Network Analysis for Mines and Mineral Engineering
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Lecture - 13
Lowest cost schedule and optimum schedule

Let me welcome you to the 13th lecture of NPTEL online certification course on Network Analysis for Mines and Mineral Engineering, this lecture title is Lowest cost schedule and optimum schedule. like every class, let us retrospect what we have covered so far.

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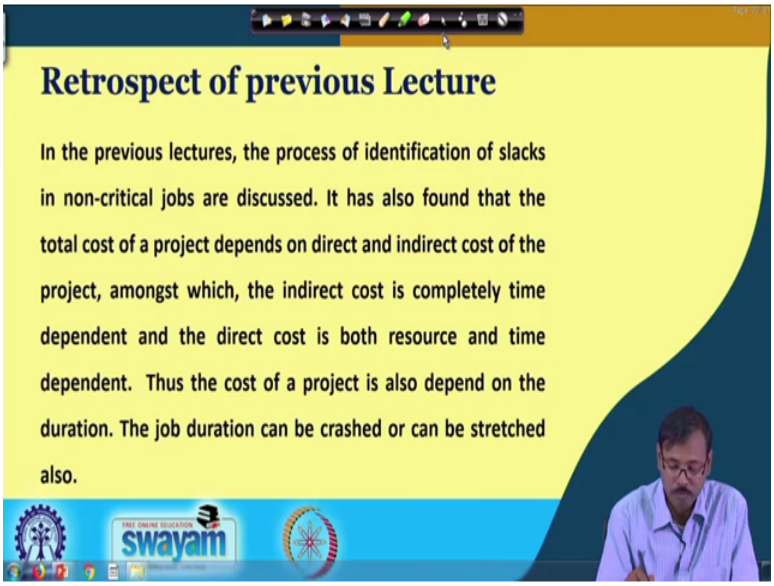

In previous lectures, the process of identification of slacks in non critical jobs are discuss. It has also found that the total cost of a project depends on the direct cost and indirect cost, direct costs directly depend on the man material machine this pricing and does not depend on the time. Actually sorry, it is depending on the time, and reduced with the time. Indirect cost is basically time dependent non not the man material machines dependent and that is why it infuses with the time and generally indirect cost compromising lighting security, etcetera.

Direct costing is the man machine material cost which is considered as the resource cost also. And indirect cost is completely time dependent and direct cost is both resource and time dependent. Thus the cost of a project is also dependent on the duration and on the material also. The job duration can be expedite or can be stretched also. So, that is also discussed in the last class.

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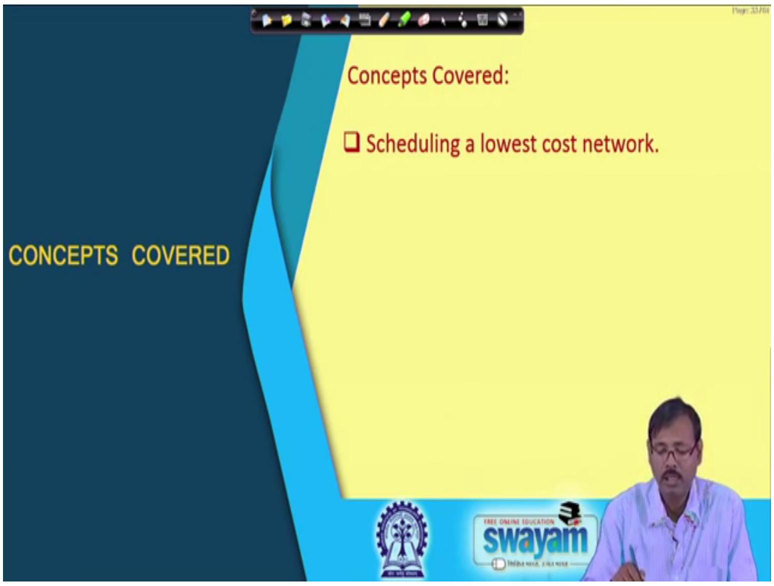

In this class, we will discuss how we can schedule a network; reschedule a network to achieve the lowest cost network by expediting or crashing the different parts of the job.

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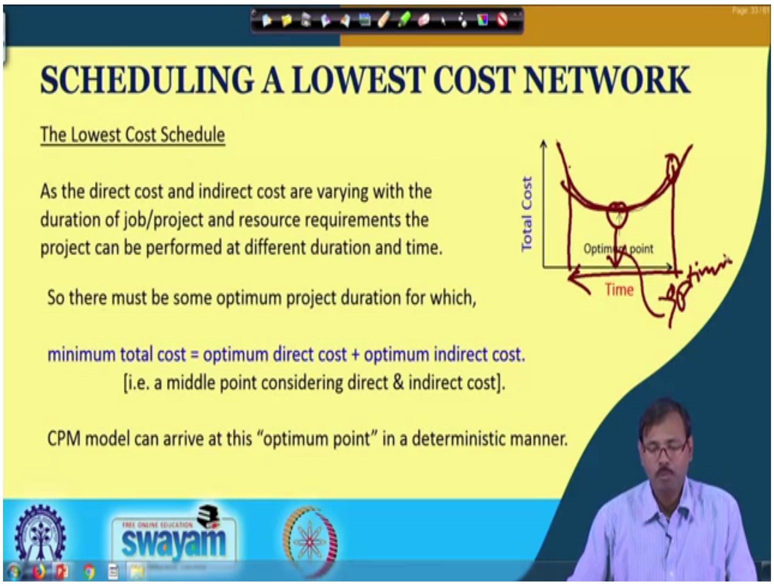

So, last class in the last slide, we are discussing around about the expediting of the job. And we are discussing how the normal time we are coming out and from normal time how the minimum time is we are considering? And then there are options how we can crash or expedite the projects. Now suppose this is the total cost curve, this is the total cost curve we are considering and say this is the duration, normal duration where we are lying at this position, this is the minimum duration which would. That means, we can reduce this time to this time, but if we look into the cost component, we can find the total cost of the project initially it is reducing then it is again increasing. So, there is a point where we can have a minimum cost for the project, but though the time is not minimum. So, this operating point is called optimum point, this is called optimum point where we believe to run our project more generously.

So, the direct cost and indirect cost are varying with the duration of the job or the project and resource requirements, the project can be performed at different duration and time. And the optimum point must be considered where the total cost is minimum, but the direct cost and the indirect costs are optimum. So, direct costs are like this, indirect costs are like this, and that is why this is the optimum point on which we are operating or more precisely, if you are considering probably it is lying this and this one is lying this. So, that is why this is the operating point.

So, the lowest cost point where as we can run our project; that means, running the project for that particular condition our total cost of the project will be minimum. The duration of that point where the lowest cost, total lowest cost is achieved that is called the optimum duration or that schedule is considered to be the optimum schedule.

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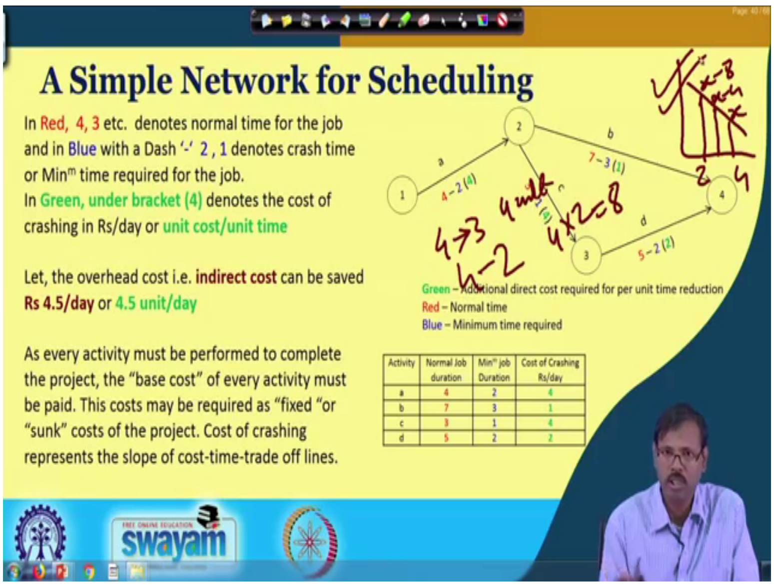

Now, to understand it more precisely, let us consider a simple network for the scheduling. We have given in red this 4, 3 these are the basically normal duration of the particular jobs and this Blue which is given in blue color these are given in blue color is the minimum time requirement. In last class we have given one example of baking this one. So, this is the minimum time requirement for completion of the job by expediting or giving the all additional resources to that job then also it cannot complete that job beyond this given hour, which is given in blue color.

So; that means, if you are considering job a, the normal duration of job a is 4 and minimum job duration is 2. And to reduce this job from 4 to 2 the cost require is 4; that means, which is given in green color here.

So, if you are looking at this, this is the normal duration, this is the reduced duration or the minimum duration and this is the cost of reduction or cost of crashing, ok. And it is expressed in unit price divided by unit duration. So, this shows us that if we are trying to reduce the job a for from 4 to 3, if it is 4 days to 3 days or maybe 4 hour to 3 hour. We have to deploy some additional man or machine or maybe material and that cost will be 4 maybe say unit US, Dollar or Indian rupees whichever it is.

If we reduce it from 4 to 2 then the cost is 4 into 2 that is 8 unit. So, that is why this cost of reduction is expressed in terms of this, that is why we have earlier also we have told we are considering it is on linear basis, say cost from this to this, it is 4, it is 2 and this is the point say this is x price, this is x minus 4 price, this is x minus 8 price. So, this is the cost of the reduction of this job for this cases and this is direct cost whatever is given here is direct costs, indirect cost is not given on the job. So, that is why the green bracket is basically, green bracket is basically the cost of crashing in Rupees per day or unit cost per unit time.

There is another part is the indirect cost which is considered separately. So, our indirect cost is 4.5 Rupees per day or 4 5, 0.5 per unit per day or per unit time we are considering.

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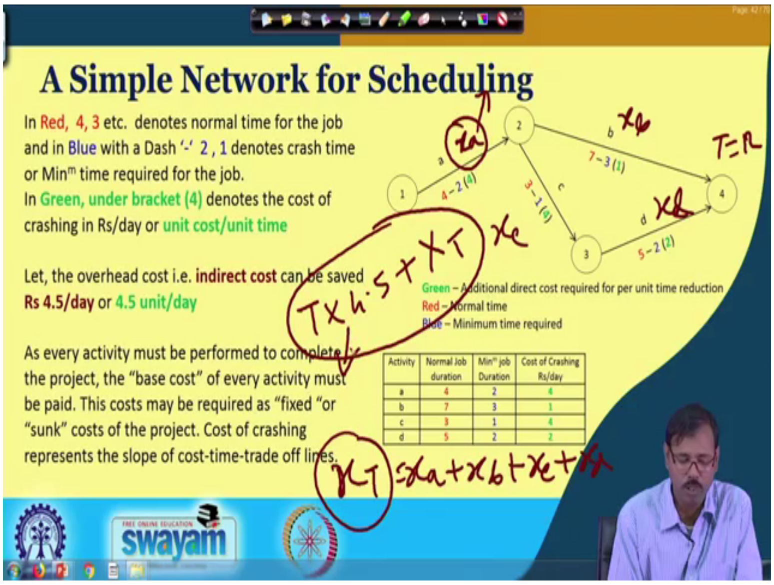

That means, the total cost of this project if you are calculating say for it is x a, this is x b, this is x c, this is x d is the cost of this project for normal time which is given in red color. So, that means, total direct costs of this project is x a plus x b plus x c plus x d is a total project cost.

Now, what is the total indirect costs? The total indirect cost say this is T is equal to 12, 4 plus 3 7, 7 plus 5 12, 4 plus 7 11. So, 12 is the total minimum time to complete this project considering the normal time. So, this T into 4.5 plus let us consider it is X T that is a total cost, direct cost. So, this plus X T is that total cost of this project, ok. So, now, if we are altering from this, suppose we are reducing this cost then this cost sorry, reducing this duration this cost will be increased, but as this is a critical job this cost will be indirect costs will be decreased.

So, basically we have to find out first the total cost of the project costs in the normal duration, then we have to find out what is the change in the duration in the other activities. So that, that can be changed. So, as every activity must be performed to complete the project, the base cost of every activity must be paid.

This cost may be required as fixed or sunk cost of the project and cost of crashing represent the slope cost time trade of lines. So, this is very- very important and if you see this table which is basically given us the activities their normal job durations, their minimum job durations and their cost of crashing.

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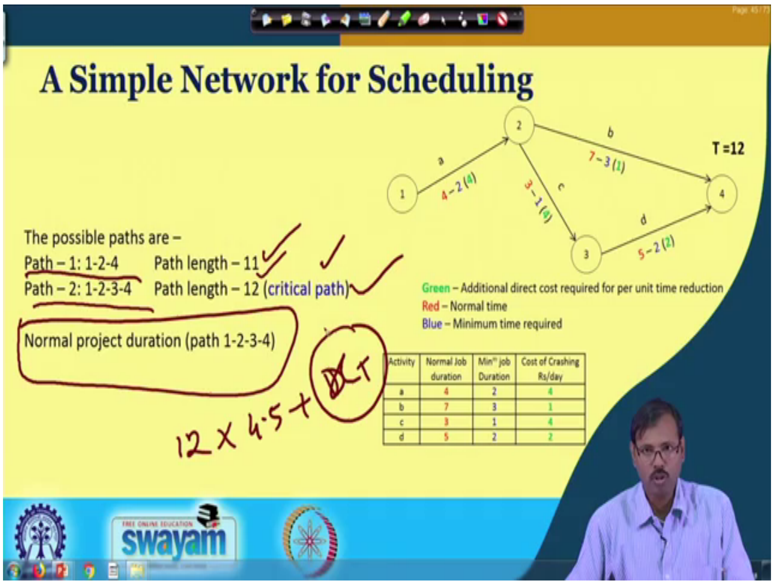

So, now, if you see there we are having two possible paths, this path 1 is 1 2 4 nodes, path length of 11. Path 2 is 1 2 3 4 node of 12, path length of 12 and that is the critical path. So, normal project duration is 12 hours and or 12 days and this is the critical path. So, the cost is now very very understood, it is the 12 into 4.5 plus whatever is the direct cost that is x T is coming. So, as it is the additive component, we can consider this is the fixed costs and any variations from these must be considered henceforth.

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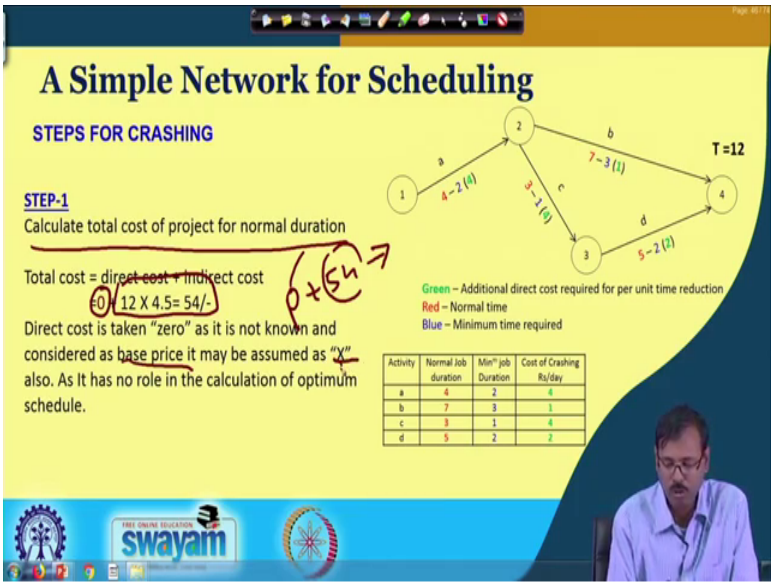

So, now how we can crash it to find out the optimum schedule where the costing must be minimum, let us check it. So, our first step is to calculate the cost for the normal duration and you know we have considered the direct cost of O, you actually you may consider it 0 also, because that is fixed cost, it is cost it will be cancelled and this is the indirect costs. So, total cost is coming basically if it is O plus 54 we have discarded this, because that is the fixed cost it is always there and the change in that will be reflected at as this minus that.

So, that is why we can discard this in the calculation or you can O plus or you may have P plus 54 as the total cost of the project. So, as it is the best price, it can be or marked as the x also, you can consider. So, now this is the first case in which we have to find out the total cost of the project for the normal duration.

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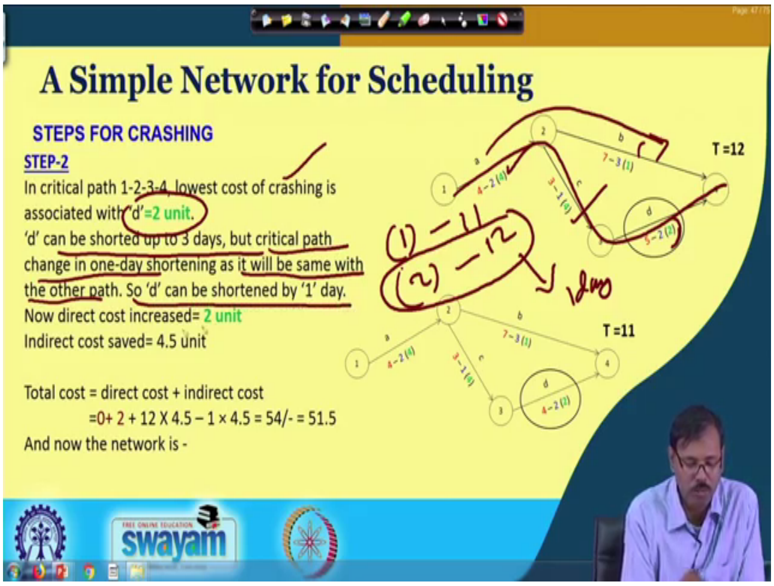

So, now let us see what we can do for the step second step. In second step, we can find out the critical path, the critical path is the critical path is this one. And this critical path the lowest cost of crashing is lying with the d; with the lying with the d which is 2 unit here, it is 4 unit here it is 4 unit it is 2 unit. That means, if we reduce this one the impact on the direct costs or the increase in the direct cost is minimum and the decrease in the indirect costs will be effective to us.

So, if we reduce we have seen there are 2 paths, path 1 is of 11 path 2 is of 12. And in the first step in this step, we can consider this one, because both the path are inequal for crashing it of 1 day to see the what will be the impact. So, this crashing it for 1 day, because d can be crashed for 3 days, but then the critical path changes if it is more than 1 day and it will be same with the other path. So, if we crash it for 2 days or 3 days, it will not change the critical path it will change the critical path to this path to this path, in that case our indirect costs will not be changed.

So, our first job is to shorten it and make it equal to the other path which is its nearly positioned so, that the both will become critical. And if we will carry out this if and only if it is giving us some cost benefit. So, now, d can be shortened 1 day; why d? Because it is having the, it is having the minimum crashing cost then other two activities in the critical job.

So, now, d can be shortened by 1 day and if d is shortened by 1 day 14 with a cost impact, the savings in the indirect cost is 1 into 4.5; that means, 4.5 unit, the increase in the direct cost is 2 unit. So, the total cost will became increase in the direct cost of 2 unit and decrease in the indirect cost of 4.5 units. So, that total cost will become 51.5 unit or this 0 plus 51.5 unit whichever you are considering. So; that means, there is the reduction in the cost, but the normal time of completion 12 is now reduced to 11; that means, that time duration is reduced.

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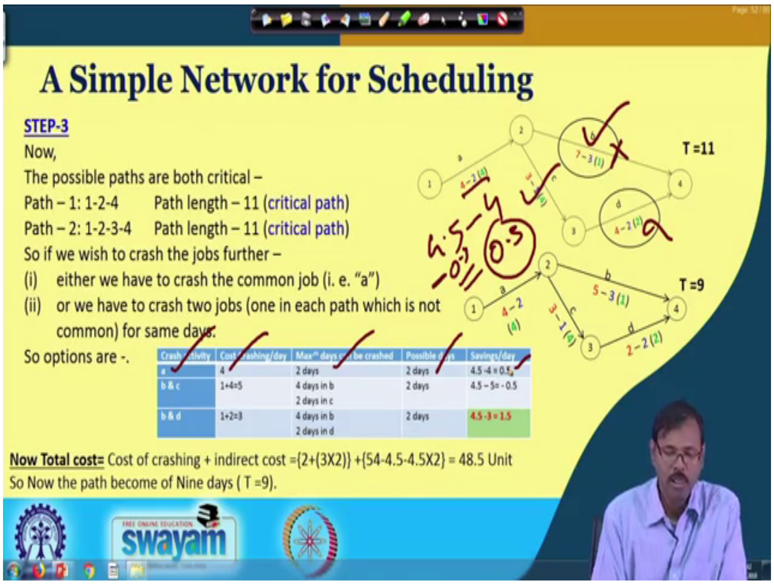

Now, let us see what will happen, because now our both the paths are critical, what will happen if we wish again wish to crash it further. Now in this case in the network T is equal to 11, you can see both the paths are critical and both the paths are critical.

Now if we wish to crash in that case either we have to crash a common job that is a so; that means, if we crash a that will reduce the total job path, we have to see whatever is the impact or our second option is that we can crash, we can crash this one and this one or we can crash this one and this one.

So, our options are either crashing of a which is a common job and in that case crashing cost is 4, crashing days maximum available is 2, 4 2 2. So, 4 2 2 crashing this available and possible days of this savings are 4.5 minus 4 per day. So, it is indirect costs will be reduced by 4.5 unit and direct costs will they are at 4 unit. So, the costing is 0.5 unit savings; that means, minus 0.5 unit is the costing.

So, savings is this one, if we are considering the second one where we will crash b and c the crashing cost will be here it is 1 unit here it is 4 unit so, 1 plus 4 5. We are having possibilities of crashing 4 days in b; 7 to 3 4 days available in b, 2 days available in c, 3 to 1 is available to us. As we have to crash both so, the minimum of that is only possible to crash. So, that is 2 days available here to crash and the per day impact is the 4.5 minus 5; that means, instead of saving, it will basically costing to us. So, this will increase the costing.

Whatever is the third option we can opt for crashing this one, we can opt for crashing this one. And you remember that we have already crashed this one from 5 to 4, so, now, our available days for crashing is only 4 to 2. And considering this we are having option the cost impact is 1 plus 2 is 3 unit. And if you see the maximum days can be crashed here for b is 4 days, for d is 2 days, because it is already reduced 1 days earlier. So, the most possible days between 4 and 2 is 2 days only. In other case it will not be possible and the cost impact you can see or savings per day is 4.5 minus 3 is 1. 5.

So, the savings if you looking at the, if you are looking at only at this savings per day column, you can find out this one is highest; that means, if you go for crashing job b and job d, you can save maximum.

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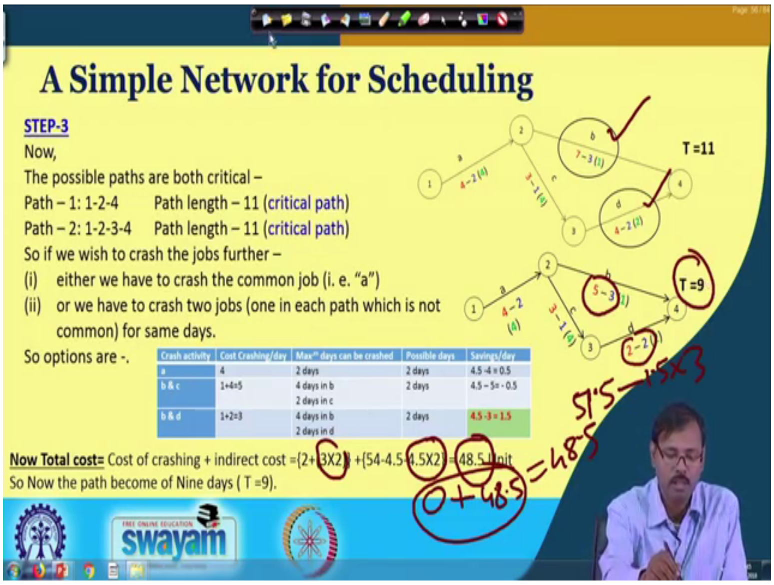

So, let us go for crashing this b and d for 2 days. Now, for crashing this one b will became 5 days normal time now at this present point and d will became 2 days that is the minimum at its minimum time from 4 to 2. And if you calculate the direct cost then again we have gone for 2 days additional reduction of this one. So, 4.5 into 2 days are the cost saving in the indirect cost 3 into 2, 6 days is the increasing in the direct costs.

So, the total cost is coming earlier it was 51.5. If you minus 1.5 into 2; that means, 3 so, it is 48.5 will be the total cost or you may have that is O plus 48.5 whichever is the total cost you are considering. But now our project duration is reduced to 9 days 4 plus 3 plus 2 9, 4 plus 5 9. So, both the paths are now critical and our now present cost is at 48.5. Should we crash it anymore? Whether we should crash it any more let us see or step 4.

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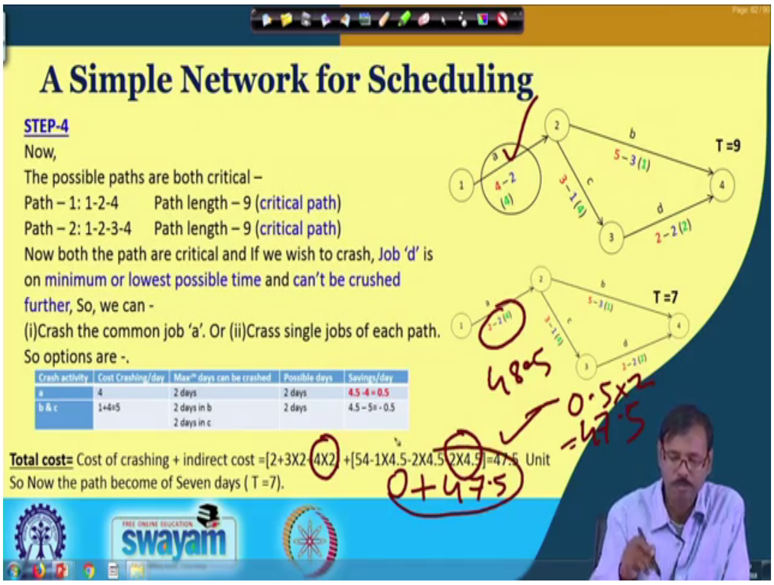

So, in the step 4, you can see the earlier, in earlier as it is shown in the earlier the duration is T is equal to 9 both the path is 9 hours or 9 days path both the path are critical. And in this case what are the options we are having to crash or expedite the job, either we can go for crashing this 1 for 2 days or we can go for crashing this 2 1 simultaneously whichever days are possible.

Now, this is at its minimum position and we cannot crash it anymore. So, the options of crashing this one and this one is no more existing. So, that is why let us see what are the options we are having either to crash a common job a or to actually this is a spelling mistake or to crash a single job of each path which is b and c.

So, let us see the cost impact. Again prepare this table a, the cost of crashing per day is 4, the possibilities is of 2 days as it is the single job the possible days for crashing is also 2 days and the savings are 4.5 minus 4 that is 5 days, that is 5 days, 5 days savings per 0.5 unit savings per day is available with this.

Now if you are considering the other option, in other option our cost impact is 1 plus 4, 1 plus 4 is 5 unit, 2 days now it is 2 days 5 2 3 2 days available for crashing in b, 3 to 1 2 days available for crashing in c. So, the total possible days are also 2 days here and if you see the cost impact is 4.5 indirect cost savings per day additional crashing cost is 5. So, the savings is basically minus 5; that means, it is actually not saving, it is increasing the costing of the term.

So, if you are comparing this, if you are comparing this column only you can see this point is giving us some saving, this is basically costing to us. So, we can opt for this one; that means, a job may be crashed for 2 days and it will become 2 and that can give us an additional costing of 4 in to 2, but an additional indirect cost saving of 2 into 4.5. That means, by calculating this earlier we had 48.5 minus cost savings is 0.5; 0.5 into 2 is equal to 1. So, it is becoming 47.5. So, it is coming 47.5 or you can expressed it in some fixed costs O plus 47.5 is the total cost.

So, now, our new network is showing this one where a is 2 hours, b is 5, c is 3 d is 2. So, that total project time is 7 hours or 7 days whichever it is.

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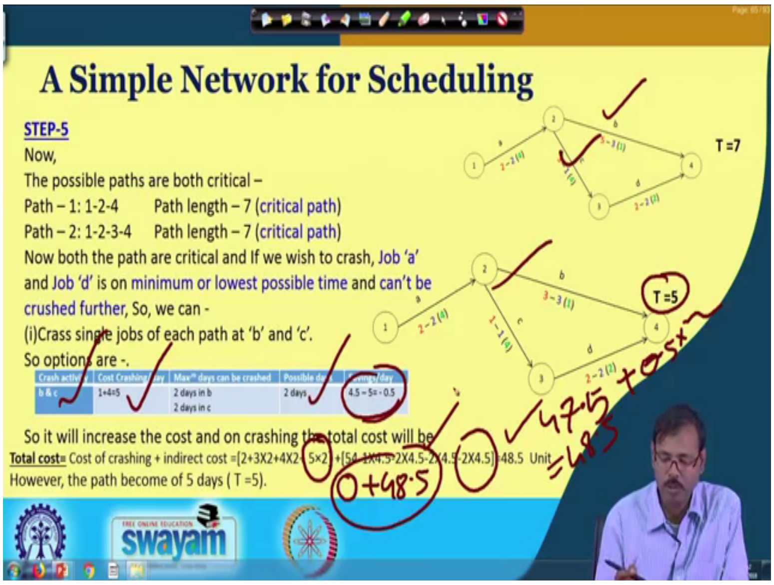

So, now should we go for further crashing whether it is giving us benefit or not? Let us check that one again in step 5. In step 5, we are now at this position and you can see both the paths are now critical and we cannot crash a job and the d job anymore, because these two are at their minimum time so we cannot crash it.

Now we are only having option either we can crash this one or this one simultaneously that is the only option we are having. So, what we are having option we are having option that b and c can be crashed together, the cost impact is direct cost impact is 1 plus 4 that is 5 possible days of crashing available is 2 days, but instead of savings it is giving us the costing. So, that is point minus 0.5 is the savings per day.

And if we carry out this one further still, if we are carrying out to achieve the minimum duration or something like that we can find out a new schedule like this, but this is costing us additional direct cost of 5 into 2, indirect costs will be reduced by 2 into 4.5. So, this is 10 minus 9 is 1. So, the earlier we had 47.5. Now that is increased to my, that is increased to 0.5 into 2 so, that is becoming 48.5. So, now, our total cost is increased, if you can have it is O plus 48.5 again, if you are considering that fixed costs or is there.

So; that means, though we are able to reduce the total duration into 5 hours or 5 day, but it is not our minimum cost, our minimum cost was 47.5 which we have achieved that T is equal to 7.

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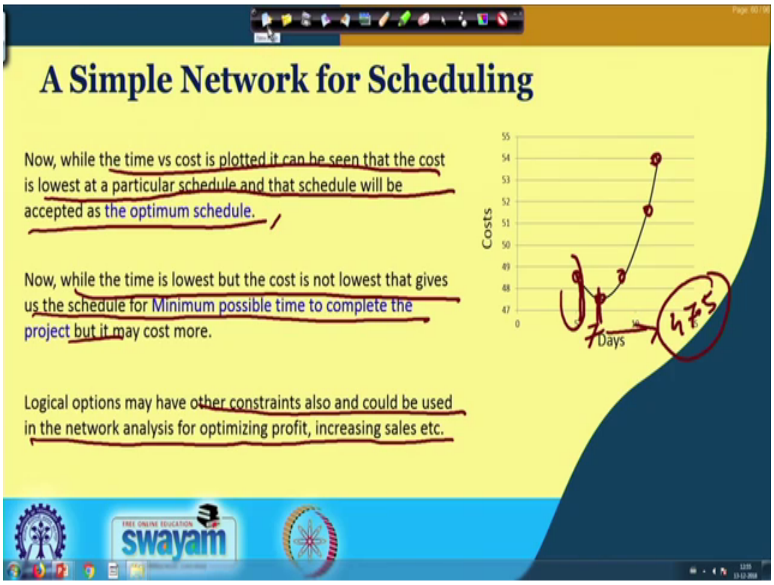

So; that means, if you plot the time versus total cost curve, if you see the; if you see that on the 12 day it was 54, on 11th day it was 51.5, on 9th day it was 48.5, on 7th day it was became 47.5 and on 5th day it was became 48.5 again. So, if you plot this, you will find out the minimum cost was obtained on the 7th day and it was of 47.5 unit and this schedule will be called optimum schedule.

That means while the time versus cost is plotted, it can be seen that the cost is lowest at the particular schedule and the schedule will be accepted as the optimum schedule. So, while the time is lowest, but the cost is not lowest that gives us the schedule for minimum possible time to complete the project that comes is at this position 5 days, but it may cost us more. So, logical option may have that other constants also and could be used in the network analysis for optimizing profit increased sales, etcetera.

In this case one particular thing matter is very important that while the revaluating the project network is going on and it has been found that a project is got delayed that time it may be possible that one has to go for this option though it is not accepted in that particular financial condition, but to address the project due date this may have to be opted for.

So, our final project will be accepted project schedule we will be of 7 days. And, that may be opted for. So, that is very very important and we are able to now go for the optimum project network that can be considered, that can be followed.

Thank you.