Network Analysis for Mines and Mineral Engineering
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Lecture – 06
Critical Path and its calculation

Let me welcome you to the 6th lecture of NPTEL Swayam, NPTEL online certification course on Network Analysis for Mines and Mineral Engineering. In this lecture we will discuss about the critical path and its calculation procedure.

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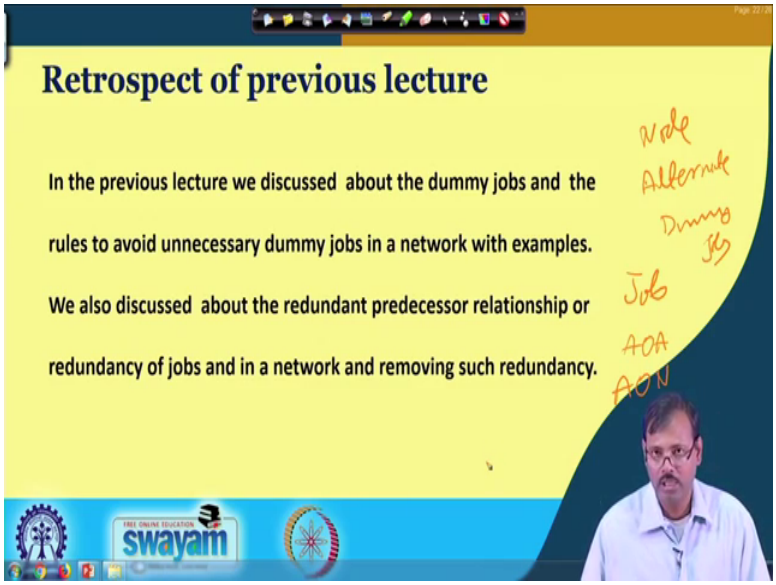

So, let us retrospect like every lecture; retrospect what we have covered so far. Now we know what is called job or activities. We know how the jobs are defined either in aero diagram activity on aero diagram or activity on node diagram, we also know what is called node, what is alternates. We know what is dummy job, how the dummy jobs has to be assigned how the dummy jobs has to be removed. Then we know what is redundancy, we know what is cycle and we are also aware about the things that a cycle has to be completely eliminated, redundancy has to be removed for the smooth analysis of the network.

So, now let us we let us carry out some analysis on the network and this analysis is called critical path method. So, we will analyze the criticality of a network for the possible optimization of the same.

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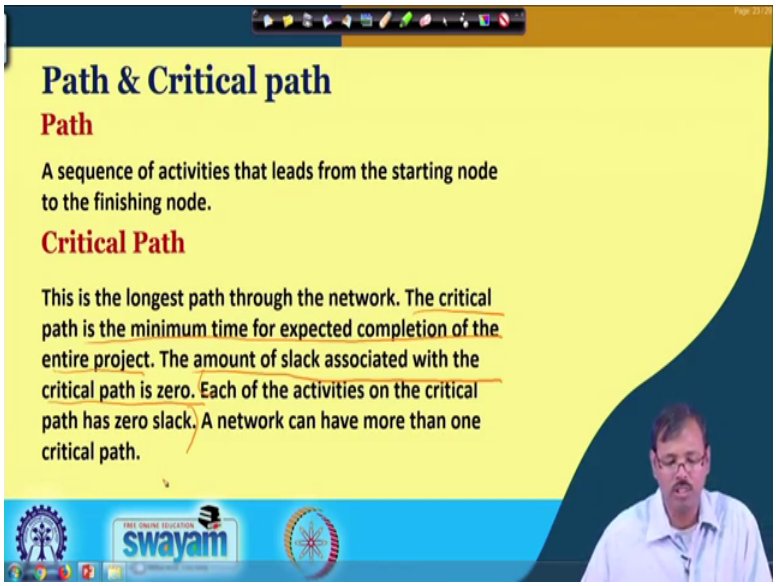

So, let us understand few more terminology here, what is path and what is critical path because we have already discussed the term critical path method. Suppose for understanding this let us define it now, path is basically a sequence of activities that leads from the starting node to the finishing node. And critical path is the longest path in the network; that means, critical path is the path which takes the longest time from reaching, starting from the starting node to reach to the finishing node is called critical path. To understanding more on this let us just read this one, critical path is the longest path which basically gives us the minimum time for the completion of the complete work.

So, basically it is the longest path, but as it is the longest path it allows all the path will be completed before the completion of this path, that is why critical path basically gives us the minimum time for completion of the total project and the amount of slack. That means, there is no relaxation time is available in the critical path, but in the other path some relaxation times are available. So, the path which is having 0 slack is also called a critical path. Let us understand this with a small example.

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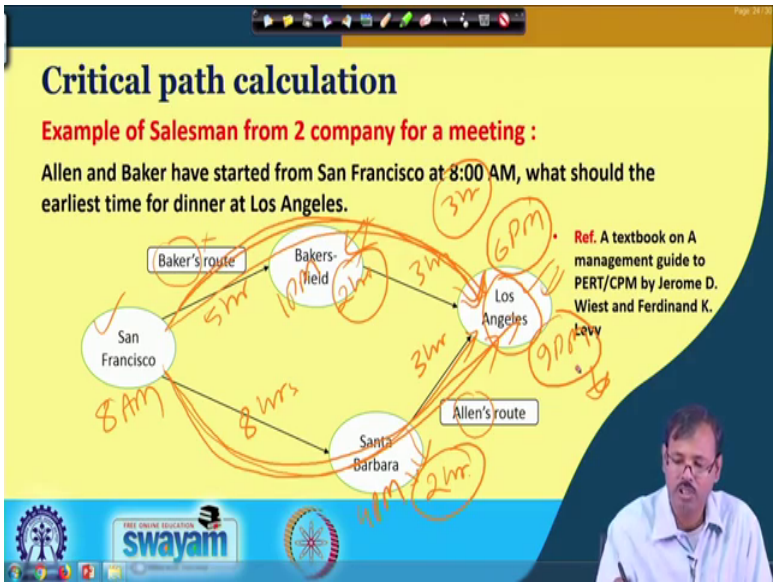

This example I have taken again from the Wiest and Levy's book, in this example let us consider the similar example we have given in the introduction lecture of also where we are starting from Ghaziabad to reaching at Delhi. But this example is telling that suppose two friends Allen and Baker are willing to move from San Francisco to Los Angeles.

But Allen's has to move through Santa Barbara and Bakers has to move through Baker field, what is the reason? The reason is that Allen wish to have a meeting with his wife or have a lunch in his house at Santa Barbara, where his wife is there. And Baker is willing to meet his friend at Bakers field and may have a lunch with the Baker field. The paths; that means, their routes are different, the paths are different and they has to have to meet at Los Angeles.

So, consider they have started at San Francisco in the morning 8 AM and they are moving from San Francisco to Los Angeles and consider that the Allen's required 8 hours driving; 8 hours driving to reach at Santa Barbara, he will take 2 hours at Santa Barbara to take lunch from his wife. Then from Santa Barbara he has to reach at Los Angeles with 3 hour driving time, where there is a fixed position in which Allen will meet Baker. However, Baker needs 5 hour to reach in the Baker field, he will also take 2 hour for a lunch with his friend or a meeting with his friend. Then he will take 3 hours, then he will take 3 hours from reaching Baker field to Los Angeles.

So; that means, starting at 8 AM Baker will reach at Baker field in 5 hours; that means, 1 PM and he will spend their 2 hours; that means, you will start from Baker field at 3 PM and will reach at Los Angeles at 6 pm. Whereas, Allen takes 8 hours, from 8 AM he will reach here at 4 PM and he will take 2 hours for lunch etcetera in his house. So, he will start at 6 PM from here and he will reach at this position at 9 PM. So, the paths are different, first path the Baker is taken will lead Baker at Los Angeles in 6 PM, the second path which Allen has taken will allow the Allen to reach at Los Angeles in 9 PM and they have a meeting there; that means, though the Baker is reached at 6 PM in the Los Angeles, but Baker cannot attend the meeting.

So, the meeting cannot be fixed at 6 PM, but the meeting can be fixed at 9 PM only when the Allen will reach there so; that means, though Baker will reach at 6 PM, but the job that is the meeting job which is required to be completed that can be completed at 9 PM only. So, the path is showing us the completion of the network time is basically the Allen's path not the Baker's path. So, Allen's path is the critical path which gives us the minimum time for the completion if the job if everything is well.

But it also allow the slack to the other paths; that means, the other path is the Baker path where it gives Baker a 3 hour slack; that means, Baker reaches at Los Angeles 6 PM; that means, he is having 3 hour slack which he can spend any whereas, per his own wish. Instead of having a 2 hour lunch he can have a 5 hour lunch there, instead of starting here at 8 AM he can start at 11 AM also.

So, all this slacks are available to Baker because his paths are shorter than the Allen's path. So, the path which is shorter is not the critical path, the path which is the longer path is the critical path. But, the longer path is allowed the earliest completion of the job; that means, the meeting is the final target of the job that meeting earliest possible time to arrange that meeting is the 9 PM not the 6 PM. So, basically this analysis reveals us that the most possible time for or the earliest possible time for completion of the job is the 9 PM not the 9 PM, not the 6 PM. So, this is called critical path analysis and from this network you are able to understood that, understand that these are the path these are the, this are the paths, this is path 1 this is another path which is path 2.

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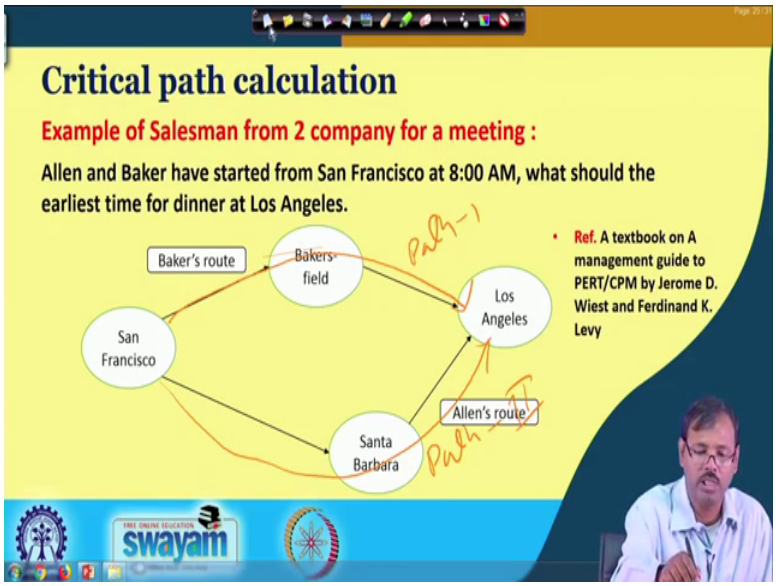

Similarly, there may be a number of paths available and we have to analyze each path and the longest path we have to find out so that we can come out with the critical path.

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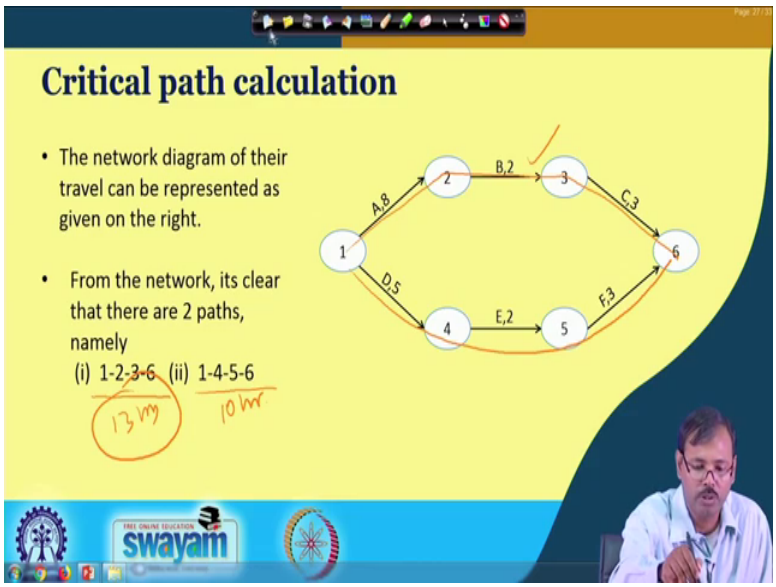

So, this analysis method is called critical path analysis. So, the same example is given here and you can see the network can be constructed like this where job A is for the Allen to move from Los San Francisco to the Santa Barbara and this is for the Baker, Los Angeles to Baker field. Then finally, a lunch in the both the places finally, in the lunch in the Baker field and in the sorry in the Santa Barbara and in the Baker field and then

Santa Barbara to the Los Angeles this is Baker field to Los Angeles same is depicted in this network.

And we can find out the two paths, two available paths are 1 2 3 6, 1 4 5 6 the duration of the 1 2 3 6 that is the node number node 1, node 2, node 1 to node 2 to node 3 to node 6 that is the path 1 which takes a duration of 8 plus 2 10, 10 plus 3, 13 hours and there is another path from 1 4 5 6, 1 4 5 6 which takes 10 hours duration. So, this is the longest path and this longest path is the critical path which happens to be the Allen's path.

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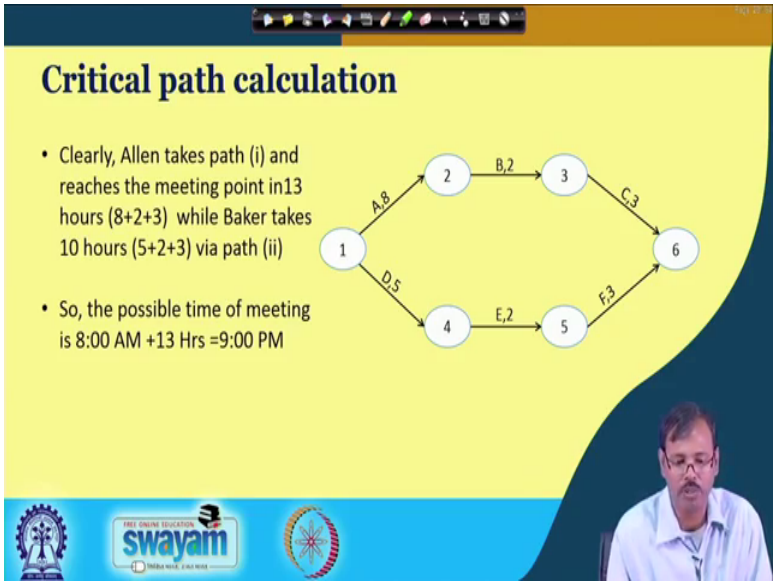

So, clearly Allen's text path one and reaches there at 13 hours. So, the earliest possible time of meeting is 9 PM which we have already discussed.

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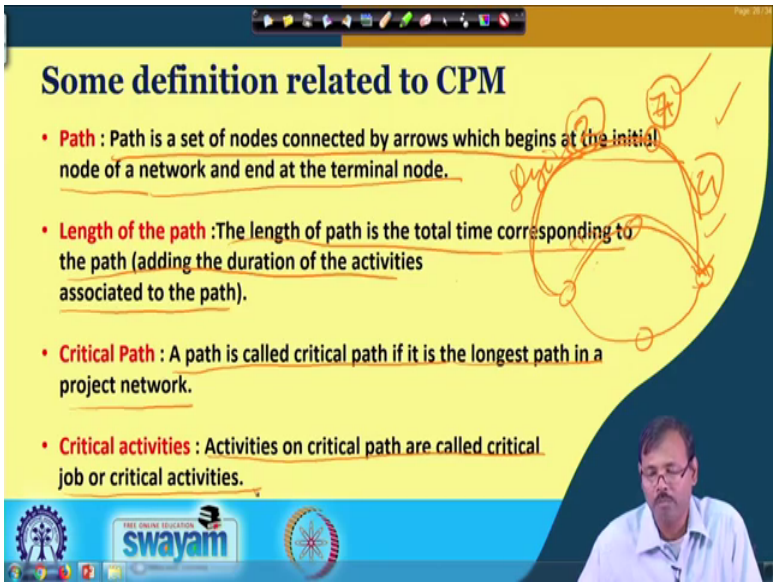

So, again let us see the definitions of this path is basically a set of nodes connected by arrows which begins at the initial node; that means, the starting point of the compete network and end at the terminal node of the network. So, starting point of the network and ending point of the networks are connected by the arrows, set of arrows is called path. A network may have n number of paths, the length of the path is the total time corresponding to the path that is actual time is attribute here, it may be anything it may be cost also, but in this in most of the cases we consider time as the main attribute in the critical path analysis.

So, while we are considering the time as the attribute. So, length of the path is the time of the is the total time corresponding to the path; that means, which takes the time for starting, reaching staring point to the ending point by the addition of the time on attributes assigned to each job. Critical path is the longest path in a project network which takes the maximum time, a project network may have more than one critical path also; that means, in the previous example if you see from San Francisco to some Los Angeles a person taking from Santa Barbara, another is from Bakers field there may be another one say someone they are driving.

So, they are taking the time for driving, another person may have taken something say he has gone to Seattle and again Seattle to he is coming to Los Angeles and he is flying instead of driving. And his time may be 2 hours for this flight then there is a 4 hours

between the gap between two flies another is the another 4 hours which may takes again sorry it have to reach it 13 hours. So, let us consider there is a gap of 4 plus 2. So, this is the gap of 7 hours at this point. So, the flying time from San Francisco to Seattle is 2 hours Seattle to Los Angeles is 4 hours and 7 hours is the gap between 2 flights.

So, that is why the time taken from this is also 13 hours time taken from San Francisco to Santa Barbara and to Santa Barbara and to Los Angeles is also driving time is 13 hours. So, both the path are taking 13 hours so; that means, both the paths are critical path. So, there may be more than one critical path, the activities or jobs on any critical path are called critical job or critical activities because the most importance should be given to them because they do not have the slack. So, that is why the critical paths are important and more man machineries money has to be invested on the critical paths, jobs of the critical paths so that they can be completed early, otherwise the complete project will be default.

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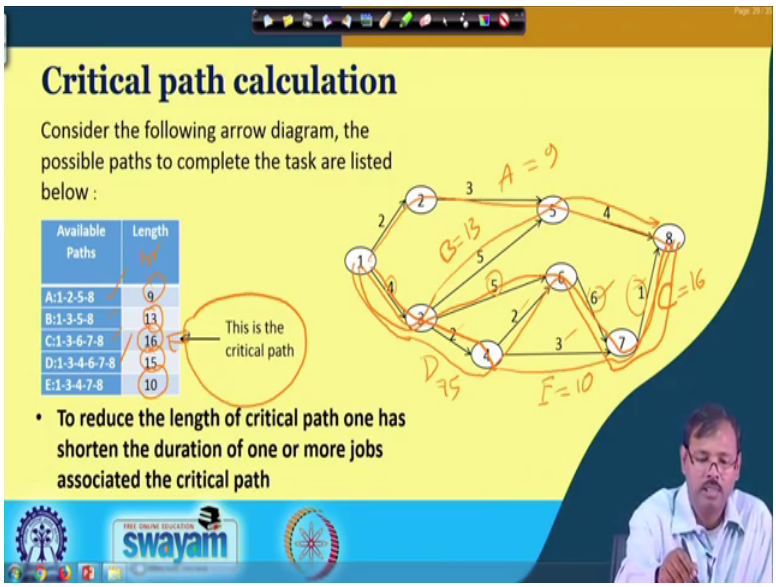

So, let us consider an example this is the example I am not given you the table, in this example you can see the nodes are only connected with the jobs; jobs names are also not given only the attributes say for this consideration this is hour say ok. So, the time of this are given and you can see the path A is taking this one; that means, 1 2 5 8 node is the path A, the path B is this one 1 3 5 8. So, this is path B and if you see the time required

for completion of path A is 2 plus 3 plus 4; that means, 9 hours path B time is 4 plus 5 plus 4; that means, it is taking 13 hours. The path C path C is 1 3 6 7.

So, path 3, 1 3 6 7 8 so, this path 3 is taking 4 plus 5 plus 6 plus 1. So, 4 plus 5 9, 9 plus 6 15, 15 plus 1 path C is taking 16 hours. Then let us consider path D which is 1, 3 then 4, then 6, then 7 and then 8. So, this is the path d which takes 4 plus 2 6, 6 plus 2 8, 8 plus 6 14, 14 plus 1 15. So, this takes 15 hours then finally, the path E which is 4 1 3 4 7 eight. So, this is path E this is 4 plus 2 6, 6 plus 3 9, 9 plus 1 10.

So that means, path A is taking 9 hour, path B is taking 13 hour, path C is taking 16 hours, path D is taking 15 hours and path E is taking 10 hours. So, there are 5 paths in this network and if we look into this path the activities connected to each path is taking in different way, A path is taking 9, B is 13. So, A is the smallest path taking 9 hours and C is the largest path taking 16 hours; that means, this is the critical path is the C path.

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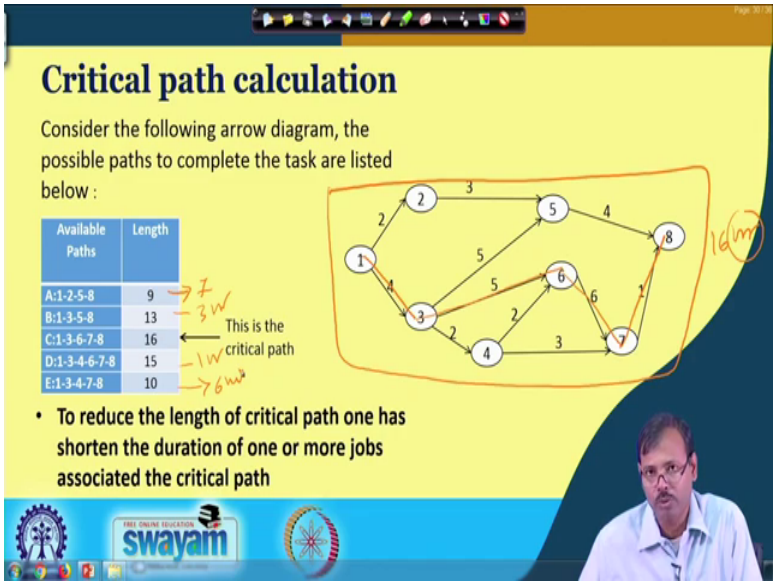

And this gives us the C path which is 1 3 6 7 8 this path is the critical path and this requires the earnest attention of the management for the possible completion of its one time. That means, to complete this total project it requires at least 16 hours for the completion of the project while it attributes a 7 hours slack in the A path, it attributes a 3 hour slack in the B part, 1 hour slack in the D part and 6 hour slack in the E part.

So, basically critical path is very very important which shows one or a number of path which are coming to the critical path needs maximum attention that has to be completed at the earliest where, the other parts are having some slack time which can be utilized in a judicial manner while the planning or optimization is being carried out. So, basically critical path is giving us the analysis where, from where we can infer which path has to be taken care more.

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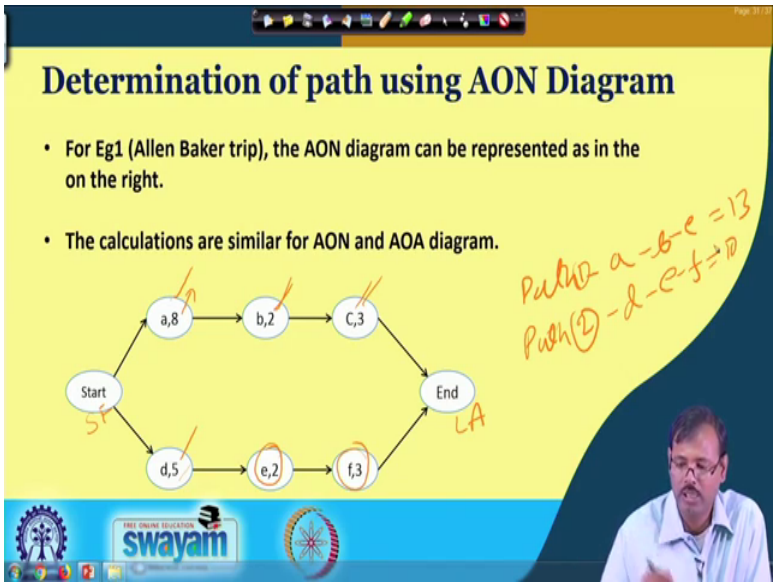

We can carry out analysis of critical path for activity on node diagram also, say this is the case for the Allen's and Bakers movement starting from the San Francisco to Los Angeles. So, in this case this is the Allen's path this is the Allen's path which takes this is the movement time from San Francisco to Santa Barbara, this is the lunch time and this is the Santa Barbara to Los Angeles time. Similarly this is the Bakers path which is at the driving time from San Francisco to Bakers field this is the lunch time at the Bakers field and this is the Bakers field to Los Angeles time.

So, in this case we say the path 1 is we have not mentioned here is a, b, c path which takes 13 hours path 2; this is path 1, this is path 2 which is basically d e f route which takes 10 hours. So, similar way you can carry out it in the activity or node diagram also; however, the construction of network in the computer in activity or node diagram is little bit difficult.

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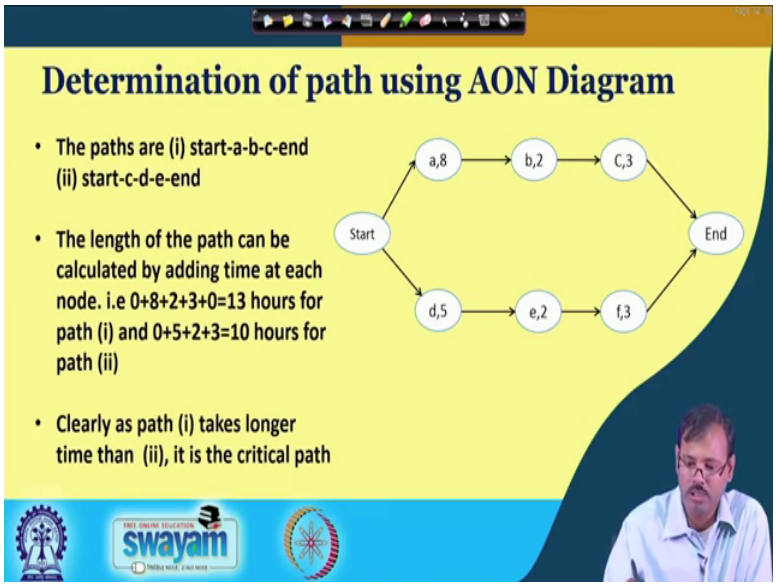

So, you can see this way you can calculate which is already discussed to you.

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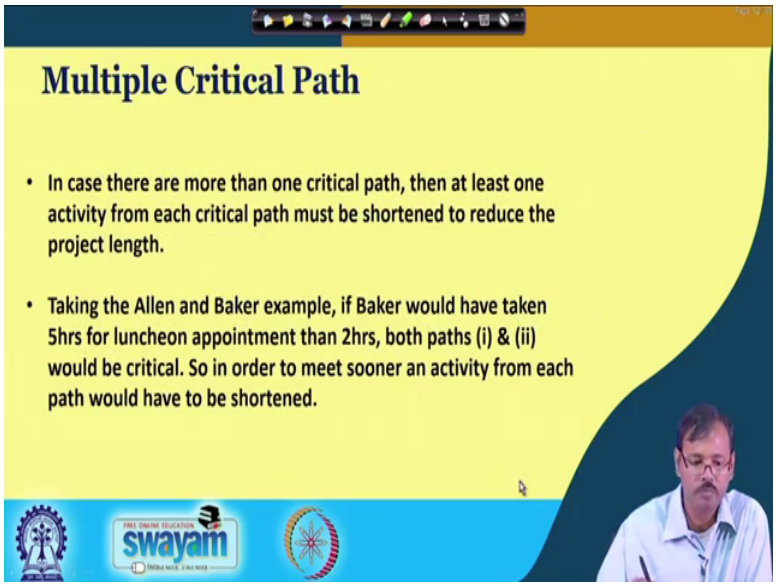

So, in any case we can have a multiple critical path also.

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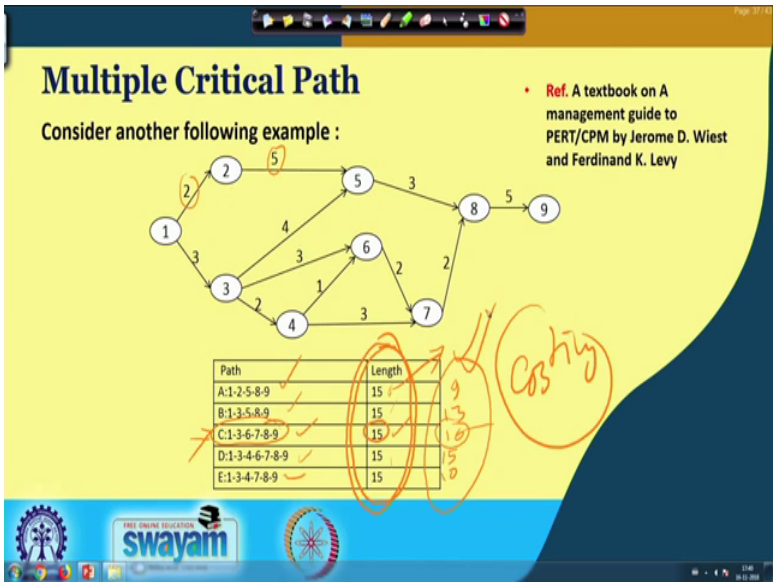

Say in this case suppose we rearrange the table little bit for the previous example which we have discussed and you can see in this case we are having path 1 this is 1, 2 5, 8, 9 and the total time requires is 2 plus 5 plus 3 plus 5. So, it is coming 15 hours the second path is 1 3 5 8 9; so, 1 3 5 8 9 that is 3 plus 4 plus 3 plus 5 so, 3 plus 4 7, 7 plus 3 10, 10 plus 5 15. Let us consider this third path 1 3 6 7 8 9.

So, this is 3 plus 3 plus 2 plus 2 plus 5 so, 3 plus 3 6, 6 plus 4 10, 10 plus 5 15. So, this is 15 we can have the D path 1 3 4 6 7 8 9. So, this is 3 plus 2 plus 1 plus 2 plus 2 plus 5; so, 3 plus 2 5, 6 7 8 9 10 15. So, again it is coming 15 the total length and if you see the last 1 e path 1 3 4 7 5 5; so, this is 3 plus 2 plus 5; that means, 3 plus 2 5, 5 plus 5 10, 10 plus 5 15. So, all the paths are now coming 15.

So, this; that means, all the paths all the paths are critical now and we have to give the intense care about the optimization of this all this works, but probably you can wonder how it is possible. Actually this is possible because of our previous example, you can see in the previous the similar problem where we had the 16 one is the maximum one and the others the smaller one is of 9, smaller one is of 9 hour length. The maximum one actually let us again write, let me write all the path durations there which we are discussed in this earlier case. In earlier case this durations were 9 here it was 13, this was 16 and this was 15 and this was 10. Now, in earlier case when we had this one, what the management

did? The management found that the demand allows us for a length of 15 which is appreciable for the management.

So, considering that they change the man, machine, material and money requirement in every jobs. By changing that one what they did say suppose they reduce the man power in some job at this position or in that this position they did earlier, where in the network and while doing that they lengthen this path to the 15 hour and reduce the cost of the production for this path. And they invest additionally in the critical path in; that means, that C which was taking 16, but the additional investment in terms of man machine material etcetera allow them to shorten the path to a 15 hours and that is allowing them to address the target time of 15 hour of the completion of the complete network.

So, suppose let us consider this is the manufacturing of TV ok, so consider this is the power board, this is the display board, this is the sound box, this is the this is the screen and this is the cabinets etcetera say. Let us consider like this these are the way we are carrying out so; that means, putting some additional man machine in the screen part and allow is to complete in 15 hours instead of 16 hours and we reduce the a requirement of man machine etcetera in the other path. So, we are providing a little bit additional, we are actually saving some amount or money on the casting o this one. So, basically we are achieving the production target of 15 hours and we are able to complete this and the costing is now acceptable to us.

So, this was when have carried out it with a natural costing which we have planned initially, then with the analysis of the critical path we are able to come out with the decision in which we need additional man powers man machines in which we can withdraw some man machines additional man power excess man power provided in that. So, some length may be extended some length may be shortened to achieve the target date and to achieve the required costing in this networks.

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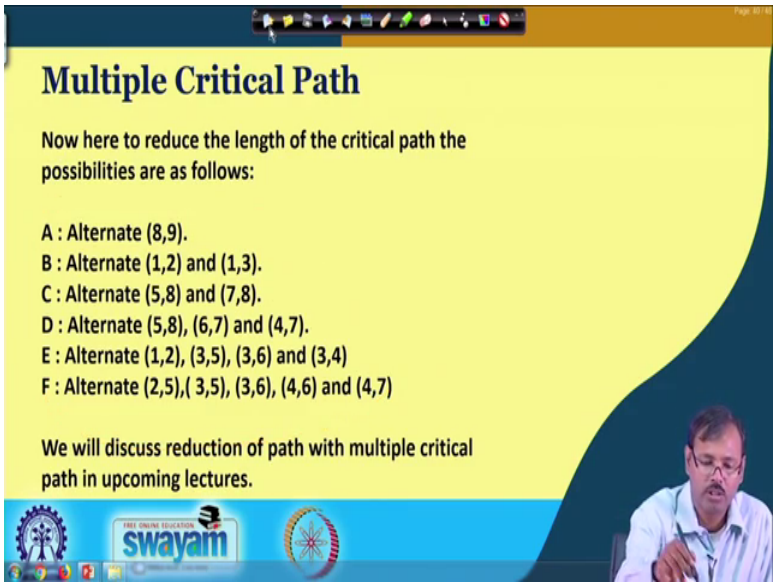

So, we are having multiple critical path here and this force us that to give the maximum, give the maximum importance to all those cases. So, basically from previous one to this one allow us to come out with this by shortening this, this, this, this points in different steps by providing additional, withdrawing the man power or providing the additional machines etcetera. Basically what this how this type of analysis will be carried out we will discuss this in our subsequent lectures so, that you can carry out this type of practices in your future. So, this is more or less, this is more or less end of this lecture.

So, I hope in this lecture you are able to analyze the different paths of a network and you can come out with which one is the critical path or which are the critical paths in a network. And, based on that the managerial people can take the decision where the additional investment or the additional man power requirement can be provided.

So, I hope after viewing this lecture you carry out some analysis, you will solve n number of critical path network problems are provided. We will search out and find out what are the paths, I always mention you must first find out what are the paths, available paths you can find out all the available paths then find out the length of those paths, then find out what is the critical path. So, by this way you go for exercising in your house that will instill confidence in you.

Thank you.