finish this job that is the early start time plus, the completion or the time required to complete this job is called early finish time. So, early finish time is the earliest possible time to finish that particular job.

So, it depends like this way early start time plus the time of completion of the activity is equal to early finish time of that particular activity. So, that is why it is also denoted as either EF of the activity name or early finish with the alternate name like this. So, this is the way we can denote the early finish.

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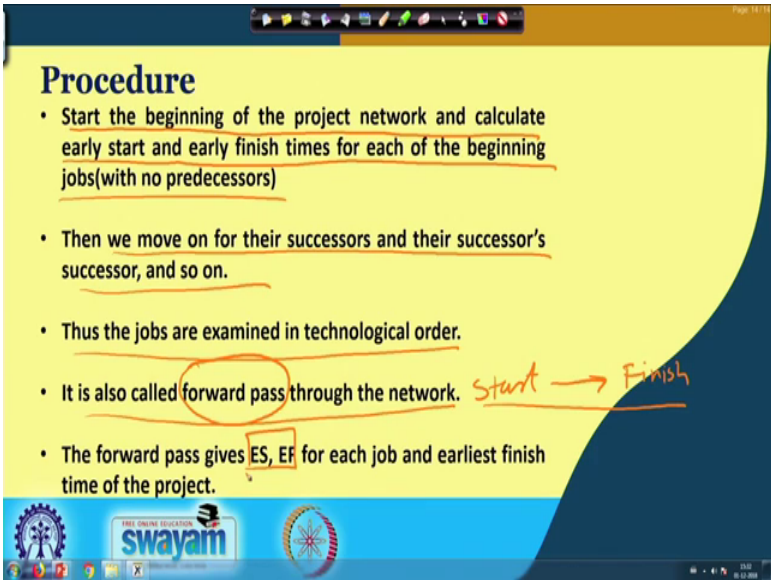

And let us start the logic behind this. So, the producer is that we start the beginning of the project network and calculate early start early finish time for each of the beginning activities with no predecessors. That means, we will we have to start with the first point then we will move on for the successors and so on.

Finally, we examine the last one we examine the last activities to finish the project. This movement is from the starting as this movement is moving from starting to the finishing. That is why this is called forward pass technique and this forward pass techniques give us the earliest staring and earliest finishing time of each and every activities of the project.

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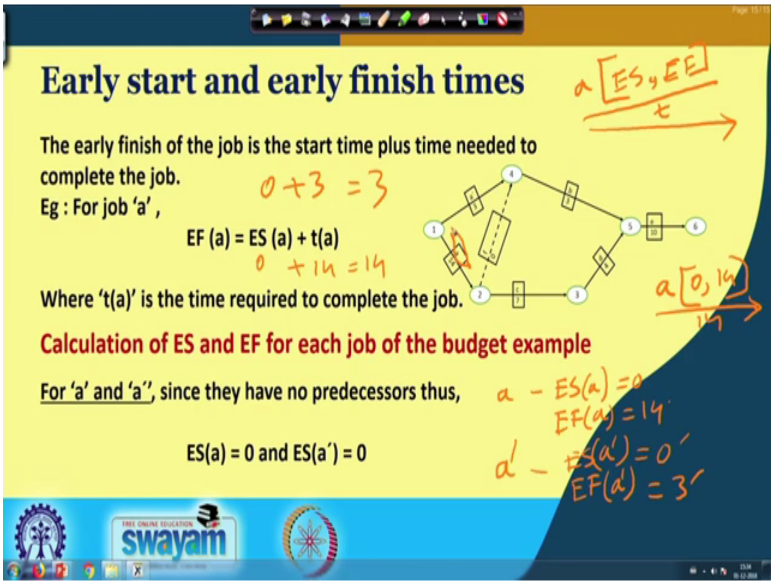

So, let us do it for a particular network which we have discussed already that the budgeting problem network. So, let us start with the first one the first job is a this first job is a and; obviously, its early start time is 0. So, early finish time of a, is early start time 0 plus t of a is 14. So, it is 14 that means, early finish time of a is 14.

Similarly, let us do it for the another initial job that is a dot whose early start time is again 0 and early finish time is again it is 0 plus 3 is equal to 3 so it is 3. That means, when we are considering a and a dot we can write here that the first activity it is a job is having the early start time of 0 early finish time of 14 a dot has the early start time of 0 early finish time of 3. Generally we express if this is a network we express our early start early finish time is like this.

This is the early start comma early finish time and this is the duration. So, basically the way of expression of this one so, in this case we can write a 0 1111114 by 14. So, this is the activity a can be prosed like this and we can write it at this position early start early finish with the separated by a comma under the third bracket can be expressed like this.

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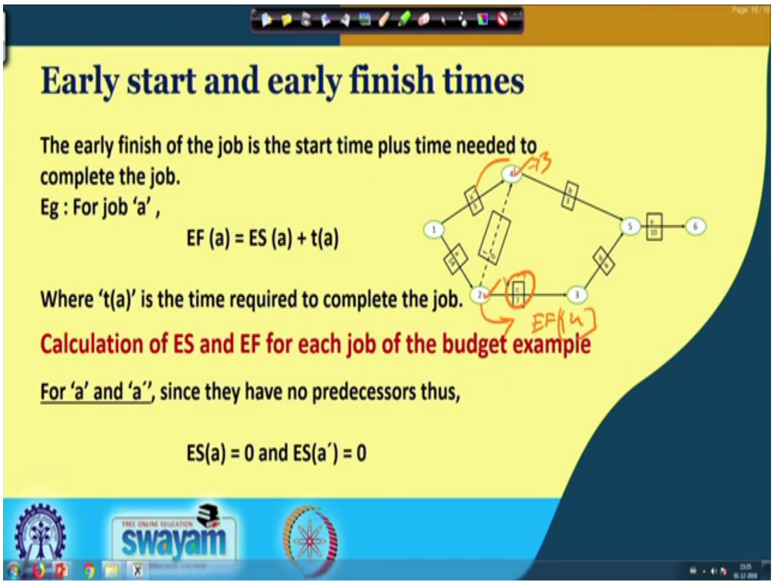

So, now we have reached up to this point and this point. And we can expect in this point our time duration early finish time is basically 14 at this point and at this point it is 3 for this network. Now let us consider this job which is dependent on the only activity a.

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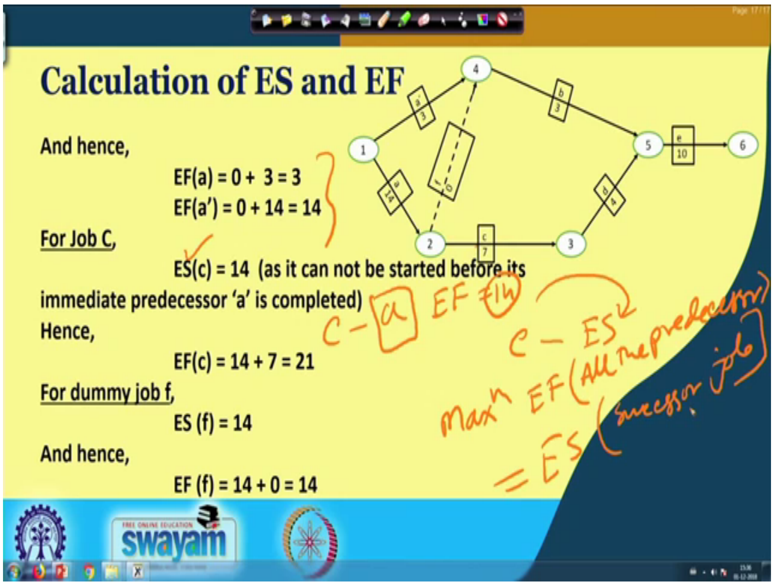

So, let us consider the c job which is dependent on activity a we can see these early finish time of a and a dots are given here. So, now, if you consider the job c this job c can be started only when its all predecessors are over. So, the predecessors of job c is basically job a only. So, its predecessors predecessors of c is a only and the earliest finish time of a if 14.

So that means, earliest finish time of the predecessors is the earliest staring time of the successor. So that means, the logic here we have to consider the maximum earliest finish time of all the predecessors is equal to the earliest starting time of the successor. So, that means, you can easily understand this c job can only be started when the a job this c job only can be stared when the a job is over.

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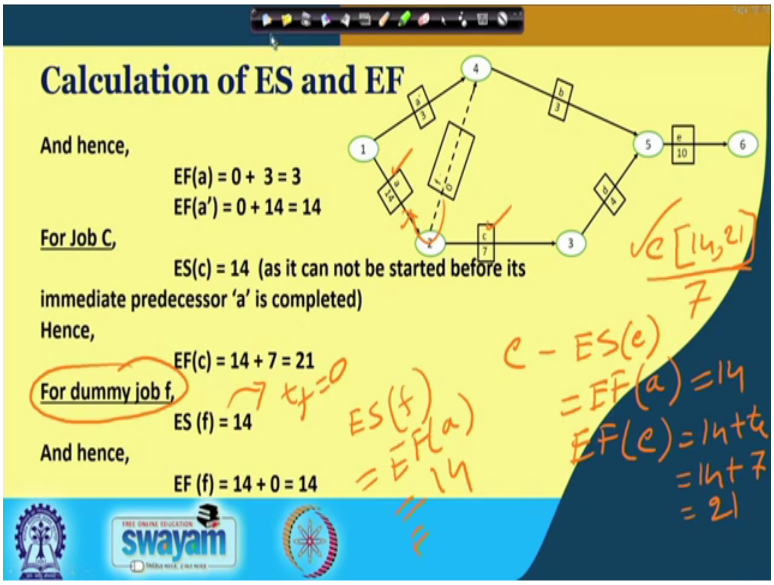

That means, the c has the earliest starting time of c is equal to early finish time of a is equal to 14. And obviously early finish time of c is equal to 14 plus time of c that is 14 plus time of c is 7; that means, it is 21.So, you can write c 14 comma 21 is the earliest finish and earliest staring and earliest finish time divided by and by 7 is showing the activity of c.

So, that is why it is easily understood that c job we can find out this is the earliest starting and earliest finish time for c. There is another dummy job it is f and f basically it is time consuming is 0 it is imaginary activity. So, the earliest starting time for f is also dummy job earliest starting time of f dummy job is also equal to earliest finish time of a job because its predecessors are a. So, the earliest starting time is again the f is again equal to the earliest finish time of a that means, is equal to 14. But as this is imaginary job and time duration is t f is equal to 0. So, early finish time.

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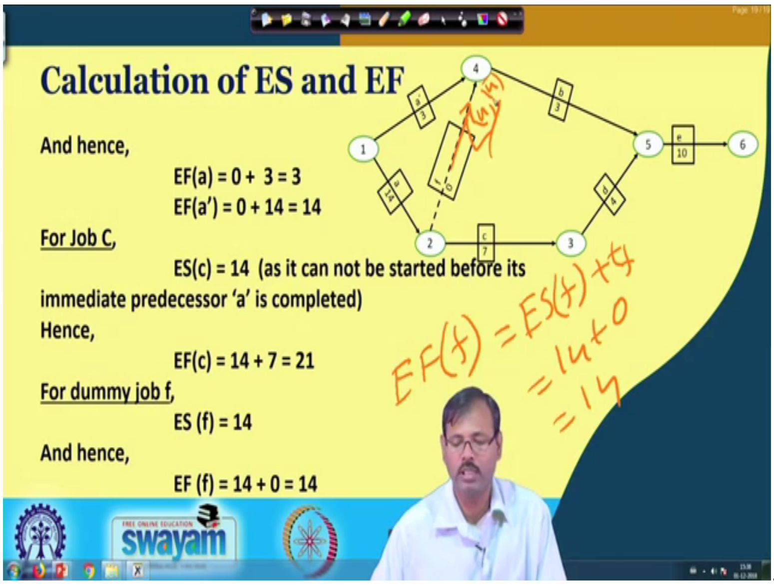

So, early finish time of f is again early starting time of f plus t f is equal to 14 plus 0 is equal to 14. So that means, this job will be finished earliest starting time of 14 and earliest starting a finishing time of 14 also and this will represent like this. So, basically this is the oa you can calculate for the cases where you are having one and only one predecessor is available. So, in that case earliest starting time earliest finishing time calculation is easy.

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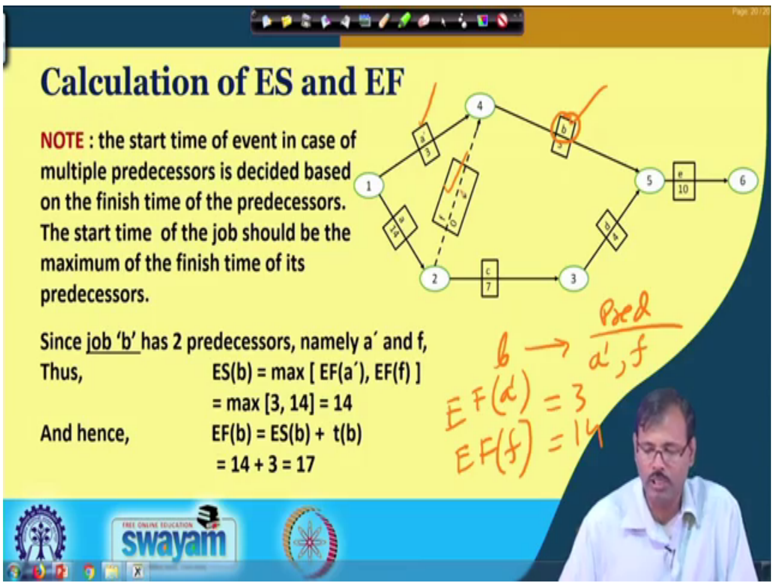

Now, let us see for job b how you can calculate the earliest starting time and the earliest finishing time. Our job b is having predecessors of a dot and f ok. So, as it is having predecessors a dot and f the earliest finishing time of a dot is three. Earliest finishing time of f is 14.

That means, what though a job is over at 3 then also as f job is not being over before 14 we cannot start the commencement of the job b. That means, we need to comments the job b only after completion of this one that means, as we have expressed the logic earlier.

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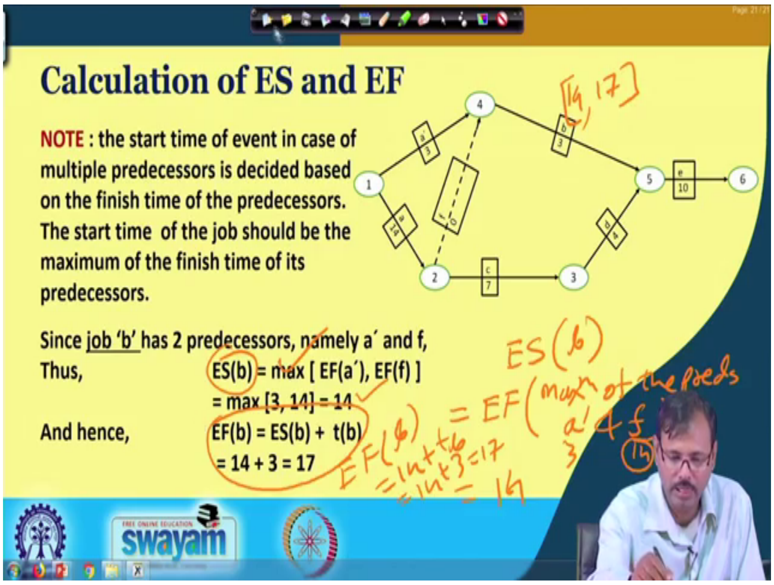

The logic is the earliest starting time of job b is equal to early finish time of maximum of the predecessors; predecessors that is a dot and f. So, this is 3 this is 14 the maximum is this one and; that means, the maximum is this one means the early starting of b is possible only at the time of 14.

And again early finish time of b is equal to 14 plus t b; that means, 14 plus 3 is equal to 17. So, early finish time is seventeen for b. So, that means for the cases where there are more number of predecessors available.

In those cases the earliest finishing earliest starting time of the successor job like b should be the maximum of them the maximum of them which is 14 and it is considered as the earliest starting time for b. Similarly, we can calculate the earliest finishing time for the b also.

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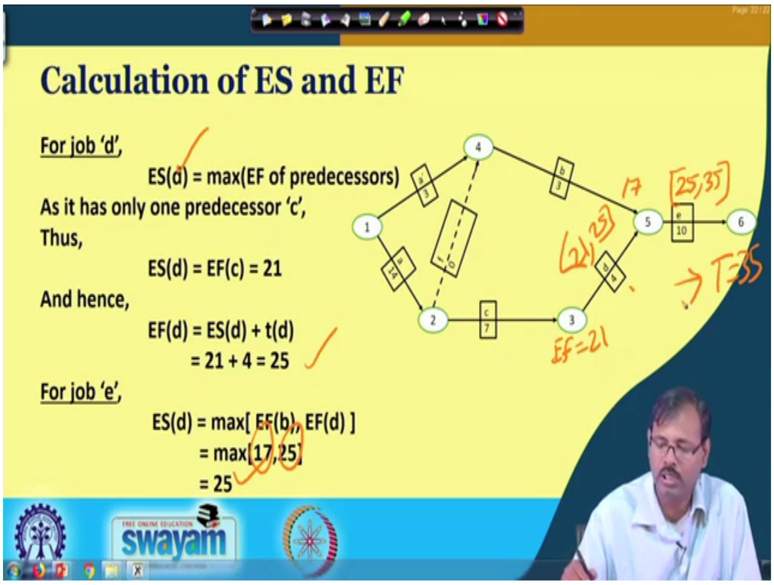

And by this way let us carry out the calculation for other jobs. For d it is again single job which is having c c is having the earliest finishing time finishing time of 21. So, its earliest starting time of 21 and this a 4 hours duration so earliest finishing time is 25. And this can be easily calculated. And again for e we have to find out this is 21 and this is 17.

So, maximum of 17 and 21 is considered. So, it is coming 25 so this is earliest starting time is 25 earliest finishing time is 35 25 plus. So, we can consider the project will be ended as three T is equal to 35 hours or days. So, that means the expected duration of the end of the project is T is equal to 35. So, now, look into the complete network now.

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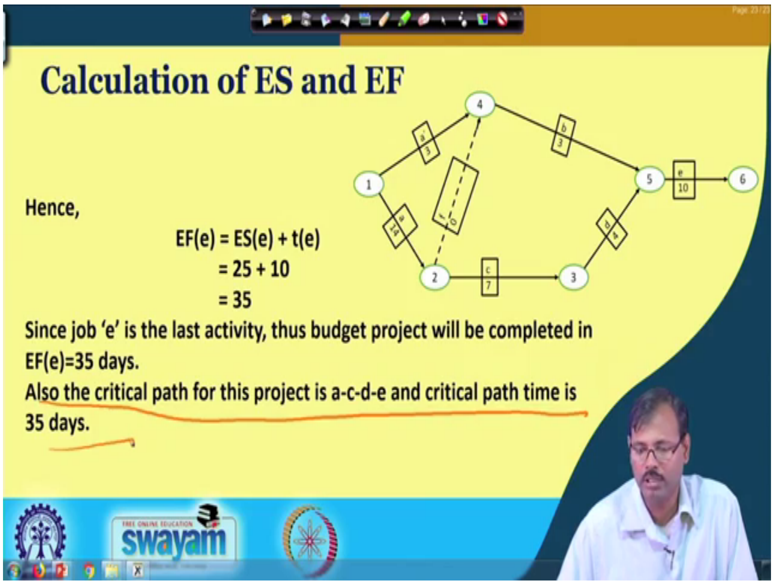

So, this is the complete network and you can find out let us consider that let us consider only this one. This job is considered as the critical path as the earliest starting time of earliest finishing time of the previous job is being considered in the earliest starting time of the next job.

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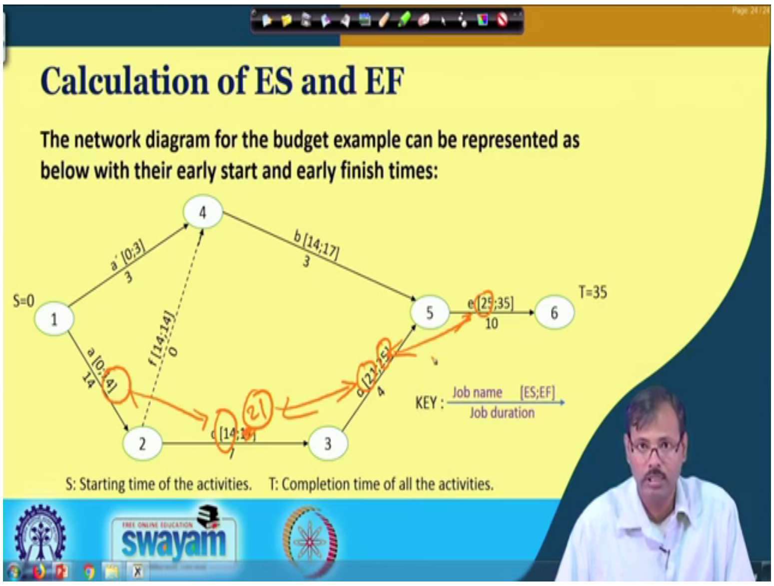

So, if you consider this in the next diagram you can find out this one and this one is matching. this one actually this is a printing mistake so this is 21. So, this 21 21 is matching this 14 14 is matching so these are following this path only.

So, actually better way of representing this is using late start late finish algorithm also we will discuss those in the later class. But you can see in this case the early start of this one is 3 a 0 finish is 3.

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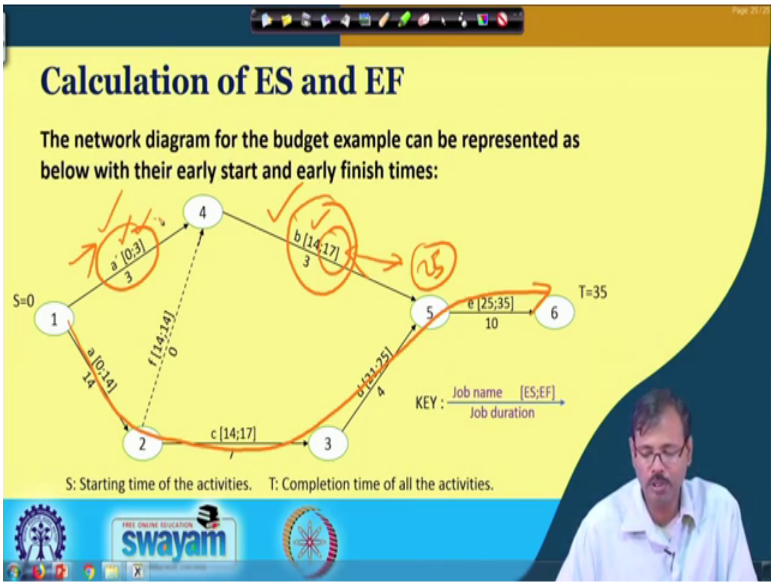

But this is starting at 14; that means, we are having the option we can go for a default start of this activities. In fact, probably you could remember in our earlier lecture when we discuss about moving from San Francisco to California that time we have considered of two path in one path. One person at the option two have some less start so; that means, in this case we are having some relaxation or slack in this case we can go for late start or we can go for a slower process. If you consider this one also here we need to reach a here at 21 sorry 25, but it is 17.

That means, we are having some slack hours available there and we may opt for commencement of this one may be at a later stage also or we can go for a slower activity in this case. So; that means, this is these are the critical path where we do not have any slack and we have to follow strictly follow the schedule as of the activities. But in these activities we are having some slacks. So, this is very important and early start early finish algorithm gives us a better understanding on this.

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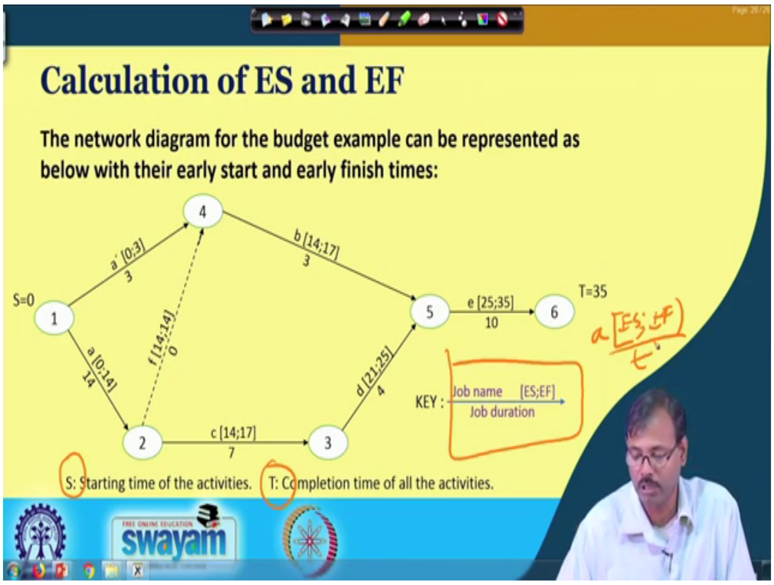

S is the starting time of the activities T is the completion time of all the activities this is the general procedure to give the nomenclature. You have to give the job name then under third bracket early start, early finish and divided by the and in the lower and the part you can mention the duration.

So, this is more or less all about our early start early finish algorithm. And I hope you can have n number of examples available with different books different in nets also. And you will solve those and in next class we will start the late start late finish algorithm.

Thank you.