Network Analysis for Mines and Mineral Engineering
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Lecture – 10
Examples of slacks and calculations of AON network

Let me welcome you to the 10th lecture of NPTEL online certification course Network Analysis for Mines and Mineral Engineering this lecture will be on example of slacks and calculation of slack for AON network.

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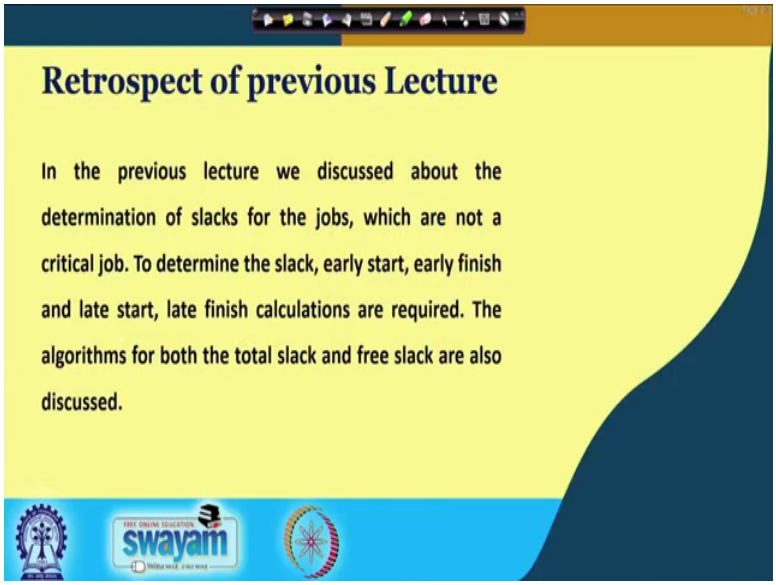

So, like every a class let us retrospect what we have covered in our previous lectures. In previous lectures we discussed about the determination of identification of the slacks in the jobs, then calculation of the slacks that slacks are possible only in those jobs which are not critical job and we are able to now calculate slack from the early start early finish late start late finish calculations.

And we also know the algorithm for both the total slack and free slack and we will discuss some example in this lecture and we will calculate the slack for the activity on node network also.

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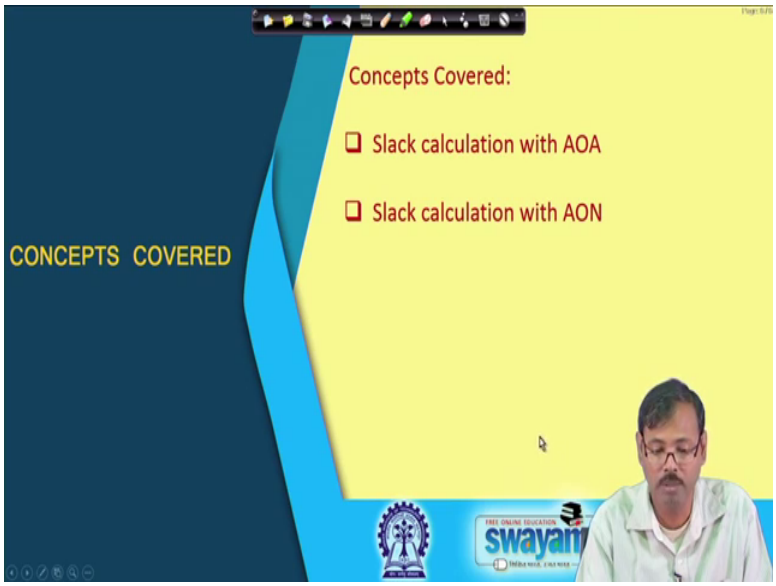

So, these are the 2 topics we will cover slack calculation with activity on diagram and slack calculation with activity on node diagram.

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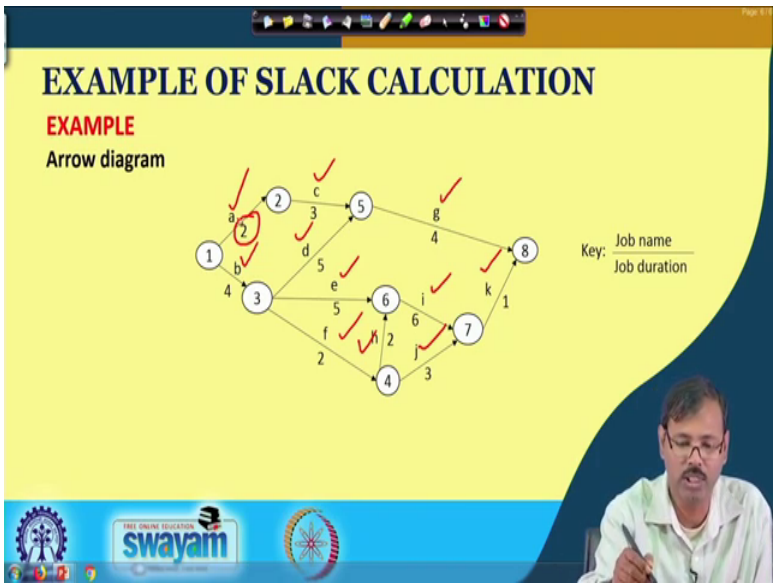

So, for that let us consider the example this example is also discussed in earlier classes and you know you have directly come with the network where a job a job b job c job d e f g h i j k. So, all these 11 activities are basically joined together to form the network and their predecessor successors are well defined and that is why the network is constructed and these are the duration of those activities and let us consider these are in hours.

So, this activity network is complete arrow diagram is made for this network and our task is to calculate the early start early finish and late start late finish for carried out further.

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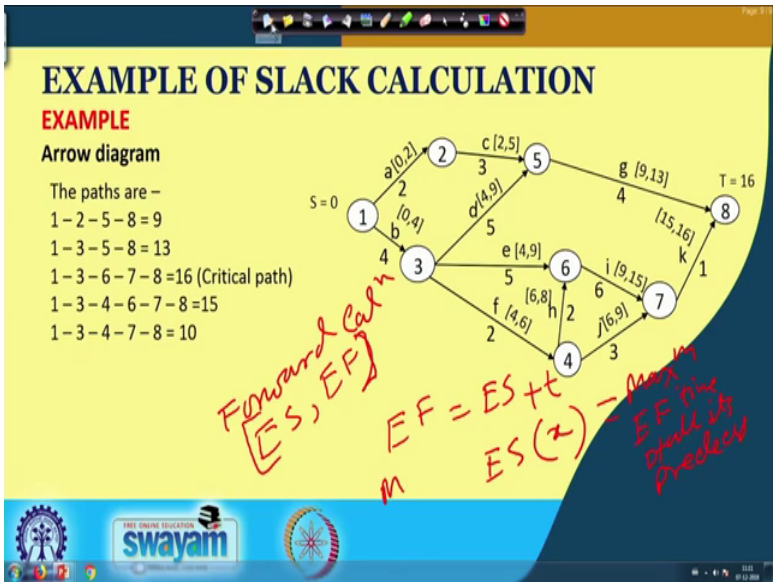

So, you can see we are having available path 1 2 3 4 5 so, this 5 paths are available in this network and in this 5 path you can see the path lengths are calculated 9 hours, 13 hours, 16 hours, 15 hours, 16 hours. So, the maximum path length 16 hours is the critical path and we have also calculated the early start early finish time for all the activities and these are computed in this network.

So, this is for kth job this is for jth job this is for h job this is for f job. So, this is actually figure is little bit distorted that is why it is looking like this. So, you can see the first consideration is the starting time is 0, considering that first job that early start times are considered as the 0 for a and b, the completion times are early completion times are considered calculated based on that.

So, you know the formula the earliest finish time early finish time is equal to early start time plus the activity duration. So, using that formula we have computed early finish time also and in the forward pass we have in forward calculation, we have computed early start and early finish time as presented as presented here under the third bracket.

And again let me refresh you that early start time for a job x is the maximum is equal to the maximum of the early finish time of all its predecessors; that means, unless and until the predecessors jobs are finished it cannot be started. So, earliest possible time starting time is the maximum of the early finish time of the predecessors so, that is calculated in this network.

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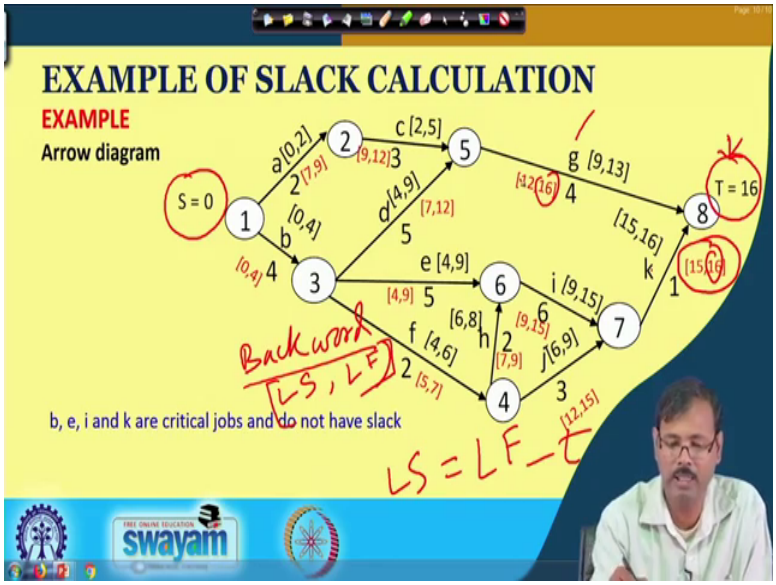

And let us carry out the backward analysis also. So, in forward analysis we have found the critical path length is 16 so; that means, the earliest possible time to finish the complete project is the 16 hours. So; that means, if the earliest starting time S is equal to 0 the earliest finish time of the project, T is equal to 16 and the earliest possible finish time is the considering the 16 is considered as the target time. And backward calculation backward calculation backward analysis is carried out for determining the late start and late finish for this network and we have found these are given in this red color under the 3rd bracket.

So, the target date is 16 the earliest finish time is 16 and for addressing this target date we have to latest by this 16 we have to complete job k and job g. So, that the late finish time of these jobs are 16 and accordingly we have calculated the late start time is equal to late finish time minus the project job duration T. So, late finish time minus 1 is 15 16 minus 4 is 12. So, that is the late start time we have achieved and for the predecessors jobs it is calculated like this.

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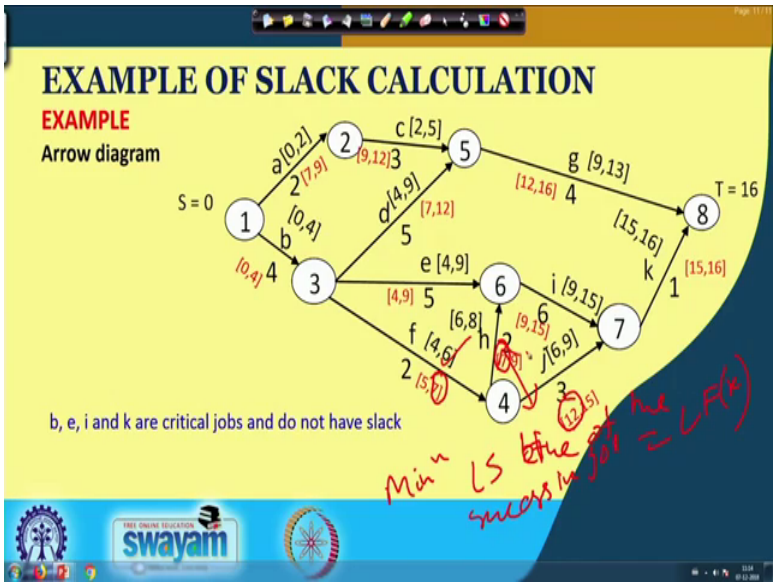

So, this is the late start time or you can say the minimum of ; minimum of late start time start time of the successive job is equal to the late finish time of the job x.

So, that is why you can see in this case there are 2 jobs the late start time of this j job is 12 late finish time of a late start time of job h is 7. So, the late latest finish time of job f which is the predecessors of that is minimum of this; that means the 7. So, in backward calculation we have to consider the predecessor job and that is why for this particular job we have to see which 1 is the minimum of the late start time of the successive activities and considering that we determine this late start and late finish time.

So, this is this are already discussed and I believe that you can you are able to understand this.

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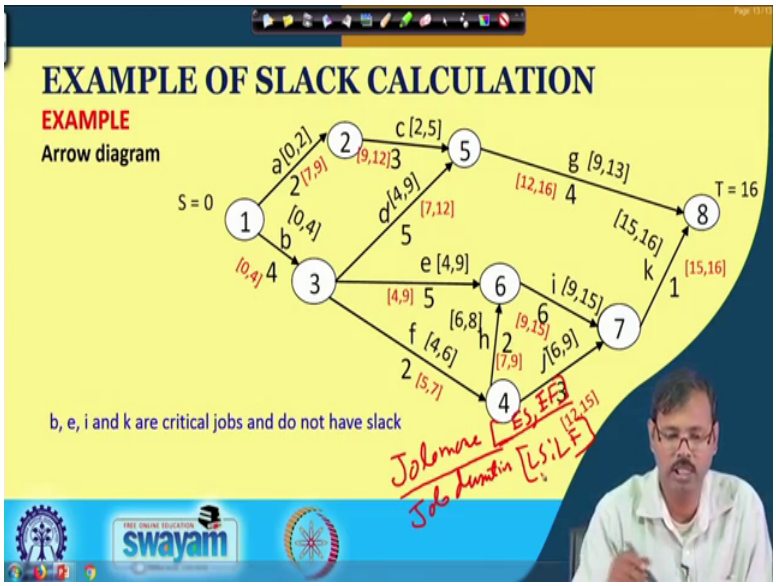

So, now at this point we are having job name; job name this is the job duration and under bracket early start early finish under bracket late start late finish. So, these are presented in all the jobs on their arrow.

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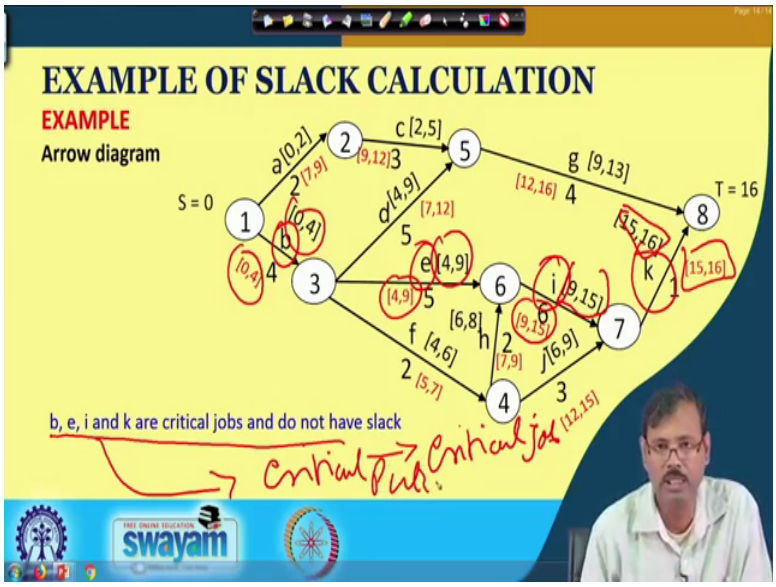

So, let us see if we are looking at this we can find out for job b job e job i and job k. So, job b, job e, job i and job k these jobs are having there early start early finish time late start late finish time same late start late finish time same. So, basically they are representing the critical job and this network is basically representing the critical path.

So, basically this job does not have any slack hours and let us then calculate the slack hours for the other jobs in this network.

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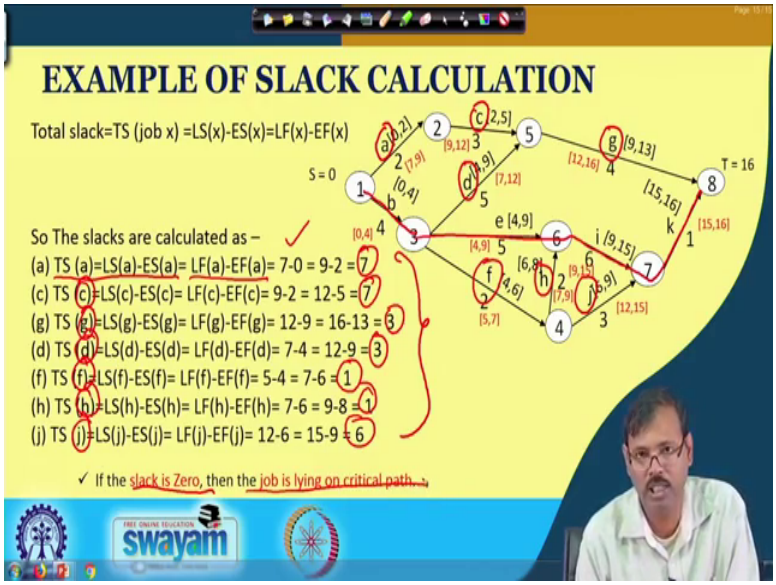

So, you can see the slack hours are available with these are let me draw the critical one in red. So, this is the critical path so, apart from this path all the other activities are having some slacks. So, we are having slacks at a, at c, at d, at g, at f, at h and at j and using the algorithm which you have already discussed in our previous class let us calculate the total slack for each activities say you if you find it out total slack of a is equal to latest start of a minus earlier start of a. And; obviously, that should be equal to the late finish of a and early finish of a and you can see it is calculated by 7 hours either 7 minus 0 or 9 minus 2 so, this is 7 hours.

So, similarly we can see for c it is 7 hour, for g it is 3 hour, for d it is 3 hour, for f it is 1 hour, for h it is 1 hour and for j it is being calculated as 6 hour. So, these jobs are having a slack hours of total slack hours for each and f jobs are represented here and if the slack is 0; that means, the job is lying on the critical path that is also known to us. Now, let us check for the free slack here.

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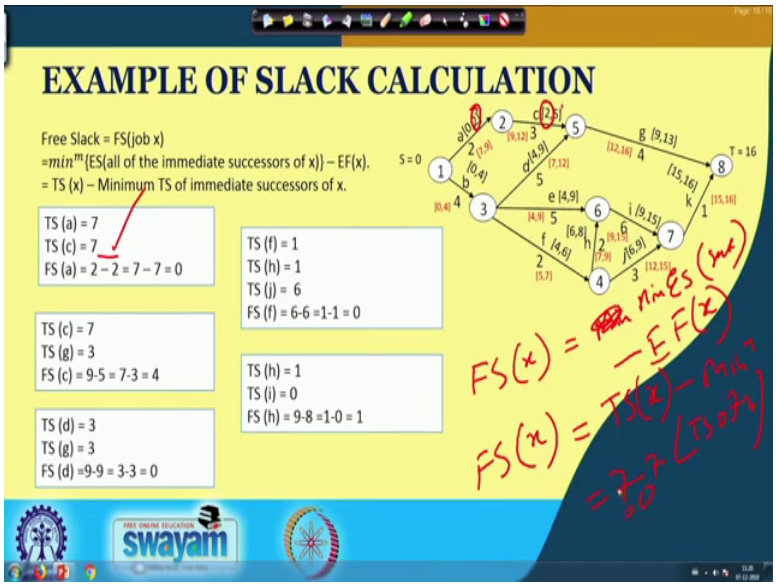

So, if you check for the free slack you can find out the free slack associated with job a is equal to 2 minus 2 that is the total slack of a minus total slack of c because, c is the only successor of job a that is coming to 0 or in other calculation the earliest starting time if you are considering based on that it also determine to be 0. So, that is why it is coming to this is the earlier starting time and this is the total slack minus total slack of a minus total slack of c is considered here and this is coming 0.

If you are considering for c if you are considering for c or let us check let us consider it once more. Say we are having free slack of two formula; one is the minimum or you can say earliest starting time of succeeding activities minus the early finish time of job x. So, if we are considering this early finish job time of x is 2 minimum of early starting time of successors activities is 2 so; that means, it is 2 minus 2 it is giving you 0.

The second formula is the free slack of x is equal to total slack of x minus minimum of the total slack of all the successive activities. So, in this case the successive activities is only one c and the c total slack is 7 total slack of a is also 7. So, it is 7 minus 7 is also 0.

So; that means, free slack for job a is 0 applying the same rule while we are proceeding we can see the free slack for job c is 4 hour because, the only successive job g is having a slack of 3 hour and the c is having 7 hour slack. So, the free slack available with job c is 4 hours; however, for job f it is 0 job d it is 0, but again h is having a free slack of h is having a free slack of 1 hour so; that means, we are having free slack at c we are having

free slack at c and at h and in though this two cases we can use this free hours as for our own. And we can carry out any maintenance etcetera related work though the activities in other part of the network is being carried out.

So, this free hours can be easily utilized in this jobs specially the man power withdrawn is one very good option utilization of the man power in other sector is one good option for the free slacks.

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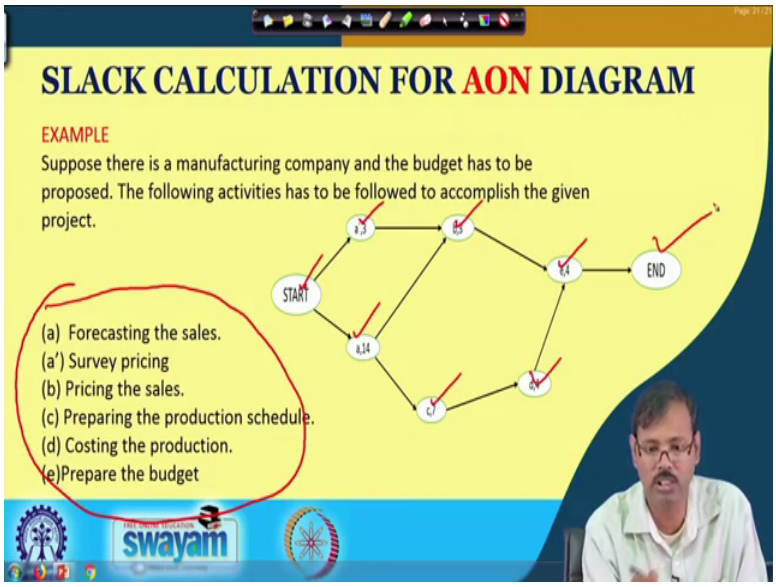

So, this is one part we have covered we have seen the calculation of total slack and free slack for the activity on arrow diagram, but so, far we have not discussed anything on activity on node diagram how you can compute that because the here the jobs are mentioned in the node.

So, let us carry out one example the example is of the same one which we have carried out that the project budgeting example and if this is already a given to you in this project budgeting we can start there is a start node and jobs are given in the nodes. So, these are the different job. So, that is why we are we are able to eliminate the dummy jobs in this case and those jobs are given in the node. And this is the representive of that network; I am not giving in the details because it is already with you. So, the network is representing this problem.

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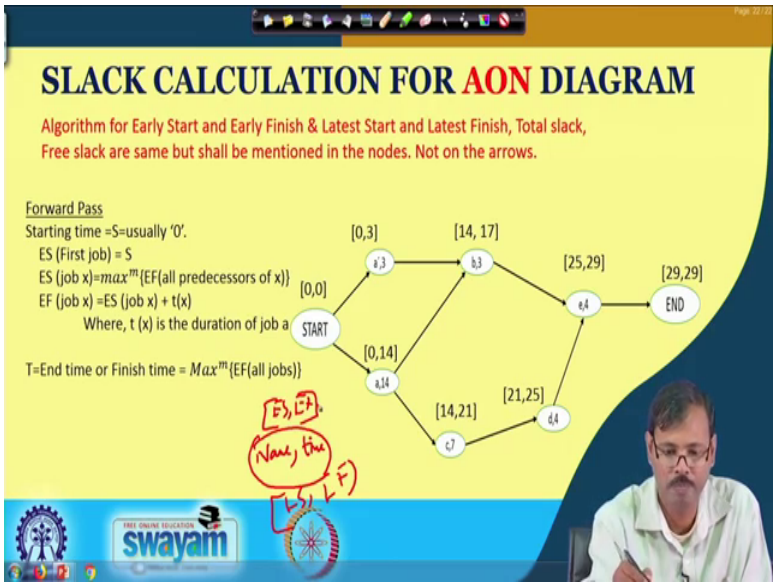

So, now let us look into the calculation of this the early start early finish calculated here are mentioned on the top of the job. So, if the node is there we give the name comma time and the early start early finish early start early finish and are available late start late finish at lower part or may be in the upper part with separated by a comma may be given in this case.

So, this is the way we are presenting here. So, let us consider we start our job at 0. So, this is the S is equal to 0 here and if it is we started 0 it can be started at 0 time will be end that 3. So, while we are carrying out forward calculation forward pass or forward calculation we carry out the same principle where the early start of a first job is equal to S it may be 0 it may be anything.

So, most of the generally we consider it is 0 then early start of a job is the maximum of early finish of all its predecessors x and early finish is the early start of job x plus time of the x. So, this is 0 so, from 0 this is duration is requirement is 3. So, that is why the early start time is 0 here and early finish time is 3 here for this job it is the early start time is 0 again early finish time is 14.

Then while we are considering this one then it is the early start time is the maximum of the early finish time of this one and this one. So, maximum of 3 and 14 is 14 so, early start time is 14 here and completion time is 14 plus 3 is 17 here. So, this is the consideration for here it is again 14 14 plus 7 is 21 here it is 21 21 plus 4 is 25 and for

this case again you have to consider the maximum of this one and this one. So, maximum of 17 and 25 is 25 so, 25 is coming here as the early starting time plus 4 is 29 is the early finish time and this is the end 29, 29 is the end.

So; that means, the T is equal to 29 S is equal to 0 with this consideration we have found the total activity total network length is 29 and some paths are of different length, but the target time is 29 hours. So, the logics are also given these are the logics and this logics are as same as with the T with the same as the activity on arrow diagram only these calculations are presented on the node not on the arrow.

So, that is why basically when we are carrying out computer analysis computer based analysis that time it is very easy to compute in activity on node diagram. The moment we place this activities as an array in the network that can easily calculated based on the values of those array corresponding values of those array as the early start early finish late start late finish calculation.

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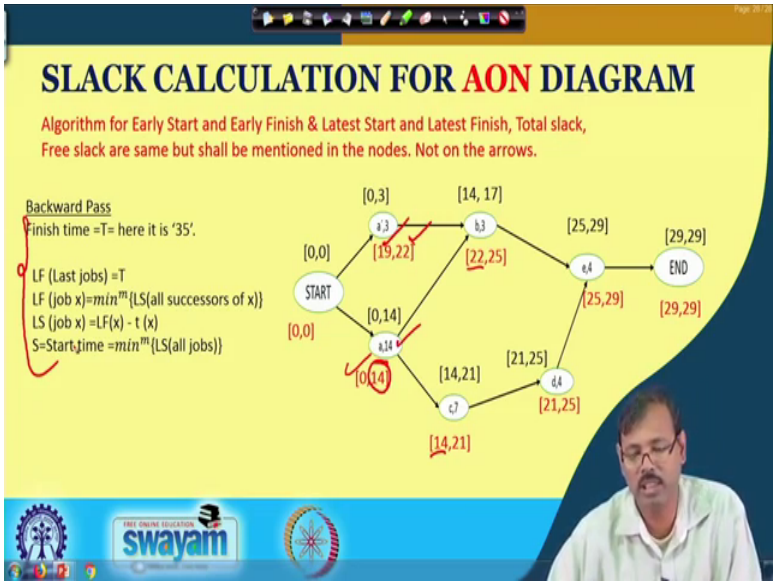

So, let us look in to the early finish also. So, in consideration of we have found the earliest finish time of the complete earliest finish time of the complete network is 29. So, T is considered as the 29 so, last jobs latest finish possible time is 29. So, you have assigned this latest finished possible time is in the last job as 29 and then we have carried out the 29 minus the time. So, late finish minus time is giving as the late start time.

So, 25 we have achieved at this position considering this is 25 we are having the algorithm like this that latest finish time; latest finish time of this job is the minimum of the latest start time the minimum of the latest start time of all its successor job is equal to the latest finish time of the particular job. So; that means, the latest finish time of this job this job is equal to the latest start time of this job while we are considering this job in that case the latest starting time we have to calculate let us calculate it subsequently.

So, that is why this is the 25 is coming at this position 25 minus 4 is 21 at this position this is the 21 21 minus 7 is 14 as the latest start time let us consider this one also the latest finish time is 25 as same as the latest start time and the latest start time for this is 25 minus 3 is 22.

So, for this job let us consider for this job the minimum the minimum of these two job latest start time of this two job has to be considered as the latest finish time. So, 22 24 so, 22 and 14 among that 14 is the minimum. So, 14 is the time which is considered as the latest possible finish time for this job and so, 14 minus 14 is coming 0 here for this case the latest possible finish time is 22. So, we are considering it is 22 and this can be latest started by 19 hour at this position without deviating the target finish time. So, this is the algorithm and who are able to understand this algorithm is as same as the activity on arrow diagram also.

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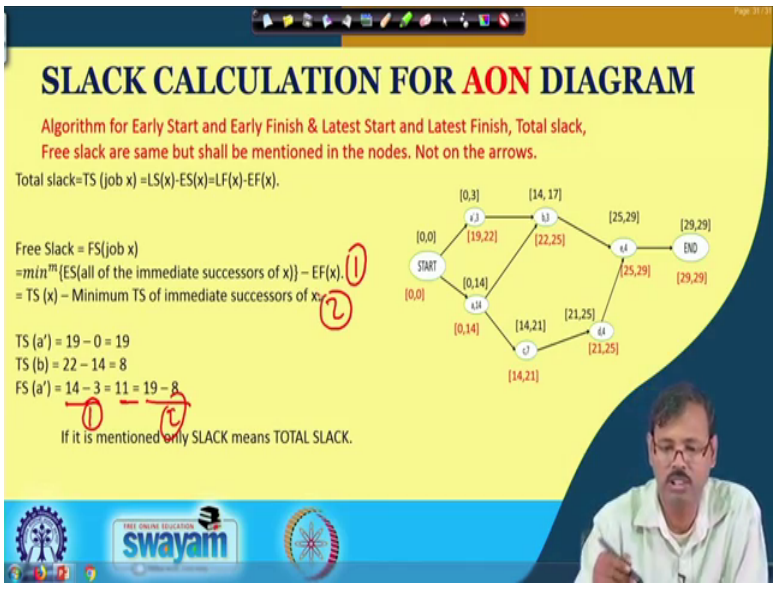

So, now let us calculate the total slack and free slack for this condition you can see the again the logics are same let start by early start time is giving as the total slack or late finish by minus early finish is giving as the total slack of the job. So, the slacks are also computed slacks in the a and b are available and this are the available we have already calculated it for the other.

And this is as this is basically the critical path this is the critical path no slacks are available in this network because all the slacks 14 minus 14, 21 minus 21 or 25 minus 25 whichever we will calculate it will come to 0.

So, that is why the total slacks are only available in this activity and this activity and this has been found 19. And 8 and as the same logic we have is used for the free slack that is the minimum of earlier starting time of the immediate successors minus early finish time of that activity gives as the free slack or the total slack of that activity minus minimum of the total slack of the immediate successors are giving as the free slack. So, 19 minus 8, 11 is the free slack as we have calculated by the formula one. So, this part is used using this one and this part is used using this formula.

So, this is the activity on node diagram which can be on which we can calculate the early start early finish late start late finish total slack and free slack. So, this is the way we can compute for the total slack and free slack for the activity on node diagram.

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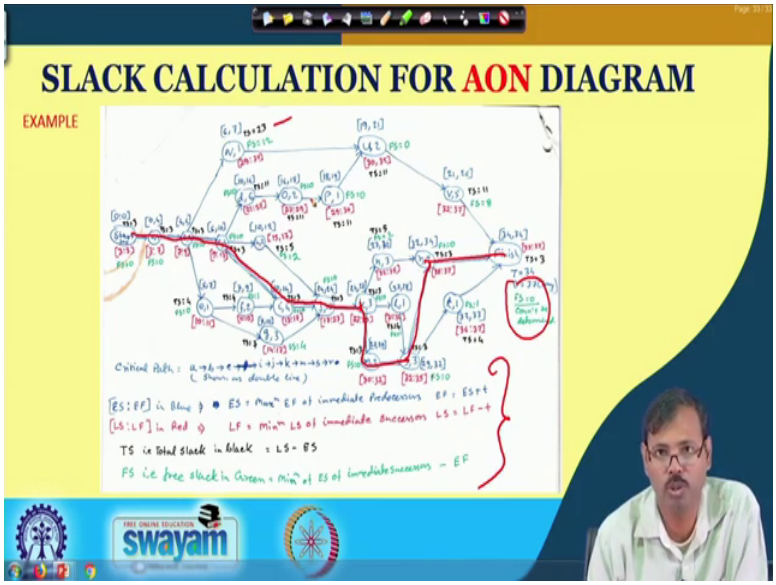

This is one example which already given to your very early classes the very big example where it is given to you for computations. So, for arrow diagram and activity on node diagram I have given with us answer in the subsequent classes also and these problems are taken from the levies book this is a construction problem.

So, the if you compute the slacks for this one I am having a another one another better figure of this. So, if you see if you see this one the total slack has given in black pen, free slack is given in green ink and these are the job durations this is the early start early finish red one is representing at the late start late finish. So, basically we have achieved from the forward pass calculation the 34 is the target days 34 day is the target day to finish the complete activities, but say though it is the earliest possible time to complete this activities 3 days relaxations are given to all. So, that is why the target date is considered as the 37 as it is given.

So, as it is given we have calculated and we have found we can start 3 late. So, that is why the late starts are considering the target date of 37 in this case. So, basically 3 hour slack is available for all the cases; however, free slacks are available only with the limited cases like in these cases.

So, all the total slacks are having a 3 days available to them, but the only this three total slack 3 are basically representing the critical path. So, total slack 3 is basically representing the critical path. So, this is representing the critical path like this way and all are other having the slacks hour.

So, in this case as 3 slack hour is already given in the target date so, 3 slack hour is already available for the critical path jobs also. So, that is a project planning carried out for this big activity in this case and it can be considered and free slacks are only available with few of the jobs where you can have free slack and you can withdraw your manpower from those activities for temporary basics.

So, this is the basically requirement of calculation of this total slack and free slack and this is a very complex example given this type of complex examples has to be solved in your as your home task so, that you can be very very competent for this activities.

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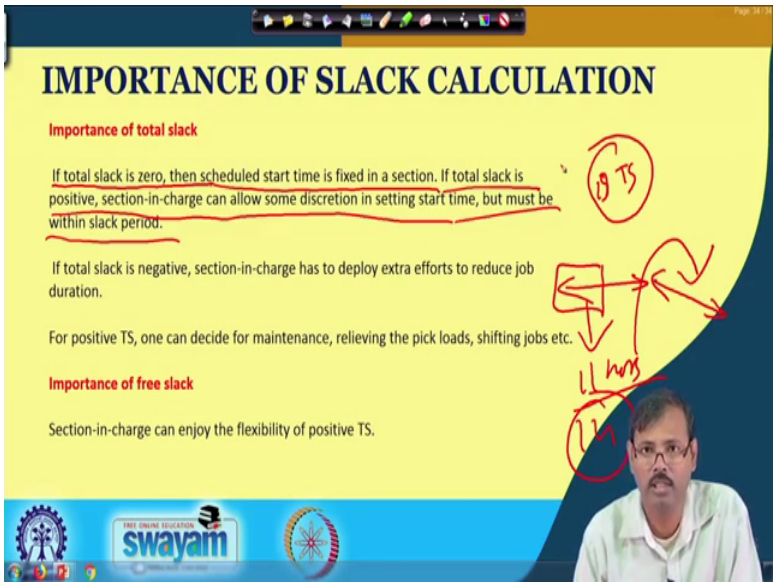

So, let us understand the importance of this total slack if total slack if total slack is 0 then scheduled start time is fixed in a section if total slack is positive section in charge can allow some discretion in setting start time, but must be within the slack period. That means, in your first example where we you had 19 hours slack, but you must complete your first job within the 22 hours; in that case you had the possibilities you can start maximum by 19 hour for that is job.

But suppose this section in charge and this section in charge is different then this section in charge is having the flexibility of only 11 hours which is the free slack available to that section in charge. If it is exceeding among the 19 total slack hours he is exceeding this free slack hour to 19 total slack hours; that means, he is unable to start it within 11 hour say he has started with 14 hour then he must inform the to the subsequent section in charge. So, that he must be ready to address that problem, but in no case this section in charge can exceed this 19 slack hours because that will then create the problem of completion of the project on time.

So, that is why this is very very important section in charge has to decide he has the liberty to play with the free slack, but he needs permission if he is exceeding the free slack and enjoying the total slack then that is must be subsequently inform to the subsequent in charge of the successive job. If the total slack is negative section in charge

has to deploy extra efforts total slack can be negative; if it is found that your target date in the previous example we have found that target date was 37 days not the 34 days.

But suppose it has been told that your target date is 31 days, then it is already negative 3 days slack in everywhere, it is not slack it is basically quizzing. So, you have to deploy you have to deploy additional man power on those cases if it is positive one can decide the section in charge can decide for maintenance, relieving the pick loads, shifting jobs etcetera.

And free slack is very very important because that is the liberty of the section in charges to utilize that hours as per his own requirement or own understanding. So, this is more or less all about the slacks calculations and slacks understanding I hope you are able to understand the importance of the slack. So, importance of the starting date, importance of the completion date, target date, slack hours, early start, early finish, late start, late finish, calculation must be understood by all of you then only you can enjoy the problems you can enjoy the solving of this a slack; slack calculation etcetera.

Thank you.