Network Analysis for Mines and Mineral Engineering
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Lecture - 17
Expected length of critical path calculation with examples

Let me welcome you to the 17 lecture of NPTEL online certification course, on Network Analysis for Mines and Mineral Engineering. In this class, we will discuss about the Expected length of critical path calculation with examples for program evaluation and review technique.

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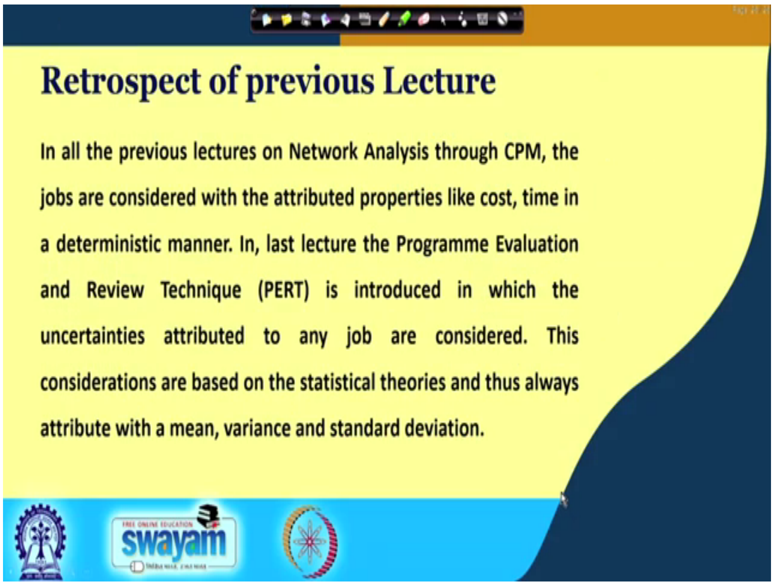

So, like very class let us retrospect what we have covered in the previous lectures. In our lecture between 1 to 15th, we have covered the network analysis through critical path method, where the jobs are considered with the attributed properties like cost time which are in deterministic manner , but in last class we have introduced ourselves into the program evaluation and review technique. In this technique the uncertainties are attributed to the job, in terms of its time and this considerations based on the statistical theories and thus always attributed with a mean, variance and standard deviation.

So, basically the problem of deterministic approach where the uncertainties are not considered and that is why whenever the decisions are made, whether the manager or the decision makers designer are approaching towards the more uncertainty or not, that is not basically determined in the deterministic approach. That is why the problemestic approach gives a better result in those in those certain conditions.

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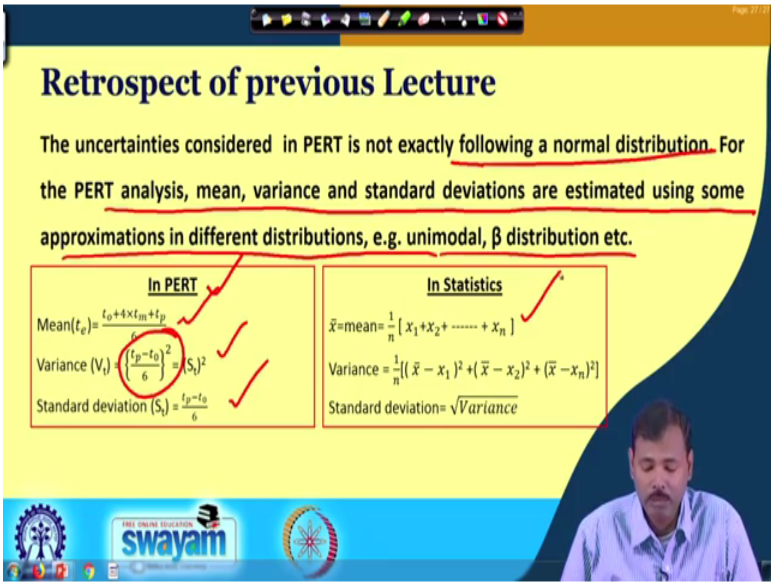

So, in this lecture in the last class we have seen that, in program evaluation and review technique analysis method of analysis, we are not exactly following the normal distribution cases. And we have seen that in PERT analysis the mean variance and standard deviations are estimated using some approximation of different distributions like unimodal distribution, beta distribution and this formula are basically very easy to remember and to follow.

So, if you see the basic difference in PERT, we are considering mean is basically the weighted average of 3 observations: one is the optimistic observation, one is the pessimistic observation and they are given weighted average of weightage of one and most likely observation that is giving the weight of 4. So, it is divided by the 6 so, t 0 plus 4 t m plus t p by 6 is basically giving us mean value or the most expected value for that particular job.

The variance is basically observed or estimated from the range, that is the most pessimistic observation and difference between the most pessimistic observation and most optimistic observation, dividing it by 6 and the square of that is giving us the variance. And standard deviation is basically the difference of the range that is the optimistic pessimistic and optimistic observations and dividing it by 6 is giving us the standard deviation.

However in statics the mean is mean is considered using the formula, where all sum of the all observations are divided by the number of observations, variance is basically the difference from the mean to the observations square of that after summing those square values dividing it with the number of observations is giving us the variance. And the square root of the variance is basically giving us the standard deviation. So, basically that is the statistics where the normal distribution is considered, but in program evaluation and review technique we consider unimodal or beta distributions for this cases.

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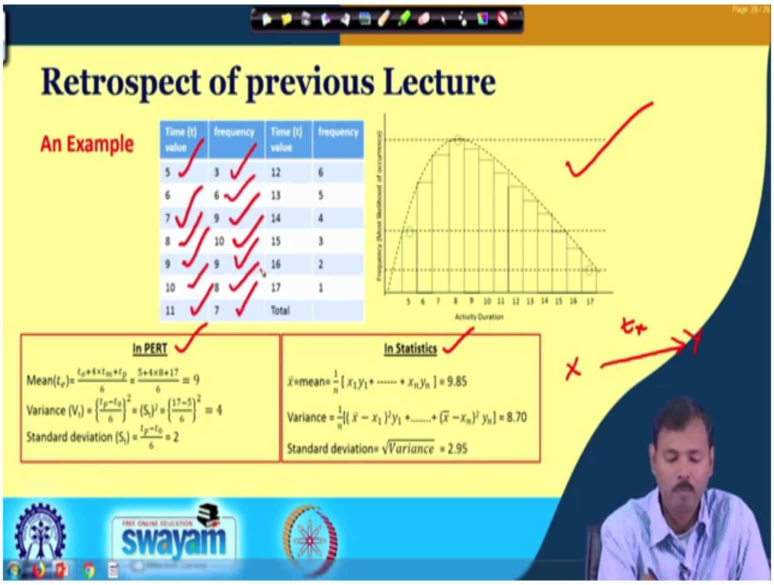

Now, in last class we have also seen for one particular example, where we have used the normal statistics and also program evaluation review technique for determining the different attributed properties. Let us consider our observations which are given depicted in this picture, if we are considering this values say value of 5 the frequency observed is 3 times means probably if we are moving from X to Y. The time required is observed 5 minutes 3 time or 5 days 3 time 6 minute or 6 days 6 times 7 minute o 7 days, whichever it is 7 days say let us say days 9 times 8 days is maximum 10 times we have observed 9, we have observed 9 times, 10 we have observed 8 times 11 we have observed 7 times 12 we have we have observed 6 times, 13 we have observed 5 times, 14 we have observed 4 times ,15 3 times, 16 2 times and 17 1 time.

So, if we have plotted the frequency versus value and this plot is we have discussed it in last class also this 5 value is observed 3 times.

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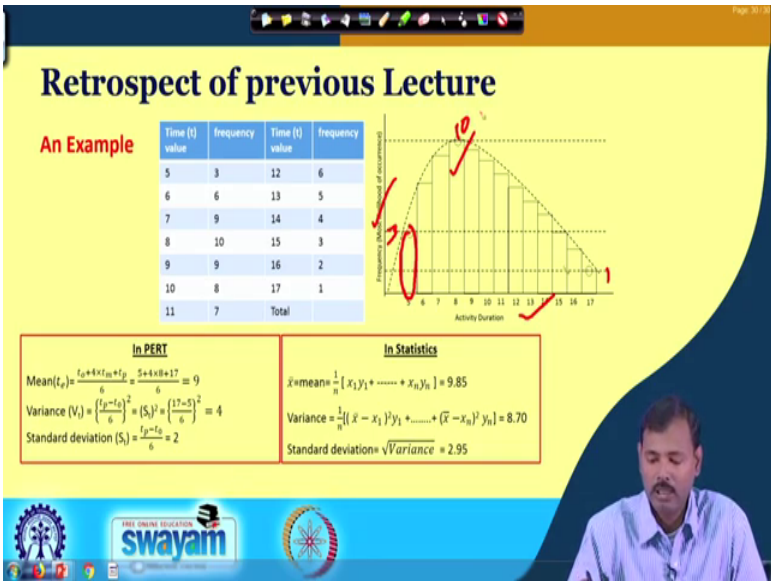

So, it is this is 3 this is one you can see and this is maximum 10, you can consider the 10 number of times we have observed 8. So, this frequency distribution is plotted with this bar chart and this is easily understood by all of us.

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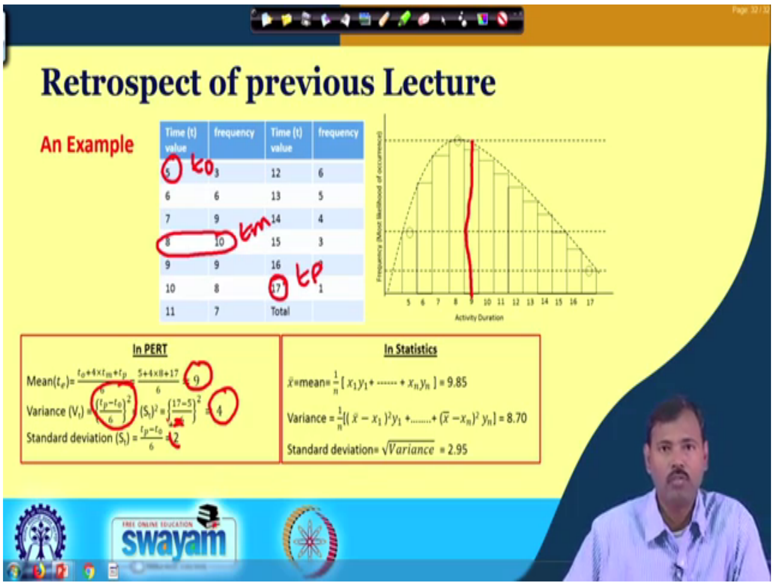

Now, if we solve this problem using program evaluation review technique, our mean is basically, or first let us understand the optimistic time. Optimistic time is 5 because this is the minimum time we have observed so far. So, if we are highly optimistic we are we can assume that in 5 days we can complete this job or we can move from X to Y.

So, that is why 5 is the most optimistic time and 17 is the most pessimistic time, because as per our experience we have observed that this job may lingered up to 17 days. So, that is why it is 17 days is the most pessimistic time and so, most likely is the maximum number of times we have observed that this job is completed within 8 days so; that means, that is the most likely consideration and we have used those as the t o t m and t p value and using this value we have found the most expected time or mean time which we have observed is 9 days.

So that means, our most expectation is that it will be end this job will be end maximum number of times or most likely in most of calculated mean is the 9 time 9 days it may take to complete this job. Using this PERT formula p programmable evaluation review techniques formula, we have found the variance is coming t p minus t 0 by 6 whole square. So, it is 17 minus 5 by 6 whole square so; that means, it is coming 2 square it is 4 and the standard deviation is the square root of variance so; that means, it is giving us 2 is the value for the standard deviation of this.

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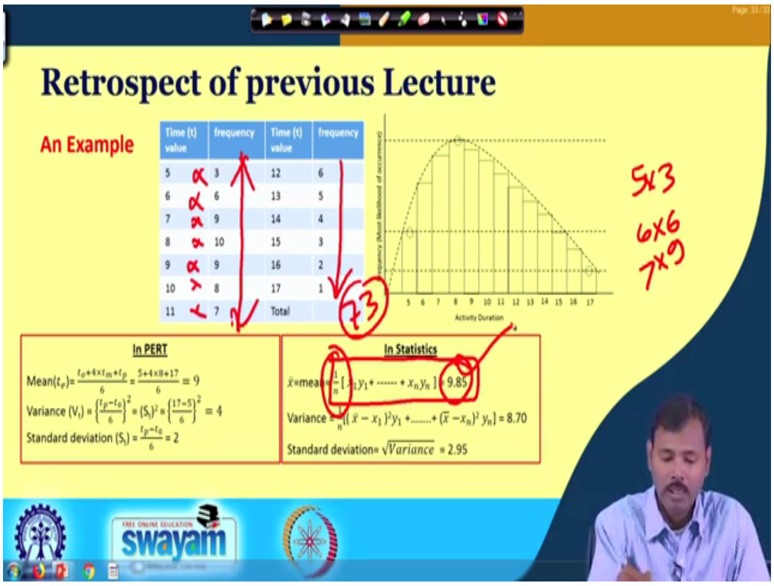

But if you try to calculate it using a normal distribution considering it is a normal distribution, the mean is can be obtained using this formula that means, the summation of 5 into 3 plus 6 into 6 plus 7 into 9 so, by summing all these and dividing it by the number of observations which is coming the summing of this one so, by this calculation you will find out this 73 so, dividing it by 73 we are getting the mean value.

And I am not given the detail calculation here you can calculate it in excel very easily, it is coming 9.85 as it is calculated. And using the same formula where similar formula, where we have obtained the difference hat is the variation of the values x minus x y x minus x 1 the sorry.

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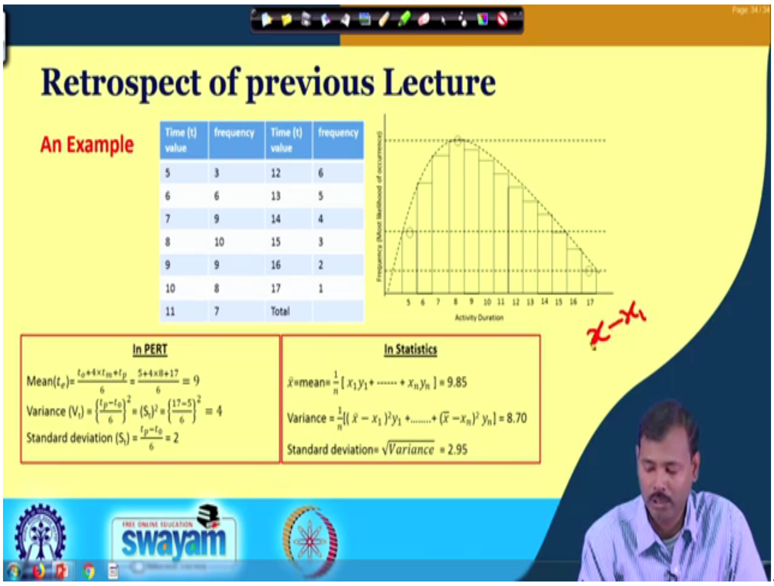

Let me consider the mean value is xX bar.

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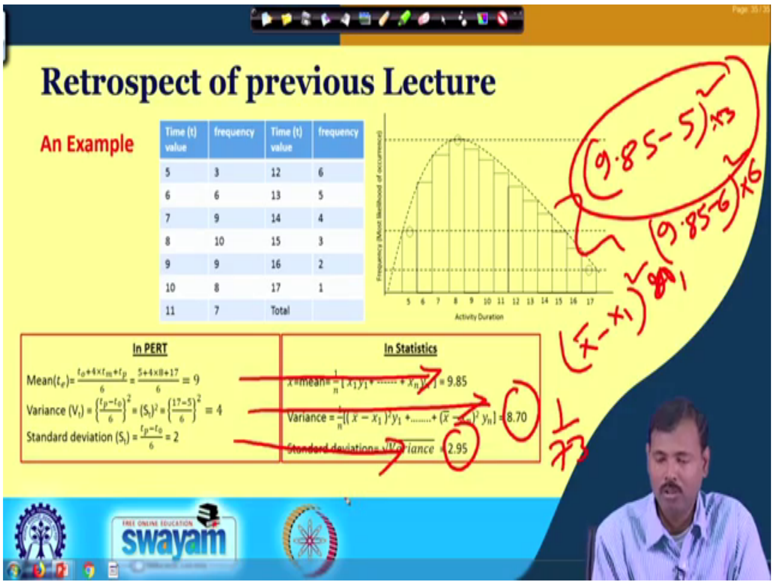

So, X bar minus xX 1 whole square multiplied by frequency say, n 1 of this one so, this is say 9.85 minus 5 whole square into 3. And by this way we are summing off all the values minus 6 whole square into 6. So, sum of all this value dividing it by 1 by 73 gives us the variance that is 8.70, we have obtained and thee square root of that that is coming 2.95 is basically the value you have obtained as the standard deviation.

So, if you are comparing this two you can find out the mean also little bit higher in statistics what we are observing, variance is also little bit higher in this case and standard deviation is also little bit higher in this case, but anyway they are not in very far away, they are in close proximity with each other. But in our program evaluation review technique we use not the normal statistics not the normal distribution formula, but the unimodal distribution formula or beta distribution formula we are using in this case.

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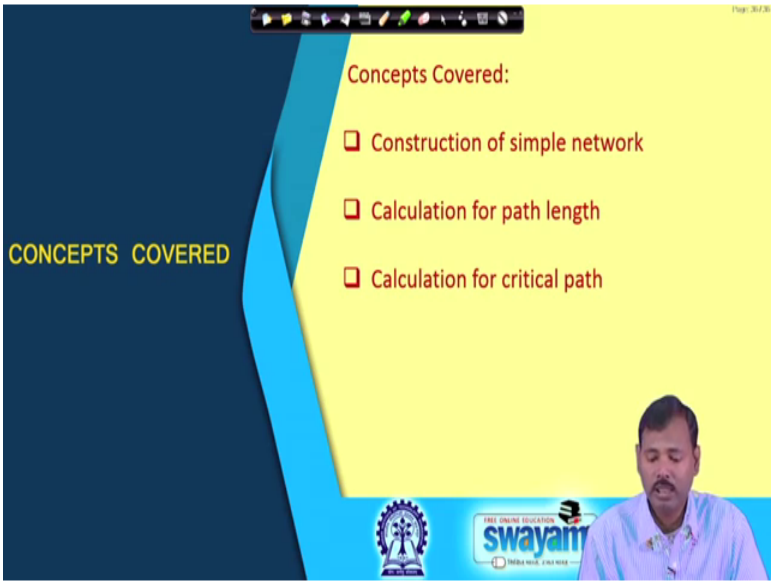

So, this we have already covered in the previous lecture in this class, we will cover the construction of simple network and the length path length calculation critical path determination we can cover in this class.

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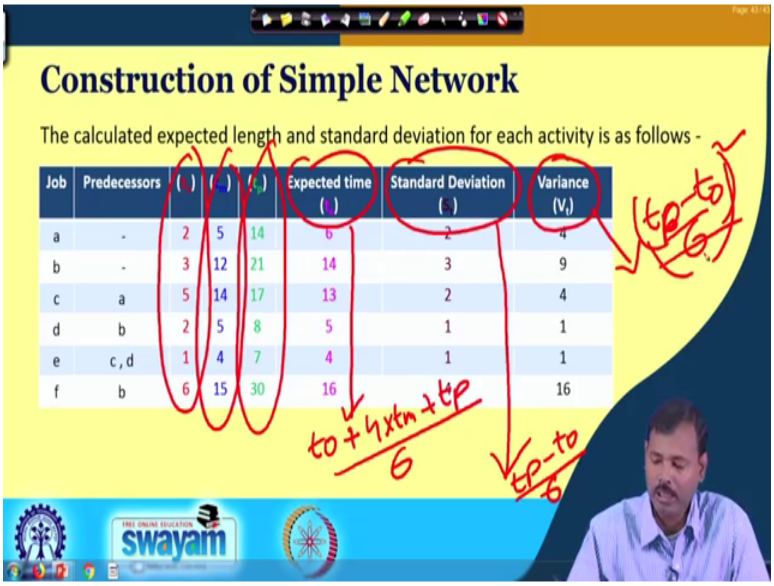

So, for construction of a simple network and its analysis let us consider one simple example. So, that we can solve and we can calculate the path length critical path can determine, say consider there is a 6 jobs and their predecessors are given here this is already, we have solved in our critical path method analysis this type of problems.

And their dependency are now basically shown in this network, if we plot this network you can see this is the a job, this is the b, job this is c, this is d this is e and this is f job. And it is also given to us these are the most optimistic time of this jobs, this is the most likely time of this jobs. And this is the most pessimistic time of completion of this job which are given in the first bracket attributed to each job in different colors red blue and green color.

So, this is the way we have presented in this network, but it need not to follow the same sequence can have your own sequence, but in this network we have explained the dependency of this jobs and activities and their respective optimistic pessimistic and most likely time. So, now, look into this when we are having this optimistic most likely and most pessimistic time.

For each and every activity we can calculate the expected time, we can calculate the standard deviation and we can calculate the variance. For this we use the formula weighted average formula four into t n plus t p by 6, for this we use the formula t p minus t o by 6 for this we use the formula t p minus t o whole square sorry divided by 6 whole square. So, this is gives us the variance. So, basically it is standard square of the standard deviation.

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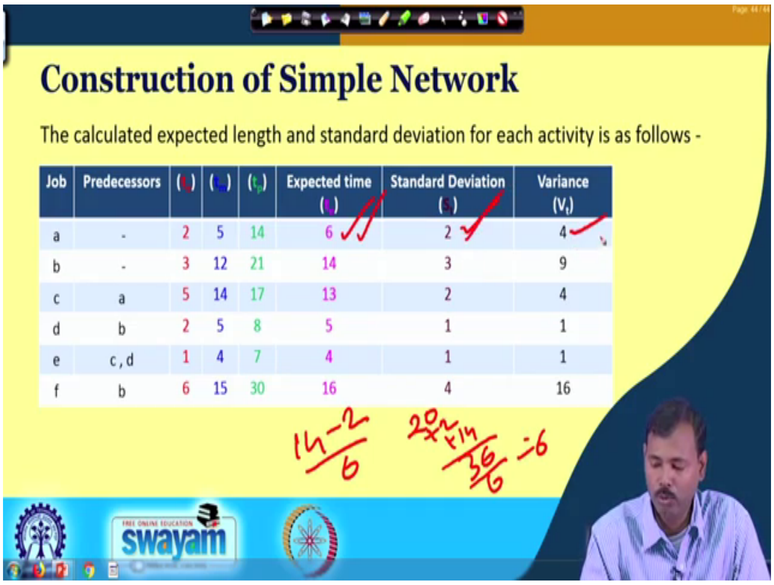

So, by this way we have calculated and we have found for job a, this is the expected time this is the standard deviation and this is the variance. So, you can see this is 5 into 4 20 plus 2 plus 14 so, this is 36 divided by 6 is 6 so, that is 6 so 14 minus 2 by 6 is the standard deviation so, 12 by 6 2 and it is 2 square. So, it is 4. so like that way it is calculated for all the jobs and these are the values we have obtained for jobs of a to f.

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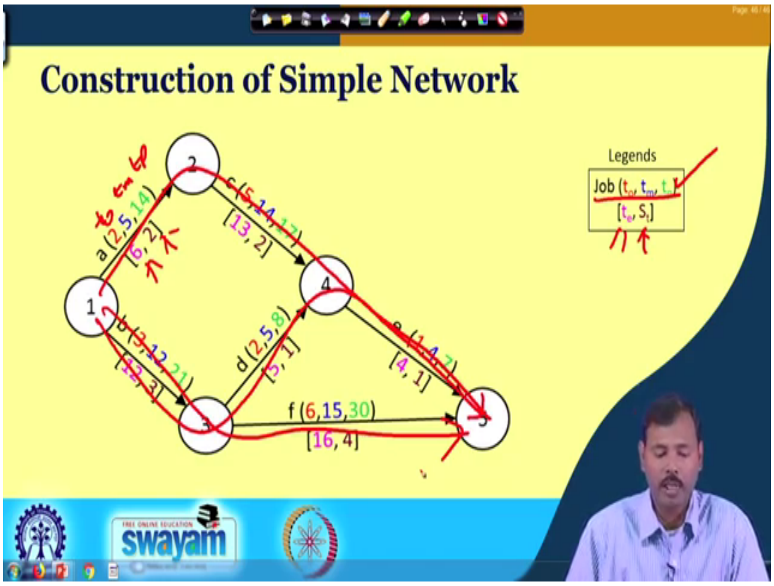

Now, let us present this on the network and you can see these are the t o t m and t p, in the under the bracket in the different color code which are legends are given here. And in this pink color we have given the expected time that is the mean time and this is the standard deviation in the brown color, this is the standard deviations are presented. So, if you look into this network now you can see this there are different paths in this, different mean times are available and different variance or standard deviations are available in the path.

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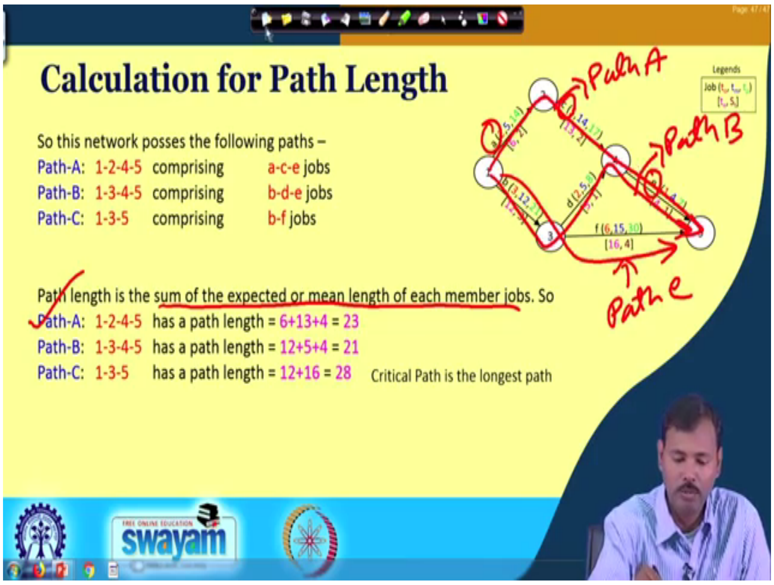

So, now let us look into this network a little bit other way. Now, you can see in this network we are having basically 3 path so, path A is basically 1 2 4 5 so, this is the path A path B is 1 3 4 5 so, this is the path B path C is 1 3 5 so, this is the path C. And the program evaluation review technique tells us that the expected path length is basically the sum of the expected or mean values of each member jobs in it.

So, if you look into the path A; that means, the member jobs are A C and E let me clear this one once. So, in path A our member jobs are A C and E and they are expected value is 6 13 and 4. So, 6 13 and 4 so the expected mean value of completion this of this path is the sum of these is the sum of this expected mean values or mean length of the each member jobs; so, 6 plus 13 plus 4 giving us 23.

Similarly, if you look into the path B if you look into the path B, in this case the expected lengths are 12 5 and 4. So, this is 12 5 and 4 it is giving 21 days. And if we are looking at the path C it is 12 and 16 this is 16 12 and 16 and this gives us the total path length of 28. And if you are looking into the complete network now, you can observe the expected path length of path B path C and path A path B and path C are 23 21 and 28 days respectively and as this is the longest path so, this is considered as the critical path of this network.

So; that means, this critical path which is taking 28 days for this particular condition and that is taking the most longest duration. So, that is why it is considered as the critical path.

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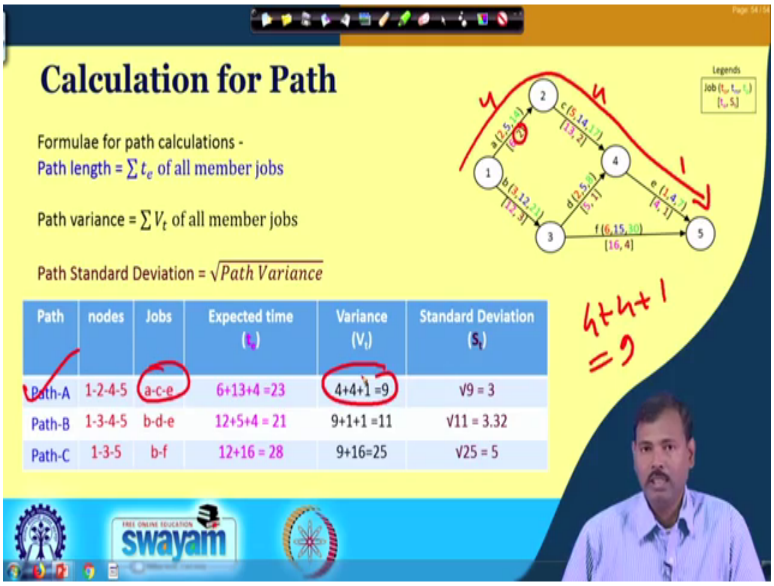

Now, let us look into the formula we are considering, path length is basically the sum of the expected times of all member jobs, this is the formula for calculating the path length. Now, let us look into whether this path length is acceptable or how much reliable it is having. So, in that case we have to consider the path variance and program evaluation and review technique gives us the formula that path variance is basically sum of the variance of all its member jobs.

So, now let us look into the path variance now, these values given here are basically standard deviation. So, let me write path variance while we are discussing the each individual path. So, let us first consider path A so, this is the path A the member jobs are a c and e and these are having standard deviation of this one so, let me write the variance; variance is 2 square 4 here variance is again 2 square 4 here variance is again 1 square 1 here so; that means, sum of the variances of path A is basically 4 plus 4 plus 1 that is 9.

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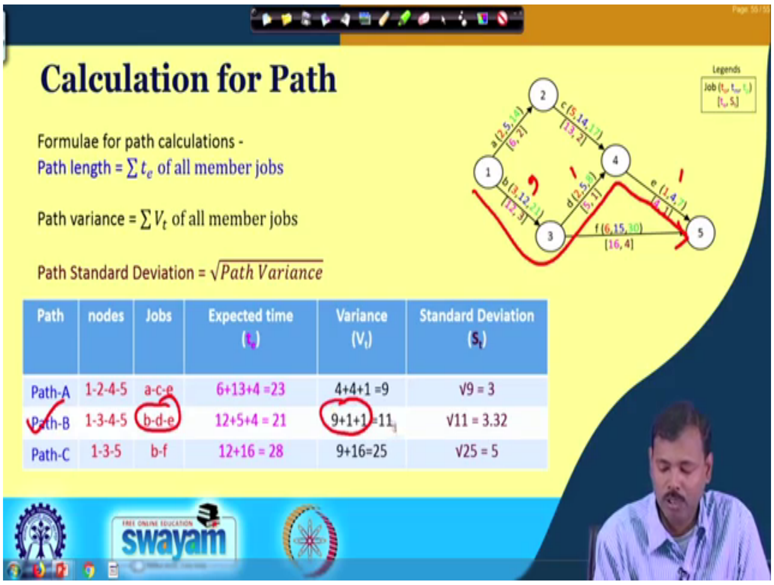

So, variance of path A is 9, now let us look into the path B, the path B is basically this path. And you can see the variance for this job b is basically 3 square that is 9 job d is basically 1 square that is 1 and job e is again 1 square that is 1. So, the sum of the variances of b d e job is basically 9 plus 1 plus 1 that is 11.

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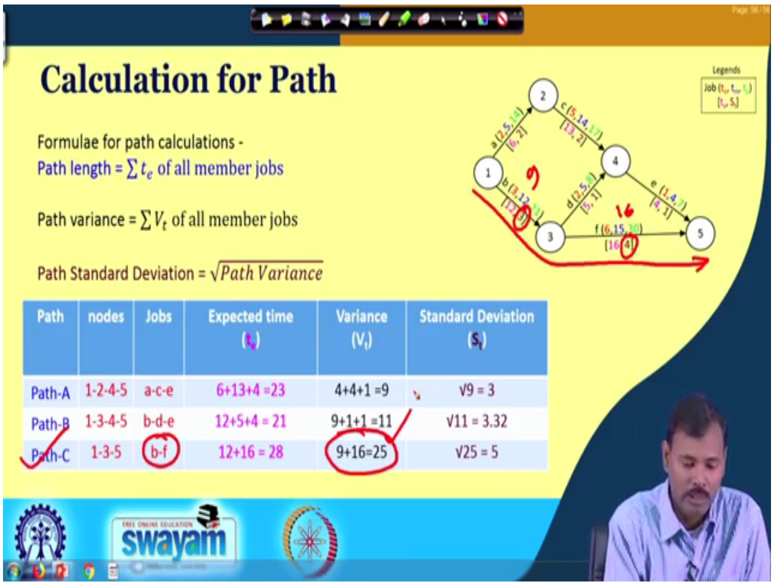

And if you are looking at the third path the standard deviations of 3 and 4 so, it is 9 and this is 16. So, there are two jobs attributed in path c b and f and their variances are 9 and 16. So, summing up them we are getting 25. So, 25 is the variance of path C which is a critical path. And standard deviation is again the square root of the variance of the path variance what we have calculated from the summing the variances of the each member jobs.

So, for path A it is coming square root of 9 that is 3, for path B it is square root of 11 that is 3.32 and path C it is square root of 25 that is 5 is 5 is the path variance the path standard deviation for this paths. So, now, if you look into this path A expected time is 23 standard deviation is 3, the path B expected time is 21 and standard deviation is 3.32 and path C is expected duration is 28 and standard deviation is 5. So, this is the critical path which is having a standard deviation of 5 days can be vary in this condition.

So, ah; that means, whenever in program evaluation review technique we are analyzing the network, the network is having an expected completion time that is coming from the critical expected critical path length, but that is also possessing some uncertainties, because we are attributed uncertainty to each of its member jobs.

So, that is why critical path is also uncertain, completion of the critical path at its expected time is also uncertain and this uncertainty depends on the standard deviational variance. And that is why there is a scope we can calculate the probability of the completion of the critical path and probability of the calculation of the complete project is also very complex one which is not within the scope of this work.

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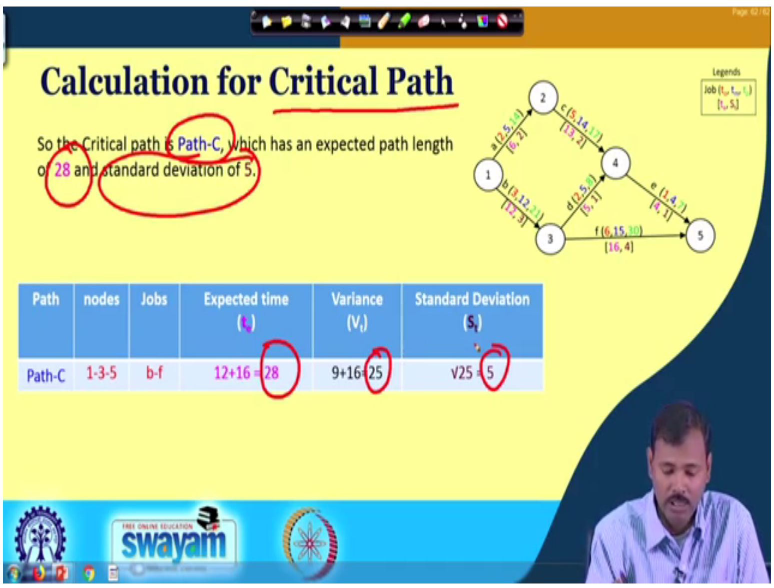

So, if we are observing the critical path now critical path is the path C which is having a an expected path length of 28 days and the standard deviation associated to that path is 5 days. So, variance is 25 days expected time is 28 days and standard deviation is 5 days. So, basically this is the critical path calculation we have we can calculate with the some given uncertainty into this.

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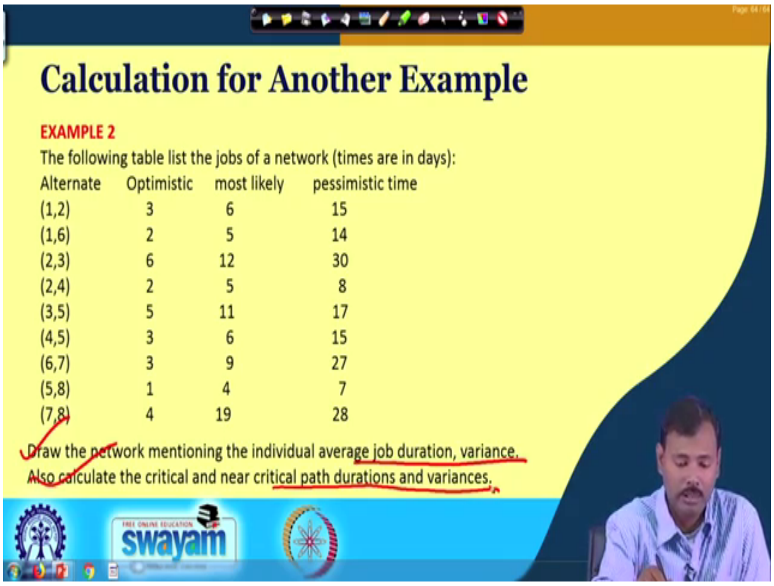

Let us observe another example for this is a very big network and all the times are given in days jobs are given in the name of alternate. So, that it can be constructed very easy and we have given the name a b c d optimistic time for each jobs are given. Most likely times are for each jobs are given pessimistic times for each jobs are also given. Now, our job is to our job is to draw the network and determine the individual job duration variance and also calculate the critical and near critical duration and variances.

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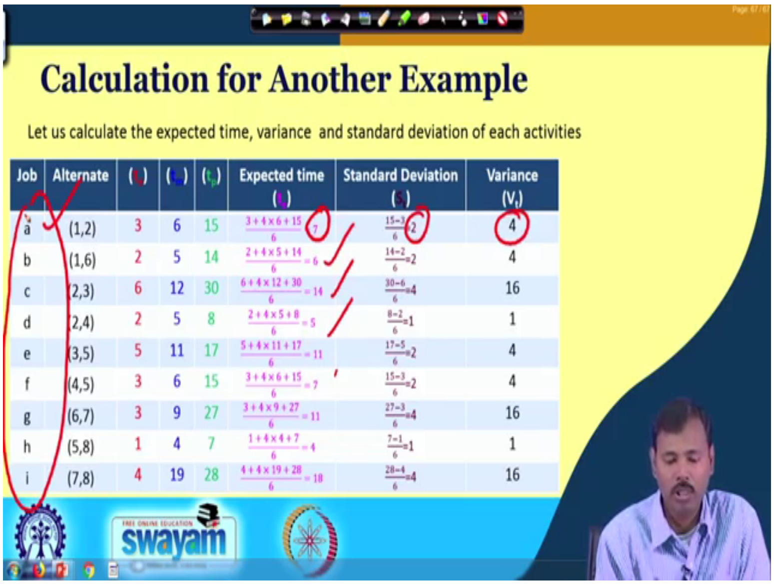

So, let us solve this problem, you can see these are the jobs and we have calculated the expected times standard deviations and variances, for each member jobs in the network. And these are already given for all the jobs, we have given the names of the jobs are also we are giving in this case.

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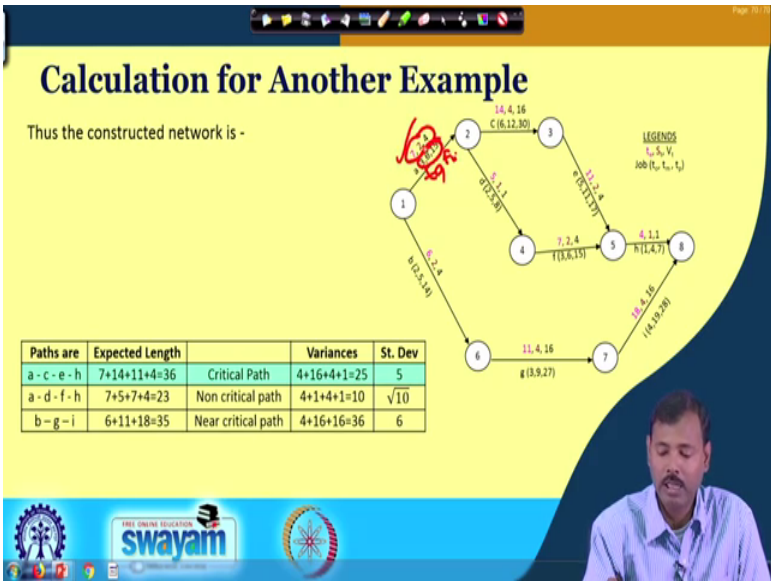

Now, let us draw the network so, this is the network as per the given condition we have drawn and you can see these are the jobs these are the t o t m and t p, these are the expected time standard deviation and variance of the job as per the calculation given in the last table.

So, this legends are given so, we can see we are having again 3 path here this is one possible path, this is another possible path and this is another possible path. And this possible path one a c e h a c e h is having the expected length of 36 with the variance of 25 and standard deviation of 5.

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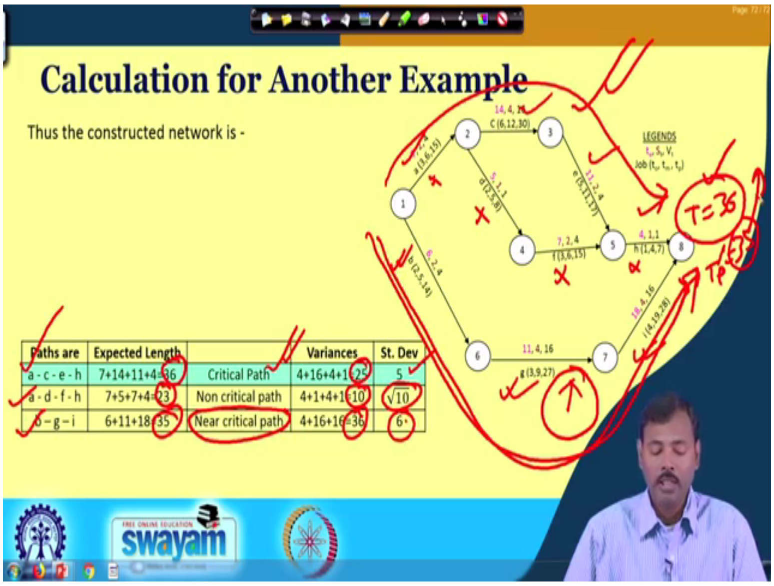

The net path a d f and h is a non-critical path, whose expected path length is 23 variance is 10 and standard deviation of square root of 10. And this is the third path that is b g and i and you can see its expected length is 35 which is very close to the critical path and with the standard deviation 36 as 6 variance of 36.

So, this path and this path are very close to each other and that is why though it is not a critical path it is called as the near critical path. And you can see this path is having a mean value of 36 which is associated with the critical path, but this is the near critical path which is expected to complete by 35 days. So, that is why this path is also a point of concern, because this path is also having some uncertainty and this path is also having some uncertainty. It may be possible in this path all the conditions are favorable and that is why it may complete at 36 or before 36, but here the conditions are may be unfavorable and that may delay or linger this path length and make it longer. So, that it may goes up and that is why there may be cases where because of the non critical path the project get delayed.

So, that is why this is the beauty of the program evaluation of and review technique that this consider the probability on the non-critical path also and that is why the project cannot be delayed, because the manager will always have the position to get warning, from the non-critical paths also that it is getting delayed and there is a likely chances that the total project may be delayed because of the delay in the non-critical path not on the critical path, which is not possible in the deterministic approach deterministic approach never consider the uncertainty in the non-critical path, but the program evaluation review technique consider the uncertainty in the non-critical path also.

We will discuss more on this in our next class and I believe you can have more example to be solve, in your in your as your home task and that will give still more confidence on solving this program evaluation review technique problems.

Thank you.