**Network Analysis for Mines and Mineral Engineering**
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**Lecture - 18**
**Probability of completion of a project**

Let me welcome you to the eighteenth lecture of NPTEL online certification course Network Analysis for Mines and Mineral Engineering. Today's our topic is Probability of completion of a project in when while we are analyzing the network using program evaluation and review technique.

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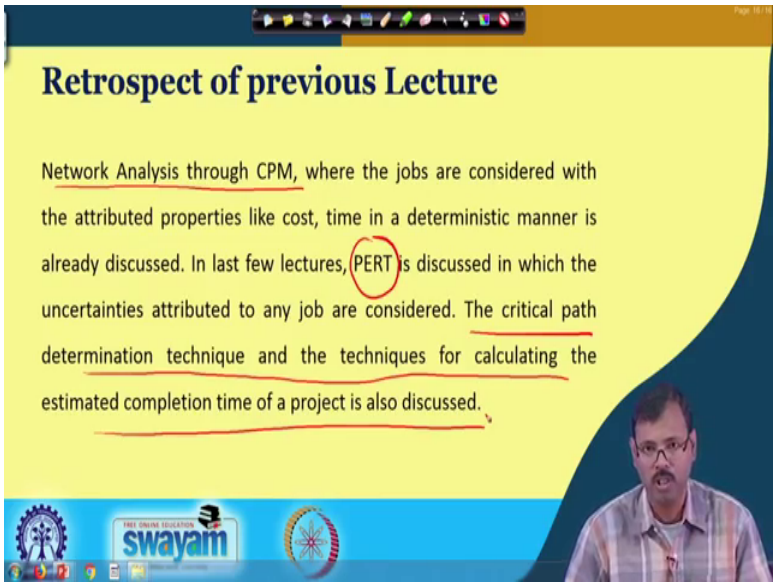


So, like every class, let us retrospect what we have so far covered in our previous lectures. In previous lecture, in first 15 lecture, we have covered network analysis through critical path method analysis and where jobs are considered in those critical path method that are deterministic manner where the cost time we have considered those are fixed and single value are considered in that case.

In last few lecture, before this class, we have already covered program evaluation and review technique which is basically a probabilistic a way of analyzing the network. And in this case, the uncertainties are considered for each job and that is why each job are attributed with three similar time values or cost values like that so that, some expected or

mean value can be arrived at. The critical path determination technique using the program evaluation review technique is also discussed in the last few classes.

So, this is more or less we have covered in last few classes.

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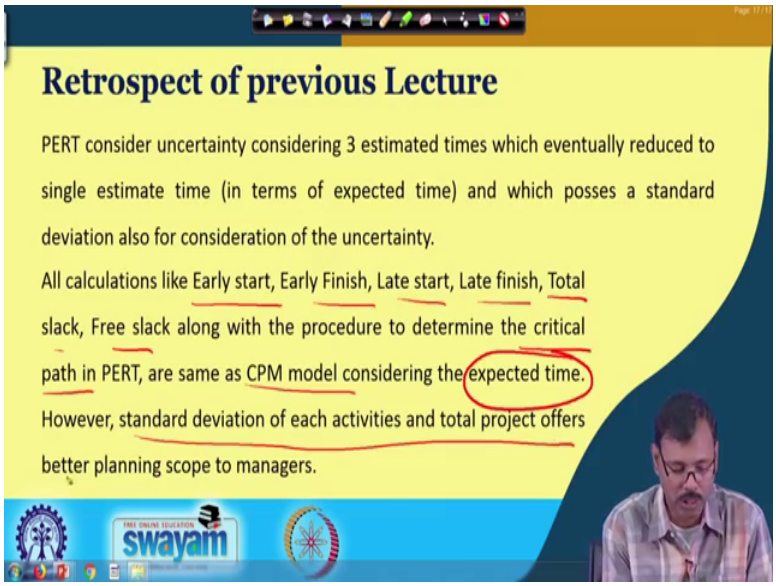


We have also considered that discussed in the last few classes that early start early finish, late start, late finish, total slack, free slack. This all calculation procedures are same in program evaluation review technique as we have discussed for the critical path method case in C P M model. And this way, we in the same way, we are calculating for the program evaluation and review technique, but instead of that single value which we have considered in the critical path method, we have considered the mean values of the jobs in these calculation procedure. And standard deviation of each activities and total project offers better planning scope to the managers as the probabilistic approach we are considering in this case.

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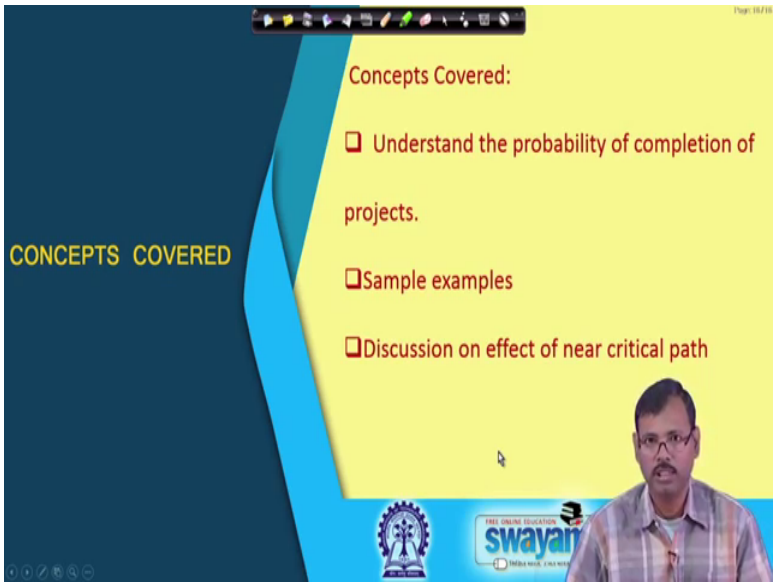


So, let us consider our today, today's topic.

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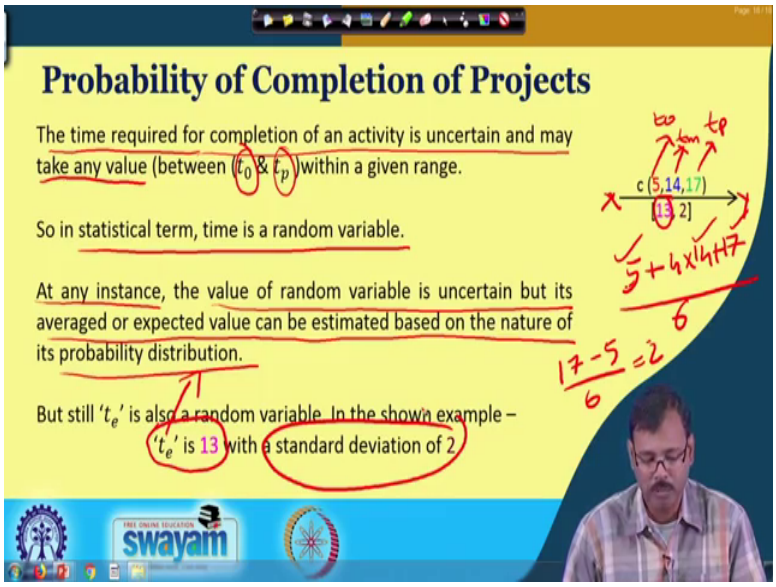


Here, we will discuss about the probability of completion of the projects. So, first let us see the time required for completion of an activity is uncertain and may take any value, may take any value between the optimistic and pessimistic value. You have seen in the last class, we have discussed the two farthest range of the values are considered as the optimistic and pessimistic values and it has been found that for a large number of cases,

sufficiently large number of cases almost 90 to 99 percent chances are there. The value which are randomly attributed to this are lying between these two values

So, that is why, within this given range, the most of the time, the values are coming between this two values and we are coming out expected time considering the mean which is the mean value of this distribution. And in statistically, we have considered the time is a random variable. Say, at any instance, the value of random variable is uncertain but it is average and expected. Value can be estimated based on the nature of the probability distribution. So, in the last class, while we have discussed, we have found that the time taken are lying between for movement from x place to y, from x place to y. The time taken are given in these figures, this is the optimistic one, this is the pessimistic one and this is the most likely which the maximum time we have observed that the this is the time considered because the frequency of this one is maximum. But while we have carried out our mean value, we have found from the distribution. The mean value is coming 13 because using the formula, 5 into 5 plus 4 into 14 plus 17 divided by 6 is giving us the value 13.

So, this mean value is considered or calculated based on these values and the standard deviation we have found is 17 minus 5 divided by 6 is 2 using this formula. So, here this T value which we have observed is also a random variable and that is why, it is associated with a standard deviation. That means, not only the value you are considering that is random variable, but also the mean value which we are considering that is also a random variable. And if we are having more number of observations related to these values, more or number of observations related to this values this value will also be changed because as we have considered here a beta distribution that is why it is not that much changing in this case. But if you were considering other distribution, this mean value will also change if you are suppose you are considering normal distribution, this will be changed.

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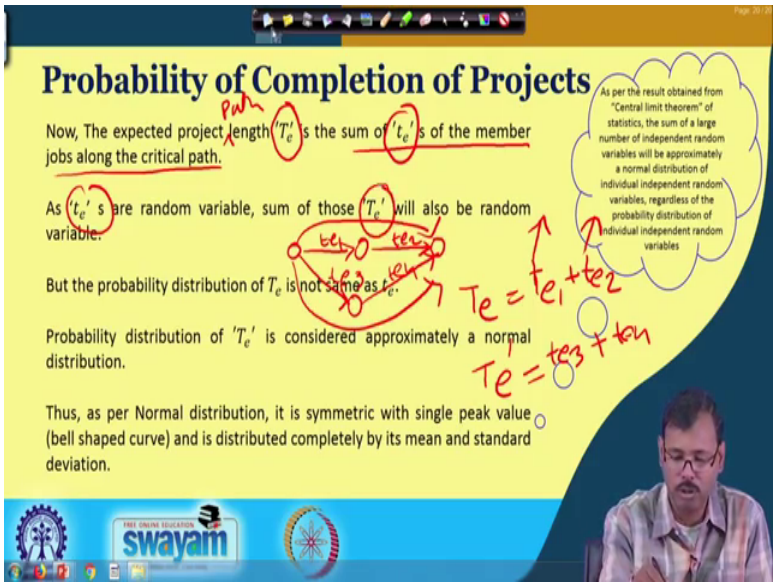


So, that is why this T value which we are coming out that is also that is also a random variable and now while we are calculating while we are calculating for a path say this is suppose this is a project path length and we have termed this the expected time of completion of this is the T which is nothing but the sum of the all expected time of his member jobs; that means, suppose we are having n number of jobs and this path is having T e 1 and T e 2. So, the capital T e is nothing but the small t e 1 plus small t e 2 and this t e. So, let me T e dot is this is t e 3, t e 4 is sum of the t e 3 and t e 4.

So, there are two paths and this path lengths are basically sum of this T s and this is also a random variable because this are all random variables. So, as T is a random variable the sum of the T e s also random variable and the probability distribution is not same as T because it is the sum of these. So, what is happening? We are considering for while we are considering a single job, that we have considered we are considering it as a beta distribution. But while we are considering for the path etcetera, we consider it is a normal distribution and it is symmetric with a single peak value, bell shaped curve and is distributed by it is mean and standard deviation.

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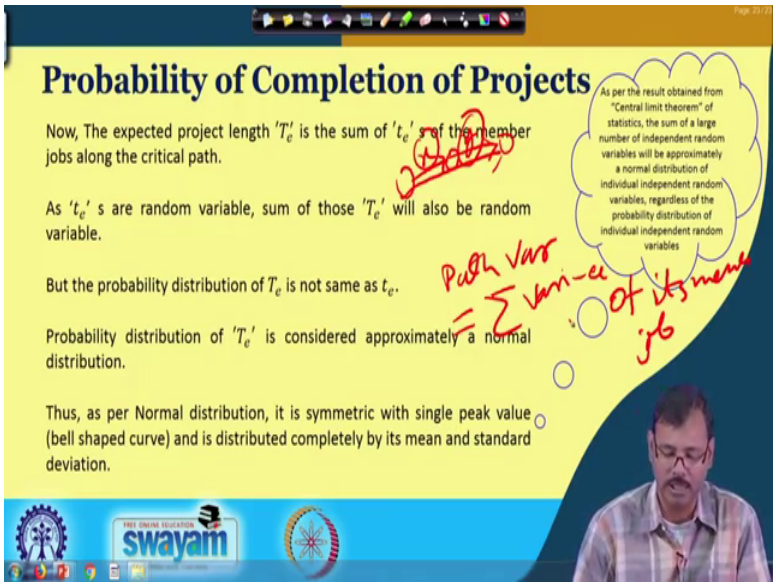


For a path, last class also you have discussed the standard deviations are achieved by summing up the variance path variance is nothing but is equal to the sum of the variances, variances of it is member job. So, that means, in last class last example which we have found the variance of this two, the variance of this one and this one, if we are adding this two variance, then the variance of this path is achieved. So, variance of this path is summation of this one and if you are square rooting the path variance, you will get the path standard deviation.

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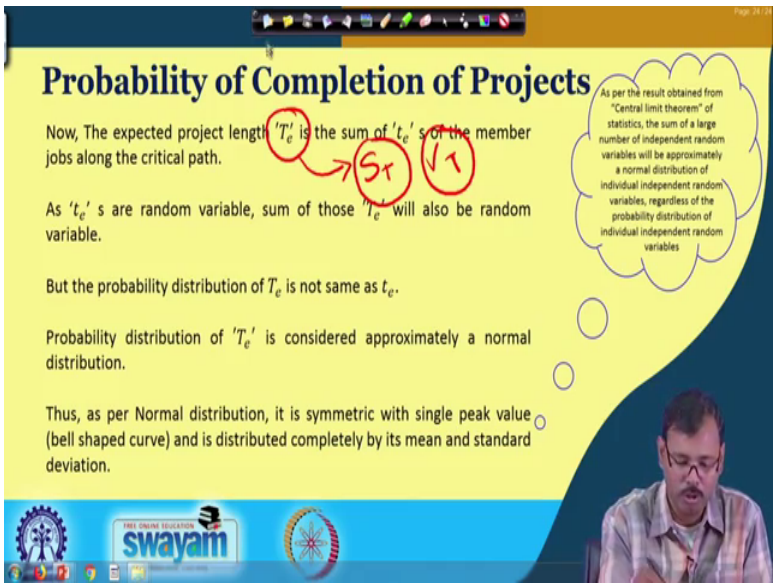

So, the path is having one random variable of expected length and that random variable is also associated with the path variance. This is standard deviation or this is the path variance we are finding out here.

So, paths are also considered as the random, randomly distributed and it is expected it will follow a normal distribution and which is considered to be a bell shaped curve to be presented to us. Now, let us just read this part to understand this normal distribution as per the result obtained from a central limit theorem of statistics, the sum of a large number of independent random variables will be approximately random a random distribution of individual independent random variables regardless of the probability distribution of individual independent random variables. So, whatever probability distribution we are considering for these, that will not effect this one we can consider that this is normally distributed and we are carrying out that assumption in this case also.

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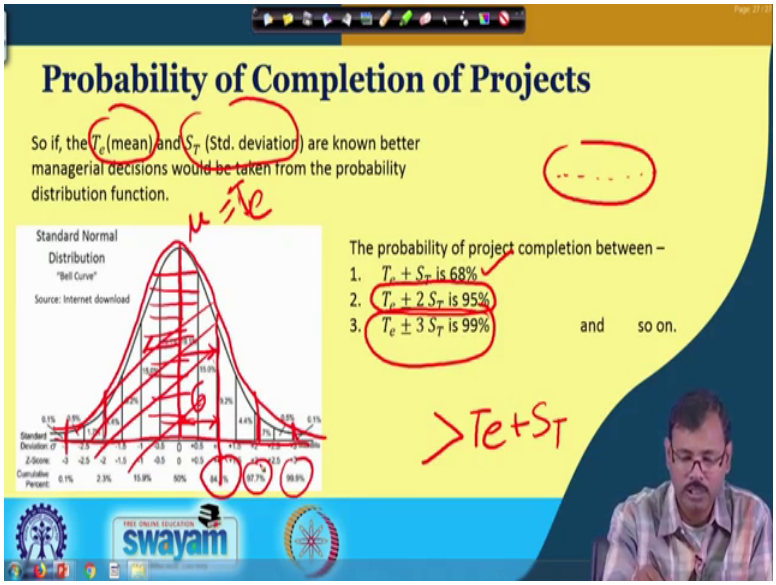


So, it is well understood that the paths or the project lengths are basically normally distributed with the consideration of mean and standard deviation of this one. And based on these, we can consider the we can take the different managerial decision this is the standard normal distribution curve in general we consider probability distribution curve for the normal distribution and the probability is given by the area under this curve. And if this is the mean, this is considered mean in general it is expressed as mu in our case,

we are considering it is we consider it is as the T e and this is the standard deviation, this length is the standard deviation one standard deviation which is called sigma.

So, the area under this curve plus 1 sigma minus 1 sigma, this area is representing 68 percent; that means, there is a probability 68 percent probability that the random distribution any value of the random distribution would lie between this zone. Similarly, we if are considering two sigma up to this up to this, then it is almost covering 95 percent; that means, if we are picking up hundred values random values here, there is a 95 percis a chance thent chance that the there at 95 percent of those hundered values will lie between this zone. And if we are considering 3, then it is 99 percent chance is there that it will lie between this 3.

So, the probability we are considering in consideration of that and if it is considered like this, what is the chance that it will lie between T e plus S T, then it is giving us this part and this part onward all this. So, this is considered as the 84 percent. Similarly, for this one it is 97 percent and this is 99.9 percent. So, this is the probability distribution along the normal distribution considerations.

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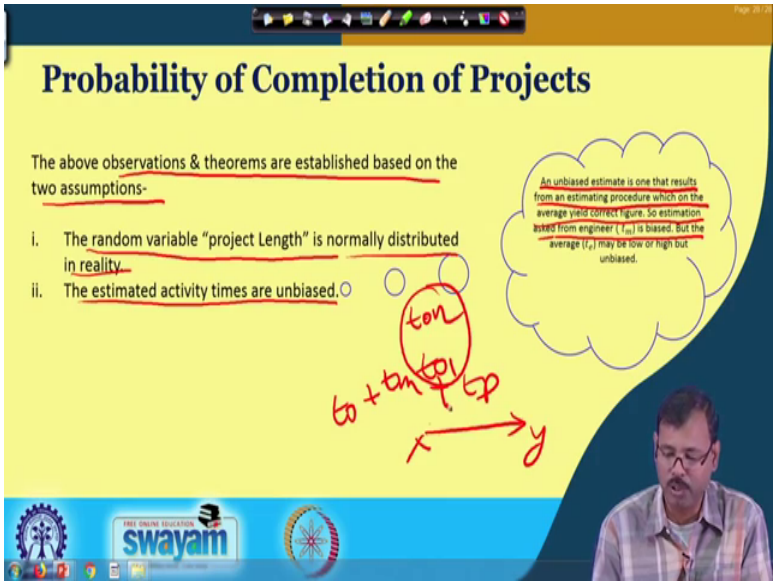


And we adopt this while we are assessing that chances of completion or the probability of completion of a project within the due date.

So, let us understand this the observation and theorem are established based on the two assumption; the random variable project length is normally distributed and the estimated activity times are unbiased. Why we are calling it unbiased because an unbiased estimate is 1 that result from an estimating procedure which on the average yield correct figure. So, estimation asked from engineer is biased; that means, when we have asked the engineer, what would be the time to go optimistic time to go between x to y, he is telling it is T 0 this is a this is a biased estimate. If you ask engineer x, engineer x will tell T 0 1; engineer y will tell it is T 0 2.

So, that is why this estimates are biased person to person specific and that is why it is biased estimate. But when we are calculating it based on T 0, T n and T p using some mathematical formula that is unbiased and irrespective of any biasness because that is coming out from some mathematical formula. That is why, it is called unbiased estimate.

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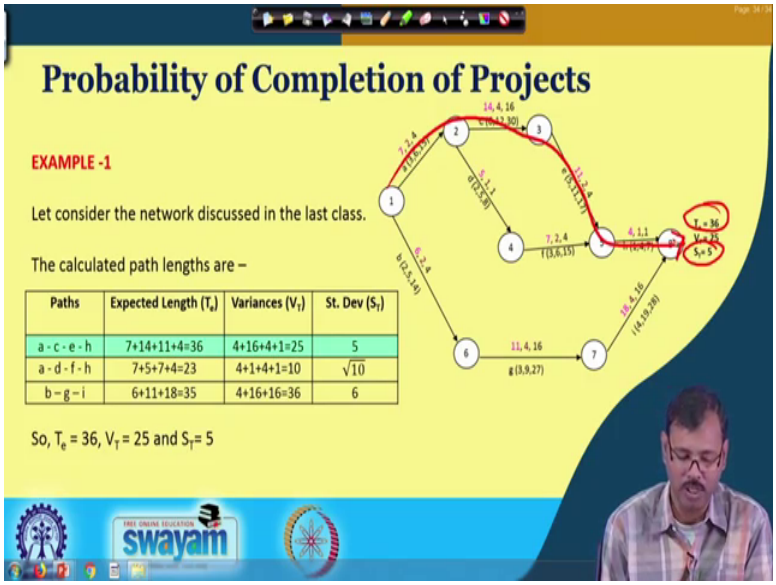


Now, let us understand this probability of completion of a project within some due date using some example. Say this is; obviously, last class also we have discussed this project. This project is already discussed this is the these are the expected time, optimistic time most likely time and this is pessimistic time based on that we have calculated which is the T e this is T e 2 is the standard deviation and 4 is the variance.

So, from this 16:17 we have calculated and you can find out in this project network. All the jobs A, B, C, D, E, F, G, H, I; for this all jobs, we have found that these are the T es,

these are the standard deviation, these are the variance which are expressed here. And we can understand very easily there are three available path. This is path A C E H path. The next path is this one which is A D which is a D F H path and third path is this one which is B G I path and the estimated path length that is the T e, capital T e is for first path is this one, second path is 23 and third path is 35. And also you have found the variances which are just sum of the variances for each that is 4 plus 16 plus 4 plus 1.

So, 4 plus 16 plus 4 plus 1 is 25 is the variance. Standard deviation is square root of that is 5 for path 1. Similarly, we have got 10 for path 2, 10 or standard deviation for path 2, 36 for path 3 and square root of that 6 for standard deviation for the path 3. So, as the longest path is 36 path 1; that means, thus means this is the longest path. So, that is why we are considering critical path is 36 with the standard deviation critical path length is 36 with the standard deviation of 5.

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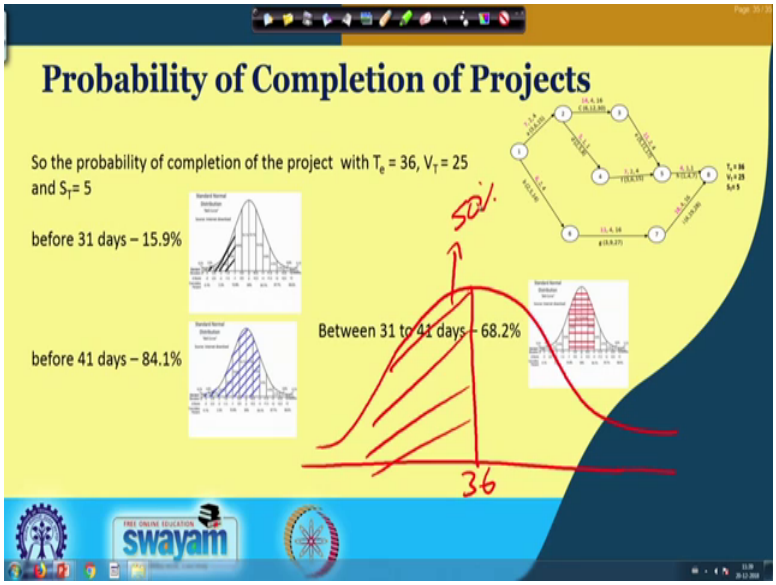


So, this example gives us the idea that these are the three paths and the expected completion of the project is 36 days or T is equal capital T is expected time is 36 or so.

Now, if any one ask we are considering only the critical path, now if any one ask the what is the probability that it can be completed within 31 days or within 40 before 31 days, before 41 days or between 31 to 41 days. So, it is basically the plotting in a bell shaped normal distribution curve. So, if you are distributing this curve, our expected mean time we have found here which is at 36. And if any one ask what is the probability

that it will complete before 36 days; that means, this part is considering the before 36 days. So, that is why this is gives us our 50 percent probability that it is lying between this two, our standard deviation is 5.

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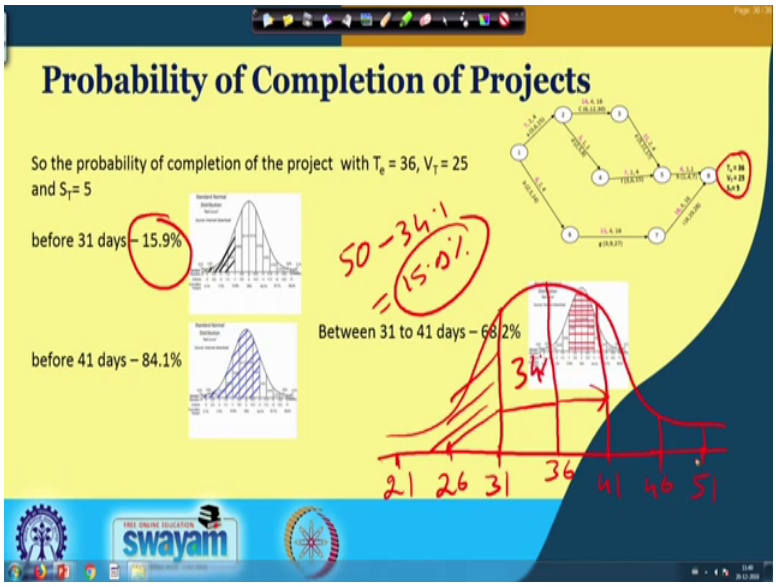


As our standard deviation is 5, if we plot this is the 36, this then mu minus sigma that is coming 31, mu minus 2 sigma is coming 26 and mu minus 3 sigma is coming 21.

Similarly, mu plus 1 sigma is 41, 2 sigma is 46, 3 sigma is 51. So, while we are considering the critical path and considering the expected project duration is 36 with mean and standard deviation of 5; that means, the probability of completion of this before 31, if the 31 days are the due date is this 1; that means, it is much less and we can see this is 15.9 percent. It is coming because you know it is this part is in the first curve you know that this is coming around 34 percent. So, this is 50 minus 34 giving us 15.9 percent.

So, this 15.9 percent is this part, this is 34 part, this is another 34 part, it is 68.

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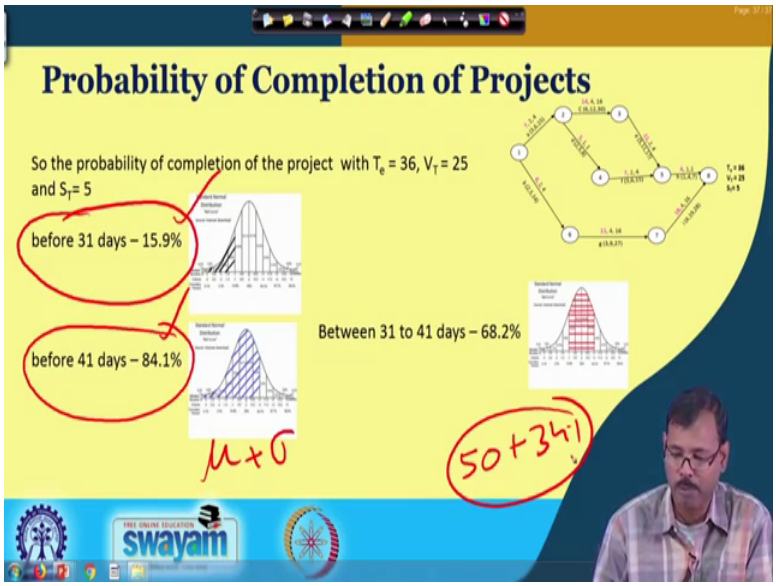


So, these are known to us from the normal distribution curve. So, that is why before probability of completion of the project before 31 days or 15.9 percent, this is before 41 days; that means, mu plus 1 sigma is 84.1 percent because it is 50 plus 34.1.

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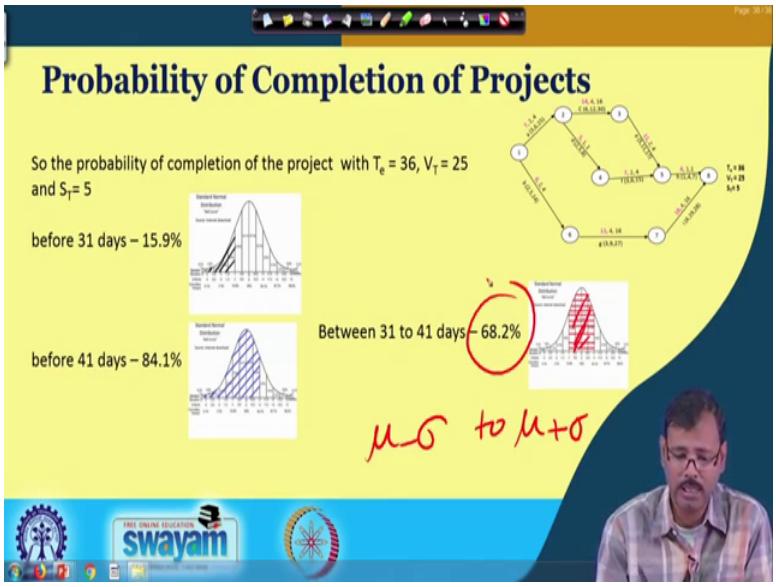


So, it is 84.1 percent and between plus minus, this is between mu minus sigma to mu plus sigma; that means, this part is giving us the idea about it is 68.2 percent.

So, basically area under the normal distribution curve gives us the probability of achieving the target within the due date.

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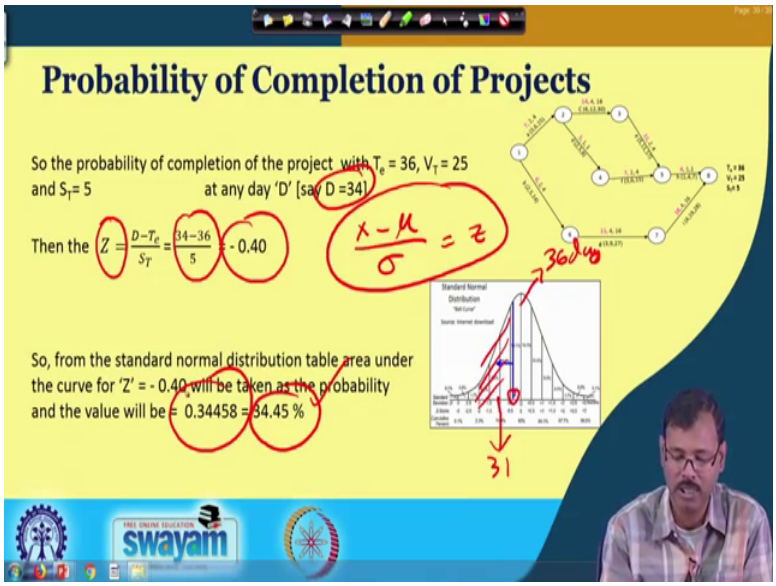


So, if any one is asked that how to see the normal distribution curve, then they have to calculate the z value. So, z value is nothing but the due date minus mu divided by sigma. So, it is value x minus mu by sigma is the variable. You have to calculate it and from the standard distribution formula you can standard distribution table you can find out the value of z value of minus 0.4 will give us the probability. So, the chances if the due date is called 34 dates to complete this 34, on 34 days where this is basically gives 36 days mu and this one is basically 31 days, so 34 days is coming somewhere at this place.

So, the area under the curve is nothing but this one and from the standard normal distribution table we have found this is representing 34.45 percent. So, the standard value is this one for minus z value of minus 0.40. So, this is one easier way to find out the probability of completion of a project by a given due date.

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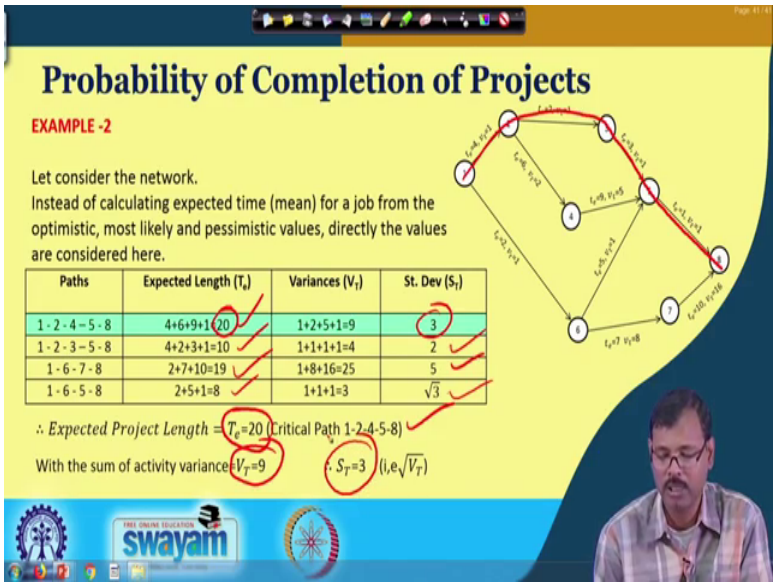


Now, let us see the another, another example. This is also discussed to you earlier. In this case, the there are four paths. Path one is this one, path 2 is this one, path 3 is this one and path 4 is this one. So, this paths are given, the we have given now directly, the mean value and variances of each job and this values are basically mean values or summed to get the expected length, variances are summed to get the variance of the path and the sum path variance is square root, rooted for getting the standard deviation for that particular path

So, by this way the four paths are identified here are considered and it has been found the critical path is this one; that means, this path is considered to be the critical path which is having the expected path length or mean value of path length is 20 and the standard deviation of 3. And these are the other path lengths 10, 2, 19, 5, 8 and under root 3. So, the expected project length is as calculated here is 20 and it is considered as the critical path, variances is 9 and standard deviation is 3.

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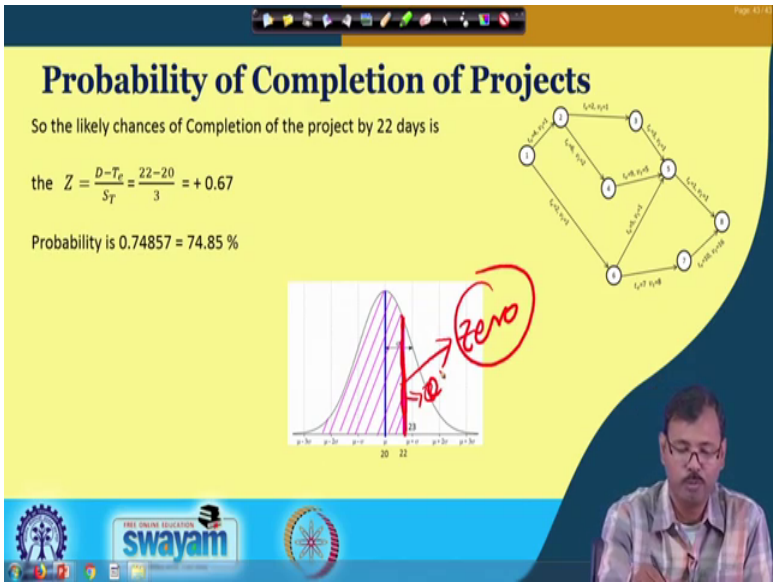


So, the likely chance chances that this project would be completed by 22 days. So, we have to find out the z value. So, as the expected due date is 22 or critical mean value of the critical path is 20 and standard deviation is 3. So, using this formula, we have found the z value is plus 0.67. And from the normal distribution curve standard table, we can determine that the probability is that the 75 percent probability is that it will lie between it will completed by the given due date of 22 days.

So, if i plot these on the normal distribution table this is the mean value 20 which we have considered as the critical path length. The standard deviation of the critical path length is 3. So, this is 23 is mu plus sigma, but 22 days is lying between these two. So, we have plotted the 22 days here. And the area it is considered that completion project due date is 22 days; that means, the chances of achieving it before 22 days is basically the area covering in this side. So, this area we have find out that the total area is 111. So, this is 0.74857. So, this area is comprising 75 percent.

Now, let me give you one point. Say, if I ask you what is the probability that this job will be over exactly on 20 second days, say the moment we are telling it is exactly at 20 second days; that means, the domain is this one only which is the line is not comprising any area. So, the practically the exact probability, probability of exact completion of this on 20 second days is basically 0 as the area under this curve is 0. So, that is why you must remember there must be two point here, up to infinity is there or if we may ask

what is the probability that it will be between this two that may be asked like this way; that means, between 22 to 20 second day. That can be, probability of that can be assessed, but the for a particular date the probability cannot be accessed because it is 0.

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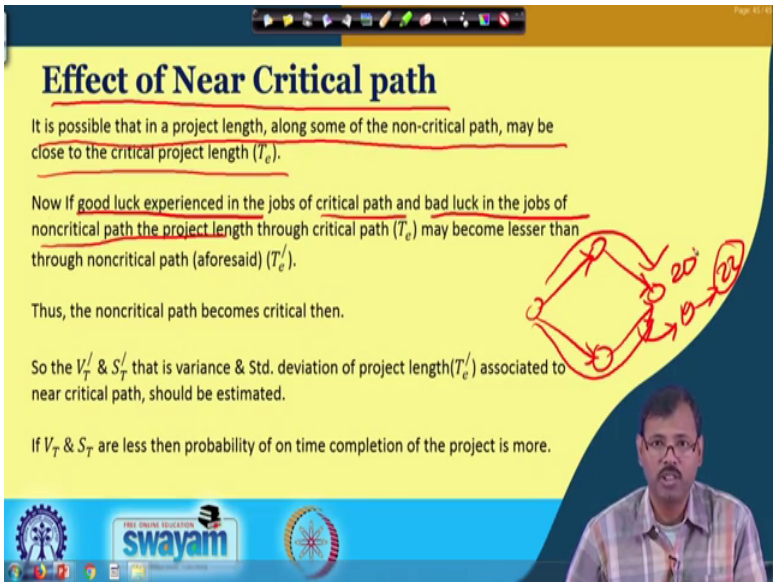


So, now let us just look into this. We will discuss the detail of that in the next class. There is effect on the near critical path also because so far we have considered the probability related to critical path. We have there are probabilities associated with the other paths also, but we are neglecting that. Now, say it is possible that the in a project length along some of the non critical path may be close to the critical path project length also. In fact, in the last example we have found there is a path of critical path is of duration 20, mean value of twenty, but there is another path which is having the expected path length of 19 also. And we may observe some good luck experience on the jobs of the critical path but bad luck experience in the job of non critical path.

That means, if this is the critical path everything goes fine everything goes fine on these and this became over wise say 20 days, but this non critical path we have experienced bad and this is instead of completing in 19 days it is longered and after that it may take 22 days or 23 days; that means, there is effect some effect or influence of the non critical path also on the probability of completion of a job, especially those path which are very close to the critical path or the that is considered as the near critical path, but all the jobs are important while we are going for the probabilistic analysis.

So, thus the non critical paths became critical in those condition where we are experiencing bad luck condition for the non critical for the non critical jobs. So, v variances and standard deviations on the project length associated to near critical paths are also important and must be considered. So, for the critical path, variance standard deviation may be less and may have a very narrow range and that is why the chances of their completion on time completion is more, but that may not be available for the non critical path because of their more variances. In fact, we will discuss more on this topic in the next class, but I understand that now you are able to believe that the all jobs are important whether it is a critical job or non critical job, all the jobs are important and the probability mean time of completion of those jobs and the variances of that job from it is mean time is also very important than must be considered while you are trying to achieve the probability of completion of the total project.

So, this is all for this class. We will discuss more on the effect of the near critical jobs on to the project network analysis in the next class.

Thank you.