Mathematics III
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Lecture - 25
Numerical Method for Laplace Poisson Equation

Welcome to the lecture series on differential equations. Today's topic is Numerical Method for Laplace and Poisson Equations. We had seen in partial differential equations that we are not getting many analytical methods, in which we can get the solution of differential equations in close form. Rather in using this Laplace equation, we had find it out that we are getting the solutions in the form of the Fourier series or in the form of infinite series, where the coefficients we are obtaining as the Fourier coefficients of the functions, which have been defined as the boundary conditions or the initial conditions and form this infinite series we are trying to give the solutions. So, finding out the close form solutions is not easy in many times, what it says is we are approximating the solutions. Approximating the solutions says is that, if we are given at certain points can be really find out the solution at that those points or that says is move to the numerical methods, so first what is the numerical method?

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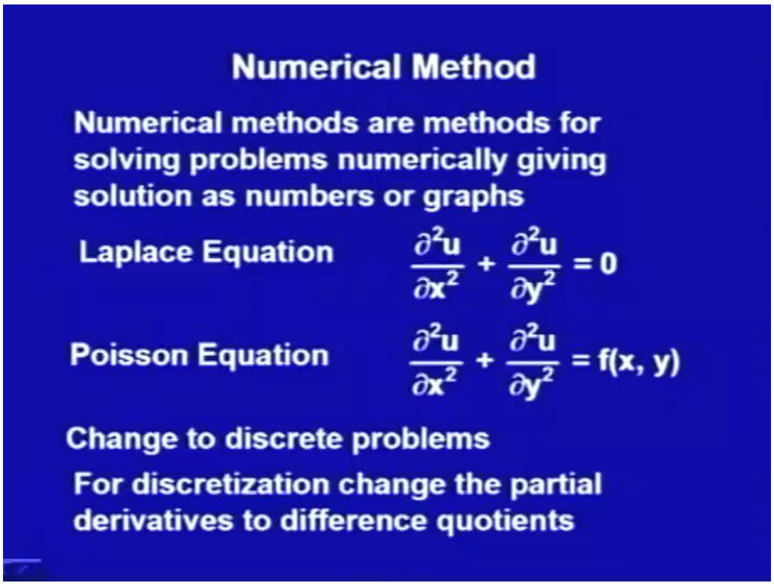

Numerical methods are methods for solving the problems numerically giving solution as the numbers or graphs, that is rather than getting those functions, we would get the solution as at particular points this is the solution at this point this is the solution or we are saying is that graph that is at those points, whatever the solutions we are getting we do join them with the showing the graphs. So this is what we are calling the numerical methods.

Now, today we will discuss two equations Laplace equation, which the two dimensional one pass will take, because in the last lectures we had seen that Laplace equation, we are getting the solutions in the form of infinite series, whether it may be two dimensional or three dimensional. So, let us first discuss with the two-dimensional del 2 u over del x 2 plus del 2 u over del y 2 is equal to 0.

Or the Poisson equation that is non-homogeneous Laplace equation del 2 u over del x 2 plus del 2 u over del y 2 is equal to f x y. Numerical method says is solving the problem numerically, what we mean by solving the problem numerically, if you see is that is here, what we are saying is the function unknown function for which we have to solve this equation that has to be a continuous function not only the continuous it must possess the second order derivatives, so that it satisfies this equation.

But, when we are solving it numerically, what we are saying is that we are knowing the function or knowing the conditions and we would actually give the solution at certain points only. So, what it says is that these things we have to actually discretized that is discontinuous problem we have to discretize, so that we can suitably get certain equations, which we can solve numerically. So, that says is change these continuous problem that is this models, which we have got from the physical problems change these models to the discrete problems.

So, this change of the models with the discrete problems to make it a discrete problem either we could says, we start our modelling with the discrete method itself or firstly modelled as here, as the continuous mathematical models from the physical conditions and then discretize it. So, we all using it here is that is first it has been modelled as the mathematical model as a differential equation and then we are going to discretize, discretize here, means is that is we are talking about the partial differential equations. So, in this case discretisation says is that we would like to write this derivatives in the forms of difference coefficient the partial derivatives we have to change to the difference quotients, what are the difference quotients and what is this let us see.

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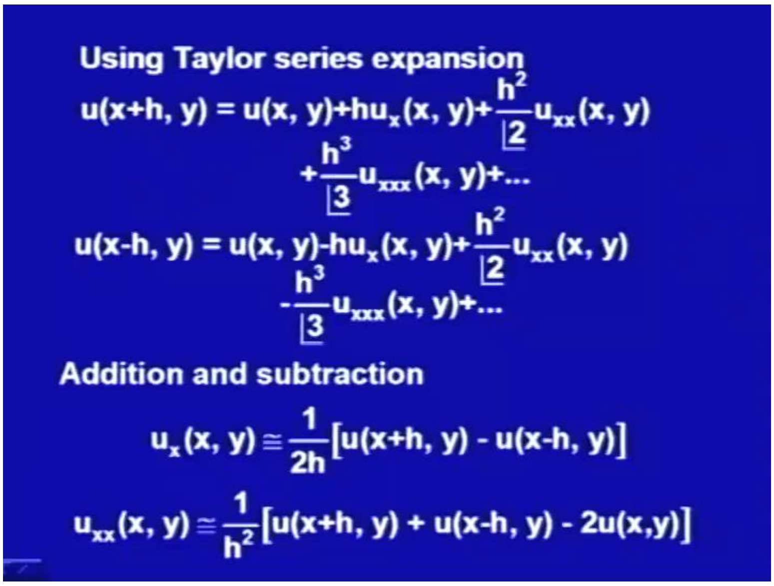

We do know that is using the Taylor series expansion for the two function of two variable that we can write u x plus h comma y as u x y plus h times u x x comma y that is the partial derivative of u with respect to x at the point x y plus h square upon factorial 2 u x x x comma y and plus h cube upon factorial 3 the thought derivative partial derivative with respect to x or of u at x y and so on.

This is the Taylor series expansion of the function u x y will the, we have taken the extension as x plus h at the point x y. Similarly, if I take u x minus h y we do get the expansion is u x comma y minus h times u x x comma y plus h squared upon factorial 2 u x x x comma minus h cube upon factorial 3 third partial derivative of u with respect x at x comma y and so on.

Now, we have to find out the different coefficient as I said that is we would actually try to find out the partial derivatives with respect to x of this unknown function u in the form of the function at the point x plus h comma y and x minus h comma y that is at certain points that is what we are seeing.

So, let us see if I subtract from the first equation that is if I write u x plus h comma y minus u x minus h comma y, what you would get u x y u x y this term will get it cancel it out. This term will get 2 times h times u x x comma y, then this again the third term would cancel it out, then we would get plus 2 h cube upon factorial 3 times the third partial derivative with respect to x into y and so on.

Now, so from here what we say that is if I take in the subtraction the h term I could take and higher powers of h I neglect, then I can approximate this u x y as 1 upon 2 h u x plus h comma y minus u x minus h comma y, what we have done is we have subtracted these two and what we are getting is the series 2 h times u x comma y plus 2 h cube upon factorial 3 times the third derivative partial derivative with respect to x of at x comma y and so on.

So, if I take h is such a small that hard powers of h that is h cube and so on, they are very small as comparison to this that they would make this stand to be 0 are negligible, we could approximate it like this one. So, we have taken this 2 h right hand side and if you to see how we have to finding our derivatives, if we just go to the analysis we define the derivative of if with respect to excess f of x plus h minus f x divided by h limit as such approach is to 0 that is, what we are doing over here.

So, what were are approximating it only with the first one rather than saying is as its approach is to 0 we just strike it as approximately with this one. Similarly, if I add these 2 I would be getting 2 times x into y this term would get it cancelled out then, we would be getting is 2 times h square upon factorial 2. The second derivative of partial derivative of u with respect x of x comma y, because two times h square upon factorial 2 I would get two only h square and so on u would be getting now.

Again, if I neglect the hard powers of x that is h square by two retain, but h to the power 4 and so on, we say is that they are very small or negligible. I would get the addition of u x plus h plus u x minus h y would be 2 x y and then plus h square times u x x x comma y of from here, who will approximate the second derivative. So, u x x x comma y we could say approximate as 1 upon h square u x plus h comma y plus u x minus h comma y minus 2 u x comma y. So, this says is that is again if you see that is what we could write it as u x plus h comma y minus u x y minus u x minus u x y minus u x minus h comma y. So, we are just having this second derivative with respect to x of u this we are approximating these we would call difference portions.

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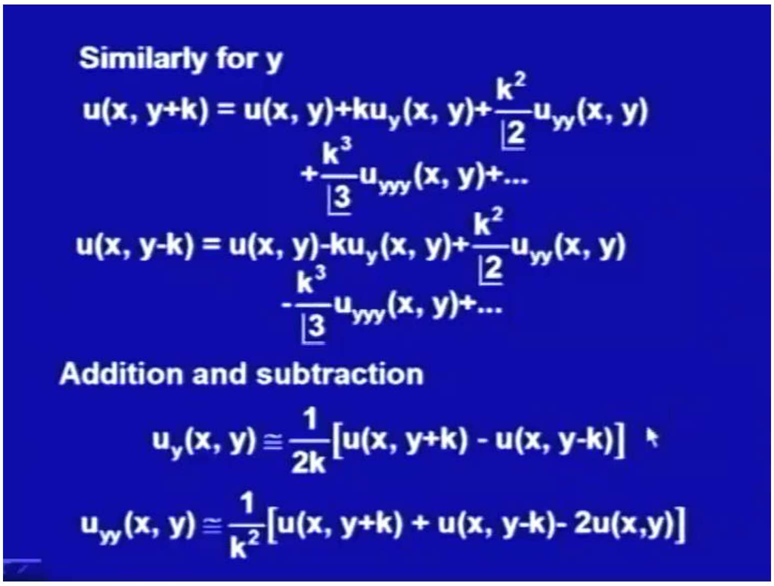

Similarly, for y again I what will use we will use the Taylor series for y that is for other than having u x plus h, we would have u u of x comma y plus k, now we would be writing the Taylor series with respect to y. So, the derivatives of y u x comma y plus k times u y at x comma y plus k square by factorial 2 second derivative with respect to y at x comma y plus k cube by 3 factorial three times the third partial derivative with respect to y at x comma y and so on.

Similarly, if I take u x y minus k I would get u x y minus k times u y at x y plus k square upon factorial times u y y at x y minus k cube upon factorial 3 times the third partial derivative with respect y at x y and so on. Now, again use the similar method that is first I subtract from u x y plus k t to u x y plus k 2 u x y minus k that would give me the approximation for the first partial derivative with respect to arrive and that would be 1 upon 2 k u x comma y plus k minus u x comma y minus k and if I add them on neglect term hard powers of k that is k to the power 4 and so on.

We would get the approximation for the second partial derivative of u with respect to y u y y x at x comma y has been approximated as 1 upon k square u x comma y plus k plus u x comma y minus k minus 2 u x comma y. Now, we have got our u x x and u y y, now first we will see the numerical method for the Laplace equation.

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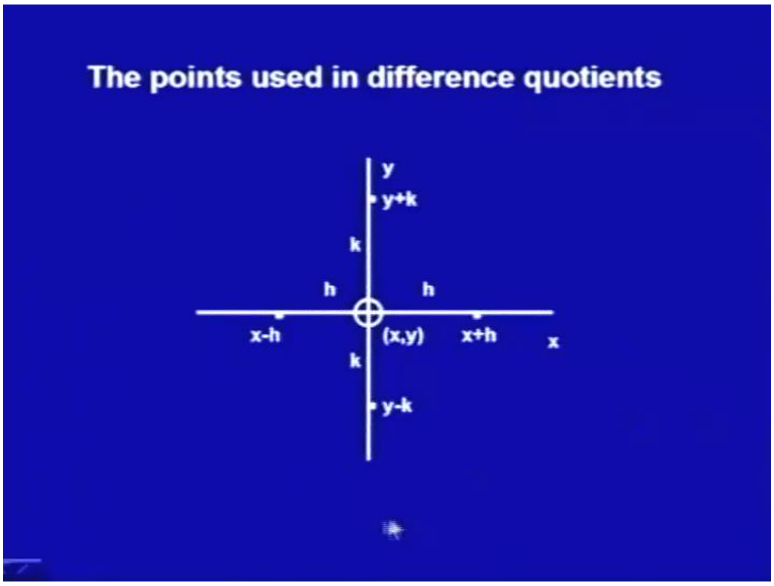

The points, which we are using in this difference equation we are having the point u x that point x y the point x plus h the point x minus h the point y plus k and the point y minus k in always difference questions. If I was using only the first difference question that is u x we are using only two point either this two points or these two points, when we are using the second derivative we are using all the one, two, three; these points and these points say if I use all the points in our Laplace equation we would be getting it all the five points say, let us see these are the points, which are we are using in different quotients difference quotients.

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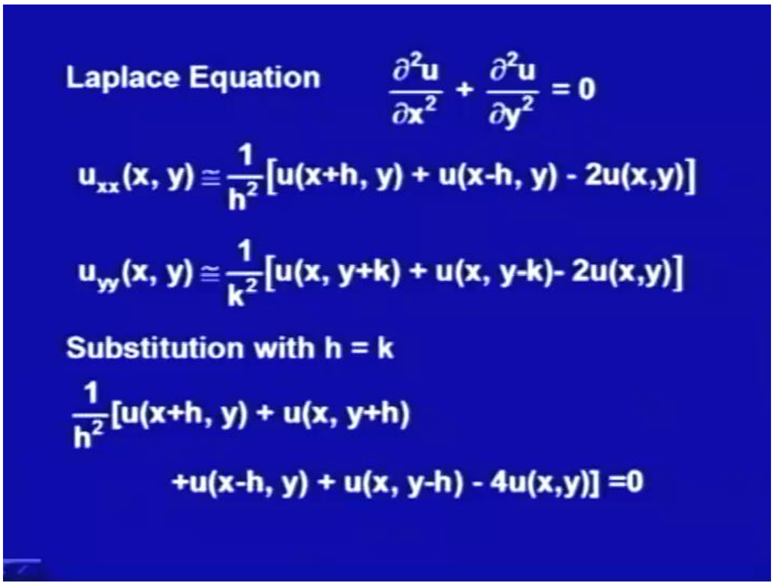

So, Laplace equation is del 2 u over del x 2 plus del 2 u over del y 2, so that is it is u x x and this is u y y. Now, change it to the difference quotients u x x 1 upon h square u x plus h comma y plus u x minus h comma y minus 2 u x comma y, u y y is 1 upon k square u x comma y plus k plus u x comma y minus k minus 2 u x comma y. Now, what you would be doing is at the first stage we have taken is that is for the x axis we have taken the difference as h and for y we have taken the difference as k.

Now, if I assume that is take the difference both the difference same that is h and k as same constant. Then, this equation we could change as 1 upon h square 2 u x plus h comma y plus u x comma y plus h plus u x minus h comma y plus u x comma y minus k minus 4 u times x comma y this is equal to 0. Now, you see I have not gone that is first writing this one and then writing this much, so we could say that is we could write this plus this plus this plus this and then minus 4 u x y.

We had used in this manner, why you see is that is we have used in this manner you find out that this is easy to move around first we are increasing this on the x axis, then we are moving to the y axis. Then we are moving to the x axis and then we that is we are following this anti-clockwise motion in this writing this different quotient and writing this Laplace equation.

Now, since it is equal to 0 this 1 upon h square term till go out, because h is not 0 say if I multiply it over here, what we would be getting is u x plus h comma y plus u x comma y plus h plus u x minus h comma y plus u x comma y minus h minus 4 times u x y is equal to 0.

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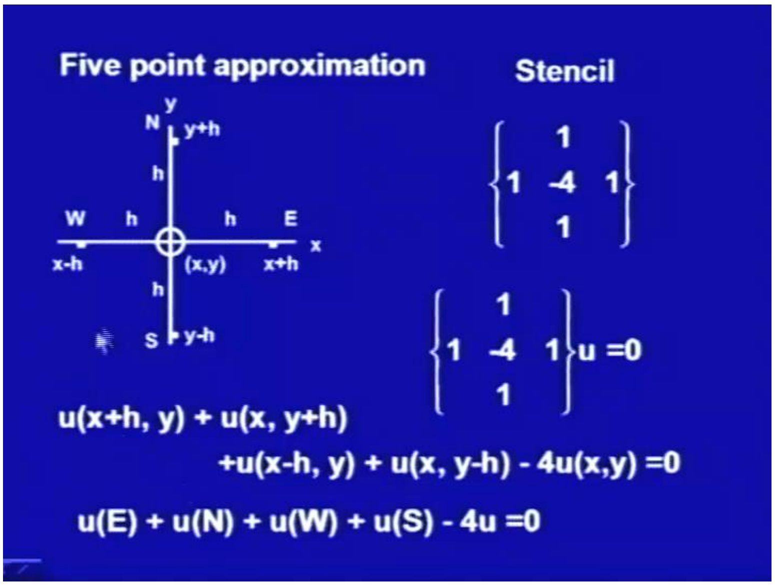

So, we are getting is space five points five point formulae for Laplace equation, what is five point approximation, again we just move as I said that is easy to find out that is right in that manner, we are starting from here we could say is if I just go in our usual motion of the maps. We are we are saying this side is east north west and south, so we say east North West south in the same direction we could find out the formula.

This five point approximation we are using for our Laplace equation, if it is Poisson equation, then the right hand side is not 0 rather it would be some h square f x y if the for both of these things we would be using this our five point approximation. This five point approximation from here, if you to see we are saying is this is one this is one this is one this is one and this is minus 4 times point at function at x y.

So, the pattern or this is also being called the stencil we could just get it from here, you see is 1 1 1 1 and minus 4 this is called a stencil or you could say pattern or the mesh points. So we are changing, now our equation at this u x plus h comma y plus u x comma y plus h plus u x minus h comma y plus u x comma y minus h minus 4 u x comma y is equal to 0. We could write it as one this matrix you say that is wherever the points have been left they are 0 0 1 0 1 minus 4 1 0 1 0 times u is equal to 0 this is at a point x y.

So, now if I had been given different point x y that is if I give this problem for many values of x and y these kinds of patterns for each x y we would get. Now, if I join all those x y we would get some or we could say algebraic equation, which would give me that is at the different points, so if it here it is at u x y. So, similarly at 1 x y then another x y, then another x y we do get our equations.

And since, this is giving as the relationship between the point x y and its right hand point left hand point upper point lower point, so we do say is that if I take the this point and the next point is this one. Then this point is also related with this point this point is related with this point and so on we do get a relationship. So, let us see is how we are going to discretize this Laplace problem over here, so as I said is that east North West south that is, what we are getting is simplified form.

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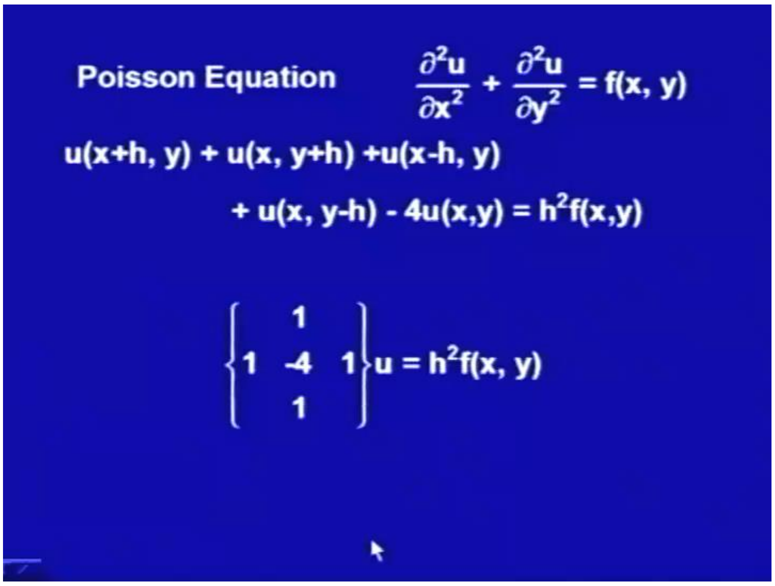

Similarly, the Poisson equation as I said is do have only non-homogeneous that is the right hand side is a function of x at x and y. So, now, whatever the point x and y we are talking about we take the evaluate the value of the function at those points and then what we do get is this Poisson equation would simply move as u x plus h comma y plus u x comma y plus h plus u x minus h comma y plus u x comma y minus h minus 4 u x y is equal to h square f x y. So, if I write in those notation of this stencil I do get it as 0 1 0 1 minus 4 1 1 u is equal to h square f x y, so from here now see that is, what we have done.

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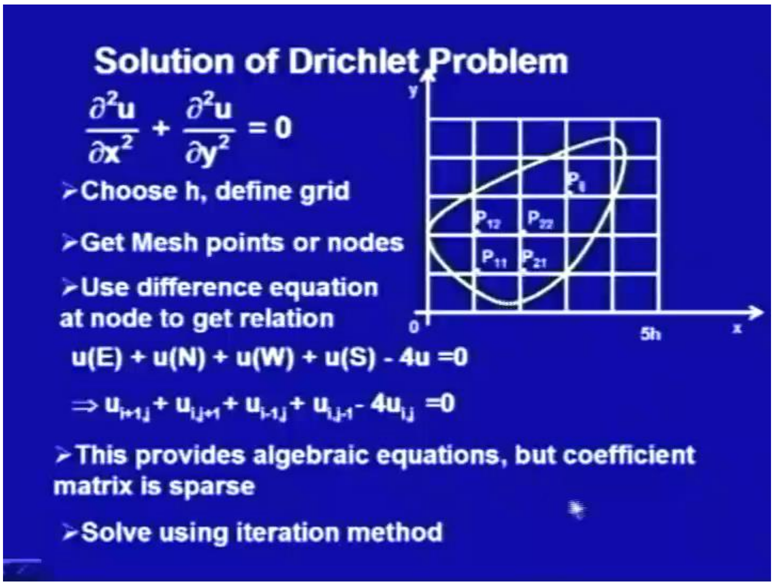

Let us move that is how we are going to really solve all Laplace equation, so we do know that practically, if we are solving this Laplace equation or Poisson equation either we are solving or problem that is using certain boundary condition. So, first we would be doing it with that two-dimensional one, so if we are moving to this boundary condition. So, whatever be these boundaries been given that is whatever reason on, which this Laplace equation has been defined. So, that the drichlet problem has been defined we first choose h and define a grid see suppose this is the plane on, which the boundary condition is being given drichlet problem. Drichlet problem says is that my boundary conditions defined on these boundary that is the card, so what we do is we choose first x here for simplicity and using a small reason.

So, let us say this if I chosen very large ones, so that I could cover it up from 0 0 we are starting, let us say this 5 h we have use, so this point you to call p 1 1, so what it is actually the point you see x plus h y plus h this point is x plus h y plus 2 h this point is x plus 2 h, which y plus h and so on. So, we write it as 1 1 1 2 and so on and now, what we do take is since h is the common we have taken. So, we were not writing it as one what we do call this point as e 0 0 this point as p 1 1.

So, what we are seeing is in this grid first we have defined this grid divided that the whole region in to the smaller regions that we are calling grid like this one small square is of the size h cross h, then these cross points of this grid these points are been called mesh points or the nodes.

And now, what we do is we take this equation defined is the equation at all these mesh points, that could actually result near as I said is if I am using this at this point. And this point then this point would use this point while as the relationship the differential equation with the different quotient would be using at this point would be using this point, so actually you would be getting is algebraic equation.

So, if I use this difference equation at nodes to get the relation and so this is how we are using it now. So, at the point pi say suppose if I am using p i j that is i h from the on the x that is u x plus i h y plus j k. So, there are the point would be u i j, so here if I use then east point; that means, from here the right point, right point means i plus 1 and j is as such, then this point i is as such, but j is moved to j plus 1 that is i times j plus 1, then this point i is as such j is as such, but i is i minus 1. So, u i minus 1 j and then the lower point wave it is u i j minus 1 minus 4 u i j is equal to 0.

So, we have got this relationship, now for l i and j inside this boundary that is wherever this problem has been defined we can find it out this will result me actually a algebraic equations. We could check in that is with a little bit inside we could see that is all those points, whatever we are getting is we would get the number of equations equalled to the number of unknowns, which we have to find out that is what you would actually find out in the numerical solution the value of the unknown function u at these points.

So, we use this relationships as well as the boundary conditions to get the algebraic equation, but the coefficient not result as you said is I will be find it out that it would be actually 0 1 0 1 minus 4 1 and 0 1 0 like that at u i j. So, if I am taking this u i plus 1 and j plus 1 like that if I write out all the points wherever, which are we interested will get more 0s than the norm 0s that kind of matrix. So, the coefficient matrix we would get more 0s than the norm 0 entries this kind of matrix will whenever we are cutting we call it is matrix as sparse.

In this kind of sparse matrix actually the method of elimination cause elimination method that is very costly, costly in the sense is that it will take too much time as well as too much work to get the solution and we may get longer division by 0 or something like that, so we may not be able to get the nice solutions. So, this kind of sparse matrices, when we do have a coefficient matrices, then using rather than using this usual elimination method we do go with the iteration method. Moreover, here in this one I have taken as small grid small grid means is x is large. Now if you do remember how we have got our this difference quotient there we have taken is that is the first derivative as f of x plus h minus f of x minus h upon h there h has to be small.

So, far getting more nice insight above the function new we required h 2 p small if h is small my grid would be more finer on that says is a half to half this system of equations very large may be sometimes in very small problems 100 cross 100 or something as 500 cross 500. So, this sparse matrix is the usual method does and go normally we would like to use the iteration method. How, we are going to solve this problem let us see it with the help of one example, so that you do get that is how we are finding out the solution and what the iteration method. Iteration method normally in this Laplace equations are being used methods, which in the earlier times used to also to be called labour man method.

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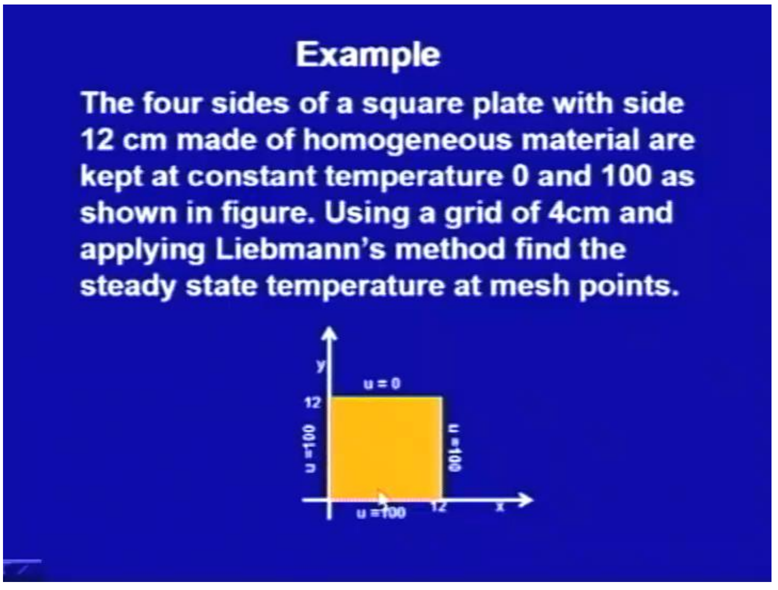

So, let us do one example the four sides of a square plate with side twelve centimetres made of homogeneous material are kept at constant temperature 0 degree and 100 degree as shown in the figure using a grid of 4 centimetre and applying liebmann's method find the stead state temperature at mesh point. So, what is this boundary condition boundary condition has been given by this, so we do have a uniform plate of a uniform material, which square plates whose size at 12 centimetre power cross 12 centimetre its temperature is been kept at 0 degrees centigrade and 100 degree centigrade that is this upper side is 0 on these three boundaries are kept at the 100 degree centigrade.

Now, what we want the steady state temperature at mesh points it says that numerical way to solve it we have already done, when we are having this steady state temperature pattern is, which is independent of time t, then our heat equation this is after the temperature out of heat flow that heat equation is changing to the Laplace equation two dimensional Laplace equation and the solution of that we have to find out using this boundary conditions with the boundaries being given here, like this one.

We had already solve this kind of problem if you do remember we have done one example, where we have kept other side’s 0 and the upper side only as some here, itself very worst kind of thing. Now, it says is that using a grid of 4 centimetre; that means, now I have to divide it into 3 points only.

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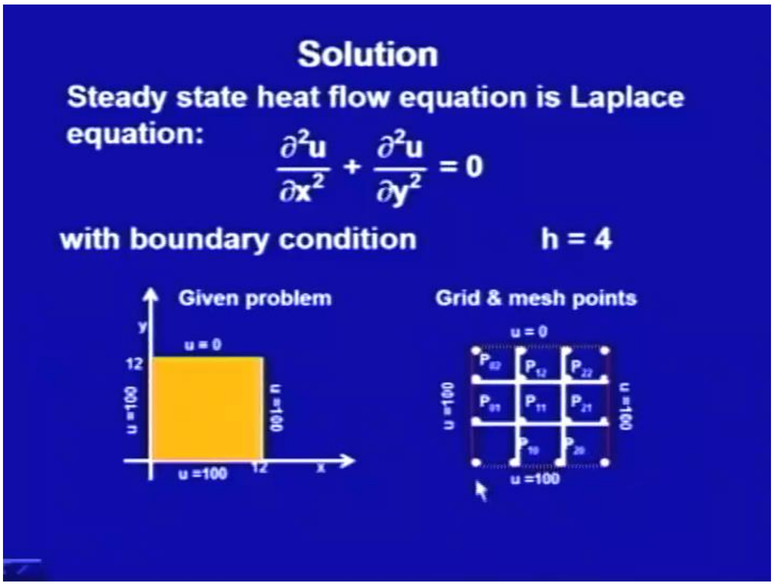

So, let us try one by one steady state heat flow equation is Laplace equation we do know that del u over del t would be 0, so del 2 u over del x 2 plus del 2 u over del y 2 is equal to 0 this is we have to solve. With our boundary condition given by this method that is what we are saying is u x 0 is 100 u 0 y is 100 u 12 y is 100 and u x 12 is 0, so this is the boundary condition, which is given to us now we have to use the numerical method. So, as I said is that the reason we have to define a grid they are saying is use this h is equal to four centimetre say if I am using h is equal to four centimetre I would get the grid of three cross three.

So, given problem this grid and mesh points we would be getting it 4 centimetre 4 centimetre four centimetre, so you would get these 4 these three cross three grid, now the points the first point would say P 1 0 P 2 0 P 1 1 this would be P 0 1 P 0 2 P 1 2 P 2 2 P 2 1 and P 2 0. And the boundary condition are been given that at these points all these points that is P 0 0 P 1 0 and P 2 0 and P 3 0 we are getting is 100 u at 100. Similarly, at P 3 0 P 3 1 and P 3 2 you would be getting it as 100 here P 0 0 P 0 1 P 0 2 we are getting is 100, while as here P 0 3 P 1 3 P 3 2 and P 3 3 would be getting it all 0.

So, we have to find out the value of this u at all the mesh points, so these are the mesh points, which we are getting while as the boundary mesh points we do know that is they have to satisfy these boundary conditions. So, finally, what we have to actually get it we have to get the solution at all the interior inside the points these four points

So, we will use the relationships, so that at all these four points and for all those four points whatever the relationship we are getting in the difference quotients there would include all these one at all these one we would use these given boundary condition. So, let us see how we are going to solve this problem using our methods

So, this is what the first step we have done, we have defined the grid and the mesh points now we have to use the difference quotient at all those nodes. So, we will take only the interior points, because at these boundary condition boundary points we do know this they should satisfy the boundary conditions, so the solution we are knowing.

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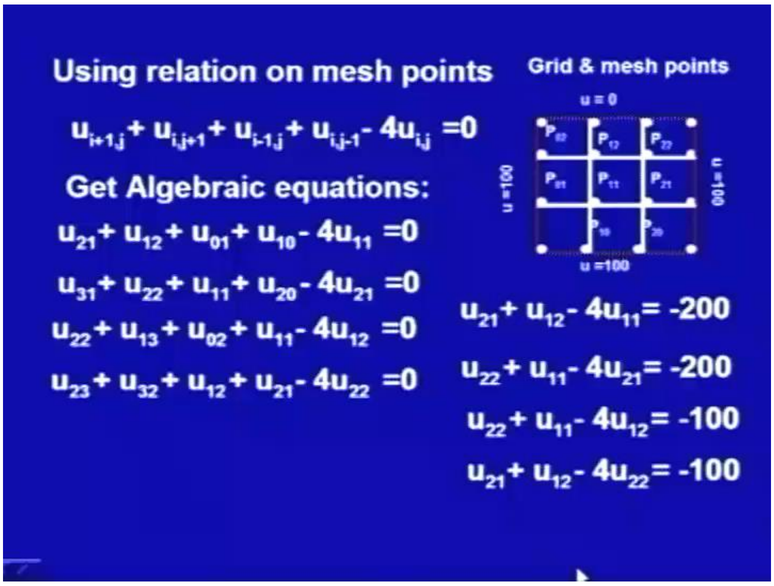

So, let us find out this relations at all the node points use this relation u i plus 1 j u i j plus 1 plus u i minus 1 j plus u i j minus 1 minus 4 u i j is equal to 0. So, the first point take u i j that is p 1 1, so at p 1 1 if I do write u i plus 1 I would get p 2 1 u i this 1 you will get p 1 2 and from here p 0 1.

So, get this at algebraic equation u 2 1 plus u 1 2 plus u 0 1 plus u 1 0 minus 4 u 1 1 is equal to 0, the second 1 at p 2 1 we do get it as u 3 1 plus u 2 2 plus u 1 1 plus u 2 0 minus 4 u 2 11 is equal to 0. Then, at u 1 2 this p 2 2 p 1 2 point u 2 2 plus u 1 3 plus u 0 2 plus u 1 1 minus 4 u 1 2 and then at p 2 2 we would get u 2 3 plus u 3 2 plus u 1 2 plus u 2 1 minus 4 u 2 2 is equal to 0.

Now, we are using the boundary point u 0 1 u 1 0 u 0 2, then u 2 3 and u 1 3 and so on, so at those boundary points because we are knowing is that the different boundaries at what are the my values are those points. So, let us rewrite this equations so here what we will get u 2 1; that means, P 2 1 this point u 1 2 is this point these two points are unknown and u 1 1; these three points are unknown, but u 0 1 and u 1 0 u 0 1 means is that is this point this u 1 0 and u 0 1 both are 100.

So, what we do get the solution as at this equation as u 2 1 plus u 1 2 minus 4 u 1 1 is equal to minus 200 that is these two I have taken to right hand side. Similarly, here u 3 1 u 3 1 means is we are getting it this point, which is 100 at another boundary condition is u 2 0 u 2 0 is u 2 0 is this one again we are getting is boundary condition that is it is 100.

So, what we do get is u 2 2 plus u 1 1 minus 4 u 2 1 is equal to minus 200, similarly for the third 1 u 2 2 plus u 1 1 minus 4 u 1 2 is equal to minus 100 and last one is u 2 1 plus u 1 2 minus 4 u 2 2 is equal to minus 100.

Now, we have got these four equations and in the four unknowns, what are the four unknowns u 1 1 u 1 2 u 2 1 and u 2 2, so we have got the four unknowns four equations and four unknowns, so we can now we solve this equation. So, now, let us move that is how to solve let us first see whether we can get a solution or not, so let us first use, because it is a simpler and smallest system we can go ahead with the Gaussian elimination method.

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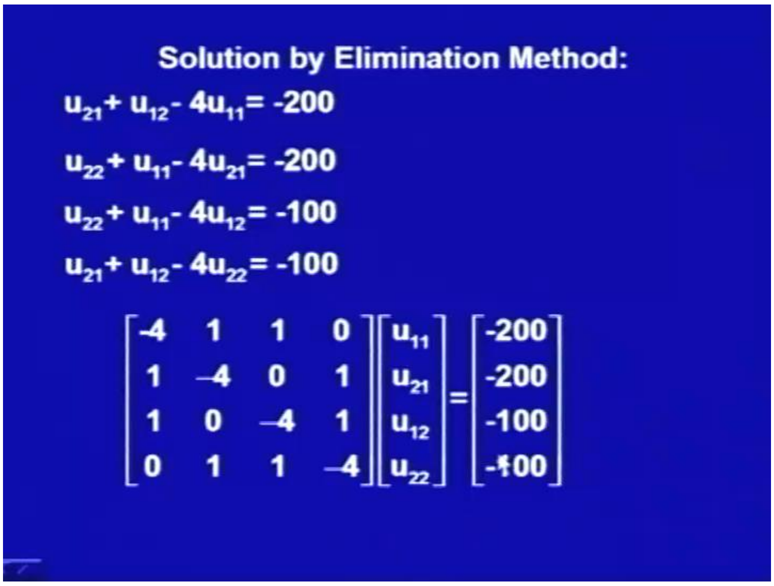

So, with the Gaussian elimination method we are having these four equations write it in the matrix form the coefficient matrix we would get, now we are using at pattern first variable u 1 1 then u 2 1, then u 1 2 and then u 2 2. So, what we get the coefficient of u 1 1 minus 4 coefficient of u 2 1 is 1 coefficient of u 1 is 1 coefficient of u 2 2 here is 0 right hand side is minus 200.

Similarly, for the second one we are getting coefficient of u 2 2 is 1 coefficient of u 1 1 is one coefficient of u 2 1 is minus 4 and coefficient of u 1 2 is 0 So on. So, we do get this system minus 4 1 1 0 1 minus 4 0 1 1 0 minus 4 1 0 1 1 minus 4 and the unknown variable that is u 1 1 u 2 1 u 1 2 u 2 2 is equal to the right hand side minus 200 minus 200 minus 100 minus 100 using that elimination method, we write this augmented matrix as this one.

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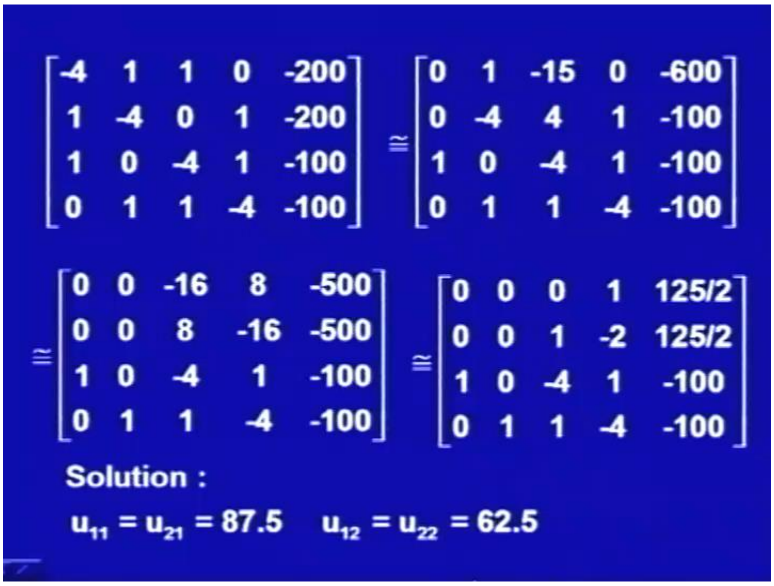

And, use the ah echelon forms are that is the row operations, so using the third one let us make this first the entry in the first column of the second row and the first row as 0, we do get it as this row is as such the last row is as such, here what we have done is we have subtracted from the second row the third row. So, directly subtraction gives me that is simply it would be 4 this would be 1 minus 1 0 that is and what we are getting is minus 600 over here that with this one would be that is multiplied 4 times and add it up up to here what we would be getting this one, so here what we do get this as the solution.

Now, again use this in try to make this equation to make this fourth row to make these two entries 0, what we do get 0 0 minus 16 8 0 0 8 minus 16 1 0 minus 4 1 0 1 1 minus 4 minus 500 minus 500 minus 100 minus 100. Now, again we just make use of a this equation to make here the entries 0 1 1, so what we do get it finally, will get first this 0 and then whatever here has been got that divided by that one we get it one entry, we do get it 0 0 1 1 25 by 2 0 0 1 minus 2 1 25 by 2 1 0 minus 4 1 minus 100 0 1 1 minus 4 is equal to minus 400.

So, from here if you do remember our first unknown was u 1 1 u 1 1 we are getting 1 25 by 2, then the last this side u 2 2 and then, what we are getting is u 2 u 1 2 and u 2 2, so again we would be getting it, so what we do get the solution u 1 1 and u 2 1 as 18 point

87.5 and u 1 2 and u 2 2 as 62.5. So, from here we will getting 62.5 and from putting all those things method we will do get. So, this method is simply one value do not address how to find out the eliminations, so I had gone a little bit shorter. Our problem example set is to not be use this one as I said is that this method is not applicable when we are actually doing the problem. So, we want this liebmann's method, what is liebmann's method that is simply Gauss-Seidel method that is iteration method iteration method say is, what is this Gauss-Seidel method.

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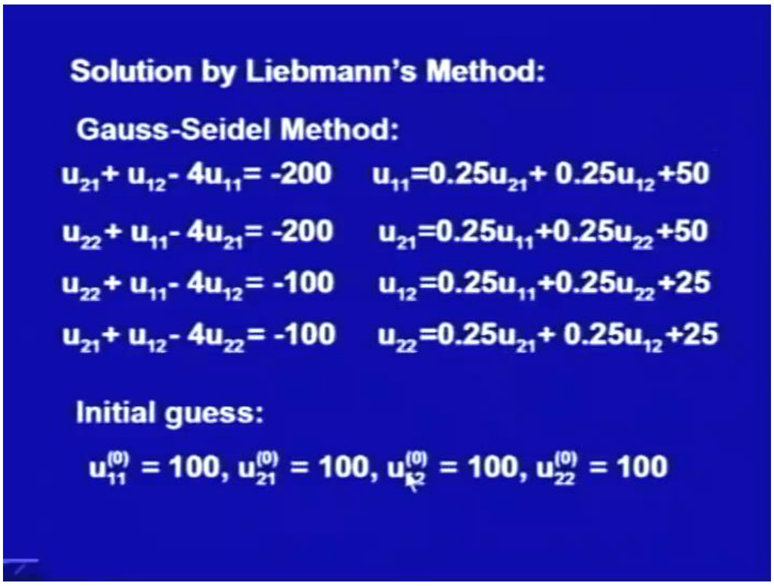

Gauss-Seidel method says is that these equations, which we were solving it we have to write it in different manner in the different manner that is we write u 1 1 in the terms of u 2 1 and u 1 2, then u 1 2 u 2 1 in the terms of u 1 1 and u 2 2, and so on. So, we write the unknown variables u 1 1 u 2 1 u 1 2 and u 2 2 in the form of these other unknown they are actually in the similar frame, so we write these equation as this one, let us say from here, u 1 1 as 1 by 4 times u 2 1 plus 1 by 4 times u 1 2 and this plus 50. Similarly, this 1 u 2 1 is 0.25 u 1 1 plus 0.25 u 2 2 plus 50 u 1 2 is 0.25 u 1 1 plus 0.25 u 2 2 plus 25, so u 2 2 is 0.25 u 2 1 plus 0.25 u 1 2 plus 25

Now, we start with some initial guess, let us say this initial guess for example, here we are taking is that all the variables at 100 it may not be very nice guess, but let us say start for the example all the ones, because we are having is that boundary conditions at the 3 points as 100s, so we are starting with the initial guess as 100. Now, what we do is that we solve get the second iterate that is the first iteration we get the first approximation u 1 1 u 2 1 u 1 2 and u 2 2 using this initial guess over here.

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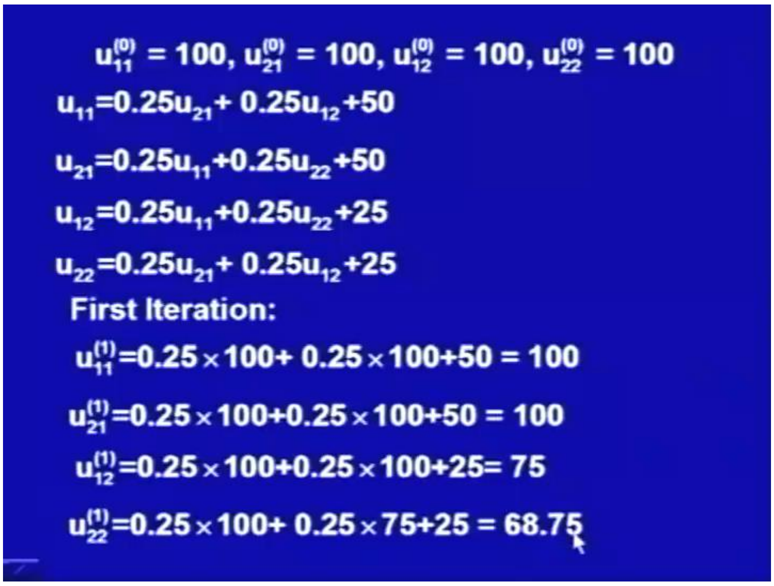

So, let us say that is how we are getting we are starting with this initial guess with all these four equations, let us write first u 2 1, u 2 1 at the initial guess is 100. So, first approximation you would be getting it as u 1 1 as 0.25 into 100 plus 0.25 into 100 plus 50 that is 100. Now, when we come to u 2 1 now u 2 1 I would not going this u 1 1 at the initial guess, because u 1 1 already we have find out in this iteration as 100 of course, it is different matter that we have got it as 100 itself, but we would use this guess u 1 1 at this point.

So, u 2 1 we would be getting 0.25 into this 100 plus 0.25 u 2 2 is 100 plus 50 again it is 100. So, the first iteration again we have got u 2 1 also as 100, then we are coming at u 1 2, you would use u 1 1 as this 100 and u 2 2 as this 100, because u 2 2 till, now we had not find it out and plus 25 we get it 75, then we come to u 2 2 u 2 2 we want to use u 2 1 u 2 1 will use this 100 and u 1 2 we will use this 75. So, we do get 0.25 into 100 plus 0.25 into 75 plus 75 as 68.75, now we have got this first iteration the first approximation as 100 100 75 and 68.75.

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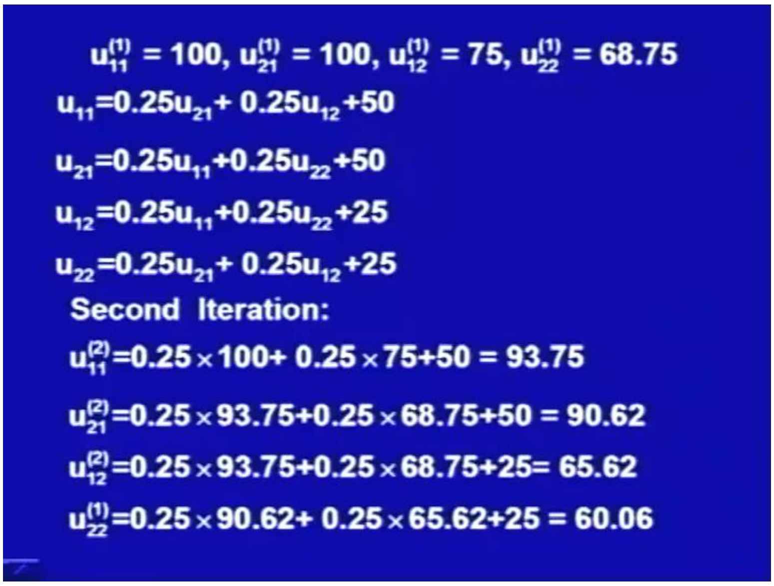

So, now this we would use in our these equations again, so now we will use these guess over these approximations to find out the second approximation. So, second approximation you do get 0.25 into 100 plus 0.25 into 75 plus 50 as this 93.75, then u 2 1 I would use this 93.75 rather than this 100, because whatever this second approximation we had already got and in the second equation the second iteration itself we can use it this one. So, that the convergence or we could say that the solution we just fast the calculations becomes fast, so 0.25 into point into 93.75 plus 0.25 into 68.75 this is 90.62.

Similarly, here we would use this 93.75 and 68.75, so we are getting 65.62 and finally, we would use it 90.62 and 65.62 over here and we do get 60.06. So, like this we would move on we try to reach to the solution how to stop it we are not knowing actual values, what we do is that will in the two iterations, whatever this approximations we are getting very close enough we say we do stop. So, this what is our liebmann's methods, which says is that how to find out this solutions in these one.

We do know already that my actual solution is 8 62.5 and 8 86.5 and 82.5, so you can this problem is little bit simpler one you already have known that is how to solve it. So, we can check it using actual Fourier series that is find it out Fourier method find out, what is the solution and we can check with there that is how much near we are reaching to this one. We can improve upon this convergence or that is reaching to the final solution if we take a better guess normally here, we are taking this that is the guess is starting from the boundary condition we are normally taking this better guess.

So, you can take this guess, so like this what we are having is the solution of our Laplace equation. Now, if I have to solve this Poisson equation both we do get you will write the different quotient and what we do get at that point x y point whatever the point we have taken we do get right inside as h square into f x y that is the value of the function at that point multiplied with that what is that h we have taken h square.

So, we do get again the algebraic system and then we can solve it and iteration method and we do get the solution. So this is, what is the numerical solution of Laplace equation using one method iterative method that we called the Gauss-Seidel method. Here we had used this because our matrix which we had got that coefficient matrix in the algebraic equations was very sparse one. So, one method we had learnt today for solving the Laplace equation using the Gauss-Seidel method as to find out the solution now for problem actually, where the boundary conditions have been given on a rectangular reason. So, we will learn some more methods in the next class.

Thank you.