Network Analysis for Mines and Mineral Engineering
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Lecture - 16
Introduction to PERT

Let me welcome you to the 16th lecture of NPTEL online certification course on Network Analysis for Mines and Mineral Engineering.

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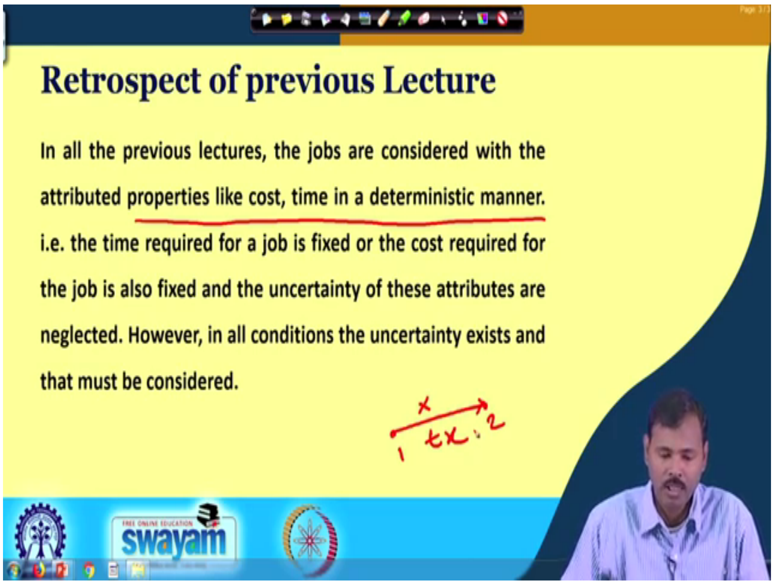

So, far we have already covered our network analysis using critical path method. So, this class will start the network analysis of program evaluation and review technique. So, before that let us retrospect what so far we have covered, in our all previous lecture whatever we have covered in those condition, the jobs are considered with the attributed properties like cost, time in a deterministic manner. That is we have considered that the time required to complete a job is fixed say we would like to transfer x material from place 1 to place 2 and we have considered the time requirement for this is t x.

So, this t x is considered as fixed, but the variability on this are not considered in the deterministic manner so; that means, there may be difference in the speed there may be difference in the traffic. So, that is why the time requirement may be different in the different conditions, but that part is not considered in the critical path method analysis what we have considered till date.

So, what happened the attributed properties like cost time are considered in a deterministic manner and that is why the uncertainty in those attributed properties are not considered, but in real life situation uncertainties are always lying with any activities. So, it is essential that we must consider the uncertainties, in our network analysis and that is covered in the program evaluation and review technique.

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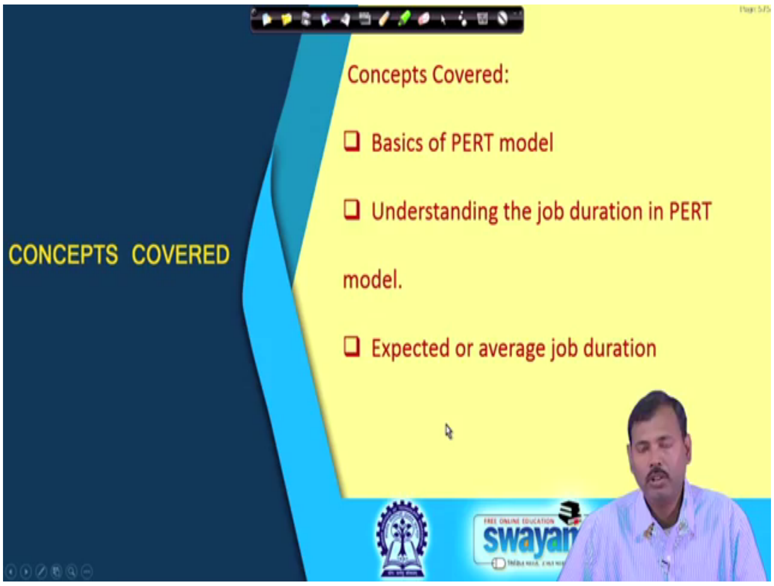

So, let us go into the introductory part of the program evaluation and review technique.

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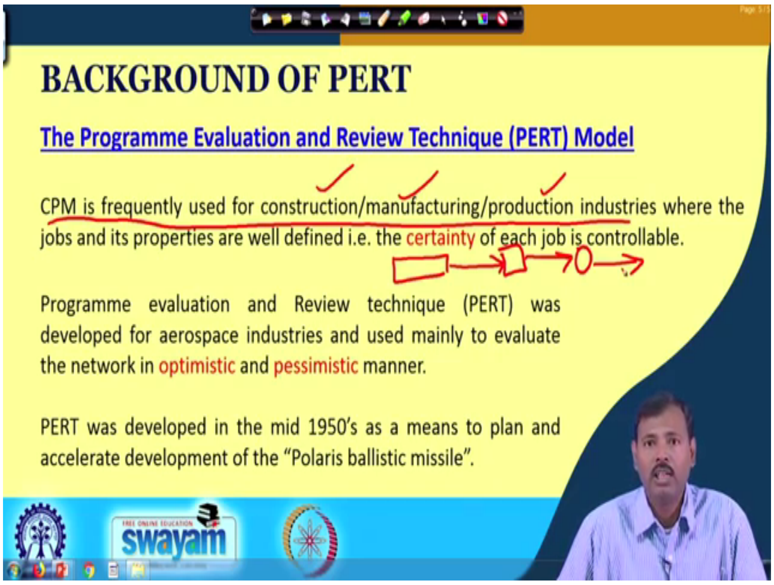

So, first let us see its domain of application generally critical path method is frequently used for construction manufacturing products and industries, which are mostly machine driven. That means, suppose we need to hit some chemical for a particular time or maybe something it is something like you are programming for microwave hitting say that time is fixed, because our all the other controlling measures machine controlling measures or artificially controlling measures are automated. So, in those conditions probably the deterministic approach may be accepted, that is why in most of the cases construction manufacturing products and these fields it is considered as the deterministic attributed properties are there and that is why critical path method are applicable to those conditions.

So, supports suppose we are having some say soft drinks manufacturing system, where some the first the bottle has been prepared which is prepared by a machine with a fixed time then it is moving, it is failed, then it is sealed, then it is leveled. So, like that way all the parts are carried out by the machine automatically and it is time bound and fixed times are consumed in those cases. So, that is why it is considered as the deterministic one and critical path method of analyzing this one probably and accepted method of analyzing the network.

However, in other cases the uncertainties are lying and program evaluation review technique which is called PERT; PERT was developed for aerospace industries and used mainly to evaluate network in optimistic and pessimistic manner. So, let us see some the historical perspective of this before going into the details of program evaluation and review technique analysis it was developed in the year 1950’s while the designing of Polaris ballistic missiles are carried out or the developmental research was carried out, that time this was first developed and first attained where the instead of deterministic approach, probabilistic approaches approach is carried out to identify the uncertainties associated with the different jobs in those projects.

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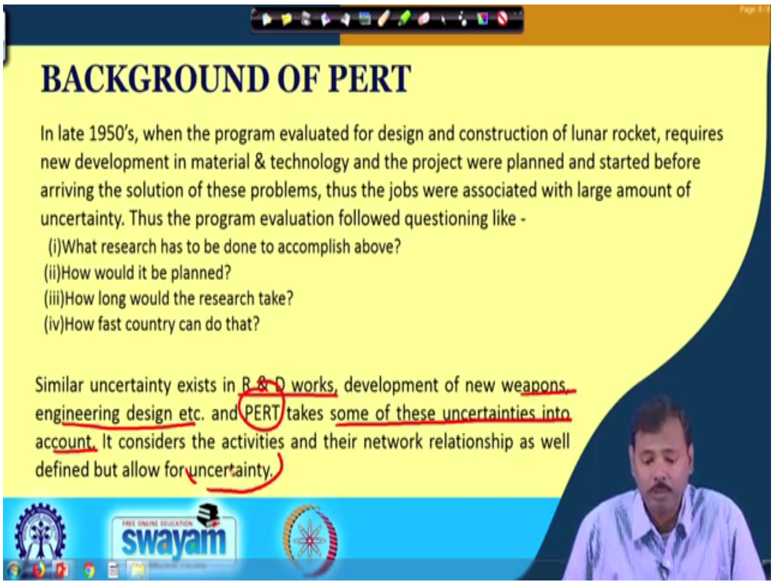

So, let us see little bit of background this of this in 1950 when the program evaluated for design and construction of lunar rocket, requires new development in material and technology. And the project were planned and started before arriving the solution of this problems thus the jobs were associated large amount of uncertainties; that means, in that ballistic missile the different materials and technologies are used that detail know how of that was not known and n number of researches, researches are associated for those development that is why it was totally uncertain. In fact, no one knows whether the research should be finally, come into its result or not.

So, that is why the questions were framed like this a what research has to be done to accomplish some how would he do it to be planned, how long would the research take and how fast the country can get this polarized ballistic missile as its weapon. So, like that way the questions are developed and gradually the time has been framed optimistic pessimistic manner and from that some probabilistic approach are taken for that so, that some analysis on the network can be carried out finally, by which time this target would be achieved.

So, this type of uncertainties works in R and D works development of new weapons engineering design and in all those cases program evaluation and review technique, is considered because some of these uncertainties can be taken into account. There is a big uncertainty in this condition is that, this uncertainty is calculated based on pessimistic optimistic, or most likely time which is considered and as it is a probabilistic approach, then the what will be the exact time of completion is basically no one knows no one can assess,.

But only the probability of achieving the target can be considered or can be estimated basically I love to use the word estimation here, not the calculation because program evaluation review technique cannot give you, the exact value by which it can be finally, the conclusion can be come out.

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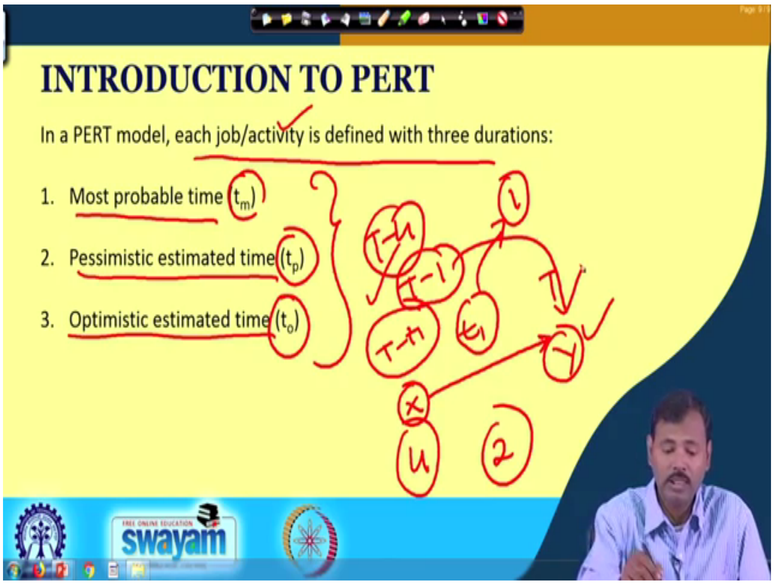

So, program evaluation and review technique is basically a probabilistic approach, in which we attribute to each job or activity, three expected times one is the most probable time, one is the most pessimistic time and another is the most optimistic time. So, suppose if time is one attribute in that condition for single attribute we consider three values.

One is most likely or most probable value, another is the most pessimistic value another is the most optimistic value. So, let me explain it in a little bit different manner suppose a person is staying at x place and he has to reach at y place. Suppose he has to take some flight or take some train from this y place and he is traveling from x plus to y place. Now, suppose he must reach there at time t at what time before this T; that means, T minus t 1 would be the what time by which this person must start from x.

Now, if you are the person and you have to move from x plus 2 y for assessment of this t 1 time, you consider few things say first what would be the likely time can be taken say, suppose you are having previous experience from of traveling x to y n number of times and we have found that probably it may take say 2 hours or something like that.

So, you can start from this place maybe 2 hours before than T. So, that you can catch the train or flight at time, but in this period your experience may be like that you have found it sometimes it may take 4 hours also, if the traffic etcetera to congested in those cases it may take 4 hours so; that means, if you are a pessimistic person in that case you can start T minus 4 hour at this place. So, that anyway if it is good condition you may reach it here at T minus 2 hour time, but you can wait at this position for 2 hours, but your flight will not be missed.

So, that may be one option you can consider, because you are a pessimistic person, but suppose you are some optimistic person or in any case you became late, but from your previous experience you have found that this t 1 time sometimes you have covered in very good condition where the traffic was to less, everything was favorable in those condition also you have covered this one in 1 hour.

So, suppose if you are led by 30 minutes or 40 minutes, then also you may be optimistic that you can if you start here from T minus 1 or one hour time, then also there is a chance you can catch the flight. So, that is the most optimistic condition. So, the conditions are different and no one knows which condition is favorable to you or you can have and your surety is not there it is uncertain, but still you can try and you can reach from x to y on time in optimistic condition also, pessimistic condition also and in most likely condition also.

Even if there may be a new case arises, where despite you are considering the pessimistic condition, but still you are not able to achieve the target suppose. So, far your experience is that if you start 4 hours before from the designated time you can reach the place safely, but there may be a new case where despite starting at four hours before from x to y still you are unable to reach y by 4 hours it would take 5 hours, because that is a new case.

So; that means, there is uncertainty even if the pessimistic condition is there, but that uncertainty condition was not known to you. So, as per your understanding you have considered you have considered a most likely time, you have considered a pessimistic time, you have considered an optimistic time, but all may fail also, but as per your understanding you are considering a pessimistic time, a an optimistic time and may be a most likely time as per your experience you are considering.

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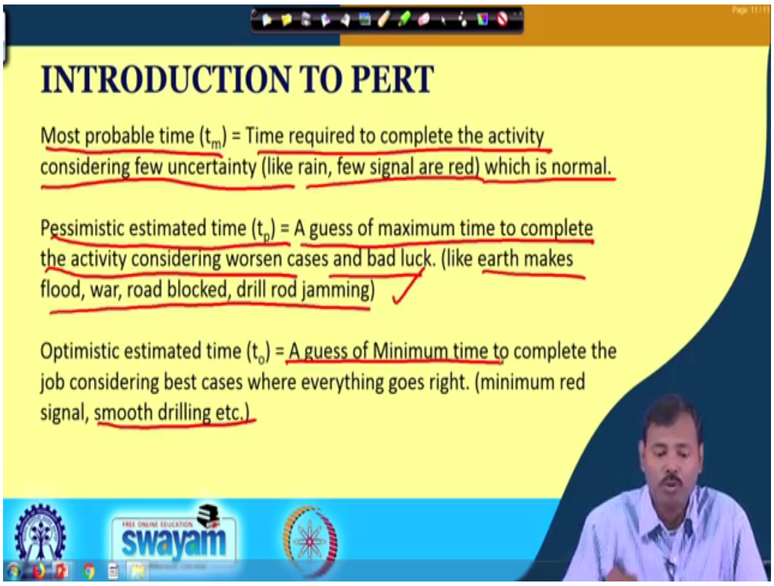

So, we can say that most probable time is the time required to complete the activity, considered few uncertainties like rain signals are red etcetera like that way you have considered and based on that assumption you are considering it is the most probable type. Or, it is basically on an average what we are considering most of the time we have experienced this is the time by which we are completing this job, that is the most likely or most probable time which we are considering.

Pessimistic estimated time is basically a guess of maximum time to complete the activity considering the worsen cases; that means, the bad luck condition like earth makes a flood wall roadblock etcetera you have considered if you are considering the drilling say, most likely is the most of the time if you have found the drills are basically penetrating a hole by that time.

More most pessimistic condition is that where the drill road jam into the hole and it takes too long time to drill a hole, that is the pessimistic condition and optimistic if the strata is very good you have found that, it is very easy smooth drilling has been carried out and it is taking very less time. So, optimistic is also another a guess of minimum time to complete the job considering best cases what we have experienced, or we have imagined that that could be the possible. So, basically these are three times we are considered for each and every activities, considering their uncertainty and these times are optimistic pessimistic and most likely times we are considering.

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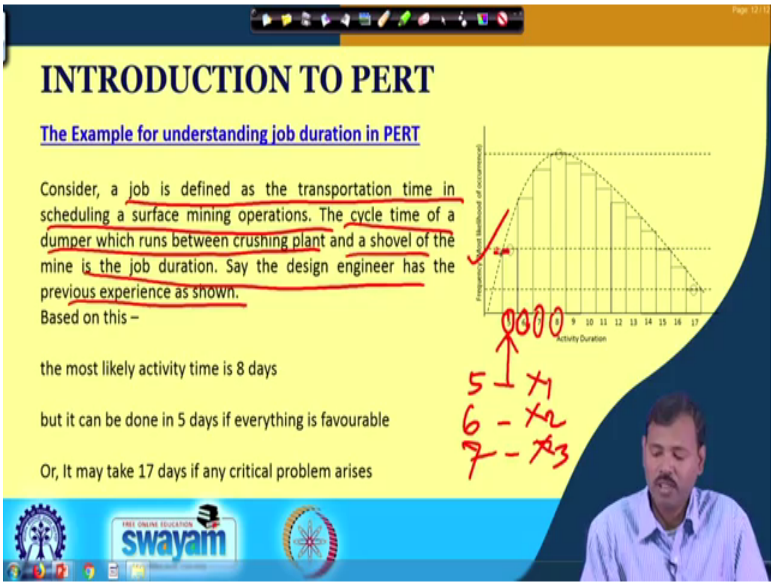

So, let me explain it a little bit more in a statistical form, suppose we have considered a job. And which is defined say transportation time or in surface mining or cycle time of a dumper, whichever you are you are considering between a crusher and a shovel and say, whatever our experience is that it is taking say 5 minute, 6 minute, 7 minute 8 minute like that way so, minimum time you have found is 5 minute. And the number of times we have experienced 5 minute is the frequency which we are considering in the y axis.

So; that means, 5 minute say X 1 time 6 minute cycle time is for X 2 time 7 minute is experienced X 3 times. So, like that if frequencies are considered and we have carried out a plot between the frequency and the actual duration say these are 5 minute duration, this is the most 117 minute maybe probably crusher has jammed or something happened to the dumper, or dumper has wait for a long time in front of the shovel.

So, like that way it has taken 17 minute time as per our experience which we are considering this is the as per our experience we are considering 5 minutes time, but so, far whatever experience we have achieved we have found this 8 minute times are taken by most of the time by the dumper operators to run between the shovel and the crusher.

So; that means, this is the time which is most of the time we are experiencing and we are considering that this 8 days or time hours whichever or minute, whichever it is that is the most likely time which we are experiencing. This is the most optimistic time maybe dumper operator has driven his dumper very fast, or the everything was fine and that is why the time taken was very less. So, this was the minimum time of what we have experienced so far.

So, we are considering that is the most favorable case and that is the most optimistic time and this is the most pessimistic time which we have experienced so, far and we are considering this 17 minute or 17 days is the most pessimistic time which we are considering as the pessimistic time. So, basically as per our understanding we are considering the most likely most optimistic and most pessimistic time while we are considering the uncertainty of the completion time of a job which we are considering here.

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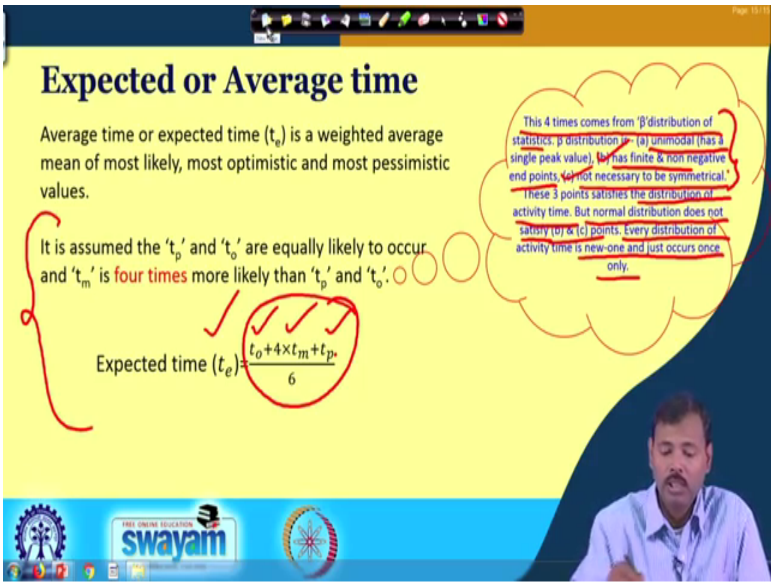

So, what we are considering that there is a most likely time there is an optimistic time, there is a pessimistic time and as per our given average time, or the expected time is the weighted average mean of the most likely most optimistic and most pessimistic values and this formula is given by expected time is equal to t 0 plus 4 into t m plus t p divided by 6. Where t 0 is the most optimistic time t m is the most, likely time and t p is the most pessimistic time and waiting averaging this one we are getting the expected time or mean time for completion of these activities.

So, this is the expected time and you may ask that from where we are getting this for most likely and divided by it 6. Basically this concept we have taken from the beta distribution of statistics and beta distribution is basically a unimodal distribution which has a single peak value and has finite and non-negative endpoints and not necessary to be symmetrical, these are the condition for which we adopt beta distribution these three point statistics, the distribute distribution of activity time, but normal distribution does not satisfy the point b and point c that is why we opt for the beta distribution, every distribution of activity time is new one and just occurs once only.

So, these are the conditions which we are considered for this for this weighted average for getting the expected time or the mean time to complete the job. Basically there are debate on accepting of this one, but the if you are if someone is moving to the normal distribution, then also the errors coming into this calculations are not significantly high. So, that is why beta distribution as it is easier to adopt basically that is adopted and its applications are tested for a number by a number of researchers already in this field.

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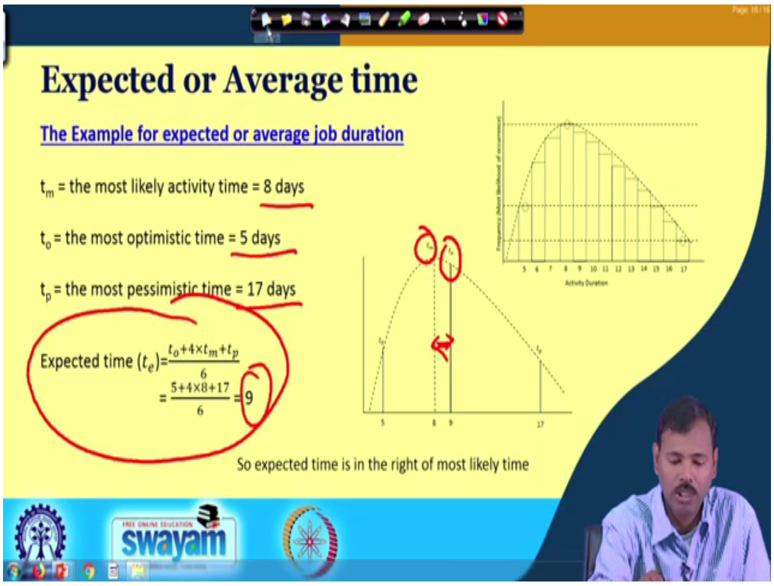

So, let us consider one this given example t m is the most likely time is given 8 days optimistic 5 days pessimistic 17 days. So, the expected mean or mean time expected time on mean time is coming 9 days which is the average, but if you look into this the mode which is coming we are expecting is 8 days, but the mean is coming at 9 days. So, that is there is a little bit difference between the most likely and the mean value or the expected value of the duration of the job.

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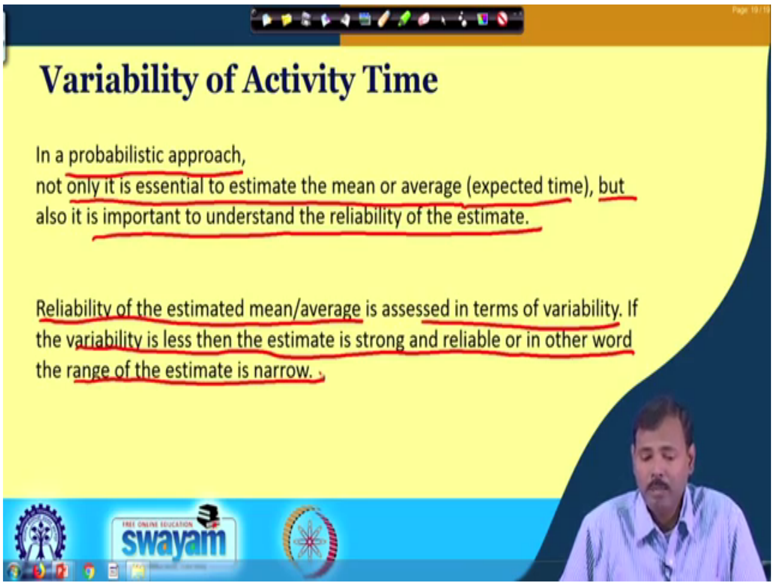

Second important parameter in this case is the variability of the activity time, in a probabilistic in a probabilistic approach not only it is essential to estimate the mean or average that is the expected time, but also it is important to understand the reliability of this estimate, because the variation may be very high variation may be less. And in this case the reliability of the estimate in of the mean or average is assessed in terms of variability, or in other word you can say it variance.

If the variability is less than the estimate is strong and reliable in other word the range of the estimate is very narrow. So; that means, the expected mean which we are considering, if the variation is very high range is very high, then the reliability of that mean is also very less, but if the variability is less standard deviation is less, then the reliability of the mean is very high we can understand these points from the statistics also.

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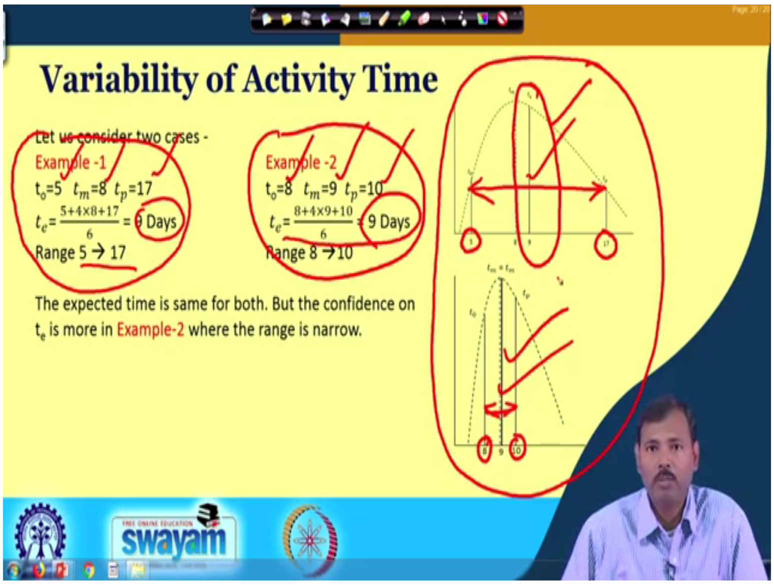

If you see the example 1, where we are having that optimistic is 5 days most likely 8 days, pessimistic 17 days. So, expected time is coming 9 days and range is varying between 5 to 17 between 5 between 5 to 17, this is the mean we are achieving. Let us consider the example 2, in which our optimistic time is 8 most likely time is 9 pessimistic time is 10; that means, our total variation is ranging between 8 to 10 all the observations are ranging between 8 to 10 and we have observed the mean is same that is 9 days.

So, basically if we see in this case our range is very wide and that is why the reliability of estimating the mean or expected time of 9 days is not that much reliable, but in this case it is very much reliable if it is deviate also, but so far our observations are within a narrow range; that means, it will not deviate a lot. In this case one another important parameter is that the number of observations; that means, our population size. If this is carried out for a large population, then only this estimation is valid because the population is different means the estimation is also not that much accurate.

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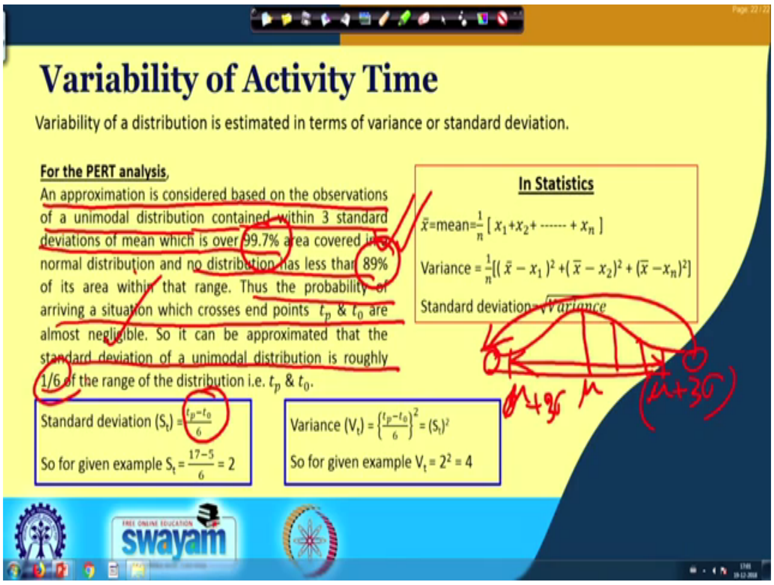

So, variability of the activity time in variability is variability of a distribution is estimated in terms of variance or standard deviation. And the formula which we use for this is in part analysis is basically the standard deviation is considered as the difference between the pessimistic time and optimistic time, divided it by 6 so, for this given case it is 17 minus 5 by 6 so, it is coming two and variance is the square of the standard deviation so, it is 2 square it is coming 4.

Now, this is also another approximation, if you see an approximation is considered based on the observations of a unimodal distribution contained within three standard deviations of mean which is 199.7 percent; that means, if your distribution is basically ranging like this and this is your mean, then it is estimated that if you go up to that if it is mu then if you go up to mu plus 3 sigma that means, in both the cases mu plus 3 sigma.

And you are basically covering 99.7 percent area within this zone, if you are considering a unimodal distribution, in case of normal distribution in case of normal distribution. And for all the cases all the distribution cases it is always lies within 89 percent; that means, the variation is not very high if we are arranging it within plus minus 3 sigma from the away of the mean. So; that means, this is well accepted range of accuracy we are considering thus the probability of arriving a situation which crosses endpoints t p and t 0 are almost negligible.

That means, probability of occurring between these zones are almost negligible and that is why we are considering the variance using this formula. So, it can be approximated that the standard deviation of a unimodal distribution is roughly 1 6th of its range and that is why it is considered t p minus t 0 by 6, if you are considering the statistics, where we are considering our distribution is a normal distribution.

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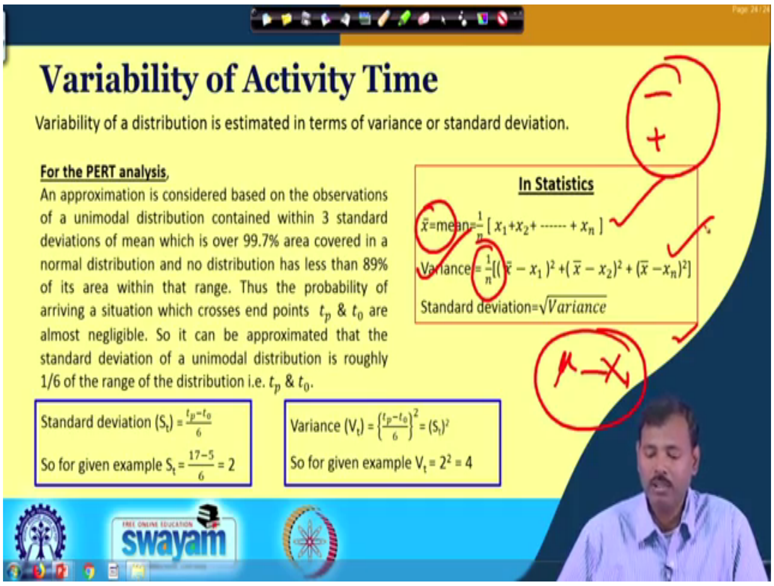

In that case in that case your x bar or mean is considered as the summation of the all the observations, divided by divided by its number of observations. The variances are considered using this formula where mean minus observations are squared. And some of those squares are divided by the number of observations give you the variance and the standard deviation is obtained by the square root of the variance.

So, basically the concept is that we are getting the mean value and from there the observations are deducted, whatever the variation is observed we are summing them by squaring them we are squaring them because the observations are negative and positive so, to overcome this we go for squaring them and after squaring them we divide it by the number of observations that gives us the variance. And by square getting the square root of the variance we get the standard deviation.

So, this is applicable for the normal distribution, in some cases where the frequencies are there then the calculation is little bit different one will discuss all those things in our next class. So, basically this is the basic of the basic introduction of the program evaluation and review technique.

In program evaluation and review technique, we use the probabilistic approach to solve the network and that is why the earlier critical path method where the deterministic approach is taken the program evaluation review technique being a probabilistic approach gives us a little bit better result. It at least you can have a reliability or they can have the probability of achieving the target can be assessed using program evolution of review technique.

Thank you.