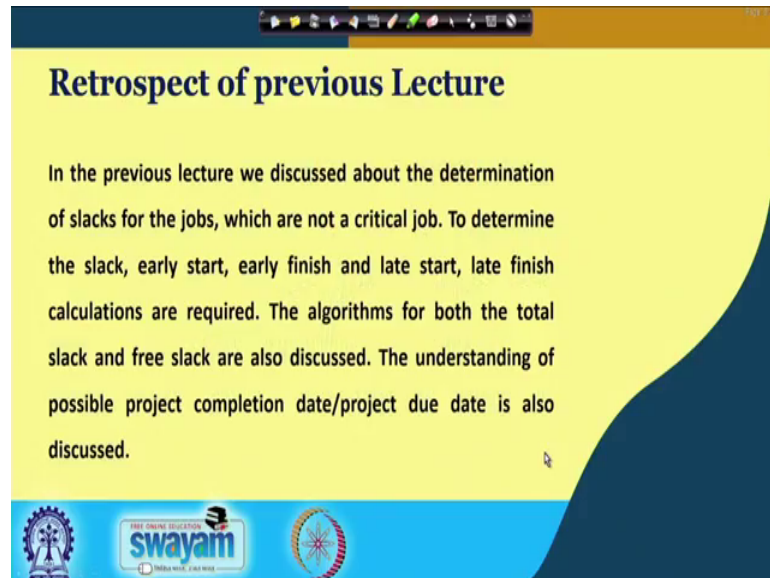


**Network Analysis for Mines and Mineral Engineering**  
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**Department of Mining Engineering**  
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**Lecture - 12**  
**CPM model and cost modeling**

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**Retrospect of previous Lecture**

In the previous lecture we discussed about the determination of slacks for the jobs, which are not a critical job. To determine the slack, early start, early finish and late start, late finish calculations are required. The algorithms for both the total slack and free slack are also discussed. The understanding of possible project completion date/project due date is also discussed.

The slide features a yellow background with a dark blue curved shape on the right side. At the bottom, there are logos for NPTEL, Swayam, and IIT Kharagpur.

Let me welcome you to the NPTEL online certification courses Network Analysis for Mines and Mineral Engineering, this is lecture number-12. In this lecture, we will discuss CPM Model and Cost Modeling. So, like every class let us say retrospect, what we have learned so far.

In our earlier class, we have already determined the slack of a job forward from using the forward pass and backward pass. Forward pass, we have determined early start, early finish time. In backward pass, we have determined late start, late finish time. And that calculation we have used late start, late finish time that calculation we have used for determining the slacks in a job. In also last class, we have discussed the project due dates, when the earliest possible time of completion of the project and the target date of completion of the project is different.

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CONCEPTS COVERED

Concepts Covered:

- Brief overlook on CPM model and the project due date
- Project cost
- Project duration

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In that case, the how the due dates has to be considered for rescheduling the job is also discussed, but how this rescheduling will be carried out, we will discuss in this lecture.

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INTRODUCTION

Retrospect CPM model with the knowledge of Project due date

CPM is a tool commonly used to solve scheduling problems and less concerned with uncertainly problems (PERT is used for this). So PERT is a "Probabilistic model" & CPM is "Deterministic model"

CPM should be more concerned with costs pertaining of the scheduled project.

With the knowledge of Project due date, it is understood that in CPM also variation of job times can be allowed (But not in from of random/statistical factors like (good luck/bad luck) but in a planned way like assigning extra or less resources (Man, Money, Machine etc.)).

A job, expedited by deploying additional resources, is called **crashed** and this process is called **crashing**

$T = 10$   
 $t = 5$

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So, let us first consider so far in critical path method of analysis of a network, whatever we have discussed that all are basically deterministic approach that means, we have considered that the time for completion of this job is  $t$ . Now, this time for completion of this job  $t$  means, suppose this job is that sending  $x$  material from a place to  $b$  place.

Now, sending x material from a place to b place depends on the velocity of that, and that may differ with the speed of some belt or any other drive which is being carried out or it may be transportation system from x place to y place, suppose it is a mine dumper has to carry the material from the shovel to the crashing plant. Now, that means it is a travelling of the dumper depends on the velocity. If the velocity is different, then the time will be different, but or maybe the traffic is congested the velocity will be different.

So, all these conditions are there that means, uncertainties are there which is not considered in the critical path method. We are considering the time taken by the dumper from shovel to crashing plant is three minute means, it is three minute, but these considerations are not considered in the critical path method. And in that scheduling problems uncertainty problems are considered in the program evaluation review technique, which will start in our 16th-lecture.

So, basically if you are considering that the program evaluation review technique is a probabilistic model and critical path method is a deterministic model, which does not entertain any uncertainty. But, we have seen in our earlier classes that they we have considered a due date, which is differing from the target date or may be the earliest possible time of completion.

So, in those cases what we are doing, we are trying to reschedule that one. And considering that we have found some jobs are having negative slacks, but some can be rescheduled within that reducing their available slacks that means, in critical path method of analysis also we are having some possibilities which in this case we can handle some level of uncertainty that means, with the knowledge of project due date it is understood that in critical path method also variation of job times can be allowed. But, it is not based on the statistical method which is considered in the pert model, but in a planned way by assigning extra or less resources like man machine and machinery, so that means it is also deterministic approach the way we are carrying out where probability is not considered, but rescheduling or readjustment is possible.

And I am giving you another example. Suppose, you are digging a pond or maybe lake; where, you have to complete it in 10 days by deploying 2 shovels. Now, you can after say two days if you see, the progress is not good enough, you can carry it out or if you

wish to complete instead of 10 days in 5 days instead of 2 shovels, you have to deploy 4 shovels.

So, this is the way in deterministic approach, you can have some adjustment in your network time by rescheduling the jobs, so that means a job may be expedited by deploying additional resources is called crashed, and this process is called crashing that means a job which is supposed to be completed in 10 or say capital T time, it can be reduced to small t time that means in 5 time, if we are deploying some additional resources in the job, so that is that consideration is possible thus that consideration is possible with the that consideration is possible with the critical path method analysis.

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**INTRODUCTION**

**Retrospect CPM model with the knowledge of Project due date**

On the other hand, there is no reason to expedite a job having generous amount of slack, and should be done at a place of lowest (optimum) cost.

So critical and near critical jobs may be required to crash/expedite with additional resources or may be additional costing also.

In case of Project due date  $D < T$ , Crashing of the critical jobs are required. ✓

In case of Project due date  $D \geq T$ , Whether crashing will be opted or not depends on financial/cost optimization. →

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Now, if you see the last class where we have discussed the project due date. We have found there is no reason to expedite a job having generous amount of slack, and should have done at a place of lowest or optimum cost that means, a job which is having huge slack in last example we have seen a dot slack a dot job has the 19-hours of slacks that means, it is having huge slack, it is not judicious to deploy additional disaster resources to reduce that job path, because it is already having huge slack, so that is there is no need to reduce that one, so which job duration has to be reduced that is the consideration has to be taken based on the financial analysis of the project.

So, now whether the project duration will be reduced, whether the project duration will remain which project which job duration will be reduced that decision is a purely basic

financial decision, and must be carried out only after the financial appraisal of the project, so that means to reduce the job path it is understood that the critical and near critical jobs may be required to be crashed or expedite using additional resources. But, this additional resources means, it is the additional costing to the organization.

So, whether the crashing has to be carried out that means, the expediting the job will be carried out or not that is a financial decision. And the impact financial impact on the project must be carried out, and the decision must be calculated, and decision must be taken on based on that..

So, in case project due date is less than  $T$ , crashing of the critical jobs are required essentially required. And if the project due dates are less than  $T$ , it may be possible some of the non-critical jobs may be required to be crashed, so that means, it is all the critical jobs plus maybe some of the other jobs has to be crashed. But, if  $D$  is greater than  $T$ , the crashing will be update or not depends on the financial costing.

If you consider the first case also, they are also whether the crashing has to be carried out in all the critical jobs or some of the critical jobs or to a particular critical jobs that decision is also must be a decision to be taken based on the financial or cost optimization. And in case of  $D$  less than equal to  $T$ , your project cost may increased than the total considerations of the project cost or it may be decreased also. If it is decreased in those case, you whether your due date is accepted or not you must go for crashing, because it is giving you the financial benefit. We will discuss those things in this, but before that we have to understand, what is the cost of a project.

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The slide is titled "UNDERSTANDING COST" and features a diagram showing "Project cost" branching into "Direct cost" and "Indirect cost". The text explains that project cost is classified into direct and indirect costs. Direct costs are those incurred due to direct involvement in the job/activity/project, such as man power wages, machine operating costs, and raw material costs. These are production dependent and not time dependent. Indirect costs are those incurred due to indirect requirements for carrying the project but do not directly depend on production, such as overheads, lighting, security, safety, and rents. These are time dependent and not production dependent. The slide also includes the Swamyam logo and a small video inset of a man in a white shirt.

The cost of a project means, we classify it in two terms; one is direct cost, another is indirect cost. Now, let us understand something what is cost. Suppose, we want to manufacture something say, there is a lathe machine, you are trying to manufacture in the lathe machine a bolt. Now, for manufacturing a bolt, what we need? We have to bring some iron part, the lathe machine has to put thread on that, and that bolt will be made.

Now, this iron part is a material, and the lathe machine is a machine, and the manpower is required to carry out this work. The electric consumption by the lathe machine is the operating cost of the lathe machine, and maybe other things are required for creating this bolt in the lathe machine. So, all this man machine, and the power consumption for running of the machines material used in the machines comes under the direct cost.

So, direct cost is the cost which incur due to the direct involvement into the job activities like manpower, wages, machine operating cost, raw material cost etcetera. And also because those direct safety cost etcetera are also coming into this, say cleaning of that place every day. If the no working is there that cleaning is not a part of that, but other things are there which is has to be considered comes under indirect cost.

The for better understanding is that if there is no job, the direct cost is zero that means, production dependent cost is considered as the direct cost, which is not a time dependent cost that means, the cost which are not time dependent if you are manufacturing a bolt, then only cost is there. If you do not manufacture the bolt, then that means, your cost on

the iron part is not there, your cost for the machine operation is not there, and your cost for the manpower requirement is also not there that cost is called direct cost. And that is considered as the direct cost, which requires which is dependent on the production only not on the time.

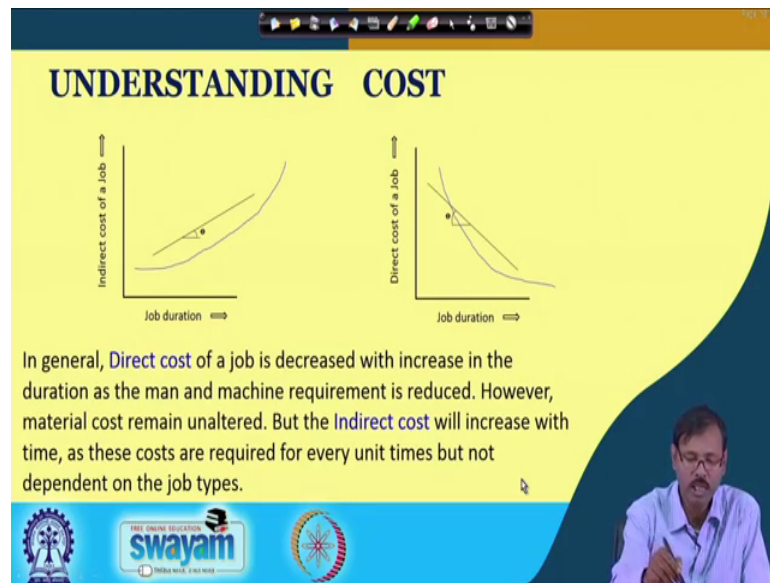
Indirect cost at those costs which incur due to the indirect requirement for carrying the project, but not directly depends on the production like overhead costs, lighting of the area, security of the area, safety requirement of the area, rents of the area. All those things are coming in to these. Say suppose if you are considering say taxes are sometimes it is a direct cost, sometimes it is indirect cost. Suppose, if you are considering the mining case, whether you are mining or not, then also you have to pay the (Refer Time: 13:34) which is basically the rent of the land has to be paid.

But, if you are paying the royalty, if it is direct royalty on the production basis, then it is the that tax is the direct tax has to be paid. So, this thing some of the parts are direct, some of the parts are indirect and that is why the other consideration is that. If there is no job, then also whatever is the cost that is the cost depends on the called the indirect cost? And if there is job that only creating that cost that is the direct cost has to be considered.

So, if no job, then also the costs are as usual same, and that so it is time dependent not the production dependent is called indirect cost, so that is very well understood that direct cost and indirect cost is there. And this direct cost indirect cost, they are having their own trend of moving with the time or with the production, and that will be considered in the due case.

But, the total cost may be classified in two groups. One is direct cost which is dependent on the production and zero production means, zero direct cost. Another is indirect cost which is dependent on the time that means zero production means that cost is there, time is not there or time is less or more is the in those case the cost is also increasing or decreasing. So, direct cost and indirect cost are the two types of cost. And let us understand little bit more on this.

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If you see in general, a direct cost of a job is decreased with increase in the duration as the man machine requirement is reduced that means, say if you are deploying some if you are doing some job, and you are allowing your man machine to work on these. If you are increasing the job duration that means, you are going for a less number of manpower. You are going for the less capacity machines, which are pricing you less, and that is why the direct costs are decreasing with the increase in the decrease in the with the increase in the duration.

So, actually this has to be reversed, but unfortunately wrongly it has placed here. If you see the trend of the direct cost, the direct cost is that with the increase in the job duration, it is going to less, it may be some straight line, it may be some curved power curve, whichever may be the trend is that it is reducing with the duration.

However, if you try to increase or expedite the job, in that case you have to deploy additional machines, additional manpower that means, it is costing you more, so that is why direct costs are inversely dependent with the time decrease with the increase in the time. But, if you see the indirect cost, indirect cost will increase with that time as it is purely dependent on the time.

Suppose, if you are carrying out 1000 volts per day from a lathe machine ok, and now you do not have demand, you are carrying out 200 volts per day that means, for 1000 volt you are taking 5 days that means, we have to deploy your security you have to



switch on the light night lighting for 5 days, you have to give the rent for the 5 days. So, those are the requirement. And that is why indirect cost increases with the time, as the time is increasing the indirect cost may increase like this, the trend may be whichever it is it is not a problem, but the tent has to be established trained for our calculation purpose.

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**UNDERSTANDING COST**

Direct Cost of a project is the sum of direct cost of each job.  
Indirect cost of a project is the total indirect cost for the duration of the project.

Total Cost of a project is the sum of direct cost and indirect cost for that project, i.e.  
 Total Cost = Sum of Direct cost of all jobs + Total indirect cost

Thus, reduction in duration of a job (Not Slack) increase direct cost, but decrease indirect cost only if the project duration is also reduced.

The graph shows 'Total cost of a Project' on the y-axis and 'Project duration' on the x-axis. A straight line labeled 'Direct' slopes downwards from left to right. A curve labeled 'Indirect' slopes upwards from left to right. A third curve, representing 'Total Cost', is U-shaped and formed by the sum of the other two. Handwritten red annotations include 'Direct' with an arrow pointing to the downward-sloping line, 'Indirect' with an arrow pointing to the upward-sloping curve, and a circled 'X' at the minimum point of the total cost curve.

So, in our simple calculation, we consider the straight line one, so that the calculation will be easy, but it may be of any equation that can be fed, and that can be considered for the time dependency of the indirect cost. So, it is well understood now that the indirect costs increases with the time, direct cost decreases with the time.

And now if we try to calculate the total cost with the time, we may have a we may have seen a trend like this, where direct cost of a project is the sum of the direct cost of each job. Indirect cost of a project is the total indirect cost of the duration of the project. And total cost is the sum of the direct cost of all jobs. And total indirect cost if we are carrying out this one, basically we are adding for a particular duration, we are adding this much length plus this much length, and by this way we suppose we are getting this value, probably there is a little bit it is not a true scale.

So, adding these two, we are getting this value that means, the total cost this is the total cost curve, this is the direct cost curve, this is the indirect cost curve. So, this direct cost

plus indirect cost gives us the total cost, and the trend is coming something like this or something like this.

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**UNDERSTANDING COST**

Direct Cost of a project is the sum of direct cost of each job.  
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Total Cost of a project is the sum of direct cost and indirect cost for that project, i.e.  
Total Cost = Sum of Direct cost of all jobs + Total indirect cost

Thus, reduction in duration of a job (Not Slack) increase direct cost, but decrease indirect cost only if the project duration is also reduced.

$\sum D_{nro}$

Direct = [ ]

Total cost of a Project vs. Project duration graph showing a U-shaped curve with a minimum point.

Logos: Swamyam, Free Online Education, and other educational institutions.

So, if you see the trend of that total cost, you can find out the total cost is initially decreasing with the time. And after that, it again started increasing with the time that means, there is a point on which it is optimum one. So, this lowest point of operation is basically considered as the optimum operation time. So, this total cost is the this curve can be obtained from the direct cost curve, indirect cost curve. The important part is that direct cost direct cost of one job; it depends on the job not on the network.

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**UNDERSTANDING COST**

Direct Cost of a project is the sum of direct cost of each job.  
Indirect cost of a project is the total indirect cost for the duration of the project.

Total Cost of a project is the sum of direct cost and indirect cost for that project, i.e.

Total Cost = Sum of Direct cost of all jobs + Total indirect cost

Thus, reduction in duration of a job (Not Slack) increase direct cost, but decrease indirect cost only if the project duration is also reduced.

$\sum_{i=1}^n a_i$

The slide features a graph with 'Total cost of a Project' on the y-axis and 'Project duration' on the x-axis. It shows two curves: one that increases with duration (representing direct cost) and one that decreases with duration (representing indirect cost). The total cost curve is the sum of these two. Handwritten annotations include a red box around the definition of total cost, a red circle around the summation formula, and arrows pointing to the curves.

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So, the network direct cost is basically direct cost of a project or network is basically the sum of the direct cost of all the jobs means, summation of all the jobs their direct cost has to be considered as the direct cost of the network. Indirect cost of a job is basically the duration of this one, but at the same time there may be two or three jobs are operating, so that is why indirect cost of individual jobs cannot give you the summation of that will not give you the indirect cost of the project.

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**UNDERSTANDING COST**

Direct Cost of a project is the sum of direct cost of each job.  
Indirect cost of a project is the total indirect cost for the duration of the project.

Total Cost of a project is the sum of direct cost and indirect cost for that project, i.e.

Total Cost = Sum of Direct cost of all jobs + Total indirect cost

Thus, reduction in duration of a job (Not Slack) increase direct cost, but decrease indirect cost only if the project duration is also reduced.

$5 \times 35 = 175$  indirect

$T=35$

The slide features a graph with 'Total cost of a Project' on the y-axis and 'Project duration' on the x-axis. It shows two curves: one that increases with duration (representing direct cost) and one that decreases with duration (representing indirect cost). The total cost curve is the sum of these two. Handwritten annotations include a red circle around the definition of total cost, a red circle around the summation formula, and arrows pointing to the curves. The text '5 x 35 = 175 indirect' and 'T=35' is written in red.

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But, indirect cost of the project, you can come with the indirect cost indirect cost for the complete project that means, in our earlier case T is equal to 35. Say per day indirect cost if it is 5 unit, then for 35 days it will be 175 unit will be the total cost so total indirect cost, so that is why the indirect cost depends on the total project duration not on the particular duration of a job.

But, the direct cost depends on the particular direct cost of a job, and direct cost of the project is basically the summation of the cost of the each and every activities of the project. Once again I am telling you must remember the dummy activities does not have any cost, does not have any time value, they are basically dummy activity to construct the network. So, in this cost calculation, you need not to include the dummy activities.

Reduction in duration of jobs say if it is not slack increases direct cost, decreases indirect cost, if the project duration is also reduced that means, in our example if you reduce the critical job, then only the project duration will be reduced, and indirect cost will be reduced. Otherwise, your cost will not be indirect cost will not be reduced. But, if you reduce the jobs which are not critical, in that case also your direct cost will be increases.

So, reduction in duration is basically increase the direct cost, and decrease the indirect cost, if it is a critical job. So, always our concern is that while we are reducing the job, we must consider, it must be a critical job that has to be reduced, so that we can have some benefit of reduction of the indirect cost.

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**UNDERSTANDING DURATION**

Any activity or job is considered to have a normal/natural time of completion. In a deterministic approach, this time is fixed and considered to be as same as given number. Say, baking time in the oven for a bread.

However, It can be reduced up to a certain range. Say, by increasing temperature below that it cannot be reduced as then it will affect the quality of bread.

In probabilistic approach the normal time and Minimum time may be the Mean time of the observed values and the lowest time observed.

The slide contains two graphs. The top graph shows a vertical line for 'Normal time' and a shorter vertical line for 'Minimum time'. The bottom graph shows a bell-shaped curve with a vertical line for 'Mean time' and a shorter vertical line for 'Lowest time'. There is a handwritten circled 'a' with an arrow pointing to the 'Minimum time' label in the top graph.

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Now, let us understand what is duration? Say every time we are giving say these activities, this activity is having a duration of say three days or three months or three hours like that. So, how we have come out with this three? This we have so far we have understood the calculation that what is cost.

Now, let us understand what is duration, the moment I am telling you the normal job duration is this. Basically, it is similar that we are expecting from x place to y place, we can reach by two hours as we have discussed. And that two hours basically depends on our velocity if we go in a higher velocity, that 2 hours will become 1.5 hours, why you then 1.5 hour is not the time we are considering. So, this normal time which we are considering here, it is called normal time.

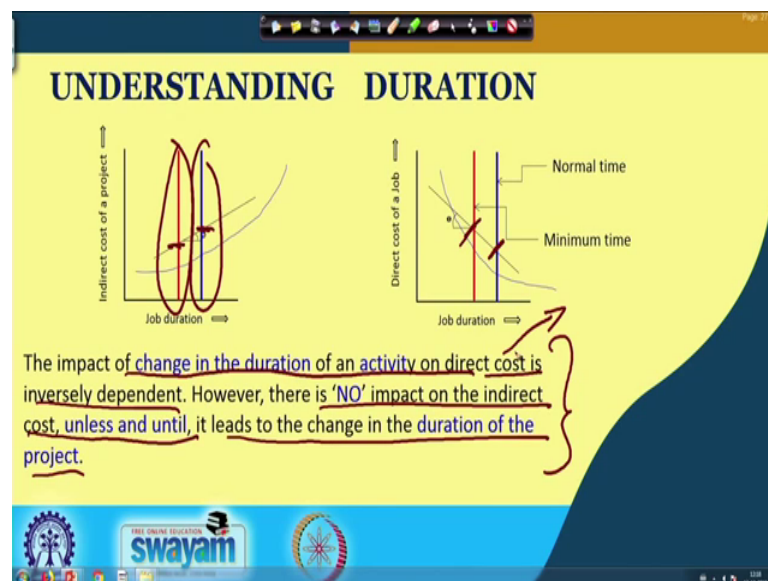
So, any activity or job is considered to have a normal or natural time of completion. In a deterministic approach, this time is fixed as we have considered here three. And is given as a number, say baking time of a in the oven for a bread. Say we placed it in oven, then we bake it for say three hours, so that it will become a bread or became a cake, whichever it is.

Now, it can be reduced to a certain range, and if we increase the temperature maybe little bit it may be a reduced or if we can change the oven that, then it may be reduced, but this may affect the quality that means, we cannot have a system where we can have a bread within say 5 minute. We can place it in the oven bake it for 5 minute, it will give the bread that is not possible, because a normal baking time must be given, then only it will become bread.

Otherwise, it will be a severe effect on the quality of the bread, so that is why every activities are having a normal time that means, the time we have placed for baking that cake or whichever it is or there must be a minimum time up to which we can adjust that minimum time may be by changing the machine, the most the most modern machine, may break it in say 1-hour instead of 3-hour, so that is the achievable reduction of the time which we can go for not for that one, so that means every activity is having a normal time and having a minimum time. And we can only reduce that activity up to that minimum time not beyond that that time, and to that that will give us some additional costing, so that is very important.

In our probabilistic manner, we can go we can have a different types of system where we may consider I think this is shifted little bit. So, this is the meantime at this position and may be lower lowest time at this position, which may be possible here. So, this actually this will be there. Generally, in probabilistic approach we consider mean as the normal time. And lowest time maybe having some plus minus one sigma or two sigma time may be considered as the lowest time up to which the crashing can be allowed. So, this is the probabilistic approach.

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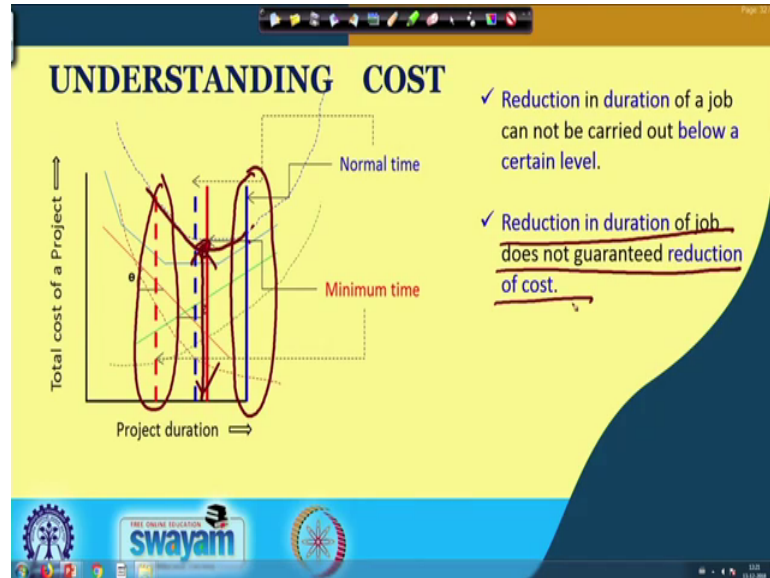


But, in deterministic approach we have to consider some normal time and some minimum time, which based on our experience or based on the expertization can be determined. So, in deterministic approach, it is essential that we have to consider the duration for a normal time case and for a minimum time case. And the cost pertaining to this in the indirect cost and cost pertaining to this in the direct cost are the point of interest.

So, impact of change in the duration of an activity on the direct cost is inversely dependent, there is no impact on the indirect cost, unless and until, it leads to change in the duration of the project, because indirect costs are not job dependent, it is the project dependent, time dependent, and that is why that completion time. If the completion time of the projects are not reduced, then the indirect cost will not be reduced. And the indirect cost will reduce, direct cost will increase that is the way one has to calculate the

financial analysis for the job, whether it will be expedite by crashing or not that has to be considered.

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So, basically we need to understand duration we need to understand the costing before expediting any job. Now, let us place it on this total curve, if we consider the normal time, and if we consider the minimum time. So, reduction in duration of a job cannot be carried out below a certain level this is fixed and their position is very very important.

Now, if we see in the first case, if we are having a normal time of completion of this one, and this is the reduction or the minimum time up to which we can reduce or crush or expedite the job, then you can see that total cost is basically coming from this place to this place that means, there is a reduction in the total cost. Though there is some increase in the indirect cost, there is a there is sorry increase in the direct cost indirect cost, those things happen, but despite that our total costs are being reduced by going from the normal time to the minimum time.

So, the total cost of this is reduced and that is why it is financially acceptable. However, if you look into the another set this one and this one where the normal time is this one, minimum time is this one, and if you are trying to shifting this one, from this place to this place this is basically increasing your total cost where normal condition it is there and the reduced condition or crushing or expedite condition it is there that means there is

a increase in the total cost. So, this increasing and decreasing of the cost in the total cost is carried out by crashing or expediting the job.

Basically reduction or increase in the day at cost or indirect cost is not important. Reduction in the total cost, reduction in the total cost is important for these cases. And it also may be possible suppose if you are considering for a particular job, this is the normal time and this is the minimum or reduced time, then what will happen initially the total cost is being reduced then it is again increasing, so that means, there is a particular point where we can achieve the reduced duration and reduced cost also. And in that context though it is not the minimum duration, but the cost is the minimum is this position that is the opt, considered as the optimum condition.

So, the reduction in duration of a job does not guarantee the reduction of the cost, reaction may be increased the cost may be cost may be increased, cost may be decreased, but there is some change in the cost whether that cost change in the cost is acceptable or not that is a different point of concern. So, this is the understanding of the cost and duration and their impact on the project network. So, this is all for this lecture. In next class, we will try to consider about the examples of crashing or expediting the job.

Thank you.