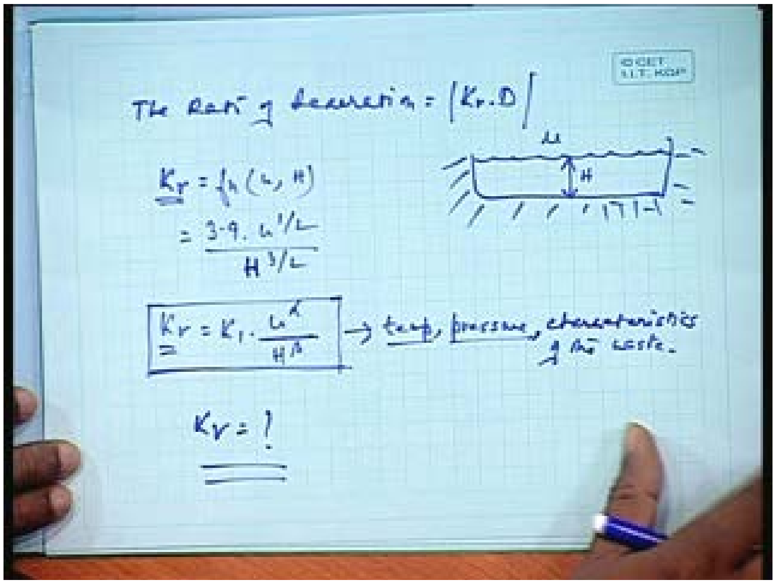
Fundamentals of Environmental Pollution and Control
Prof. Jayanta Bhattacharya
Department of Mining Engineering
Indian Institute of Technology, Kharagpur
Lecture No. # 11
Oxygen Demanding Waste in Streams
Part-II

Okay, so will carry on with this, the study that we have been undergoing at present. So, if you see this, the earlier slide that I have shown you is that temperature, pressure and the characteristic, characteristics of the waste.

(Refer Slide Time: 00:00:51 min)



The Rate of Reaeration = $|K_r . D|$

$K_r = fn(u, H)$

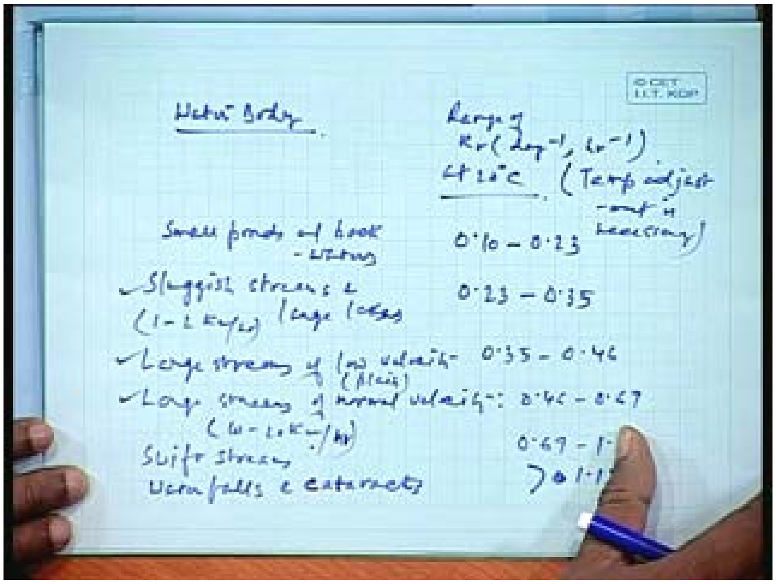
$= \frac{3.9 . u^{1/2}}{H^{3/2}}$

$\boxed{K_r = K_1 . \frac{u^{\alpha}}{H^{\beta}}}$ → temp, pressure, characteristics of the waste.

$K_r = !$

So, this one is K_r essentially also depends on temperature, pressure, characteristics of the waste and various other things also in sometimes, so it cannot be so simply examined like that but nonetheless this one is this particular function that is this particular function given is generally suitable for almost all various of kind situations and for all kind of problems that we will see in our exams or anywhere would mostly use this equation if unless otherwise told, okay. In case if it is some other things are given in such cases we have to find out that way only okay but before we go into this, you know before we go into this, this there are few things that I would like you to see what is the value of, how this, what is, what are the likely values of K_r, what are the likely values of K_r, how K_r essentially is the value of K_r one can experimentally what will be the value of K_r. Say this type of water body, water body, range of K_r remember the same way that we have seen in the case of K or in the case of you know in the Kd.

(Refer Slide Time: 00:02:13 min)



© CET
I.I.T. KGP

Water Body	Range of K_r (day^{-1}, hr^{-1}) at 20°C (Temp adjustment is necessary)
Small ponds and back-waters	0·10 – 0·23
✓ Sluggish streams & large lakes (1-2 km/hr)	0·23 – 0·35
✓ Large streams of low velocity (plains)	0·35 – 0·46
✓ Large streams of normal velocity (10-20 km/hr)	0·46 – 0·69
Swift streams	0·69 – 1·
Waterfalls & cataracts	> 1·1

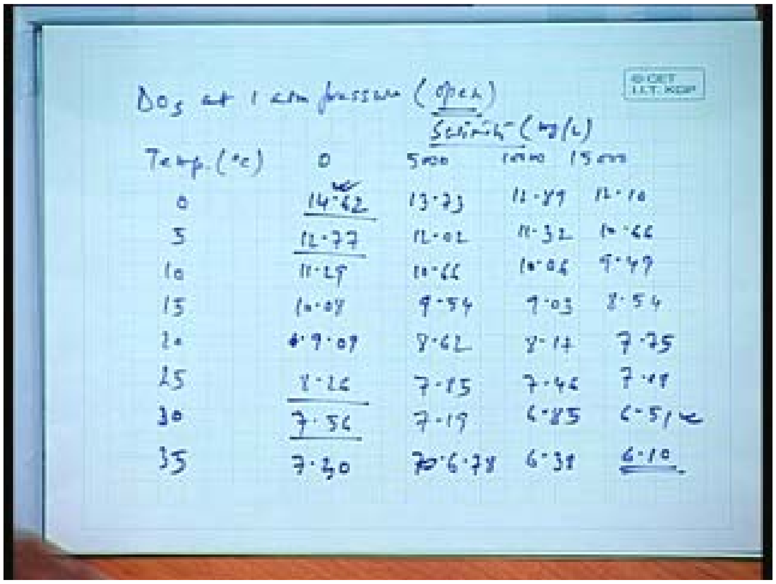
This is essentially a time inverse function, this is day inverse or hour inverse or you know some times say any other parameters like this, hour inverse like this you know this is the range that we generally observe for that and at 20 degree centigrade. So the adjustment, the typical adjustment that we have done in the case of fixed water body that also has to be done here, if it is given for a different temperature, right. You remember that function that I have given that you know K, K, K theta is equal to K_{20} into you know theta into another function t minus 20 that is you know temperature 35 minus 20, 15 so that value has to be given. Theta is 1.047 that we explained, so that function has to be implied. So at any temperature, temperature, at temperature adjustments is necessary, so remember this temperature adjustment has to be is necessary.

So, here we can see the small ponds, small ponds, small ponds and back waters, back waters. The back waters essentially where the sea enters the land and there is various little flow in the, in the water body, there is little flow in the water body and the water does not generally remains still say in a small pond also there are some, some you know some waves that you see generally in the pond. So this kind of, under this kind of situation when the water flow is very, very small water flow we can see that the value is of is 0.23, this is the value for sluggish streams, sluggish streams and large lakes sluggish streams you know having a very little speed, you know very little speed say about say 1 kilo meter or 2 kilo meter per hour kind of say you know 1 to, 1 to 2 kilo meter per hour velocity streams and large lakes, large lakes. So you can see the value is about 0.23 to 0.35 large streams of low velocity, large streams of low velocity is 0.35, 0.46 large streams of low velocity, large streams of normal velocity, normal velocity of water in most cases should be about say you say in 10 to 20 kilo meter per hour large stream of normal velocity.

Here we can see that 0.46 and 0.69, we can see this swift streams particularly that generally comes out of the mountains, generally comes out of the mountains across you know is that first one is this large streams here we have discussed these are only mostly in the plane. So this is mostly in the plane, you can see the plane they are in the plane area wherever is mountainous region the swift stream that we generally see 0.69 to 1.15 and rapids you know waterfalls,

waterfalls and cataracts, cataracts where generally the water comes out drops from a particular height. See you know if those of you who have seen in mostly in India most of the cases we do not see waterfalls a lot, we see mostly the cataracts. So this cataracts are zero, this is mostly 1.15. This is these are the values you know that would be generally useful for you in the K_r unless otherwise say it you know depending on the type of the problems given you have to understand, you have to find out the value of say the mostly the large streams or swifts streams you know generally we of our cases we would be discussing mostly these two kind of streams of velocity and also sluggish streams here sluggish streams as well. So we can see these three you know mostly you what you have to understand and remember okay. This is about the water body you know having the K_r, range of Kr that is you know this a reaeration rate coefficient K_r that we generally observe like this here, okay.

(Refer Slide Time: 00:08:22 min)



DOs at 1 atm pressure (open)

Temp. (°C)	Salinity (mg/L)			
	0	5000	10000	15000
0	14.62	13.73	12.89	12.10
5	12.77	12.02	11.32	10.66
10	11.29	10.66	10.06	9.49
15	10.08	9.54	9.03	8.54
20	9.09	8.62	8.17	7.75
25	8.26	7.85	7.46	7.08
30	7.56	7.19	6.85	6.51
35	7.30	6.78	6.38	6.10

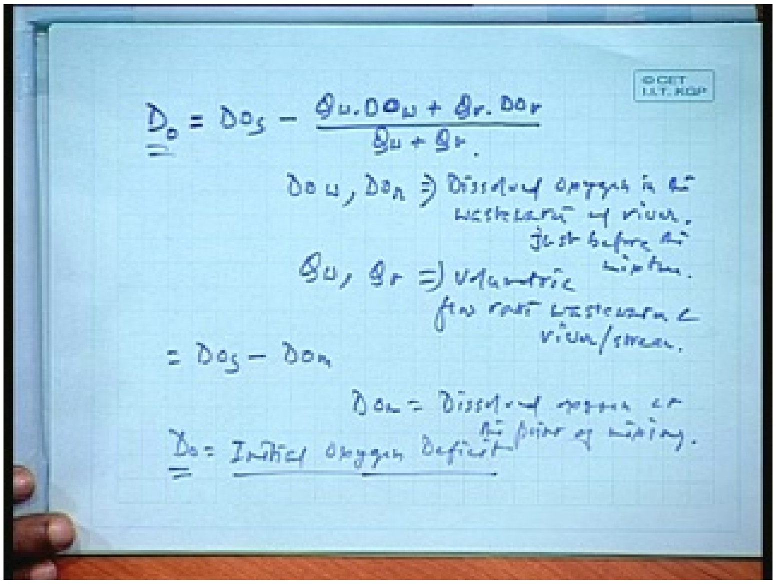
Another important thing is say Do saturated, Do saturated, the value of Do saturated is also of great importance for us is a Do of at, Do as at 1 atmospheric pressure. One atmospheric pressure means what it is open, all right nothing else it means you know it is Dos at open, it's an open stream. So here it is temperature, if you see the temperature at degree centigrade this is salinity, the salinity is a 0 to 5000, this is 10,000 and this is so this salinity is in milligrams per liter. If you just observe these milligrams per liter understandably at 0 degree centigrade and 0 fluoride concentration, we find the maximum value of oxygen, dissolved oxygen in water, the saturated dissolved value of oxygen, this is the maximum that you can see. And here as if you can further go on, you see this a 13.73, 12.89 and this is 12.10, this is at 5 degree centigrade, this is 12.77, 12.02, 11.32 and 10.66, 15 is 10.08, 9.54, 9.54, 9.03, 8.54, 20, 25, 30.

See this is 20, 25, 30 is ten point say the 20 is 9.09, 8.26, 7.56 right. We can see this, this one is 8.62, 7.85, 7.19, 7.19, 8.17, 7.46, 86.85, 6.85, 7.75 is this value 7.08, 6.15. One thing you can very well see with temperature with these, with these temperature we can see the rate of change also differs. See here it is about say 1.5 degree, 1.5 you know the milligram per liter change is there wherever here it is almost about 0.3.

So, as such you know the depletion increases with temperature and the rate of depletion decreases, oxygen getting depleted with higher temperature and but the rate of depletion decreases with higher temperature. Increase, yeah increasing the pressure would increase the solubility, essentially always in all cases essentially increase the solubility right. So, here as you can see this is the highest value that we get, this is absolutely the lowest value under this case. Say you know here if you can further extrapolate this say here it would be 7 point say 7.3. This in is you know 7.3 that you can see here it is about 7 point, no 6 point, 6 point something you know 6 point say 78 say then it is say 6 point, this one is 6 point, say 6.78 from 6.78 it would be about say 40 is about 3, 38 it's like that rather or this is about 6.10 something like this. So you can see this is you know this is the value that we generally obtain here.

So just to understand one thing you know this is what I have been kept saying also that is where the rate matters, the rate here says that the temperature with higher temperature the, this solubility decreases essentially but the rate of decrease of the solubility essentially the, this is you know the decreases, decreases. Rate of decrease of solubility decreases but the deficit increases isn't it? Deficit essentially increases with temperature. So these two things, this thing, this particular, this particular table would be of great importance as in many cases in various situations you would require this about coming into this, Dos you know here in this case.

(Refer Slide Time: 00:14:49 min)



$$\underline{\underline{D_0}} = Do_s - \frac{Q_w . Do_w + Q_r . Do_r}{Q_w + Q_r}.$$

Do_w, Do_r ⇒ Dissolved oxygen in the wastewater & river, just before the mixing.

Q_w, Q_r ⇒ Volumetric flow rate wastewater & river/stream.

$= Do_s - Do_m$

Do_m = Dissolved oxygen at the point of mixing.

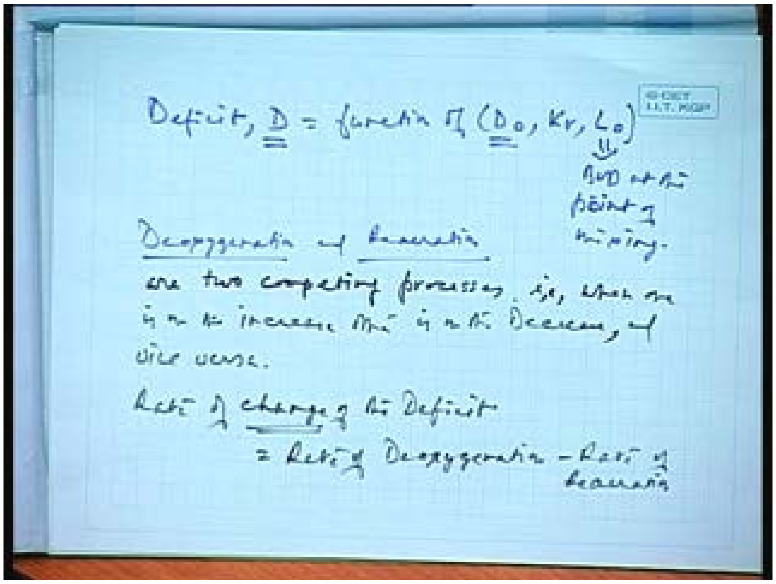
$\underline{\underline{D_0}}$ = Initial oxygen Deficit

let's say you know what is the deficit, initial deficit we generally say a term initial deficit is initial deficit, initial deficit or initial deficit D_0, D_0 is essentially Do_s minus, minus dissolved oxygen Q_w D_w, dissolved oxygen sorry, sorry Do_w plus Q_r Do_r divided by Q_w plus Q_r. See that, at that point of this is Do, Do_w, Do_r is equal to, is equal to dissolved oxygen, oxygen, dissolved oxygen in, in the waste and the river or the stream. We write it as river or stream whatever river or stream, dissolved oxygen in the waste water and river, dissolved oxygen in the waste water and the river just before the mixture, just before the mixture.

Similarly Q_w and Q_r I have said that you know volumetric flow rate say typical mass balance equation say basically typical mass balance equation mostly in all cases volumetric flow rate of waste water and the river or stream all right. Now here worth is, what is we generally or we can see is Do_s is equal to Do_m, Do_m, this is Do_m is dissolved oxygen at the point of mixing. And this D_o, D_0 is D_0 remember this D_0 don't confuse it D_o this is D_0, this is D_0 is initial, initial oxygen deficit, this initial oxygen deficit. Why this initial oxygen, why you have brought in this initial oxygen deficit, where you have brought in D_o, the initial oxygen deficit is to be understood now.

The reason is, the reason is that initial oxygen deficit you know the deficit, the term deficit throughout through the time and distance in the stream from the point of mixture would essentially depend on the initial oxygen deficit, initial oxygen deficit and the BOD, this BOD of the, BOD of the mix waste water and BOD of the waste water and the water, the river water itself combined and also depend on the rate of, rate of change I mean let me tell you this.

(Refer Slide Time: 00:18:53 min)



Deficit, $\underline{\underline{D}}$ = function of ($\underline{\underline{D_0}}$, K_r, L_0)

$\Downarrow$ BOD at the point of mixing.

Deoxygenation and Reaeration are two competing processes, i.e., when one is on the increase, the other is on the Decrease, and vice versa.

Rate of change of the Deficit = Rate of Deoxygenation − Rate of Reaeration

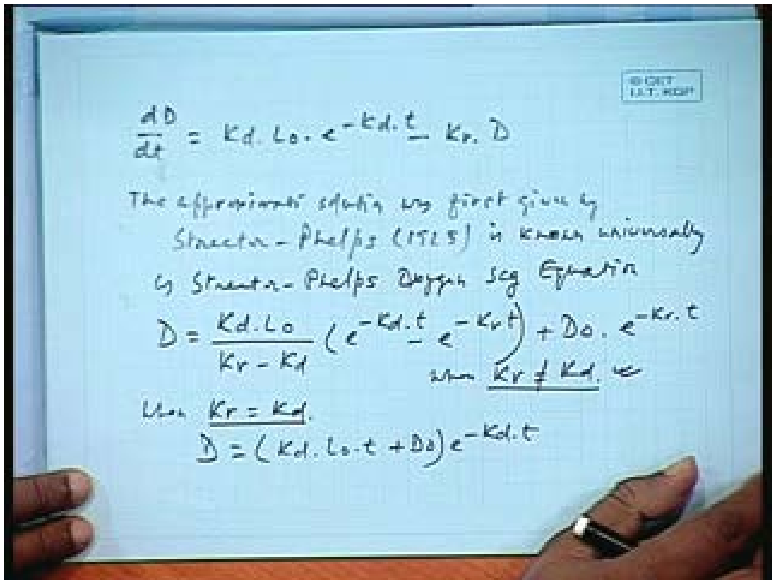
So, here you can see this that if you can see the deficit, deficit D, deficit D. This deficit D is a function of a, function of, D_o is a function of D_o, is a function of K_r, is a function of K_r and also as I have said you know that function of K_r that you already know and also that you know this the proportion that you know it has you know it is the BOD at the point of mixture, BOD at the point of mixture, BOD at the point of mixture, BOD at the point of, at the point of mixing, at the point of mixing or BOD of the mixture at the point of mixing, right.

So this is why you know these things are necessary to be, necessary to be found out. So we have says this is we can we understand now at this point of time that deoxygenation, deoxygenation and reaeration, deoxygenation and reaeration are two competing process, two competing processes, two competing processes, 2 competing processes that is when one is on the increase other is on the decrease, when one is on the decrease and the other is on the decrease and vice versa.

So, we can see now you know that you know from this we can find out we can rate of, remember in some books it is written as even in the book by masters it is written as the rate of increase of the deficit. The word should be rate of change, rate of change of the deficit, the rate of change of the deficit is not necessarily that you know it should be always increasing or it would be always decreasing. So here we should write you know rate of change of the deficit, rate of change of the deficit is, rate of, rate of deoxygenation minus rate of reaeration, rate of reaeration okay the rate of reaeration, rate of deoxygenation and rate of deaeration.

Remember again as I have said you know this is the as you can see if it is so and is rate of deoxygenation is higher than the rate of reaeration it should be the, this should be positive, this should be negative, on the other hand it would be negative. So, if you can leave at this point of time you know you can understand this as a function we should not be interested in which one is higher at this point of time, we would just try to see the difference, difference is important for us than the absolute value of H. In such cases, in such cases we can write a, the, we can write a typical function where it is the rate of change of deficit, this is dD by dt is equal to Kd into L_0 into e to the power minus Kd into t minus K_r into D okay minus K_r into D.

(Refer Slide Time: 00:23:06 min)



©CET I.I.T. KGP

$$\frac{dD}{dt} = Kd.Lo.e^{-Kd.t} - Kr.D$$

The approximate solution was first given by Streeter - Phelps (1925) is known universally as Streeter - Phelps Oxygen sag Equation

$$D = \frac{Kd.Lo}{Kr - Kd}\left(e^{-Kd.t} - e^{-Kr.t}\right) + Do.e^{-Kr.t}$$

when $\underline{Kr \neq Kd}$.

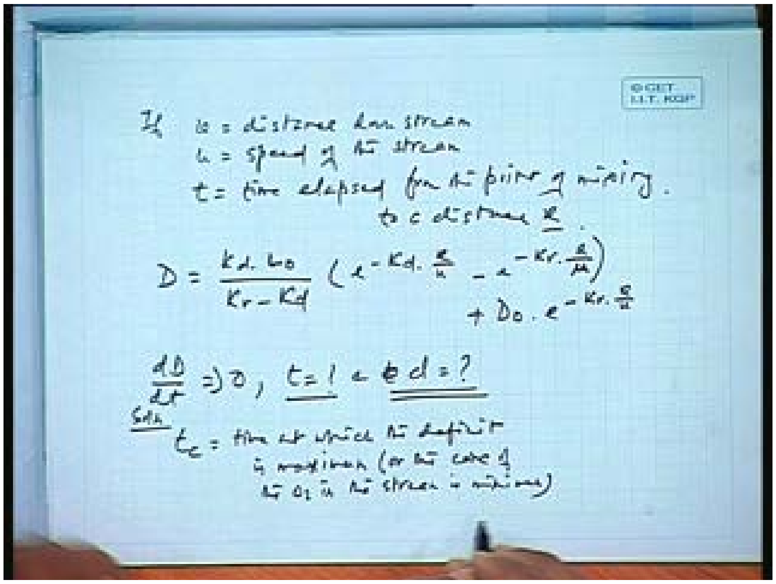
when $\underline{Kr = Kd}$.

$$D = (Kd.Lo.t + Do)e^{-Kd.t}$$

The approximate solution, the approximate, the approximate, the approximate solution was first given by Phelps, Streeter and Phelps, Streeter and Phelps in 1925, Streeter and Phelps in 1925 and is known as. Now if it is was given by Streeter and Phelps and is known as known universally as Streeter Phelps, Streeter Phelps oxygen sag equation, Streeter Phelps oxygen sag equation right. And this is the approximate solution was given and is known as this. This is what is the solution is, what Streeter Phelps an approximate solution that it gave this Kd into L_0 divided by K_r into Kd, K_r into Kd e to the power minus Kd into t minus e to the power minus K_r into t, K_r into t plus D_0 e to the power, e to the power minus K_r into t, K_r into t.

Remember this you know here this is the typical approximate solution of this differential equation and approximate solution of this differential equation and considering and considering that this D is a function of, this D is a function of D_0 initial deficit, initial deficit is a function of L_0, is a function of L and K_r, K_r obviously K_r that we have seen okay. So considering this you know this is the, this is an approximate solution that we generally get in case of an even case of K_r and Kd when condition is K_r is not equal to Kd. When K_r is equal to Kd, when K_r is equal to Kd we find another solution for this which is should be written as Kd into L_0 into t plus D_0 e to the power minus Kd into t, okay. This is what you know when K_r is equal to Kd, we find its different solution so you know you can see this in such cases like this but mostly now we would, we would see that you know these two are, these two are essentially different things. So K_r and Kd not being same would be the case when we would mostly consider them and having said this, having said this you know we can find out that you know if it is, if we just generally write x as if, if x is the distance downstream, this x is the distance downstream, u is the speed of the stream speed, speed of the stream and t is the time from the point of mixing, t is a time from the point of mixing to a distance x, to a distance x, distance x.

(Refer Slide Time: 00:27:24 min)



©CET
I.I.T. KGP

If x = distance down stream
u = speed of the stream
t = time elapsed from the point of mixing.
to a distance x.

$$D = \frac{Kd.L_0}{K_r - Kd}\left(e^{-Kd.\frac{x}{u}} - e^{-K_r.\frac{x}{\mu}}\right) + D_0.e^{-K_r.\frac{x}{u}}$$

$\frac{dD}{dt} \Rightarrow 0$, $\underline{t = ?}$ & $\underline{\underline{d = ?}}$

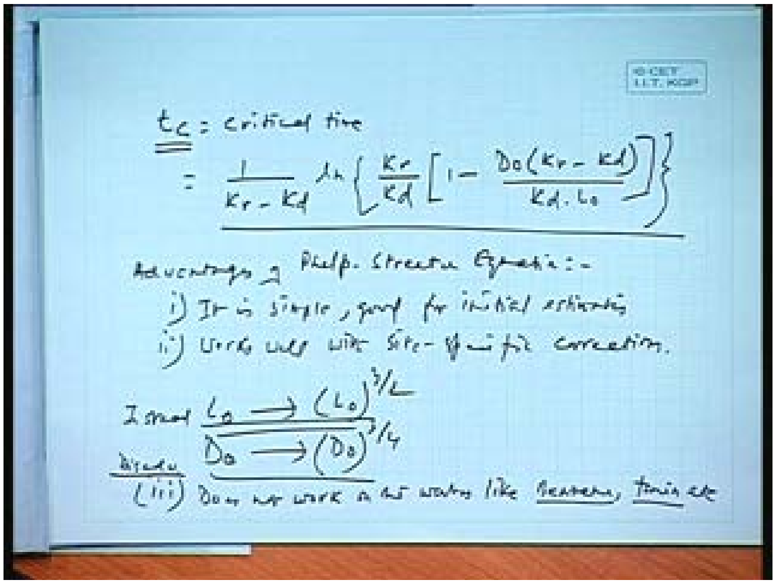
Soln

t_c = time at which the deficit is maximum (or the conc of the O_2 in the stream is minimum)

Then we can, this particularly this equation is just can be a simply modified is a simply modified as, can be simply modified as K_r into Kd. This one does not change much but here it changes a little bit Kd Dt is essentially x by u, x by u, this is x by, x by u minus e to the power minus K_r x by u sorry u, only u don't write mu I mean sorry it is only u and plus this 0, e to the power minus K_r into x by u, x by u okay. This is the function, this is the function that we generally observe you know this is what is known as Streeter-Phelps function you know converted from that we have said you know initially this is the basic function we have merely made a simple change in the function considering that you know x and u are, x, u and t are related. Considering x, u and t are related so you know we have made a simple change here. So if you just can find out now if it is for that the when this, the deficit would be maximum, when the deficit would be maximum can be found out from D is equal to and these two zero. Equating these two zero, we can find out this function we can find out the time, we can what is the time, we can find out the time and if you can find out the time we can also find out the distance, okay.

So this is, this is you can you should try it yourself, now it’s very simple you know you will generally obtain the value very easily. So in such cases you can such cases this the, a simple solution of that is basically t critical that is t critical is the time solution, is a time at which time at which the deficit is maximum, the deficit is maximum or the concentration, the concentration of the oxygen in the stream is minimum is known as is generally can be found out by, can be generally can be found out like this.

(Refer Slide Time: 00:31:52 min)



t_c : critical time

$$= \frac{1}{K_r - K_d} \ln \left\{ \frac{K_r}{K_d} \left[1 - \frac{D_o (K_r - K_d)}{K_d . L_o} \right] \right\}$$

Advantages of Phelp. Streeter Equation :-

i) It is simple, good for initial estimating

ii) Works well with site-specific correction.

I stead $L_o \longrightarrow (L_o)^{3/2}$

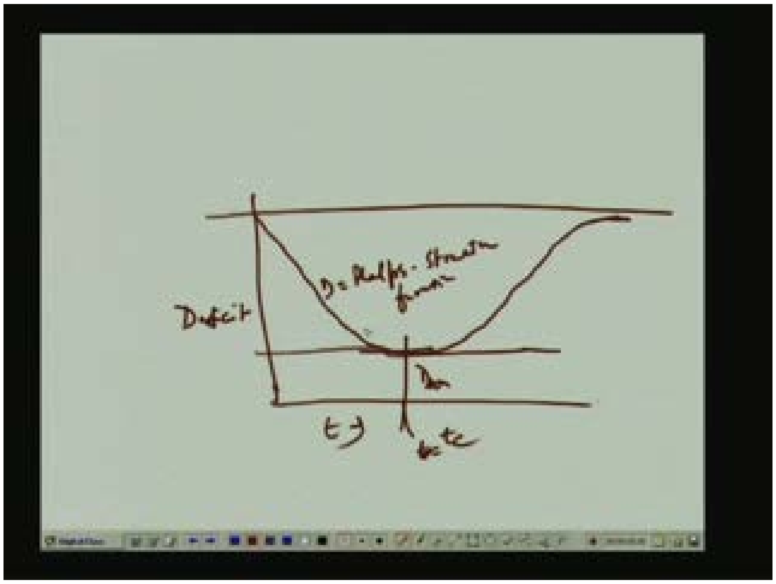
$D_o \longrightarrow (D_o)^{3/4}$

Disadv

(iii) Does not work in all waters like Estuaries, tidal etc

This t_c would be this is equal as critical time, we generally also known as this as critical time also known as critical time. This t_c is nothing but 1 minus K_r into Kd log, log K_r into K_r into Kd K_r into Kd 1 minus 1 minus 1 minus d initial oxygen deficit minus K_r minus Kd divided by Kd into L_0. So this is how you know we can find out the value of, we can find out the value of this function you know I would say you know please carry it out in your, in your halls try to see that you know this deficit, this differentiation equation you can derived. This can be easily derived I mean there is no problem, I mean just you know we have to transfer t from one place and finally we will find that in the form of exponential. So again it’s integrating that you know you will find out the value in the terms of log, so it’s this can be, this can be the solution for t, solution for, solution can be solution for t is like this. So, the same thing and as I have said if you remember as we have said in the classes in the class like you know where we have said first of all you know in cases like this it’s okay is I think, so here you know just so here as we have said you know this is particularly we can see here this should be the typical function like this.

(Refer Slide Time: 00:34:02 min)



This is where is the, this is where the function is so you know the function that the typical function that you are finding is the deficit function D is this, this is the D function d that we have known so far that is the Phelps D is the function and this is where this is where the tc is, tc, this is where is the time, this is where is the deficit, this is where the deficit value, this is where the deficit value or you can say that you know this, this is where the deficit value that one can obtain and so this is how it would generally of go into this Phelps. The first initially the combined curve that I have shown it is quite a similar to that curve, is just similar to that curve, so we can find a plot like this. This one is what is known as the maximum deficit, the maximum deficit would take place here.

So this deficit you can say this is the maximum deficit D_m where would be the maximum deficit would take place. This is nothing but the tangent here, the tangent that you get due to the differentiation itself. So here this one you would be, would be going to know about all this and this is how this Phelps Streeter equation is generally used. See one important thing about this Phelps Streeter equation is there are is a great, is a, of great use. Phelps Streeter equation is you know used throughout the world for different purposes. The reason is the advantages of Phelps Streeter equation is advantages number one it is simple, simple and good for initial estimates, initial estimates works well, works well with site specific, site specific corrections, with site specific corrections, with site specific corrections say you know some cases like site specific corrections should be say I here as we have seen say instead of L_0, if we are instead of L_0, instead of L_0 we can use a correction we can make as this, okay.

This is the correction that we can make say sometimes also in place of initial deficit, if we just see this initial deficit can also be a function, can also be used for the initial deficit like instead of this 3 by 4. So, if this you know with an, it gives a basic frame work of a function very importantly because you know in many cases you need to have, you need to start with something. Phelps Streeter equation gives that start I mean you know where, you don't know much about the, much about the characteristic of the waste, you do not know much about this you know the other criteria’s like, the other criteria’s in such cases to start with a Phelps Streeter equation is perfectly all right. And throughout the world it is accepted, it is very much a very popular form of equation still being used particularly for modeling of BOD related waste, biochemical waste or biochemical or even organic wastes, biochemical and organic waste in particularly in stream, wastes in stream.

This is particularly a quite efficient equation and is used universally people but is not suddenly not, does not work I mean remember this, this is you know this is a disadvantage, is disadvantage is you know this, this particularly doesn’t work, work on, does not work I mean it’s not applicable does not work on wastes like benzene, tannin, etc does not work for like in case of benzene, tannin, etc right. So this is where you know this Phelps Streeter equation can be used and you know this is what it does not work in many cases as I have said, it is too simplistic at times you know but you know there is no kind of only another important thing is to you know this is, this is where, from where the environmental engineering modeling started.

Actually this is where at an engineering waste water modeling started so that way this has a starting formula it is a, it has a good application but suddenly with time the characteristic of a, of the waste and all other things have changed the world itself, the pollutants different kind of pollutants have changed say you know previously whatever the pollutants in water say in 1950’s has gone, considerable change now if you see the way pollutants in the water so that their degree has changed, their composition has changed and their variability has changed a lot. So, if this equation may not be very good at this present point of time but for initial estimate it is still very much used, which has to be used with caution but generally for all kind of waste where it is biochemical and organic waste Phelps Streeter equation is still very much used but well when one is using that one has to use it caution, okay. So, with this you know we have almost we have concluded the particularly the waste water in, in a fixed body of water, waste water in a fixed, fixed system and as well as where we have considered as waste water stream. So we’ll move from this surface water is closed I mean surface water is complete will move to ground water from the next class. In the next class will start with ground water modellling, okay.

Spelling Error	Number of Occurrences

Summary	Total
Number of Unique Misspellings	0
Total Number of Spelling Errors	0