Network Analysis for Mines and Mineral Engineering
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Lecture - 15
Crashing and stretching of jobs (Contd.)

Let me welcome you to the 15th lecture of NPTEL online certification course Network Analysis for Mines and Mineral Engineering. The topic of this course is Crashing and stretching of jobs.

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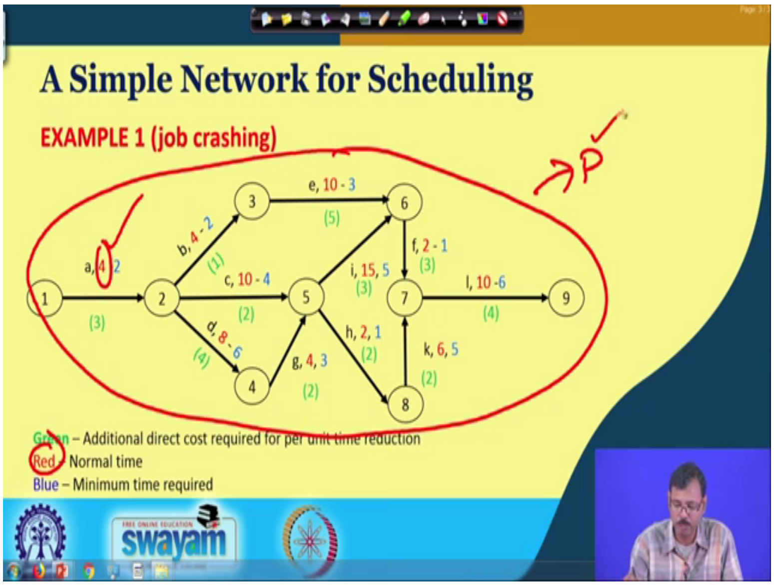

But let us tell you that so, far we have covered all the parts which are related to the deterministic approach of network analysis that is critical path method. So, we have covered everything related to critical path method so, far what was in the there is in the syllabus of this course. Now, we will solve two more examples on these and will understand the or basically you can say you can summarize or conclude the network analysis for critical is in critical path method.

So, whatever we have covered so, far is that we know how to draw the network, we know how to identify the critical jobs in the network, we you know we know now how to find out the early start late start and early in early finish late finish for a particular job. And based on that we can go for optimization of the costs also by crashing or stretching the jobs we can optimize the cost and we can deliver the job within a stipulated period, that is also possible in this critical path method analysis. So, let us go for the network analysis where we will go for crashing the jobs up to a certain distance to identify which will be the optimum cost for that particular network.

So, let us consider one example here and the network is already drawn here it is given this network and this whatever given in the green color is basically the additional direct costs required for per unit time crashing though it is the jobs are given in consider it is in this. So, per unit time crashing or per day crashing this is the cost incurred for every jobs. These are given here particular to the name of the jobs which are given in a b c d e f g h i j k l up to that it is given actually k l up to that it is given. A red is the normal time, this red one which is given is the normal time that is the expected time for which the total costing is already carried out.

And let us consider that total costing is P that is the direct total cost required for completion of this total activities is hereby assumed as P. And while we are considering this P we are considering the activities has to be completed in the normal time. And this normal time of each jobs are marked in red color, the blue color what we have marked is the time minimum required to complete that particular job which are given in blue color. That means, this is the normal time which we have considered for which we have calculated direct costs and that direct cost for all the activities are summed and that we are considering as the P.

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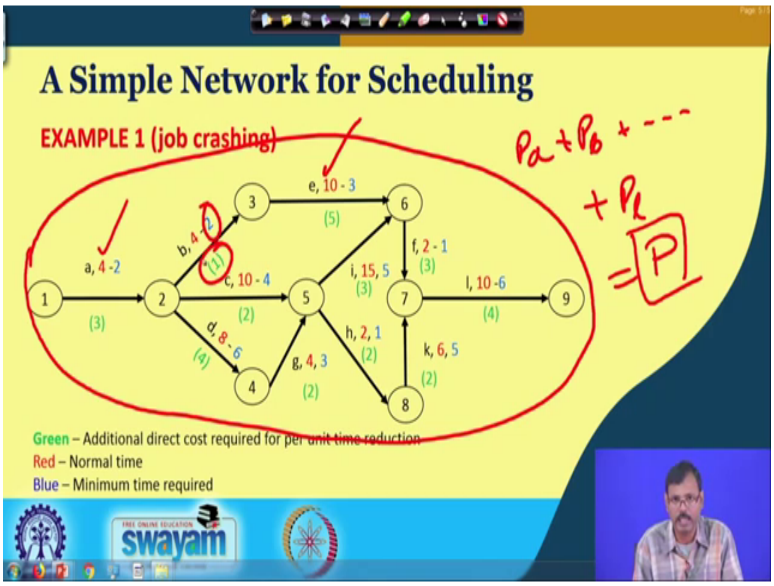

So, the direct cost of this total network in consideration that the jobs will be completed in the normal time in that consideration the direct cost is P and the minimum days required to complete those jobs are given in blue color and the cost of crashing per day is given in green color.

So, network is given so, you know the what are the predecessors what are the successes like b c and d are having predecessors job of a, e is having predecessors job of b, i is having predecessors job of c and g, d is having predecessors job of a g is having predecessors job of d, f is having predecessors job e and i, h is having predecessors c and g, k is having predecessors are h and l is having predecessors f and k. So, this jobs dependencies are given in the network. So, you have directly draw the network so, that we can carry out the calculation as per our requirement.

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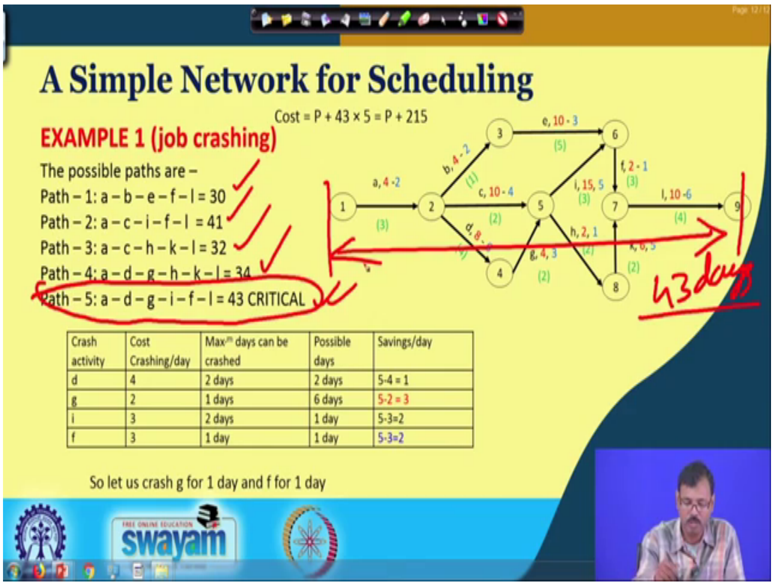

So, for optimizing the cost let us find out which one is the critical one. So, we have identified 5 paths are available, the path 1 is this one comprising job a b e f and l a b e f and l job. Path 2 and if you sum up this durations 4 plus 4 8 8 plus 10 18 18 plus 2 20 20 plus 10 30, so, the total path length this 30 days. And if you are considering path 2 which is comprising a c i f and l so; that means, this is the path. And if we are calculating the total path length 4 plus 10 14 14 plus 15 29 29 plus 2 31 31 plus 10 41 is the path length.

And this is mark like this, considering the path 3 a c h k and l this is path length is 4 plus 10 14 plus 2 16 plus 16 22 plus 10 32 is the path length. Considering the path 4 we can see a d g h k and l. So, this is 4 plus 8 12 12 plus 4 16 16 plus 2 18 18 plus 6 24 plus 10 34 is the path length which is given here and this is the path we are considering in the path 5 4. And this is the path 5 which is a d g i f and l and if we are considering this, this is coming 4 plus 8 12 12 plus 4 16 16 plus 15 31 31 plus 2 33 plus 10 43. So, path 5 is taking 43 days to complete this one; that means, the expected time to complete this project complete project as per the normal duration of this is 43 days.

So, this is the 43 days is the requirement for completion of this project and that is why the path 5 is considered as the critical path. Other paths this is having 13 day slack, this is having 2 days slack, this is having 11 day slack and this is having 9 days slack associated with this path. So, this is the critical path.

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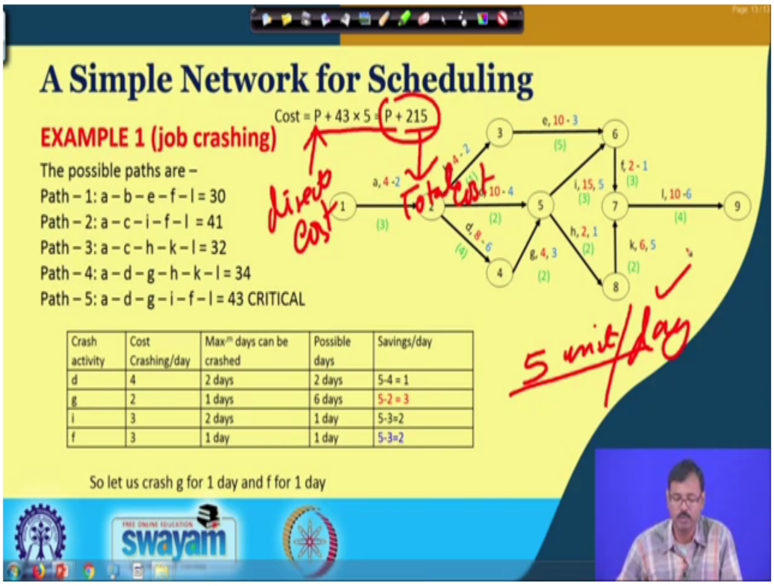

And now, if we consider that per day indirect cost is 5 unit it may be dollar, it may be rupees whichever it is. If 5 unit is the per day indirect cost then the total cost of this project is P plus 43 into 5; that means, P plus 215 is the total cost of the project where, P is basically the direct cost total direct cost and 215 is the total indirect cost for the completion of the project; considering the 40 43 days are the required operating days for completion of the project.

Now, let us optimize this normal schedule for which we have calculated; in consideration with that crashing of the jobs within a stipulated period and the cost of crashing; that means, the increase in the cost increase in the direct cost for that particular job is given here and we have to consider this for optimization of the same.

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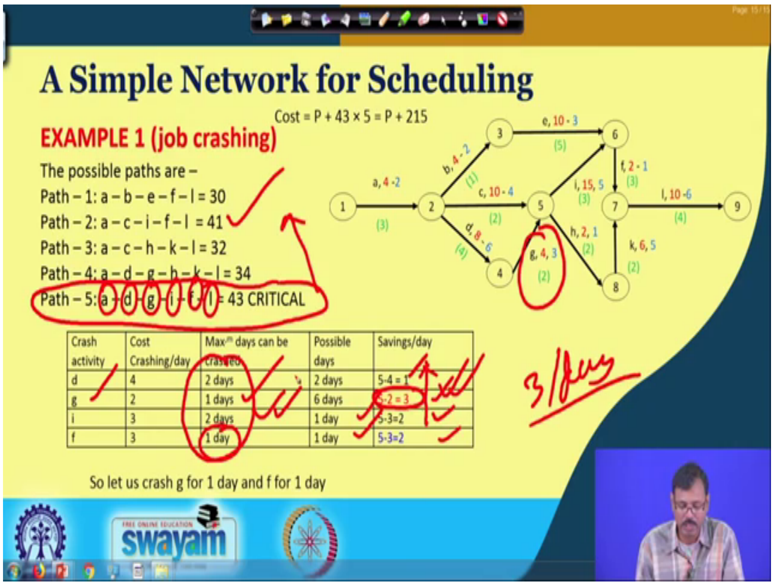

Now, if you look into this, this is the critical path and the near critical path is this one. So, to make it this one to 41 days we are having option we can crash these all those associated jobs here for 2 days and if we crash them what are the cost savings incurred to us, that is given here. These are the cost savings and if you see the cost saving is maximum in job g; that means, if we crash this job our cost saving is maximum that is 3 3 unit per day is the savings which can be found here, but the problem is that it can be crashed for 1 days. So, let us observe for the next options among that this one and this one gives us 2 unit savings in the cost per day and this is available for 1 day, this is available for 2 days.

So, let us consider in our case the first one this has to be crashed because, this is giving us the maximum profit. So, along with that either we can opt for crashing 1 day here or crashing 1 day here. In this particular calculation we have considered the crashing at this place in case of f. So, that actually the matter which influencers to crash this one because, if we crash this one this will come I think f will come into its minimum duration of 1. So, further crash option of further crashing f job will be eliminated and influenced by that we have crashed f job. But, there is no harm you can crash the job i also because the costings are same to us.

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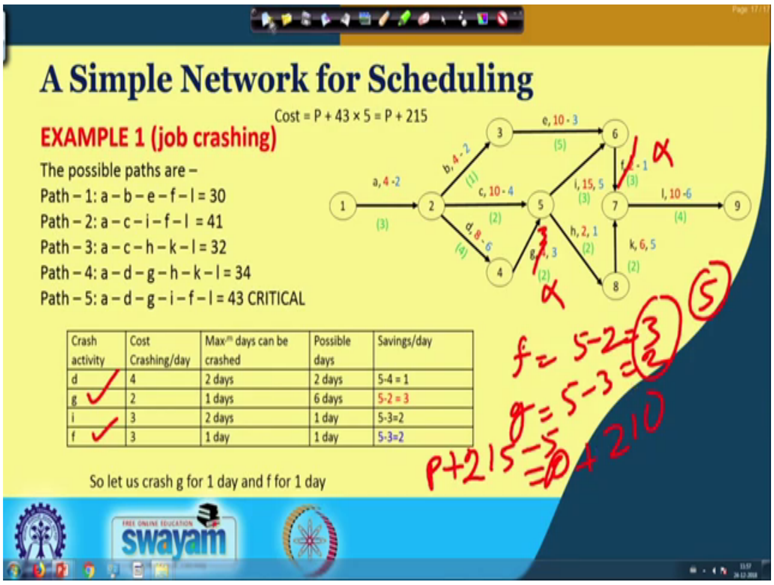

So, what we did we have crashed job we have crashed job g and job f. So, that job g will become 3; that means, no more crashing is possible here job f will become 1. So, no more crashing is possible in this case and our savings will be for f, for f job our saving is 5 minus 2 that is 3. And for g job our saving is 5 minus 3 that is 2; that means, total saving is 3 plus 2 5 unit and that is why our total cost will be reduced to P plus 210 unit and we have represented that in the next slide.

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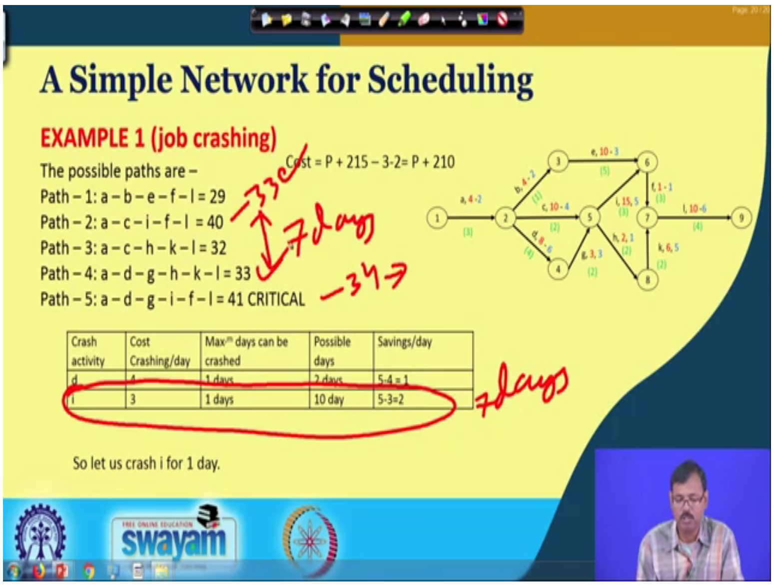

If you see now here our total cost is now coming to P plus 210; our new schedule you can see this g job no further crashing is possible, f job no further crashing is possible. Now, if we look re look into the path you can see the path 1 is now 29 days, path 2 is 40 days, path 3 is 32 days and path 4 is 33 days, path 5 is 41 days which is critical. Now, earlier our try was to reduce path 5 to make it equal to path 2, but we cannot do it because the moment we crash this job this job and this job this f job is also associated with this path which is path 2 and that is why this is also reduced to the lower one.

So, we can if we look into this we can see that to make it equal both the path our options are like this either we can crash d for 1 day; either you can crash d for 1 day which is not associated to this or we can crash i for 1 day. But, i is the common job between this two and profit is more in i. So; that means, if we crash i whichever is the day it is, but that will reduce this duration also and this duration also. But as per rule we need to crash i to make it equal to the near critical path, but the moment we crash the i for whichever it does it is it will also reduce the path length of path 2.

So, as per our option we need to crash i for 1 day, but we have found that as it is the common job between these two we are having a gap of 7 days between the next near critical path of path 2. That means, if we crash i for 7 days; if we crash i for 7 days also then also it will become 33 days, it will become 34 days but, then also it will remain critical the near critical one will be this one and this one. So, in consideration of that it is decided that let us go for crashing of the common jobs.

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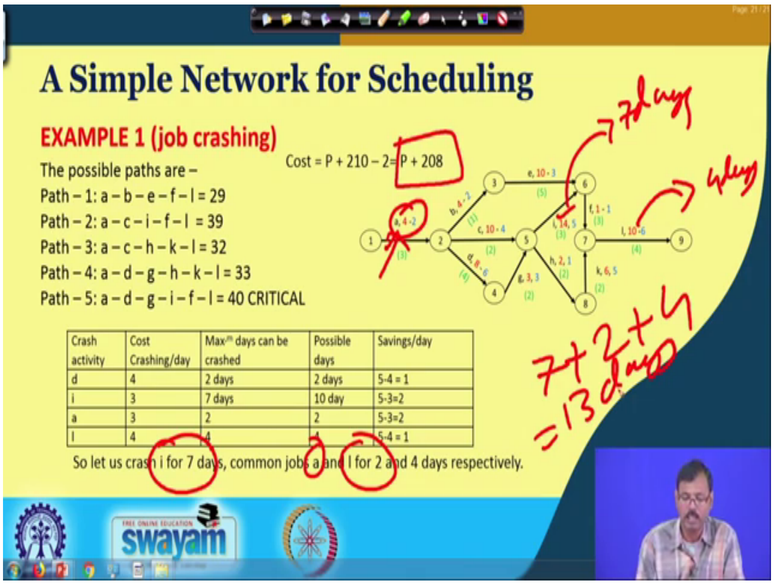

And this common job i is crashed, common job a is crashed common job l is crashed. So, now, i we have crashed already once so, it becomes 14 15 to 14 and that savings to us is reflected here. And in consideration of that we are now reducing i further to 7 days, this also will be reduced to 4 days; reduction of 4 days will be done. So, that the 10 become 6 and it will be crashed for 2 days, so, that it will become 2 days. So now, we are crashing 7 days plus 2 days plus 4 days; that means, we are crashing 13 days together.

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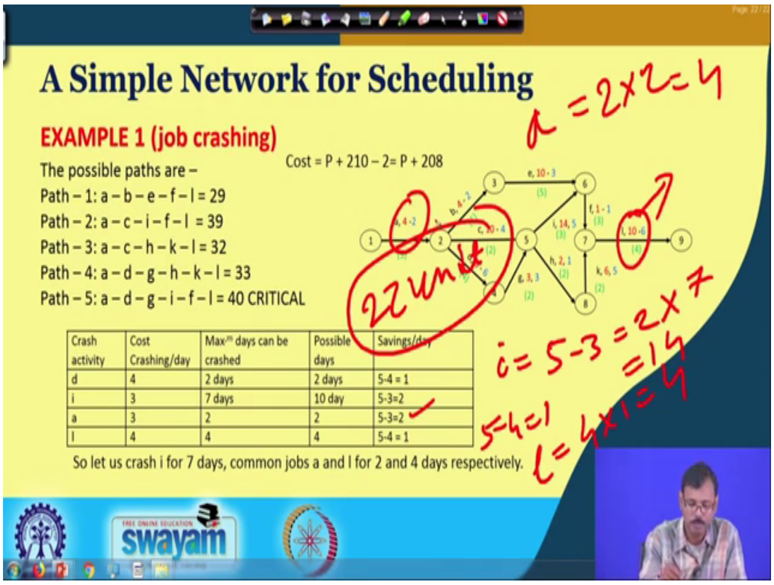

And if you see what is the profit we are getting by crashing this then you can see, the profit for crashing i is 5 minus 3 that is 2 days which we are crashing for 7 that is 14 unit profit is there for crashing the i for 7 days. And there is 1 unit profit 5 minus 4 that is 1 unit profit for crashing l and we are crashing it for 4 days. So, the profit is profit will be for l the profit is 4 into 1 that is 4 unit profit. Similarly, for a the profit is it is for a our benefit is 2, so, 2 into 2 that is again 4 unit profit is there. So, altogether we are having profit of 14 plus 4 18 18 plus 4 22 unit profit is there we can achieve here.

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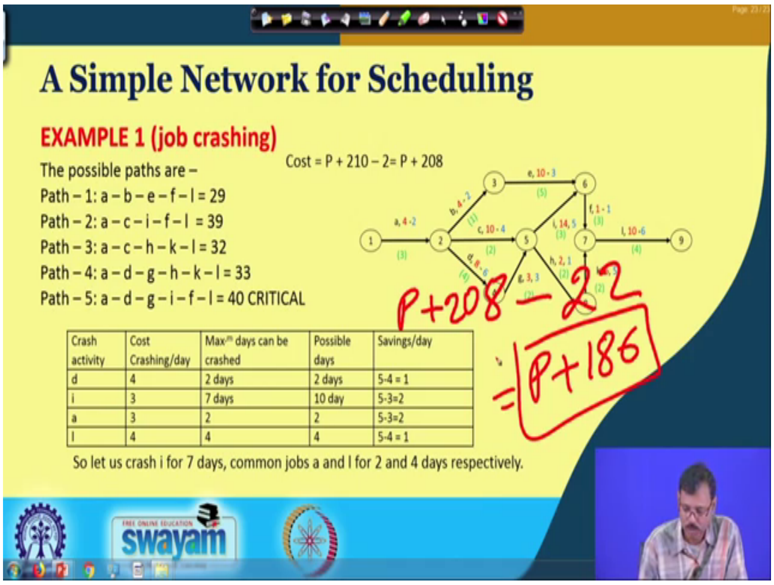

So, our new cost will be P plus 208 minus 22 unit that will leads to P plus 186 will be the cost.

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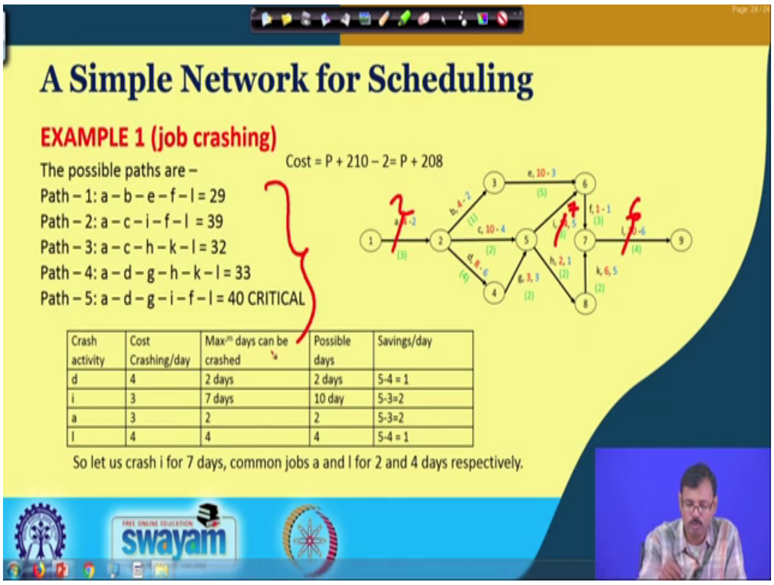

Now, we have reduce; we have reduced this to 7 unit; that means, it becomes 7, we have reduced it to 2, you have reduced it to 6 and then all these will be changed. Now, let us look what is the new network we are getting at this place.

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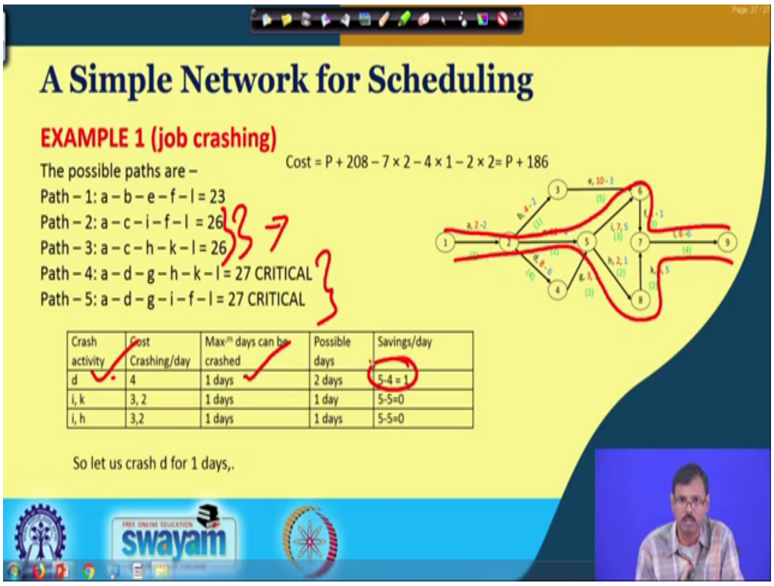

The new network you can see the a is now 2, l is now 6, i is now 7 and if you determine the critical path now you can see the path 4 and path 5 is becoming critical. That means, this path and this path is now becoming critical and both are taking 27 days. Path 2 and path 3; that means, this path; that means, this path and this path are now becoming near critical of 26 days; obviously, path 1 is a little bit away from this. Now, if you look into our options which are possible now for further crashing of these jobs to get the optimization cost optimized cost; you can see the d can be now crashed for 1 days and d is the common job between these two.

So, if by alone crashing d we can crash it for 1 day, after crashing 1 day it will become same with this. So, the new consideration has to be taken care of. So, for this step we can maximum crash only 1 day. So, d is d can be crashed here and this savings is now rupees 1 5 minus 4 1 unit per day.

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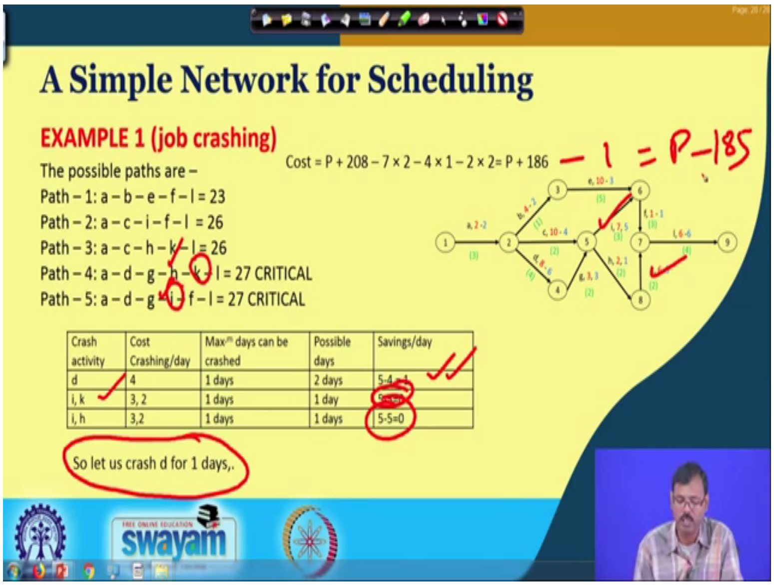

Our options are there to crash both i and k, if we crash this one and this one for 1 days then also this will be reduced to 26 and 26 or we can do it for i and h this one and this one and this one instead of i and k. Then also it will become 27 days 26 days, but you can see our cost savings are mean in this case. So, it is best that we can opt for only crashing single job d because, that is the giving us more profit and that is for available for 1 day for this case. So, this will be the new cost will be P minus 1 that is 185 will be the new cost and we have drawn it for the next session also.

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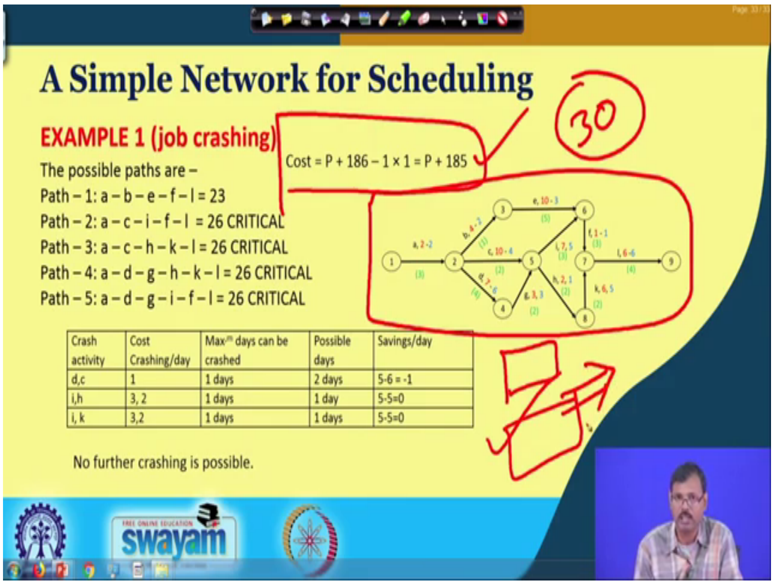

So, now you can see the new path is that all the jobs are critical, we have crashed d for 8 to 7 days and as all the 4 jobs are critical does common jobs a and l are they are common jobs for all 4 which are we cannot crash this anymore. So, either our options for further crashing is that either we can crash this one and this one; that means, d and c for 1 day, but if you see the cost there is no profit of crashing this one. It is basically increasing the total cost or other way we are having i and h that is i is here i is here h is here h is here. So, either if you crash i and h both 1 day then also the cost will be the days will days can be crashed, but our profit is 0. That means, crashing the job will not give me any profit for this particular case.

If we look into this i and k also you can see i is available here k is available here i is available here k is available here; that means, crashing i for 1 day and k for 1 day will reduce this all for 25 days. But, then also our profit is 0; that means, further crashing of this jobs taking these options further crashing basically will not give us any cost profit; our cost will remain same P plus 185. But, you may opt for adopting this crashings because it is having the reduced length, but it is not giving us any further profit. That means, our optimum cost for completing this schedule is P plus 185 which is less duration than the normal schedule and that is why it is giving us some additional profit what we have earlier calculated in the normal duration.

That means, in this case crashing of jobs gives us around 30 unit profit additional profit and by this way we have neither not only the reduce the duration of the project, but also we are gaining some additional profit. So, maximum profit as it is observed because the total cost is reduced. So, that is why it can be considered as the optimum cost or optimum duration of the project.

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So, this is one full example we have discussed as the concluding of this critical path method analysis of the network. Now, let us see some example of the network in a mining and mineral processing cases in general we can observe. Suppose, we are having a coal handling plant and the coal handling plant is basically supplied coal by my trucks or mine cars. Then that is dumping the material onto the tripper, then it is coming to hopper, from hopper it is through reciprocating fried feeder it is coming to the conveyor, conveyor to vibrating screen the oversized material either to send to the ground or to the loading hopper. And it is from loading hopper it can be going to the wagon loading or truck loading.

Similarly, undersize material can be taken from this position and from picking conveyor it can come to stacking conveyor, then to ground stock. Similarly, the oversized material of the ground stock may again going for the crashing unit and taken to the underground stock and from there it can be transferred to the steel plant and here it is going to the power plant or maybe something users we are sending. So, this is a common type of networking coal handling plant.

Similarly, we can we are having n number of processing network in the mineral processing units, their classifications, then prothpotations, then ball mills all those are there desaltings are there. All those are associated in those processing plant and they have to be considered with due importance due weightage while, the network are being analyzed. So, this is one example is given for a real life situation.

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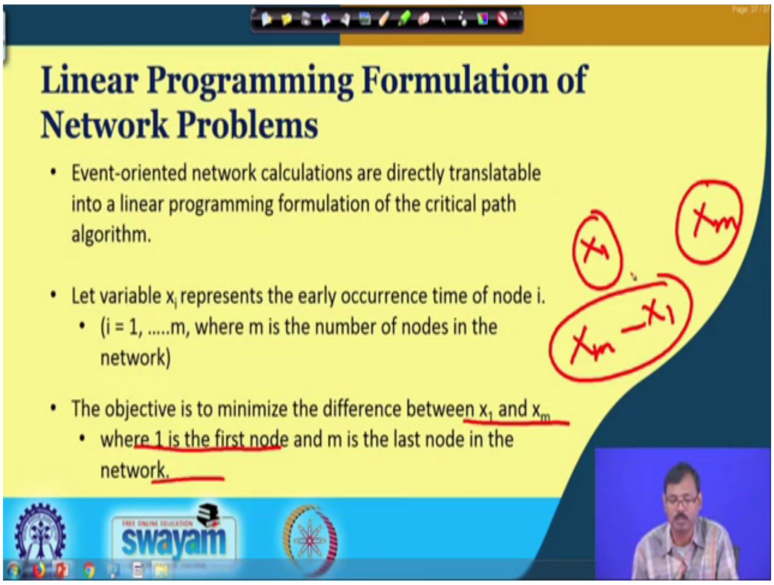

This network may be optimized using linear programming method also and in this additionally we are giving in this lecture how we can form a network linear program in the network analysis case. This event oriented network calculations are directly translable translatable to linear programming formulation for the critical path algorithm.

And what is our let us consider the variables are x i represent by the early occurrence time of the node i, where i is varying from 1 to m, m is the number of nodes which are considered in the network. That means, first note that is the i is equal to 1 is considered as the start node and i is equal to m is considered at the end node in between it is varying from i to m. And this is the considerations are carried out here and our options our intention is that early occurrence time if x 1 is given x m we have to calculate through the different path. And our objective is that x m minus x 1 must be minimum that gives us the earliest possible time for the completion of the project.

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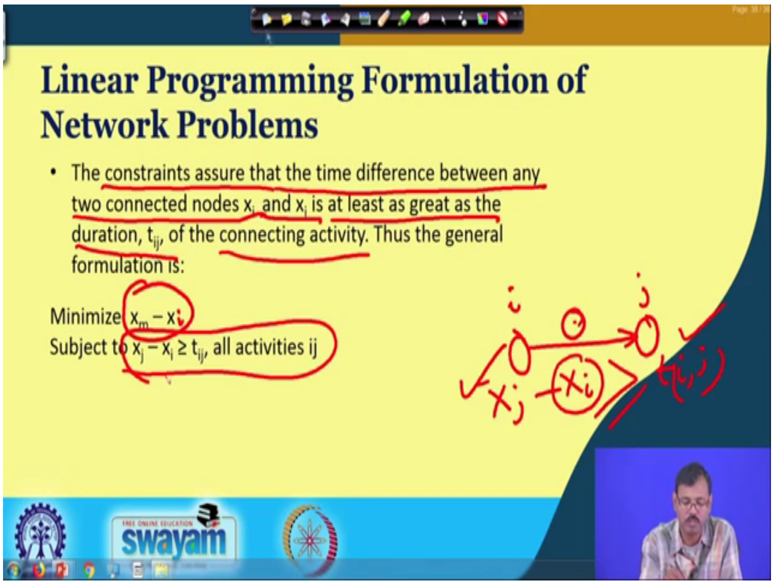

So, in that case if you formulate it our objectives are the constants given here the time difference between any two connected nodes i j x x i and x j is at least as great as the duration of the particular job t ij of the connected activity. So; that means, if this is the network, this is the ith node, this is the jth node then x j minus x i must be greater than is equal to t ij which is basically the activity duration.

So, activity duration the normal duration given here activity that must satisfy this one; if x j and x i is exactly equal to this then its a critical job. If x j is much more greater than x i such that it is greater than t ij; that means, there is a slack associated with that particular job. So, what is our objective? Our objective is to minimize x m minus x i; x m minus x i subjected to x j minus x i is greater than equal to t ij this is the constant given to this.

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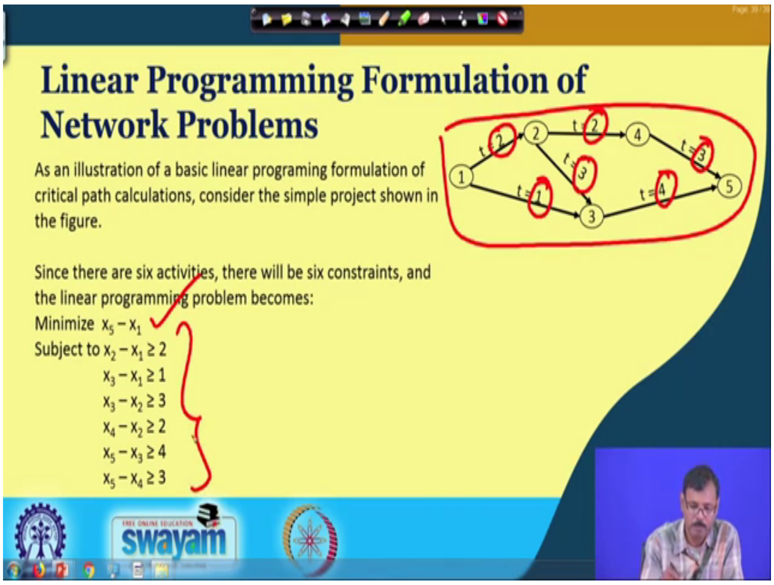

Now, let us consider one particular example here, this is a very short example. If you consider this very short example you are having x 1 x 2 x 3 x 4 x 5 only and we have to minimize x 5 minus x 1 subjected to x 2 minus x 1 x 2 minus x 1 must be greater than equal to 2, x 3 minus x 1 must be greater than equal to 1, x 3 minus x 2 must be greater than equal to 3, x 4 minus x 2 must be greater than equal to 2, x 5 minus x 3 must be greater than equal to 4 and x 5 minus x 4 must be greater than equal to 3. So, we have formulated this one.

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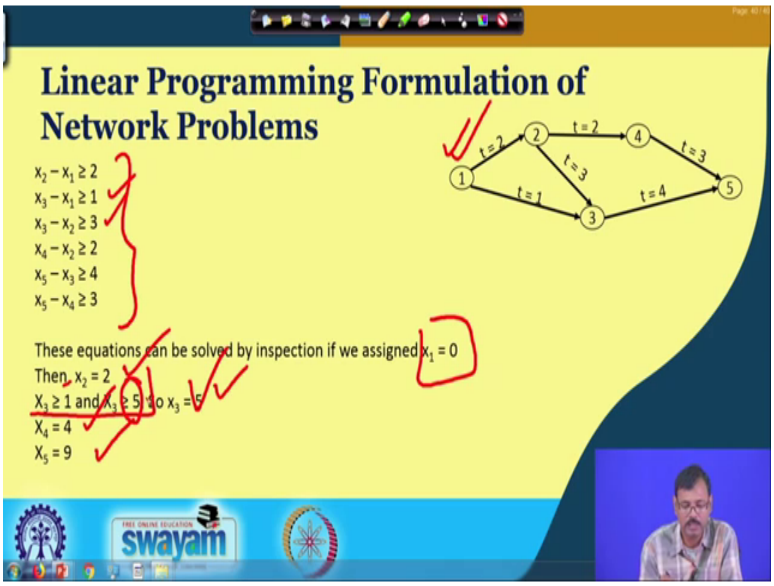

Now, we can solve this problem using linear programming as that is not within our syllabus, but you can opt for that that is why in this case in your 0 assignment we have given some LPP problems. So, you can understand those things, but you can solve it by your hand also. In this case if you are assigning x 1 is equal to 0; that means, you are starting this as 0th time in this total given formula all are considering as equal to; so, you can get x 2 is must be greater than equal to 2.

So, that is the minimum value is coming here. So, similarly you can get x 3 is equal to 5 because, x 3 must be greater than equal to 1 considering this equation and x 3 must be greater than equal to 5 considering this equation. So, in consideration that as both the condition conditions are existing we have to take x 3 must be greater than equal to 5; that means, x 3 is having the value of 5 only.

And considering x 3 value we can get x 4 value; similarly x nine value we can obtain and by that way we can finally, come out with the solution that x 9 is the final node and that is coming to x 5 is the final node that is coming to the 9; that means, the maximum minimum time requirement for completion of the project is 9. So, this is the end of the network analysis is in critical path method, from next class onward we will start the program evolution and review technique for the analysis of the network where, the probabilistic approach are being followed.

Thank you.