Mathematics - III
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Lecture - 8
Higher Order Non Homogeneous Linear Equations

Welcome to lecture series on differential equations for under graduate students. Today's topic is Higher Order Non Homogeneous Linear Differential Equations. Till now we have done homogeneous linear differential equations with constant coefficients and with non-constant coefficients, and we had learned about their solutions. Today we will learn about non homogeneous equations of nth order.

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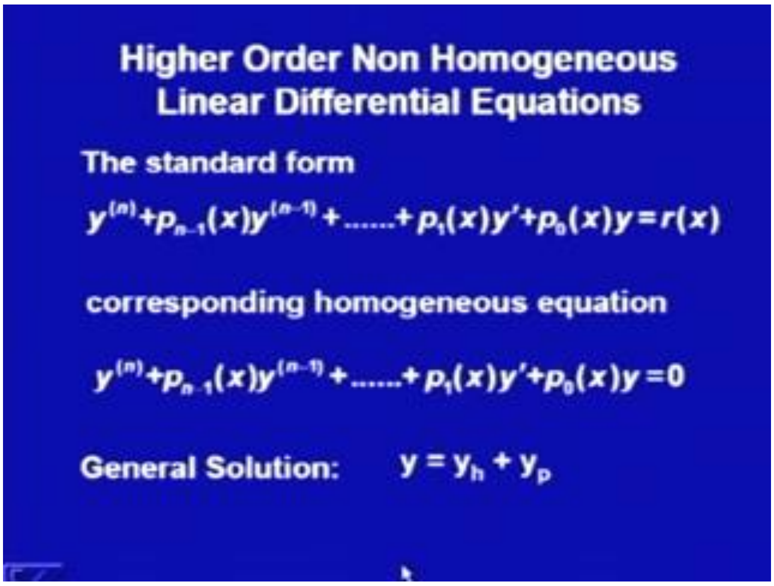

The standard form of this equations is as y n plus p n minus 1 x y n minus 1 plus so on, p 1 x y dash plus p naught x y is equal to r x. Here this p n minus 1 x p 1 x p naught x they are all function of x, we are not now differentiating with the constants and non constant we will just treated as simultaneously. Y n as usual is the nth derivative of the unknown function y, y n minus 1 is the n minus 1th derivative of the unknown function y and so on, and r x is all also a function of x. Now here because it is non homogeneous one we are assuming that this r x is not 0.

So, we have learnt that is it is corresponding homogeneous equation would be simply y n plus p n minus 1 x y n minus 1 plus so on, p 1 x y dash plus p naught x y is equal to 0, that is the right hand side we will make the 0. This is the corresponding homogeneous equation of this.

Now, as we have done in the second order non homogeneous equations, here also the general solution of this non homogeneous equation is of the form y h plus y p, what is this y h, y h is the general solution of this corresponding homogeneous equation. And y p this is a particular solution of this non homogeneous equation.

Now, this homogeneous equation which we are seeing here, we had learn whether this coefficients were constant or they were function of x we had learn that method how to solve these homogeneous equation, that is this part y h we can calculate for the homogeneous equation. So here, what is left to learn is to find out y p that is a particular solution for this non homogeneous one.

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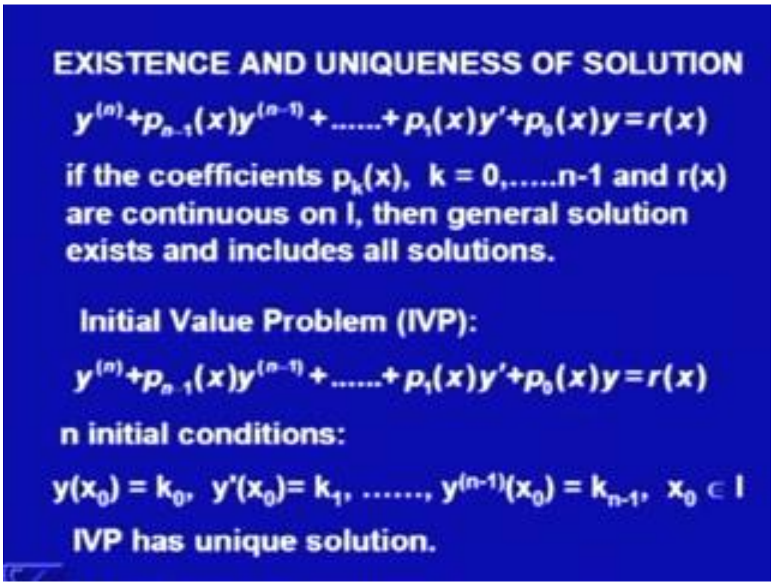

So, first we will see the existence and uniqueness of the solution, if I do have a non homogeneous linear differential equation of the form y n plus p n minus where x y n minus 1 and so on plus p 1 x y dash plus p naught x y is equal to r x, where the coefficients p k x that is this p n minus 1 x and so on. They and this right hand side r x, they are continuous on some interval i.

Then, the general solution exist and includes all solutions, that says is the solution of this equation would be exist. If I do have this coefficients as continuous on some interval as well as the right hand side is also continuous on that interval, then the solution will exist on that interval. Here, you see in this standard form we are taking the coefficient of y n as that is the nth derivative as 1.

Moreover, it says is the general solution includes all solutions that is all the solutions of this differential equation can be obtained from the general solution. We do not have any singular solutions here, what is the initial value problem of this one? Initial value problem consists this non homogeneous linear differential equation with n initial conditions.

What are those initial conditions? Those initial conditions are y at x naught is equal to k naught y dash at x naught is equal to k 1 and so on y n minus 1 x naught is equal to k n minus 1 for some x naught in that interval i where we are talking about the function is continuous. So, we are saying is we are giving some initial values of the function and its n minus 1 derivatives that is first derivative second derivative till n minus 1th derivative at some point x naught we are giving the value.

If this initial conditions have been given, then we can find out from the general solution the values of the constants and that solution would be unique, that is initial value problem does has a unique solution. So, what we had learn we had learn if the coefficients on the right hand side is continuous on some interval, then the general solution will exist for this equation and the general solution as we had just learn in the previous slide that it would be of the form y h plus y p. And, if initial conditions any initial conditions are being known, then they would be a unique solution for this one.

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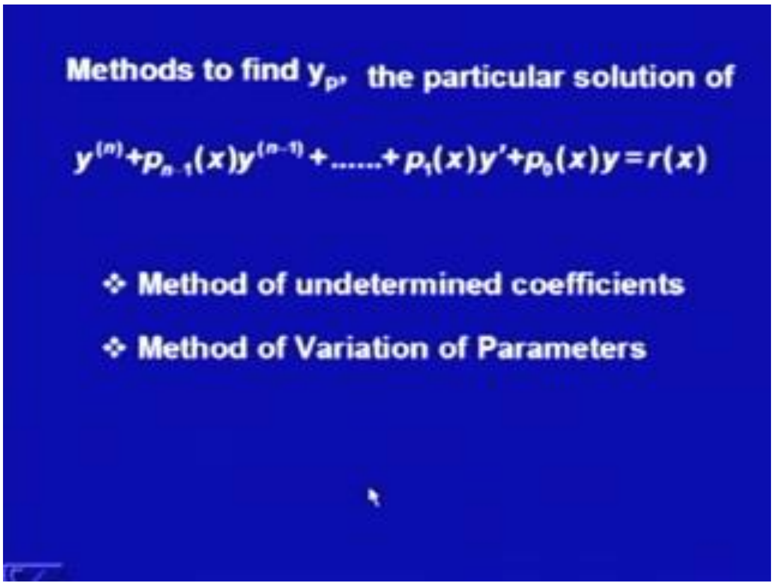

So, now we would learn how to find out this y p, so methods to find y p as in that done we have done this finding out this particular solution of this non homogeneous equation as we have done in the case of second order equation here also we will learn two methods. The first method of undetermined coefficients and second the method of variation of parameters. Do you remember that we have already done these methods in the case of second order linear differential equations non homogeneous one. Here, we are having nth order equation again we would learn these methods.

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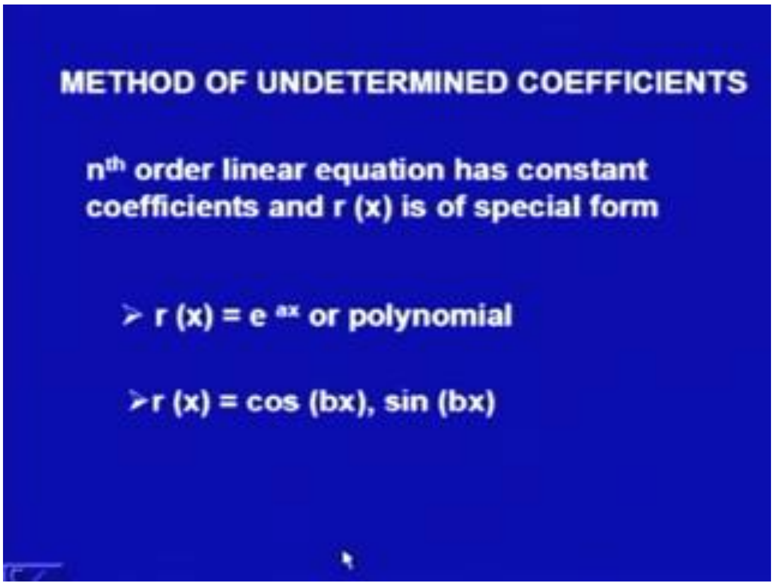

So, first learn this method of undetermined coefficient, this method is applicable in certain special cases, what are those special cases? First is that nth order linear differential equation has constant coefficients. And, that is those p n minus 1 x p n minus 2 x and p 1 x p naught x, they should be the constant they should not contain any function of x.

And, r x is of the special form, what is that special form? Either it is e to the power a x or polynomial such as x to the power n plus b 1 times x to the power n minus 1 and so on plus b naught times x or it is of the form of cos b x sin b x, that is either it is of form sin b x cos b x e to the power a x or of a form of polynomial. And, the coefficients p naught p 1 x all these are constants, only then this method is applicable this is as usual as we had learn what is this method says.

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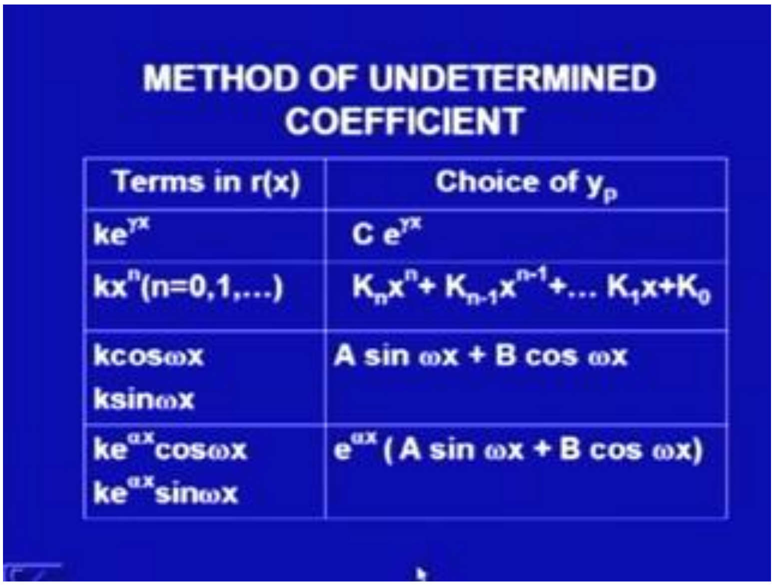

This method says is that if r x contains the term of e to the power x kind of thing, that is here we are saying is k times e to the power gamma x. Then, we choose the y p as c times e to the power gamma x, you see that is same a constant as we are having is in e exponential function. But here, this constant is changing c this constant is called undetermined coefficient.

And, we are determining this coefficient by putting the this function as a solution of non homogeneous equation. So, the function and its derivative will keep in the given equation and from there we will determine this coefficient c. Similarly, if my left hand side that is if my r x is of the polynomial form. So here, I have written only k times x to the power n it is not necessary, it may be simple polynomial that is k n times x to the power n plus k n minus 1 times x to the power n minus 1 and so on.

But whether, it is having here the complete polynomial or not it does not matter, the choice of y p must be the polynomial of degree n, that is k n. Here, you see this is capital K capital K n x to the power n plus K n minus 1 x to the power n minus 1 and so on plus K 1 x plus K naught that is I do have n plus 1 undetermined coefficients.

This we will determine by putting this function and its n derivatives in the given equation. And then find out from the equating it from the right hand side we would find out what are these coefficients. Similarly, if my r x is of this form cos omega x or sin omega x, whatever it may be whether it is having both the things or it simple one, we just choose this function. Similarly, if I do have is that special form is e to the power alpha x into cos omega x or e to the alpha x and sin omega x we would again have here this y p as e to the power alpha x times a sin omega x plus b cos omega x. Here, again this undetermined coefficients are A and B.

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So, what is the method of this rule of this method.

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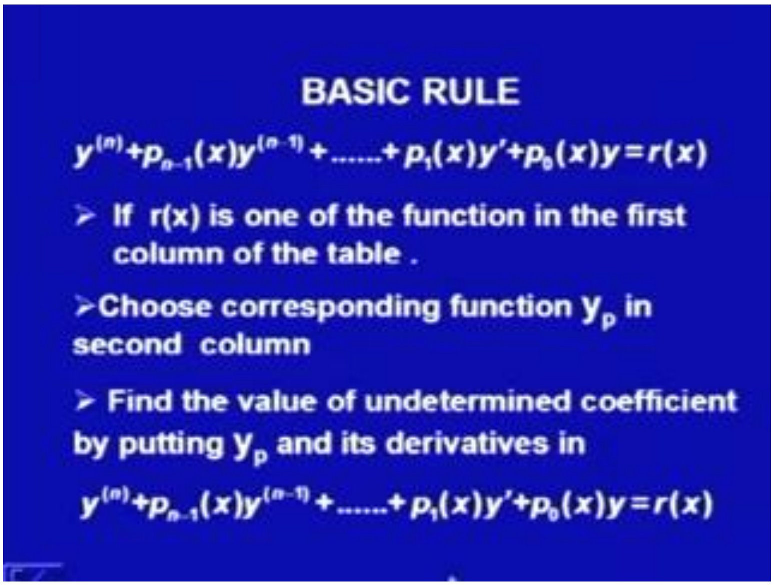

Basic Rule just now I have defined the basic rule which says is if this equation for this equation y n plus p n minus 1 x plus into y to the power n minus 1 and so on plus p 1 x y dash plus p naught x y is equal to r x, where my p n minus 1 x p 1 x and all these p naught x they are constants not the function of x.

And, r x is of any form which is been given in this previous table, that is just now I had explained you the first column of the table. Then, choose the corresponding function y p in the second column that is what is my basic rule and find the value of undetermined coefficient by putting y p and its derivative in this given equation.

Now, it may happen, that is when we are choosing this y p I may be having that that choice of y p is already in the solution of the homogeneous equation of that is the corresponding homogeneous equation of this equation, that says is then that y p will not be linearly independent of those homogeneous equation solution. While we require y p to be linearly independent of that one, that says we require certain modifications.

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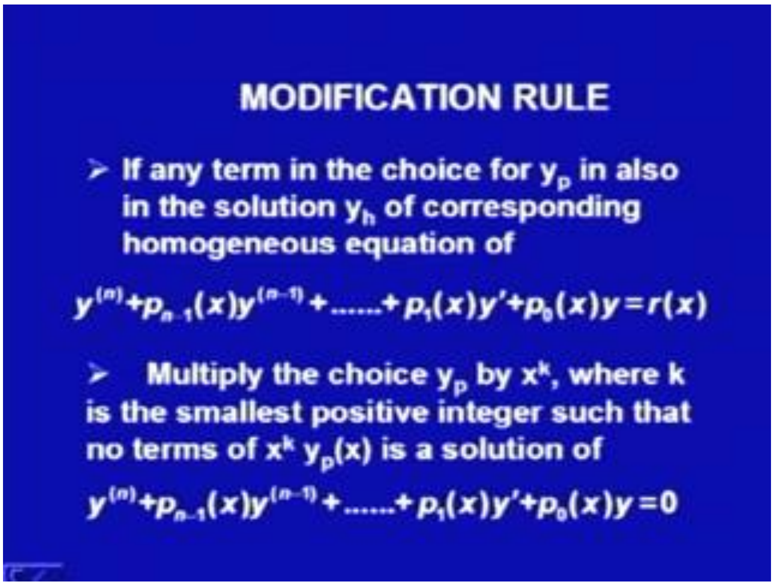

So, the second rule is called modification rule, what is this modification rule? If any term in the choice of y p is also in the solution y h of corresponding homogeneous equation of this non homogeneous equation. Then, multiply the choice y p by x to the power k where k is the smallest positive integer such that no terms of x to the power k y p x is a solution of this homogeneous equation.

Why we are saying here, if you do remember the second order 1, there you said is that is multiply it by x or x square depending upon whether it is a single root or the single solution or the double solution of the corresponding to the single root or the double root of the homogeneous or the characteristic equation of the homogeneous one.

Now, here since I am having is here, a nth order equation and we had learn in the homogeneous one that we will have actual multiple roots, that is roots are not only single and double why we could have a multiple rule of root of multiplicity k. So, if suppose I do have a root of multiplicity k minus 1.

Then, we must multiply with x to the power k times y p x, that will make it sure that is if the root has k we would have that is x times and so on we by the method of variation of parameter, that is they would though all those would be in the solution. So, this will not be in the solution of this corresponding homogeneous equation, this is the modification rule.

Again, we may have that the right hand side is consisting of those special functions, but they are a sum of those things.

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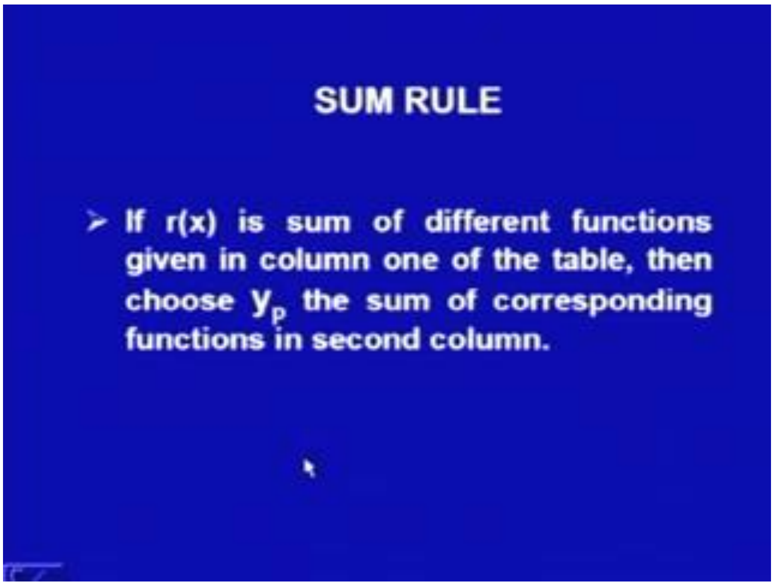

So, the third rule is the sum rule again this is similar to that what we have done in the second order, if r x is the sum of different functions given in the column one of the table then choose y p the sum of corresponding function in the second column. Now, we had learn that is the rules are similar to that what are in the second order differential equations. Now let us try one example to understand that how to apply this method.

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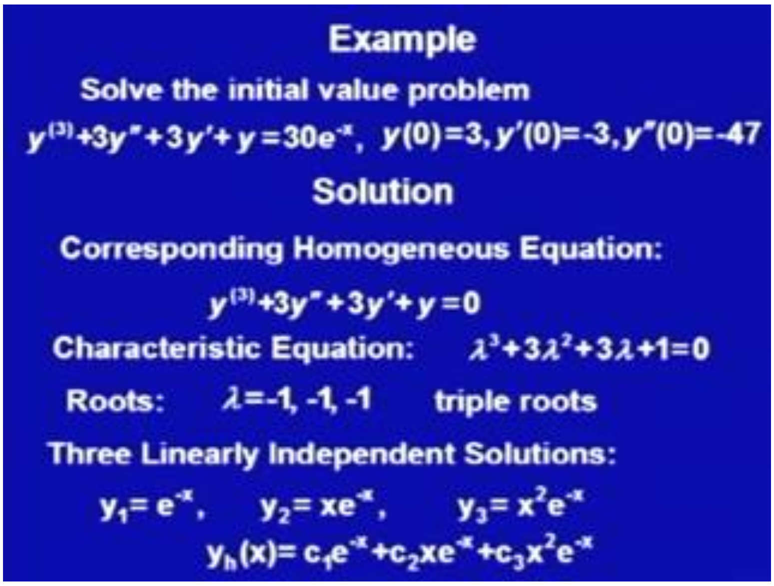

So, let us do one example solve the initial value problem y 3 plus 3 y double dash plus3 y dash plus y is equal to 30 times e to the power minus x. Given initial conditions are y at 0 is 3, y dash at 0 is minus 3 and y double dash at 0 is minus 47. We see here, we are having a third order differential equation, the coefficients again we are having is that the function y and its derivatives they are occurring separately. So, it is a linear differential equation.

Then we are having the coefficients of y y dash and all those things they are just constants 1 and 3. So, this is a linear differential equation with the constant coefficient. Right hand side is of a special form 30 e to the power minus x this is not 0, but this contains a function which is of a special form of the exponential form, that says is we can apply the method of undetermined coefficient. So, let us just try the solution.

First we will find out the homogeneous equation the, so the corresponding homogeneous equation would be what y 3 plus 3 y double dash plus 3 y dash plus y is equal to 0, that is we just make it right hand side as 0 that is the corresponding homogeneous equation. So, first we will find out the y h for finding out the solution of this homogeneous equation, we require the characteristic equation of this one. Characteristic equation for this equation as we know would be lambda cube plus 3 lambda square plus 3 lambda plus 1 is equal to 0.

If we see what is this lambda cube plus 3 lambda square plus 3 lambda plus 1 this is nothing but lambda plus 1 whole cube. So, what will be the roots? The roots are lambda is equal to minus 1 minus 1 and minus 1 that is we are having a multiple root rather a triple root, this is a third order equation we are getting a triple root.

So, what will be the basis of the solution for this homogeneous equations as we do know earlier the linearly independent solutions would be corresponding to this e to the power minus x. Then the second solution will have x times e to the power minus x, then the third solution we will have x square times e to the power minus x. These are3linearly independent solutions

So, y h would be c 1 e to the power minus x plus c 2 x times e to the power minus x plus c 3 x square e to the power minus x. Now, we see right hand side is of this a special form 30 e to the power minus x, if you remember in our table we had is that is if it is a times e to the power gamma x we should choose e to the power gamma x; that means, in the y p we have to choose e to the power minus x. But, we are finding out e to the power minus x is a solution corresponding to a triple root.

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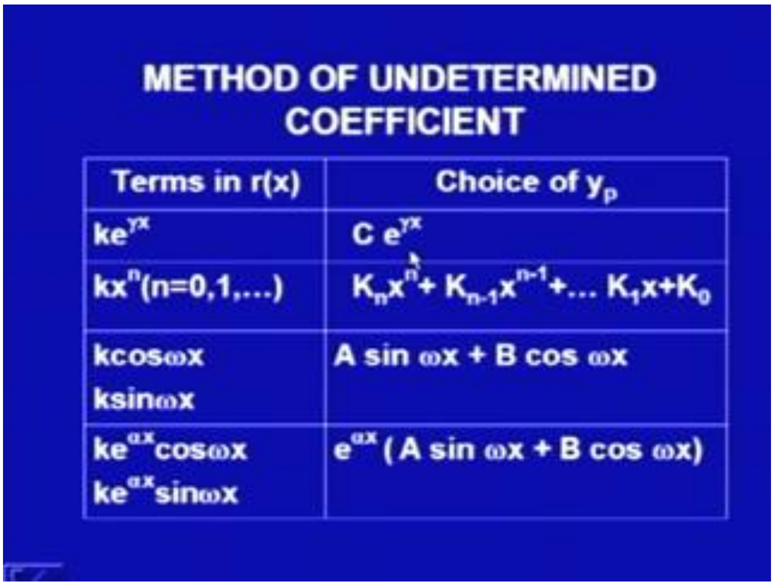

So, let us see it this method of undetermined coefficient says, now we are having in the right hand side 30 times e to the power minus x; that means, I have to choose c times as a times e to the power minus x. But, already we have seen that e to the power minus x is a solution corresponding to this thing.

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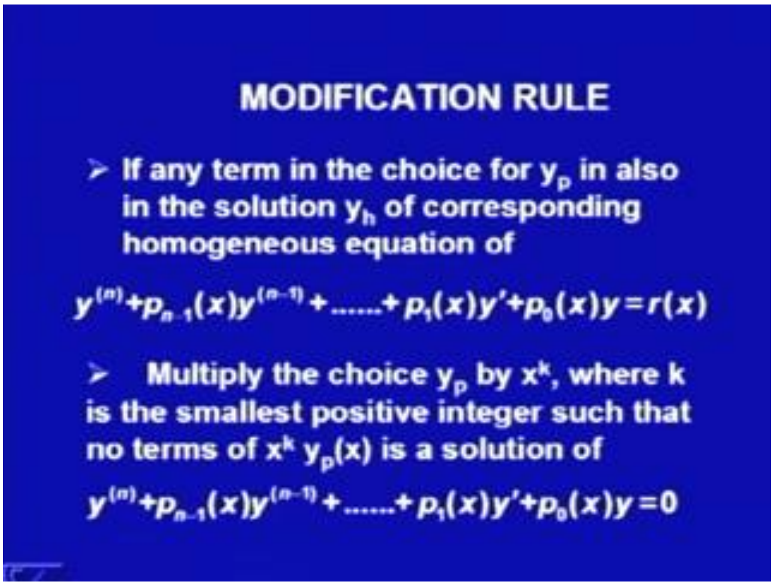

So, we have to use the modification rule, modification rule says is that I have to multiply the choice of y p by x to the power k such that x to the power k y p x is not a solution is no term in this is a solution of this homogeneous equation. We have seen that in however we have got e to the power minus x x times e to the power minus x and x square times e to the power minus x all are the solution of y h.

So, we will choose here, x cube e to the power minus x, so you see I am having in my y h e to the power minus x x times e to the power minus x and x square times e to the power minus x. So, what will be my choice of y p? This y p I will choose a times x cube e to the power minus x, according to the method of undetermined coefficient with respect to the modification rule.

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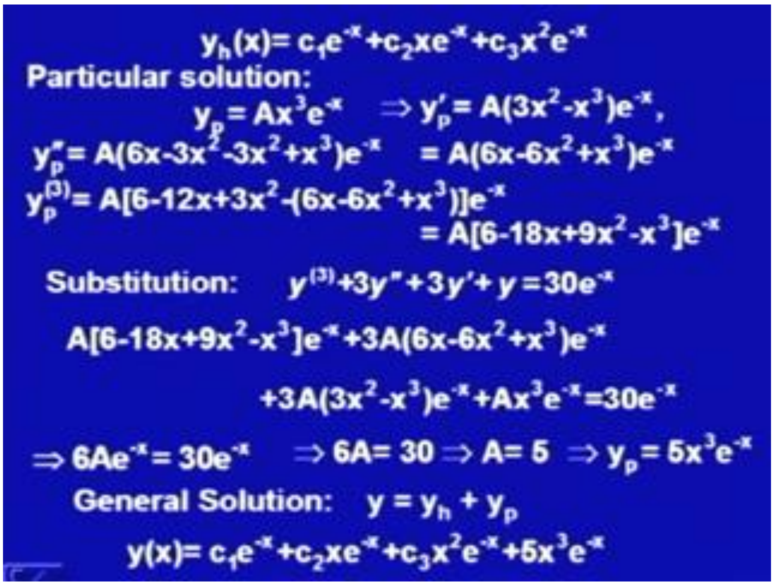

Now, how to find out this A we will put this y p and its derivative in the given equation. So, what will be its derivative y p dash would be A times 3 x square minus x cube e to the power minus x, y p double dash would be A times now here 6 x minus3x square that is into e to minus a to power minus into e to power minus x.

Then, this derivative have e to the power minus x is minus e to the power minus x and this coefficient is as such. So, that is what here, what we are getting is that it is equal to a times 6 x minus 6 x square plus x cube e to the power minus x, what will be the third derivative, again first we will make the derivative of this function and write this function as such and then derivative of this function and this function as such.

So, we are getting is 6 minus 12 x plus 3 x square minus 6 x minus 6 x square plus x cube into e to the power minus x which is equal to 6 minus 18 x plus 9 x square minus x cube times e to the power minus x. Now, the given non homogeneous equation if it was y 3 plus 3 y double dash plus 3 y dash plus y is equal to 30 times e to the power minus x.

Now, substitute this y p 3 y p double dash and y p dash and y p in this 1, so what we will get substitution A times 6 minus 18 x plus 9 x square minus x cube e to the power minus x plus 3 times y double dash, that is 3 times A 6 x minus 6 x square plus x cube e to the power minus x plus 3 times y, that is 3 times A 3 x square minus x cube e to the power minus x plus y, that is A times x cube e to the power minus x. This is equal to this must be equal to 30 e to the power minus x.

So, what we are saying is, this we are saying is that is a particular solution of this homogeneous. This non homogeneous equation that says is the function and its derivatives must satisfy this equation, so now we want that this must satisfy this equation. So, if we simplify this left hand side and say that it must be equal to this one, we are getting is 6 A times e to the power minus x must be equal to 30 times e to the power minus x, what does implies, that my 6 A must be equal to 30 or a must be equal to 5. So, what I got the particular solution? Particular solution I got as 5 x cube times e to the power minus x.

So, what will be my general solution of this non homogeneous equation, that is y h plus y p y h is this one and y p is this one. So, what I would get, c 1 e to the power minus x plus c 2 x times e to the power minus x plus 3c c3x square e to the power minus x plus 5 x cube e to the power minus x.

Now, you see this is a general solution we are getting the terms e to the power minus x x times e to the power minus x x square times e to the power minus x and x cube times e to the power minus x they are all linearly independent, so this is a general solution. Now, we want the solution we have to solve the initial value problem that says is we have to find out in this general solution, what are these general constants, c 1 c 2 c 3, what are the values given those initial conditions, so that must have a unique solution.

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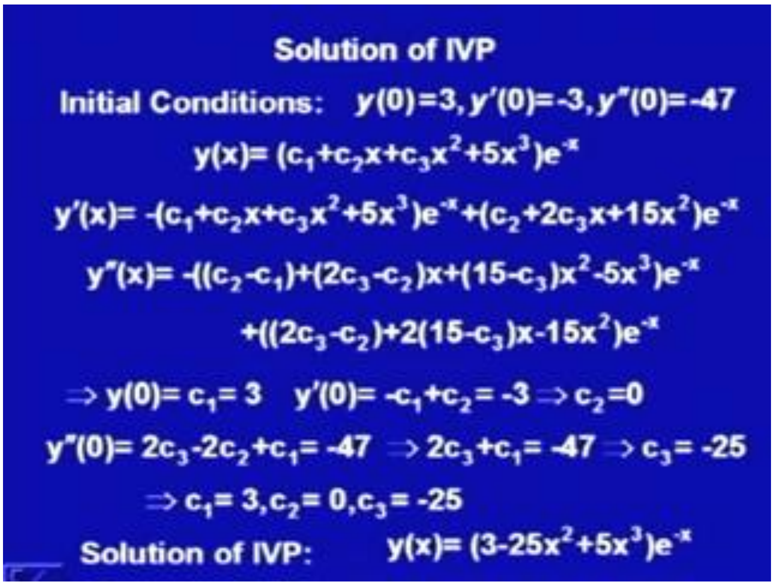

So, solution of initial value problem, the given initial conditions are that y at 0 is 3 y dash at 0 is minus3 and y double dash at 0 is minus 47. And our general solution is again I have written it c 1 plus c 2 x plus c 3 x square plus 5 x cube times e to the power minus x. Now, find out it is because we have to put the initial conditions for the first derivative and the second derivative, what are the first derivative and second derivative? They are minus c 1 plus c 2 x plus c 3 x square plus 5 x cube times e to the power minus x, that is differentiating this function first and keeping this as constant. Then differentiating this function and keeping this as such we are getting c 2 plus 3 c 2 c 3 x plus 15 x square times e to the power minus x.

And, y double dash again with the similar method we would be getting is minus c 2 minus c 1 plus 2 c 3 minus c 2 times x plus 15 minus c 3 x square minus 5 x cube times e to the power minus x plus 2 c 3 minus c 2 plus 2 times 15 minus c3times x minus 15 x square into e to the power minus x.

Now, put the initial conditions y at 0, that is when I put x is equal to0 if here I am putting x is equal to 0 all these terms would become 0, this term will become 1. So, I would be getting is y 0 as c 1 which is given as 3, so I am getting c 1 is equal to3. Now, y dash at 0 is minus 3 it is given, what will be y dash at minus at 0?If I put 0 in place of x in all these things I would get from here minus c 1 and from here plus c 2. So, I would be getting minus c 1 plus c 2 which is equal to minus 3 as given. Now, since c 1 is we had already find out is 3, so if I substitute this 3 over here I would get c 2 is equal to 0.

Now, the third condition is about the second derivative in the second derivative if I try to put x is equal to0, I would get from this here minus c 2 minus c 1 and from here plus 2 c 3 minus c 2. So, I am getting is 2 c3minus 2 c 2 plus c 1 which is minus 47. We had already find out c 1 is equal to 3 c 2 is equal to 0, so this gives me 2 c 3 is equal to 50, you could get it or c 3 is equal to minus 25.

So, now we have got the three constants c 1 as 3 c 2 as 0 and c3as minus 25, so what will be the solution of my initial value problem? I will substitute these constants in this general solution, I will get 3 minus 25 x square plus 5 x cube times e to the power minus x. This is the unique solution of this initial value problem, you are getting is that single values you are not getting the general constants c 1, c 2, c3 here. Let us see, how it looks like so I will take some other values for c 1 c 2 c3so what will be the other solutions.

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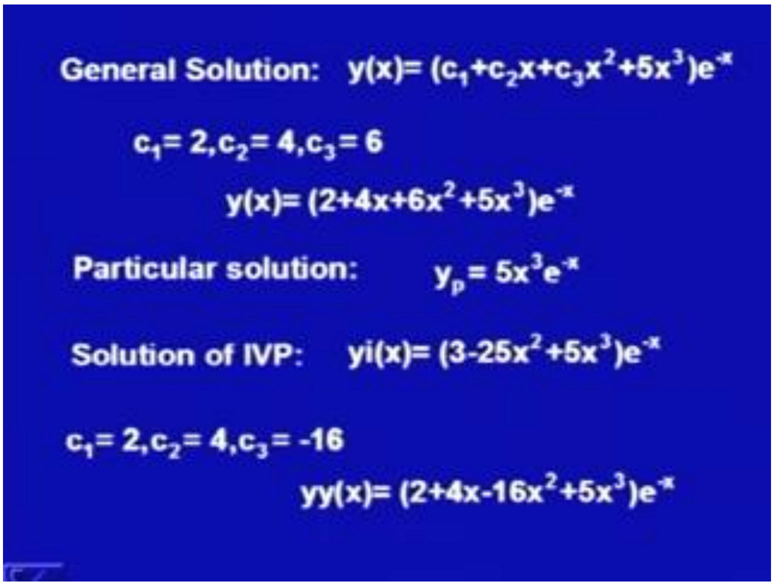

Since, the general solution is this one, if I choose c 1 as 2 and c 2 as 4 and c 3 as 6 I will get a solution as 2 plus 4 x plus 6 x square plus 5 x cube times e to the power minus x. And the particular solution, we are already having is 5 x cube e to the power minus x of this of that homogeneous equation non equation and the solution of i p initial value problem that unique solution we had already got this one. Let us, a take some other values for c 1 c 2 and c3, then that is 2 4 and minus 16. So, other solution I am denoting it here by y y, this would be 2 plus 4 x minus 16 x square plus 5 x cube is equal to e to the power minus x.

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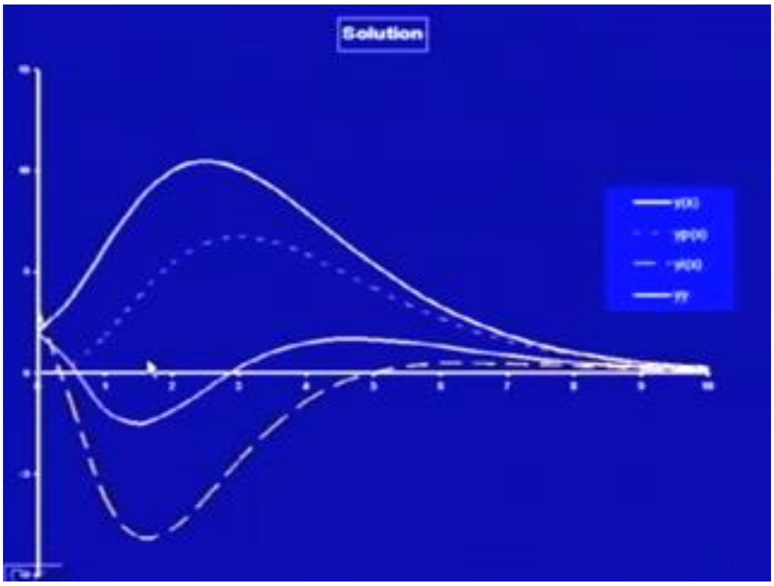

Let us, see in the graph how they are looking like, you see here, we are having is this is y x that is when I have taken 2 4 and 6 this is y p that is the particular solution, particular solution when I have taken c 1 as 2 you see is that is my graph would start from at 0 the function is 2.

Similarly, in the other condition, other solution which I have taken is there also I have taken c 1 as0and c 2 as 4 and c 3 as minus 16. So, this graph y y you are seeing is again it is starting from 0 this two. Initial condition said that c 1 is 3, so you see is this y i, that is the solution of initial value problem, this is starting from 3.

And y p was the particular solution, particular solution means is that is the general solution part is 0; that means, my c 1 must be 0. So, you are saying is that it is starting from 0, you are seeing is that by different conditions we are getting different solutions and this is the solution of your this yellow line this line is the solution of initial value problem and the solutions would contain all these kind of solutions. So, this is what this example says.

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Now, let us learn the other method, method of variation of parameters, again we will go in the similar lines as we have already done in the second order one. First this method is applicable to all those cases where the method of undetermined coefficient is not applicable, that is I may have general non homogeneous linear differential equation.

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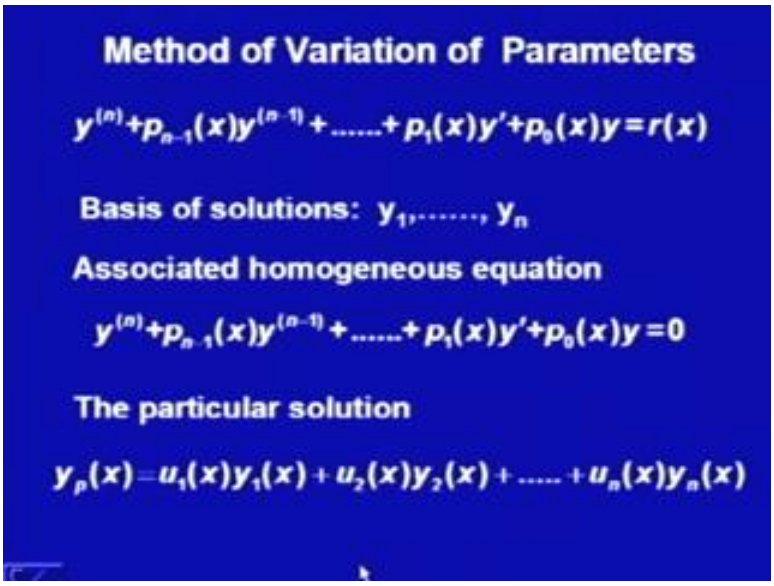

y n plus p n minus 1 x y n minus 1 and so on plus p 1 x y dash plus p naught x y is equal to r x, where this p n minus 1 x p 1 x p naught x they need not to be constants and r x need not to be of any special form is still this method is applicable, what is this method?

This method says is if I assume that y 1 y 2 y n are the linearly independent solutions of the corresponding homogeneous equation of this given non homogeneous equation.

Then, we by method of variation of parameters says we can find out the particular solution as u 1 x plus y 1 x plus u 2 x y 2 x and so on plus un x y n x do you remember that in the second order one we have taken u 1 x y 1 x plus u 2 x y 2 x. Now, here since we are having is n solutions of this corresponding homogeneous equations, so we would have here n unknown functions u 1 u 2 u n this by method of variation of parameter. We have to determine that is how to find out this functions u 1 x u 2 x and u n x they can be obtained as the solution of this system.

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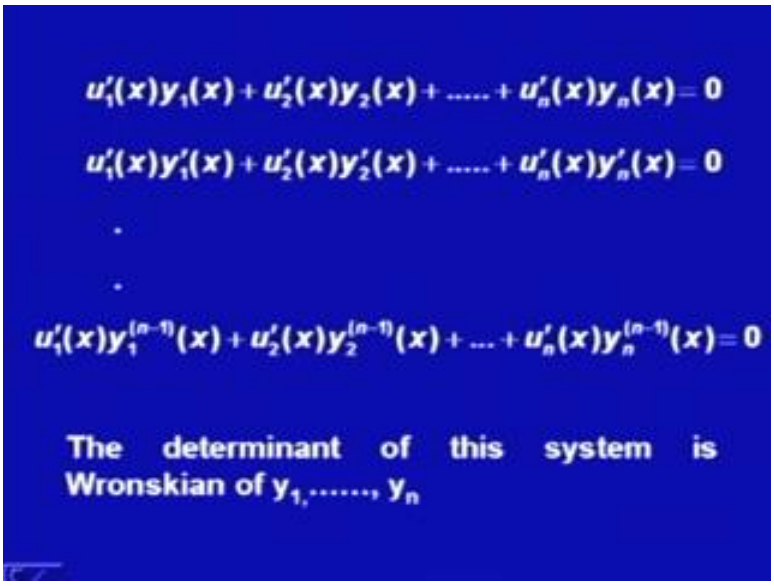

u 1 dash x plus y 1 x plus u 2 dash x times y 2 x and so on plus un dash x y n x is equal to 0 u 1 dash x y 1 dash x plus u 2 dash x y 2 dash x and so on u 2 un dash x plus y n dash x is equal to 0 and so on we will have the nth equation as u 1 dash x times y 1 that is n minus 1th derivative of y 1 plus u 2 dash times n minus 1th derivative of y 2 and so on u n dash times the n minus 1 th derivative of y 1 y n is equal to 0, how we are getting these one, this what we are doing is, that is the choice of this y p which we have taken as u 1 x y 1 x plus so on un x y n x, that and its derivatives we are putting in that non homogeneous equation.

And, from there we are trying to satisfy that is it should satisfy the solution, so we are getting since each y 1 y 2 y n is the solution of corresponding homogeneous equation.

So, from there we are getting this system of equations. Now, if we see the system of equations, what the matrix we would be getting the determinant of this system is nothing but the Wronskian of y 1 y 2 y n. So, what will be basically the solution of this one.

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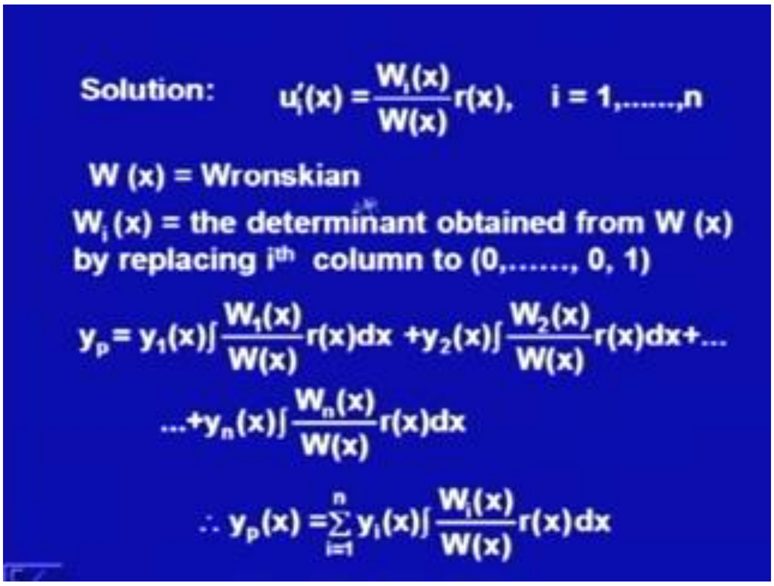

The solution would be that u i dash x would be W i x upon W x into r x for all i 1 to n what is this W i x and W x? W x is nothing but the Wronskian of the solutions y 1 y 2 y n, since y 1 y 2 y n are the basis of the corresponding homogeneous equations. So, this will never be 0 that is we can always divide it by. And, what is W i x? This is the determinant obtained from W x by replacing the ith column to 0 0 0 1. This is I am just giving you the method, how to find out the solution with the system of equations.

Then, what will be my y p x? y p x would be simply y 1 x integral of W 1 x upon W x r x d x plus y 2 x integral of w 2 x upon W x r x d x and so on y n x times integral of W n x upon W x r x d x that is u i dash x are this 1. So, u i x would be simply the integral of this with respective x and that would be so by this method of variation of parameter this is the method how to find out the particular solution of the given differential equation.

Now, let us do some examples to find out this, in other words y p also we could write as summation i is running from 1 to n y i x times integral y W i x upon W x r x d x since this is the notational form to write this lengthy expression is in the simpler form.

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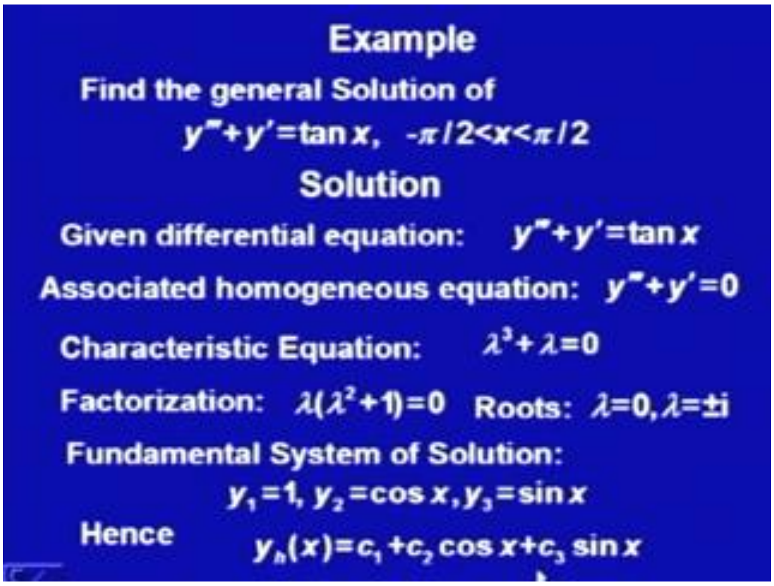

Example, find the general solution of y triple dash plus y dash is equal to tan x in the interval minus pi by 2 to plus pi by 2, we see here this is linear differential equation the coefficients are constant. But, the right hand side is of not of the those special forms, so we have to use the method of variation of parameter.

Let us see, how we go for the solution first we will find out the general solution of corresponding homogeneous equation. So, the given differential equation is y triple dash plus y dash is equal to tan x, so associated homogeneous equation would be y triple dash plus y dash is equal to 0.

Its characteristic equation would be lambda cube plus lambda is equal to0. Factorization lambda times lambda square plus 1 is equal to0, the rules would be that lambda is equal to0 lambda square is equal to minus 1 that is the complex conjugate. So, the roots would be 0 and plus minus i, so we are having 1 real root and a pair of complex conjugate roots. So, what will be the three linearly independent solution that fundamental system of the solution for this homogeneous equation that would be 1 cos x and sin x. So, what will be y h? y h would be c 1 plus c 2 cosine x plus c 3 sin x.

Now, for finding out the y p we would use the method of variation of parameter for that we require the Wronskian of these three solutions and the different Wronskian.

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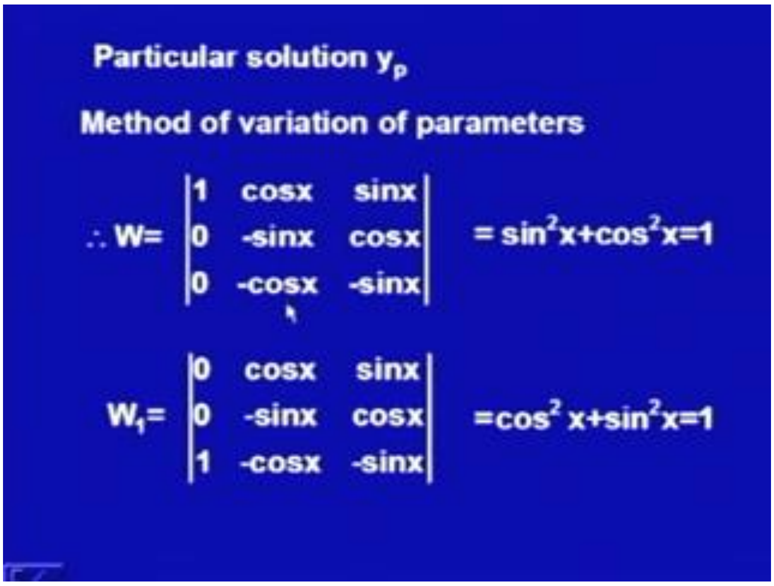

So, for particular solution y p this method of variation of parameters the Wronskian of the three solutions 1 cosine x and sine x its derivatives0minus sine x cosine x derivative of 0 is 0derivative of minus sine x is minus cosine x derivative of cosine x is minus sine x. This determinant if I solve, I would get sin square x plus cos square x which is equal to 1.

Then, the different determinants that is W i, so first W 1 W 1 says is that I have to change this first column by 0 0 1. So, the first column is being changed by 0 0 1 rest of the columns are as such. If we see what will be this determinant, this determinant would be cos square x plus sin square x that it is again 1, what will be W 2 W 2 means this second column I have to change to 0 0 1.

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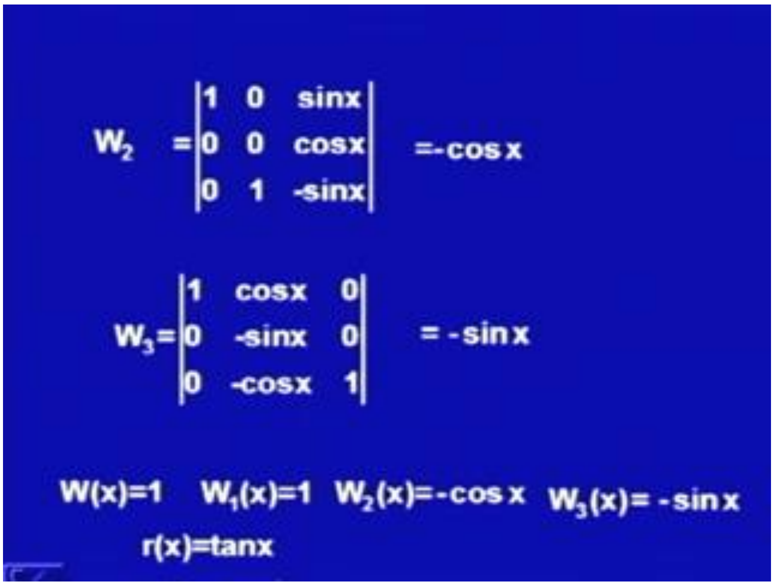

So, the second column has been changed to0 0 1, and the first column and the third column is as such. Now, if I just open with respect to the first column I would get it as cosine minus cosine x, W 3 the first two columns as such the third column 0 0 1, again I am opening with respect to expanding it with respect to the first column I would get it as minus sin x. So, what we have got we have got Wronskian as 1 W 1 x as 1 W 2 x as minus cosine x and W 3 x as minus sin x. So, what should be our u i is and r x is tan x.

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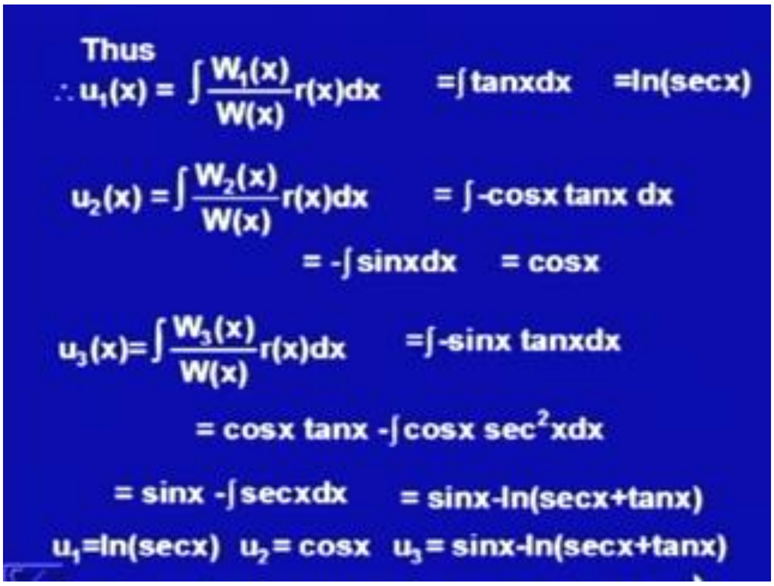

So, my u 1 x would be W 1 x upon W x r x d x W 1 was 1 W x was also 1 r x was tan x. So, what I would get tan x d x, its derivative is its integral is log sec x, then u 2 x W 2 x upon W x r x d x W 2 x we have got minus cosine x W x is 1 r x is tan x. So, I would get minus cosine x into tan x d x, minus cosine x tan x gives me minus sin x, minus sin x d x its integral is cosine x d x. So, we have got u 2 as cosine x, u 1 as log sec x u 2 as cosine x.

u 3similarly it should be W 3 x upon W x into r x w3minus sine x w 1 r x is tan x, so we are getting minus sine x tan x d x this will indicate by parts. So, we would get is cosine x tan x minus cosine x sec square x d x, cosine x tan x is nothing but the sin x integral of this is integral of sec x d x integral of sec x is log sec x plus tan x. So, what we are getting is u 1 as log sec x, u 2 as cosine x and u3as sin x minus logarithmic of sec x tan x.

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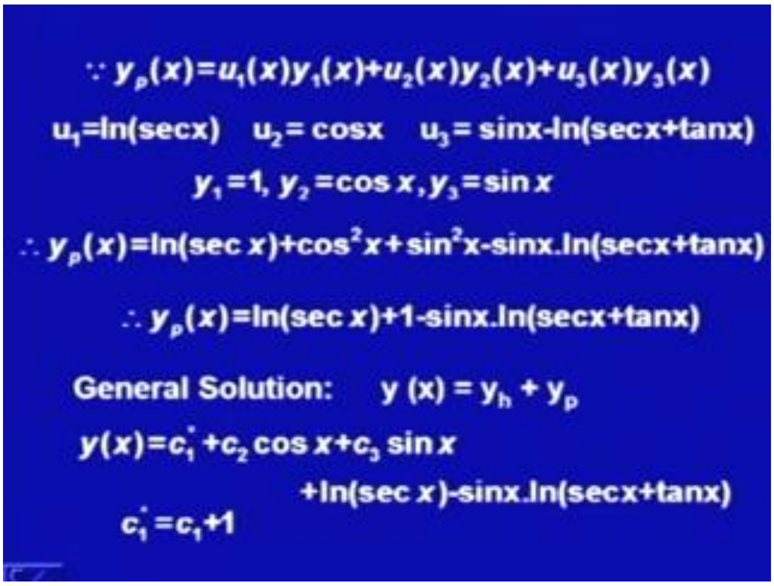

So, y p u 1 x y 1 x plus u 2 x y 2 x plus u 3 x y 3 x, since we are having is third order on. So, we will have only 3 ones u 1 and u 2 and u 3 just now we had obtained as such y 1 if you remember was 1 y 2 was cosine x and y 3 was sine x so write it, u 1 y 1 log sec x u 2 y 2 cosine is square x, u y 3 sine square x minus sine x times log sec x plus tan x. Now, this cos square x sine square x this will become as 1.

So, finally, y p I am getting is log sec x plus 1 minus sine x log sec x plus tan x, this is what is the particular solution of the given non homogeneous equation. The general solution we had already find it out in the terms of c 1 plus c 2 x cosine x plus c 3 sine x.

So, the general solution of the non homogenous equation y h plus y p would be c 1 star plus c 2 cosine x plus c 3 sine x plus log sec x minus sine x log sec x plus tan x.

You see here c 1 is star I have chosen as c 1 plus 1, because 1 we are getting from here in the particular solution and c 1 was from here. So, that is what we had put it in the constant terms that is all, so this is what is the solution of general solution of the given non homogeneous equation y triple dash plus y dash is equal to tan x.

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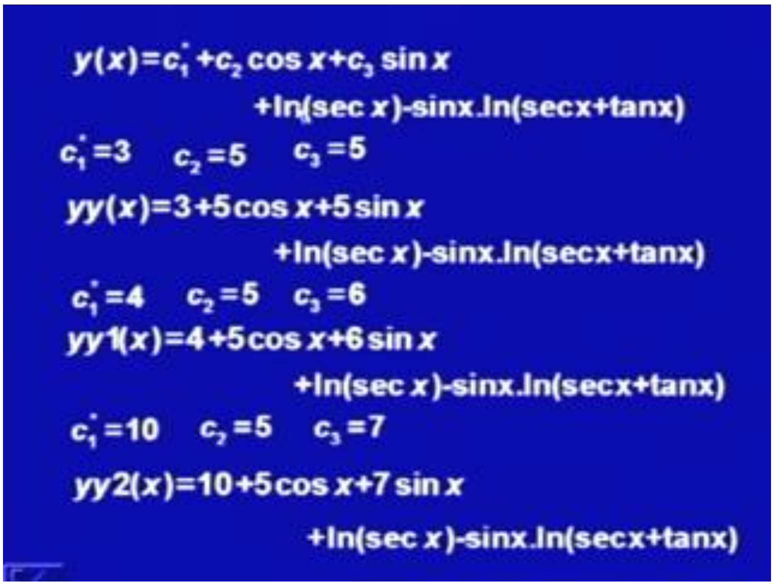

Again put certain values and see how they are looking in the graph, this is what is our general solution, if I choose c 1 star as 3 c 1 star is c 1 plus 1 so on, I am choosing that is whatever it may be the value so c 3, c 2 is 5 and c 3 also has 5. So my I am denoting this solution by y y x, so this should be 3 plus 5 cosine x plus 5 sine x and plus this terms log sec x minus sine x times log sec x plus tan x.

Let us, choose another values for this 1 that is 4 5 and 6, so what will be this, I am denoting by y y 1, so 4 plus 5 cos x plus 6 sine x and then the particular solution this part is as such. Another set of values 10 5 and 7, so what this y y 2 I am denoting, this is 10 plus 5 cosine x plus 7 sine x and the particular solution part as such. Let us, see how this graphs of these three solutions looks like that is I have chosen this y y, y y 1 y 2 and this y p, y p means is that log sec x plus 1 something.

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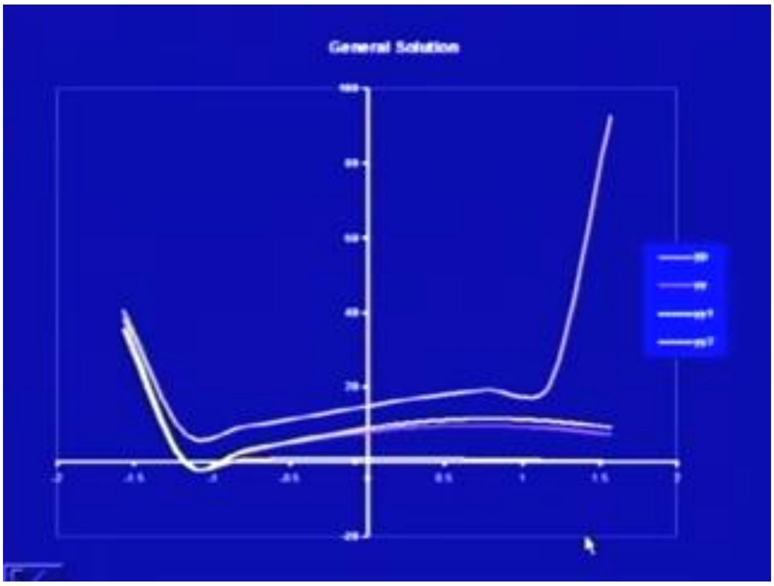

So you see here, we are having is this y p, y p is this solution which you are having is that is this yellow line this is y p that is log sec x. And this then we are having is that y y is the solution which we have chosen as at 0 that is we have taken the value as 3. So, c 1 star c 3 star is that c 1 plus 1 that is my c 1 would be 2 that is what we are having it here is that it should be somewhere going like up and like that we are having this different solutions y y 1 and y y 2. So, this are the graph it is looking like that is between minus pi by 2 to plus pi by 2, we are not having any values more than that the because the solution is existing only over there.

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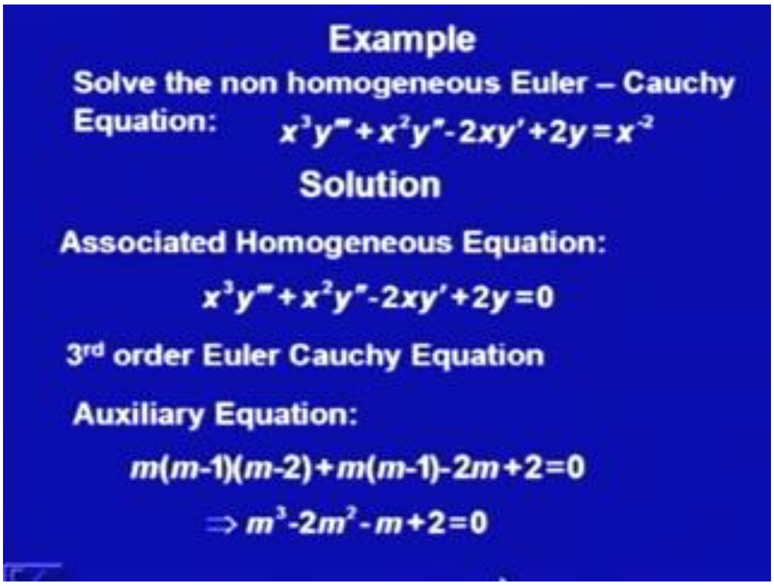

Let us see 1 more example, because in this example we had the homogeneous part as constant coefficients. So, solve the non homogeneous Euler-Cauchy equation x cube y triple dash plus x square y double dash minus 2 x y dash plus 2 y is equal to x to the power minus 2.

Now, we are having is that this is not in the standard form first thing, so when we would apply the method of variation of parameter we have to check it that we have to change it to the standard form. Let us, first start from here we are having that coefficients are also not the constants. And, the right hand side this is not of the form of polynomial this is x to the power minus 2. So, again we have to use the method of variation of parameter, see how we solve it.

The associated homogeneous equation corresponding to this non homogeneous equation would be x cube y triple dash plus x square y double dash minus 2 x y dash plus 2 y is equal to 0. To solve this we require the this is nothing but the third order Euler-Cauchy equation, we do know that in the Euler-Cauchy equations, the solution is of the form y is equal to x to the power m.

And, when we are putting this y as x to the power m and its derivative in the solution we are getting some auxiliary equation in the terms of m, what is that auxiliary equation that is I have to put here y triple dash. So, what will be the third derivative of x to the power m of course, m into m minus 1 into m minus 2 times x to the power m minus 3 and so on. So, what the auxiliary equation? We are getting is m into m minus 1 into m minus 2 plus m into m minus 1 minus 2 m plus 2 is equal to 0 let us, simplify this equation. It gives me m cube minus 2 m square minus m plus 2 is equal to 0. So, what we have got this is the auxiliary equation we have to find out the roots of this equation. So, that we can determine what will be the solution of this Euler-Cauchy equation.

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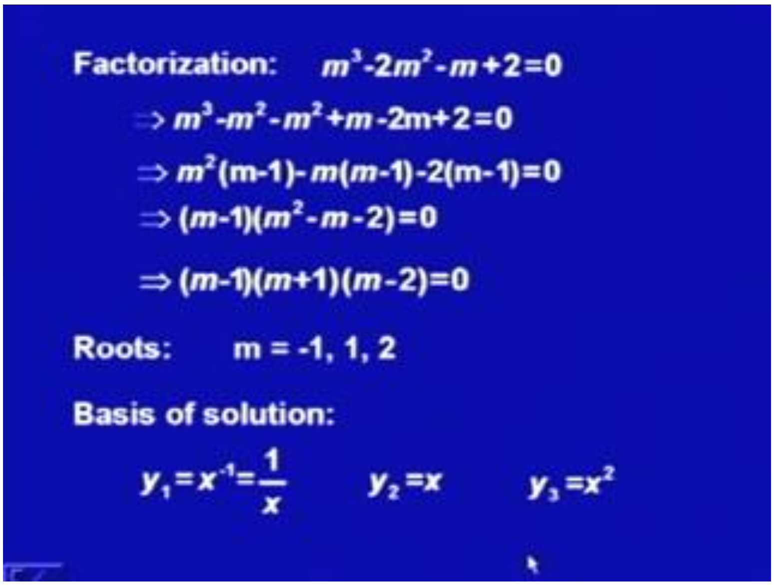

So, factorization gives me we have to factorize this we see it from here that is 1 would be 1 root. So, we just make it as m cube minus m square minus m square plus m minus 2 m plus 2 is equal to 0. Taking in from the first two terms the m square common from the second two terms minus m common from the last two terms minus 2, we would be getting is m minus 1 into m square minus m minus 2 is equal to 0. Again factorizing this, second term we get is this is m minus 1 into m plus 1 into m minus 2 is equal to 0.

So, the three roots of this auxiliary equations would be minus 1 corresponding to this 1 and 2, what will be now three solutions the basis of the solution, we do know that it should be x to the power m. So, what we would get here, y 1 as x to the power minus 1 which is as 1 by x y 2 as x and y 3 as x square we do know that these three solutions are linearly independent.

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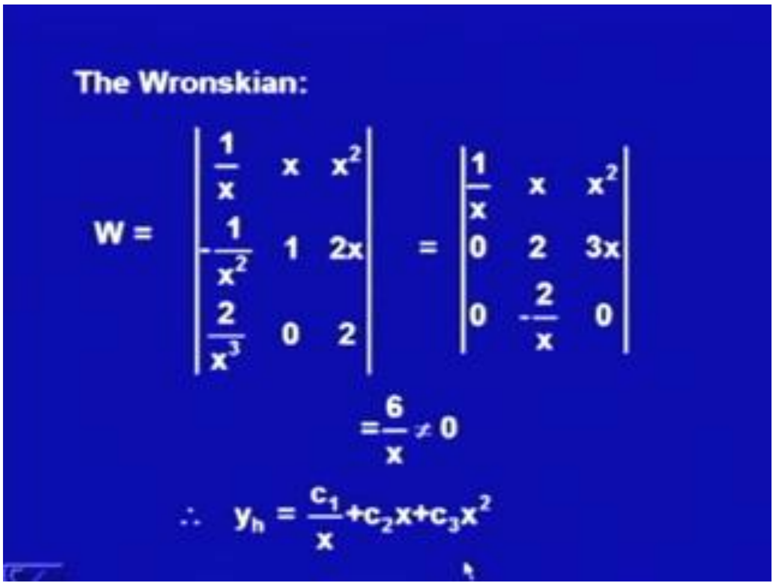

We want to find out this particular solution for the non homogeneous one for that we require the Wronskians and the different determinant. So, Wronskians of this three solutions 1 by x x x square its derivative is minus 1 by x square 1 and x squares 2 x again its derivative that is second derivative of this one or the derivative of this one 2 by x cube its derivative is 0 this derivative is 2. If I evaluate this determinant, so, I am just doing little bit a determinants, so we are making these two entries in the first column as 0 by multiplying the first row by 1 by x and adding to the second row.

Then, multiplying the first row by minus 2 by x square and then adding it to the third row we do get this determinant. This determinant we do know the by the method of determinants that this will not alter the value of the determinant, what we do get from, here if I expand this determinant with respect to first column. I would get 1 by x times or 6 by 6 so I am getting is it is 6 by x which is of course not 0. So, they are the three linearly independent solution, we have already knowing it they are not 0 . And this is what is my W now let us calculate my W 1 W 2 and W3. So that, we can find out that u 1 u 2 u 3 y h of course c 1 times 1 by x plus c 2 x plus c3 x square.

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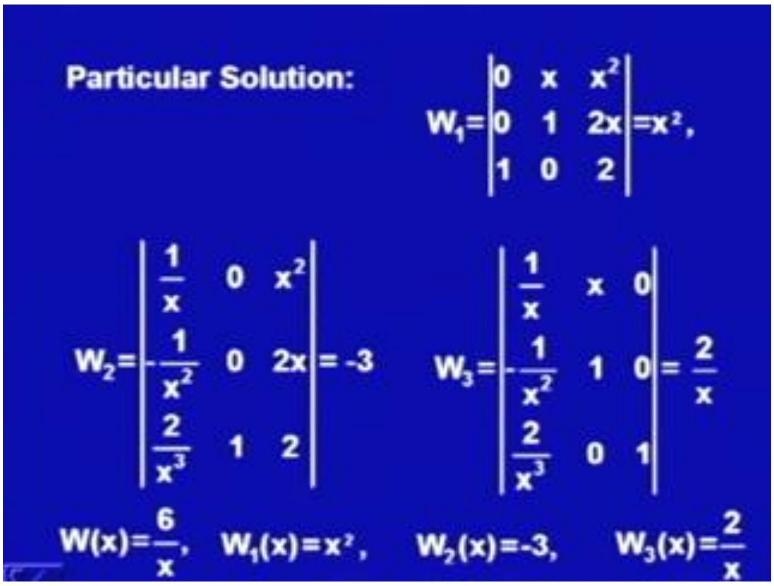

For particular solution W 1, the first column has been transformed from whatever is being there that is 0 0 1 and the rest two columns are as such, if I expand this with respect to the first column I do get is 2 x square plus minus x square that should be x square.

W 2 the first column is as such the second column has been transformed to the 0 0 1 again if I open it up with respect to this second column. So, the minus sign should be somewhere here and if I take these two things I would get from here 2 and from here minus 1 plus 1, so that is minus 3. Similarly, W 3 the third column we have changed to the 0 0 1 with respect to this if I am expanding this one, I would get 1 by x plus 1 by x that is 2 by x. So, we have got W x as 6 by x, W 1 as x square, W 2 as minus 3 and W 3 as 2 by x.

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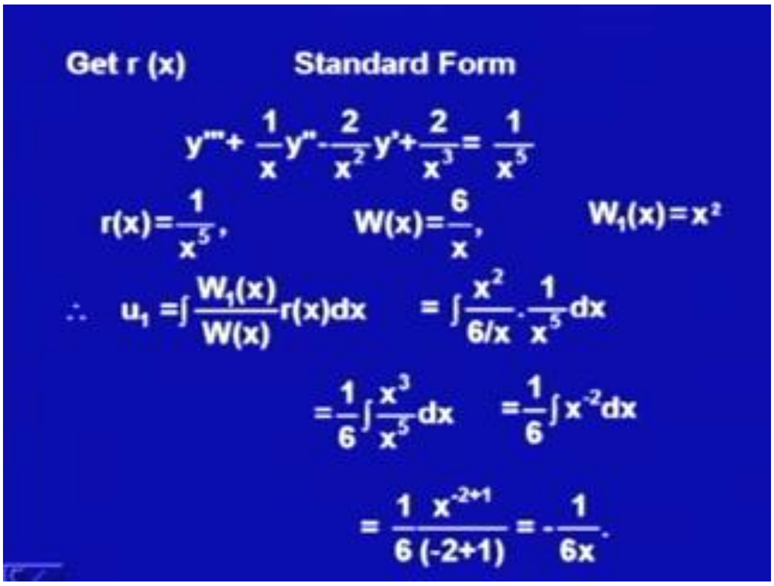

Now, we are ready to find out those functions u 1 u 2 u3 for that as I said the method of variation of parameter requires that the my non homogeneous equation must be in the standard form, then I can take that r x to be the special one.. So in the standard form, if I write that is I will divide my equation by x cube to make this part that is the coefficient of highest order to be 1, if I am dividing it by 1 by x cube I would get y triple dash plus 1 by x y dash plus minus 2 by x square y dash plus 2 by x cube is equal to 1 upon x to the power 5, that is now my r x is this x to the power minus 5 it should not be x to the power minus 2.

Since, the method of variation of parameter where we are taking is this W i upon W x into r x that r x must be from the standard form it should not contain any coefficient of the highest order derivative, that is why we have to make it sure that is this we are getting is x to the power minus 5.

So, r x is 1 upon x to the power 5 and W just now we had find out as this one, so w and W 1 are this one. So, what will be u 1 W 1 x upon W x times r x, now substitute this, x square upon 6 by x into 1 by x to the power 5 d x this would give me 1 by x times x to the power 3 divided by x to the power 5 d x that is x power minus 2 d x its integral I would get it as again I have explain it here all the process of the integration finally, we will get minus 1 by 6 x.

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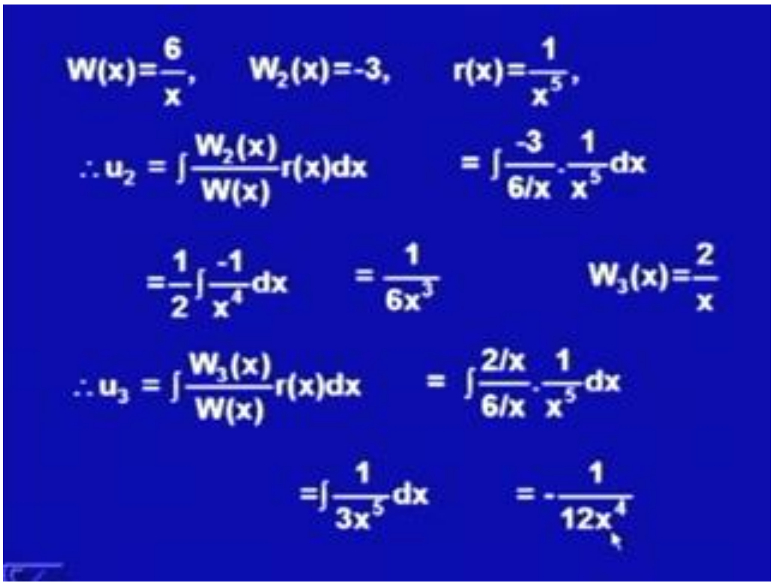

Now, what will be u 2, W x is 6 by x W 2 if you do remember we have got minus 3 so and r x is 1 by x to the power 5. So, W 2 by this formula we would get minus 3 divided by 6 by x into 1 by x to the power 5 d x which will simplify to me, as 1 by 2 minus 1 by x to the power 4 d x integral of this would be 1 by 6 x cube. Then u 3, where W 3 that is 2 by x, W x is 6 by x, by the formula we will get W 3 x upon W x into r x. So, substitute these values 2 by x upon 6 by x into 1 by x to the power 5 d x and getting it as 1 by 3 times x to the power 5 d x its integral gives me minus 1 by 12 x to the power 4.

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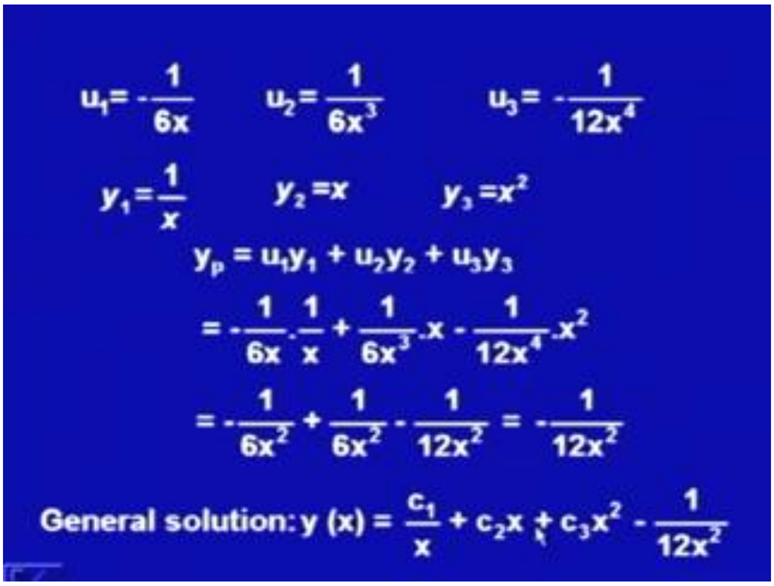

So, what we have got u 1 x as minus 1 by 6 x, u 2 x as 1 by 6 x cube, u 3 as minus 1 by 12 x to the power 4 my y 1 was 1 by x, y 2 was x and y 3 as x square. So, what should be my y p u 1 x y 1 x plus u 2 x y 2 x and plus u 3 x y x, so let us substitute minus 1 by 6 x into 1 by x plus 1 by 6 cube into x minus 1 by 12 times x to the power 4 into x square. Simplify it minus 1 by 6 x plus 1 minus 1 by 6 x square plus 1 by 6 x square and from here again minus 1 by 12 x square, what is up finally, it gives minus 1 by 12 x square. So, this is what is my y p.

So, what will be the general solution, y h would be c 1 by x plus c 2 x plus c3x square and y p is minus 1 by 12 x square. We are getting this as the general solution of the given non homogeneous differential equation that was the third order non homogeneous Euler-Cauchy equation. We were getting it this solution we to see is that is all my solutions 1 by x, x x square and 1 by 12 x square, all are the linearly independent. Let us again see how the graph is look like so for graph we have to give some values for c 1 c 2 c 3 u h first will choose y p as this 1 there is the particular solution of that given equation.

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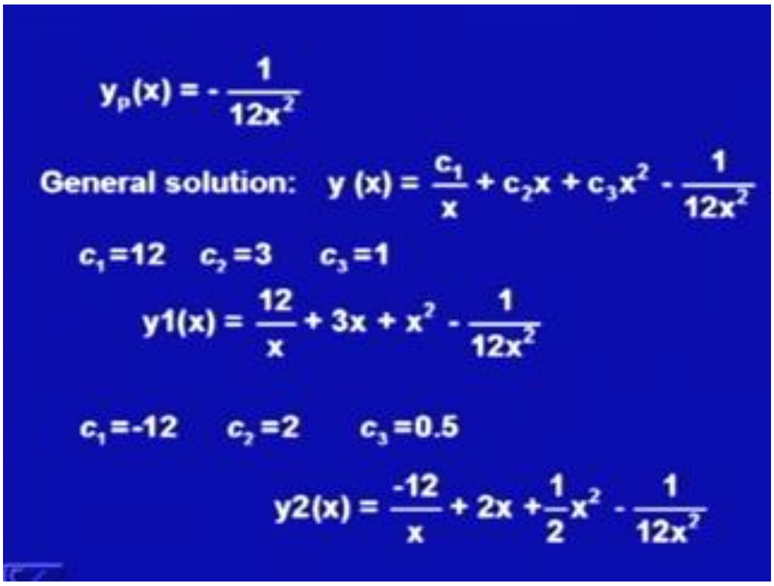

The general solution is this one, so here if I choose c 1 as 12, c 2 as 3 and c 3 as 1, remember here this is not going to give me the this equation is not solvable at 0. We have to take the interval after the 0 of the solution this function is not continuous at actually at 0. So, y 1 x would be this I am denoting the first solution that is where I am putting these particular values for c 1 c 2 c3as this one. Similarly, let us take another 1 that is c 1 I have taken as minus 12 c 2 as 2 and c 3 as 0.5, then another solution I am writing it.

So now, I am just showing you the graph of this y p that is one particular solution in which the solution of homogeneous equation has been taken as 0 only. And here, the solution of homogeneous equations we have taken some particular values just like that I have chosen it is not with any initial values.

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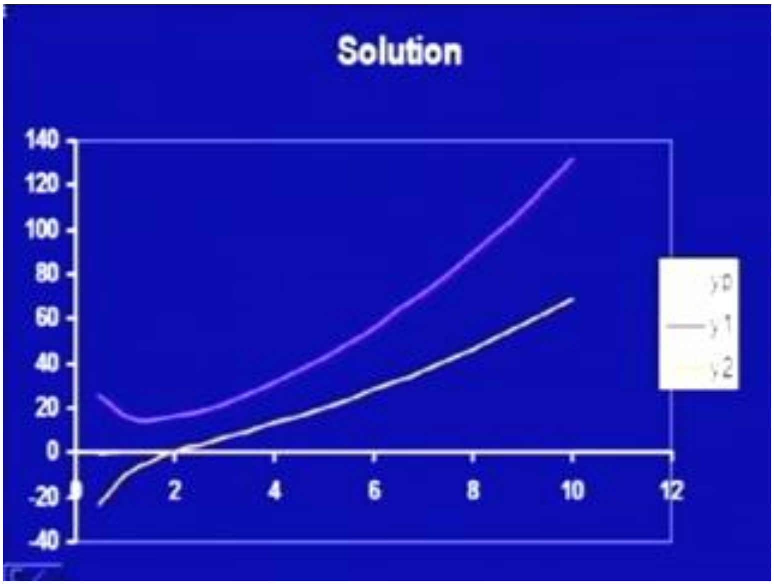

So, we do get this solution here you see is that minus 1 by 12 x square this is this what we are getting is that line which is completely coming on the x-axis showing, but its little bit slightly down to this one and then it will go up. And here, this is the solution y 2 where I have taken c 1 as minus 12 and here, this is with c 1 as plus 12, so that is on these sides we have taken these two solutions. So, what today we had learnt the non homogeneous differential equations linear one for the nth order in general 1 that is.

So, we had learn the second order linear differential equations homogeneous non homogenous, then we had proceeded for the further higher order one and we have done in general the nth order linear differential equations. Homogenous as well as non homogeneous we had covered the coefficients to be the constant and the coefficients to be of the form or the general form that function of x, but they has to be continuous for the solution to exist.

So, we could solve all those equations we had seen in between also that where are the applications in the engineering modelling I have taken very simple applications. Just to show that I show we can model those simple systems by the terms of differential equation and we can get the solutions we can check it with this one. So, that is what we have done with this linear differential equations that is all for today.

Thank you.