Mathematics - III
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Lecture - 3
Differential Equations of 1 st Order & Higher Degree

Welcome lecture series and differential equations. Today's lecture is an Differential Equations of First Order and Higher Degree. Till now we had learn various methods to solve the differential equations of first order and first degree. Now, we will learn again only in the first order, but of the higher degree, so let us first answer the question.

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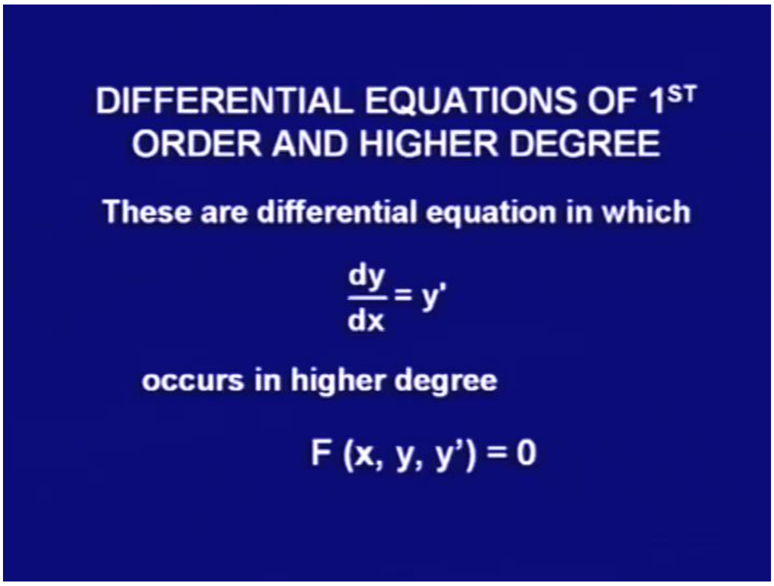

What are the differential equations, which we call of first order and higher degree, all those equations in which d y over d x are which we call y dash occurs in higher degrees. So, general form of this equations would be f of x y y dash is equal to 0, this is called implicit form.

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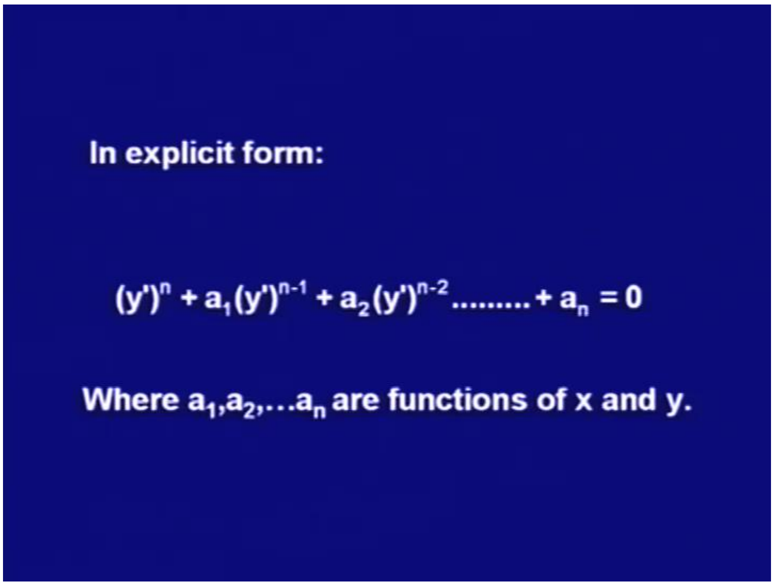

In explicit form when we do write what we do write it the general from would be, y dash to the power n plus a 1 times y dash to the power a n minus 1 plus a 2 times y dash to the power n minus 2 plus so on, plus a n is equal to 0. So, here we are having is that is the order is only first we are having only the terms y dash, and we do have it is different degrees, all the coefficients a 1 a 2 a n are functions of x and y.

We will learn three cases of this kind of equations the case one, equations which are solvable for y dash, case two equations which are solvable for y. And case three equations which are solvable for x, we will discuss these methods one by one, so let us discuss the first case.

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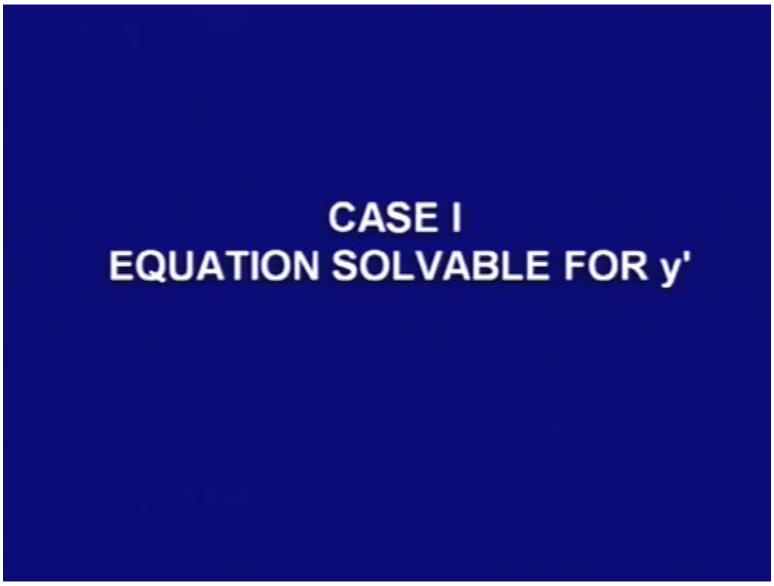

Equations solvable bar for y dash.

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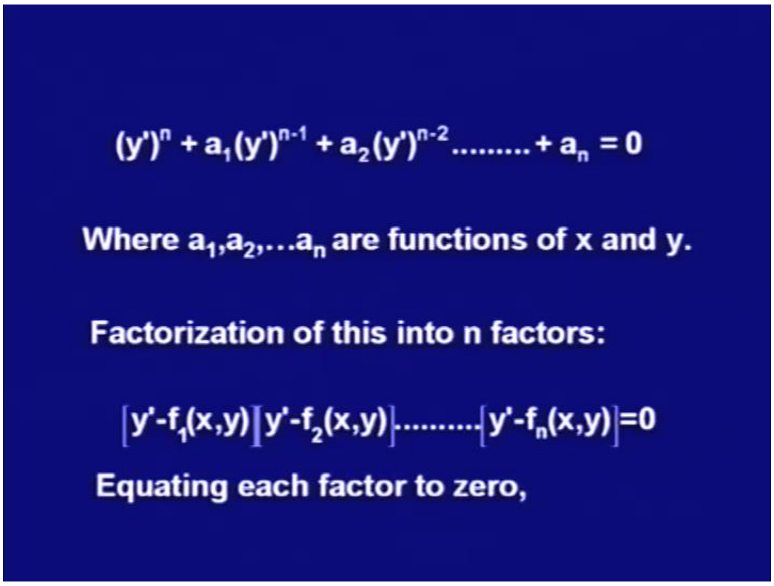

Again, we will write the general form of the equation is y dash to the power n plus a 1 y dash to the power n minus 1 plus a 2 y dash to the power n minus 2 and so on, plus a n is equal to 0, where this a 1, a 2, a n can contain both x and y. Since this is of nth degree equation we can factorize it into n factors. What how could we factorize it let us see that is let us say, we do get the factors as y dash is equal to y dash minus f 1 x y into y dash minus f 2 x y and so on, y dash minus f n x y is equal to 0. Now in this if I equate each factor to 0, what we would get we would get actually n differential equations where each differential equation is of first order.

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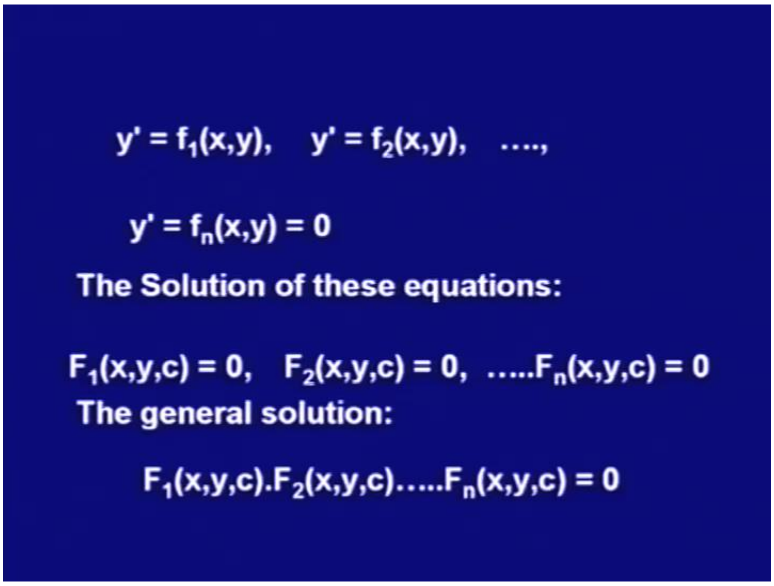

So, we would get the equations as y dash is equal to f 1 x y, y dash is equal to f 2 x y and so on, y dash is equal to f n x y. Let us say the solution of these, because we had already learnt the solving first order equations, so let us say this equation solution of this equations is f 1 x y c is equal to 0 f 2 x y c is equal to 0 and so on, f n x y c is equal to 0. This solutions here are written in the implicit form that is they would be some function of x y and of course, they would contain c, then the general solution we would write as the product of all these functions equate it to 0, that is if 1 x y c times f 2 x y c and so on, multiplied till f n x y c is equal to 0.

What we had learnt, if we do have a differential equation which has first order and degree n, we can factorize it into n factors, each factor then would give me equation of first order and first degree those methods we had already learn how to solve it. And whatever, the solutions we get for these equations, we write them in composite form as the multiplication and we get this as the general solution to understand this method let us try to do one example of this kind, so let us see the example.

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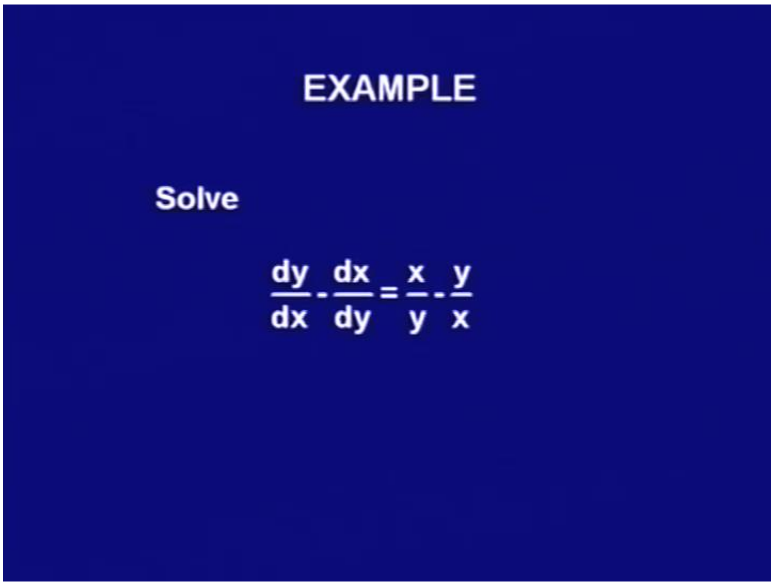

Solve d y over d x minus d x over d y is equal to x by y minus y by x, see how we will solve it.

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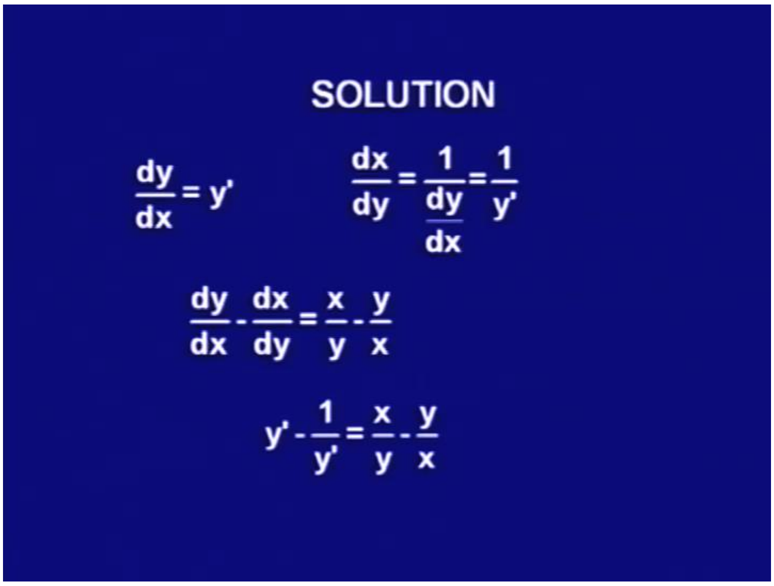

Let us first write d y over d x as y dash, then what would be d x over d y it would be one upon d y over d x, so we can write it as one upon y dash. Now instead of, d y over d x and d x over d y in the given equation we will write it as y dash and 1 by y dash, and thus got we would get in this equation y dash minus 1 upon y dash is equal to x upon y minus y upon x, now let us simplify at little bit more, so what we do get.

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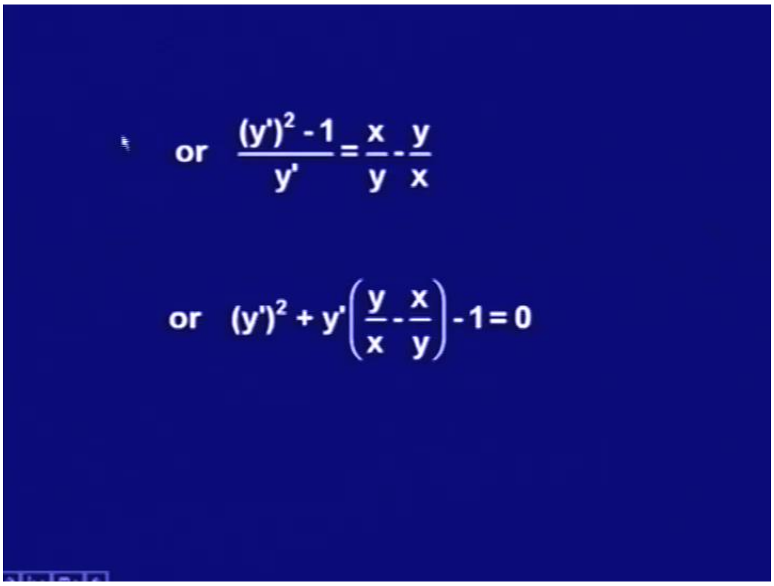

Y dash square minus one upon y dash is equal to x upon y minus y upon x, again take the cross products what we do get, y dash square plus y dash y upon x minus x upon y minus one is equal to 0. Now, we see this is the differential equation which is having terms of involving y dash only that is the first order, but degree is two here we have got the second degree equation, now if I factorize this equation it would give me two factors see what those two factors are...

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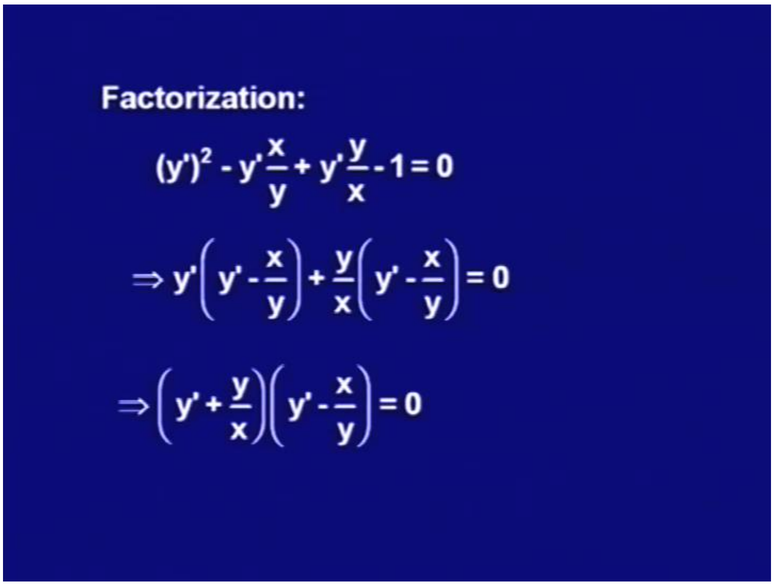

So, for factorization I am rewriting this given equation as, y dash square minus y dash times x upon y plus y dash times y upon x minus 1. Now, from the first two terms we will take the y dash as common what we do get, y dash minus x upon y from the second two terms you will take y by x as common we would get, y dash minus x upon y at we would get that, y dash plus y upon x and y dash minus x upon y as the two factors of this equation. Now, equating each of these two factors to 0 we will get two linear of first order equations.

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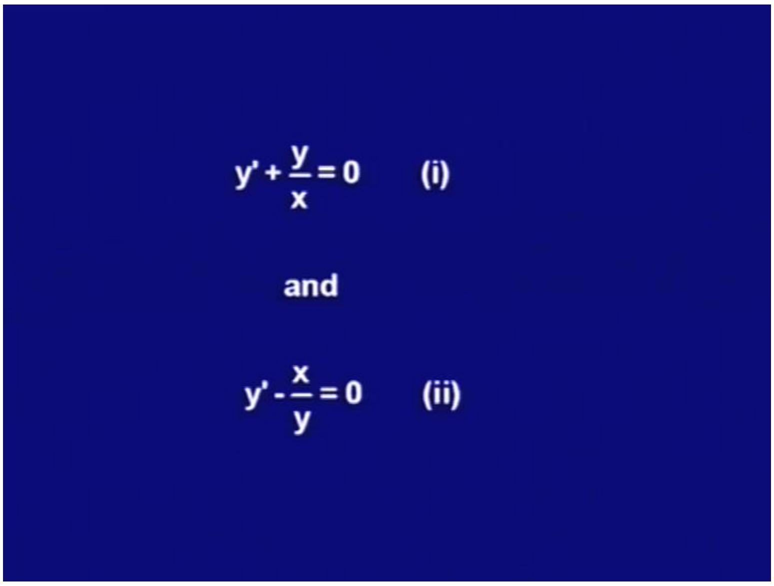

The first equation, y dash plus y by x is equal to 0 and second is, y dash minus x upon y is equal to 0. Now, we will find out the solution of these equations one by one. Let us consider the first equation first.

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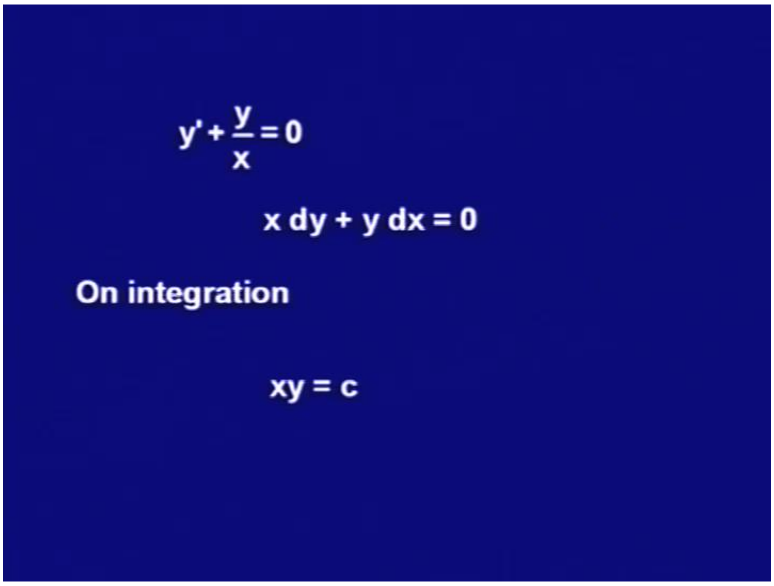

Equation y dash plus y upon x is equal to y, now y dash is d y over d x we would use the method of variables ((Refer Time: 07:55)). So, we can write it as x d y plus y d x is equal to 0, now integrate on both the sides we do get, x y is equal to c this is nothing but, the derivative of x into y. So, right hand side would give me the constant c, so we have got test the solution in the first equation.

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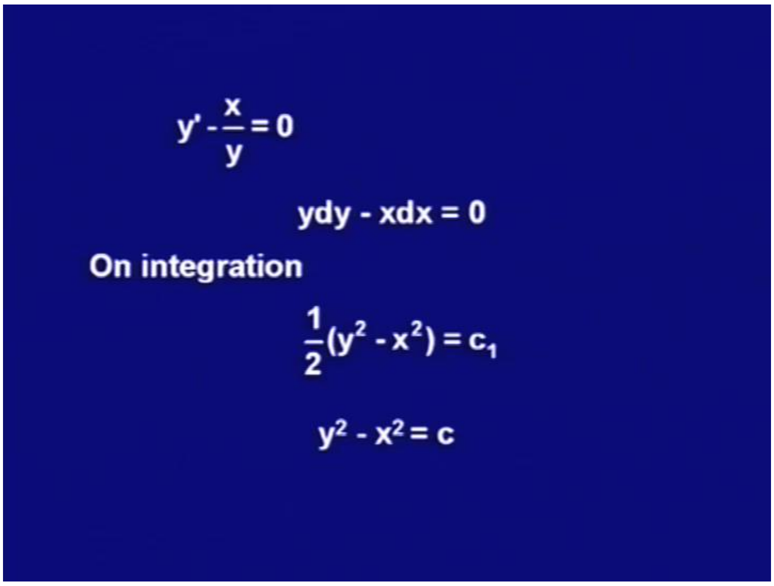

Now, consider the second equation, y dash minus x upon y is equal to 0, if I write it as y dash as d y over d x we could rewrite it as y d y minus x d x is equal to 0. Now, integrating on both the sides what we do get see the y d y over d x would give me, y square by 2 x d x would give me, x square by 2 on them right hand side just a constant or in other words we could write it as, y square minus x square is equal to c. So, we have got the other solution as y square minus x square is equal to c, now we have got the two solutions for our equation.

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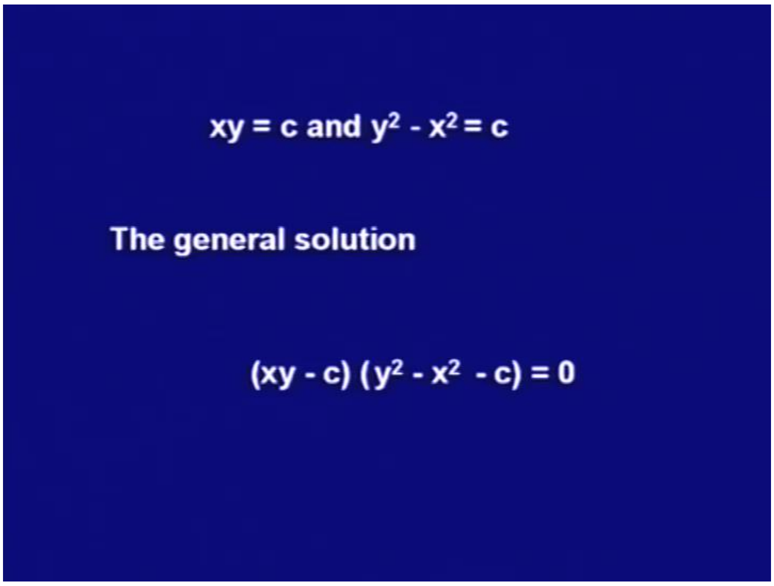

First x y is equal to c and other y square minus x square is equal to c, now combine these two to get the general solution what we do get, x y minus c into y square minus x square minus c is equal to 0, this is the solution of the given differential equation. Let us try to do one more example, so that we do understand little bit more about this method.

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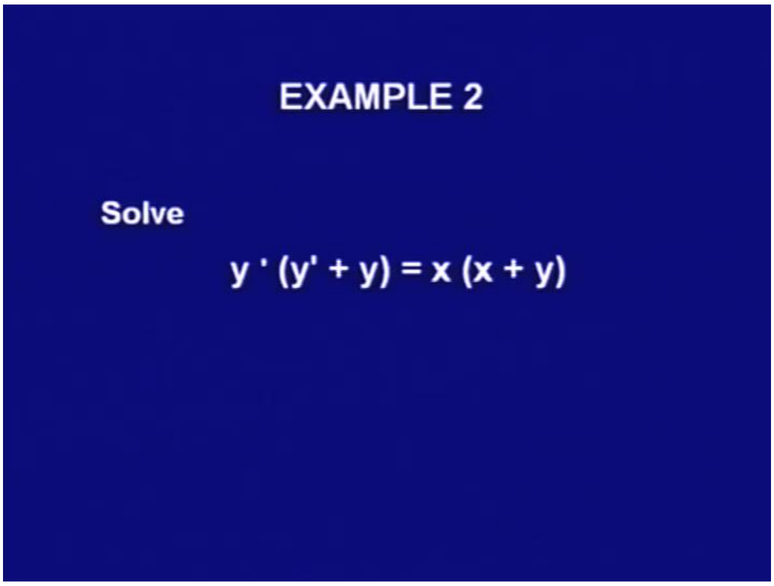

The second example is solved is y dash into y dash plus y is equal to x into x plus y.

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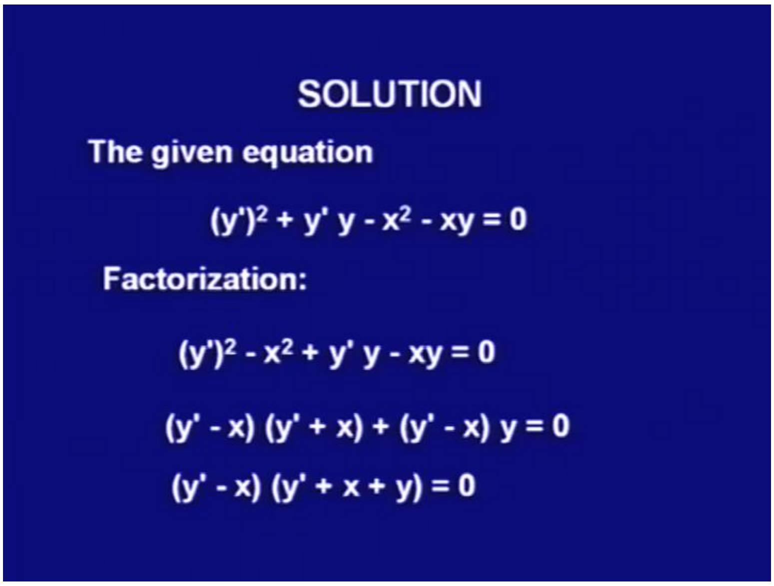

See first we would rewrite this equation as, y dash square plus y dash y minus x square minus x y is equal to 0. Now, this is equation of first order and second degree we do have that derivative of y that is y dash it is taking the power 2, so again if I do factorize I would get two linear two equations of first order, now let us try to factorize it. So, for factorization I would again rewrite it, y square minus x square plus y dash y minus x y, now the first two terms y dash square minus x square the factor gives, y dash minus x into y dash plus x plus.

The next two terms will take common y we would get y dash minus x into y, now this would give me a final factors as y dash minus x into y dash plus x plus y is equal to 0. So, what we have got these two factors if I do equate them with the 0 separately we would get two linear equations.

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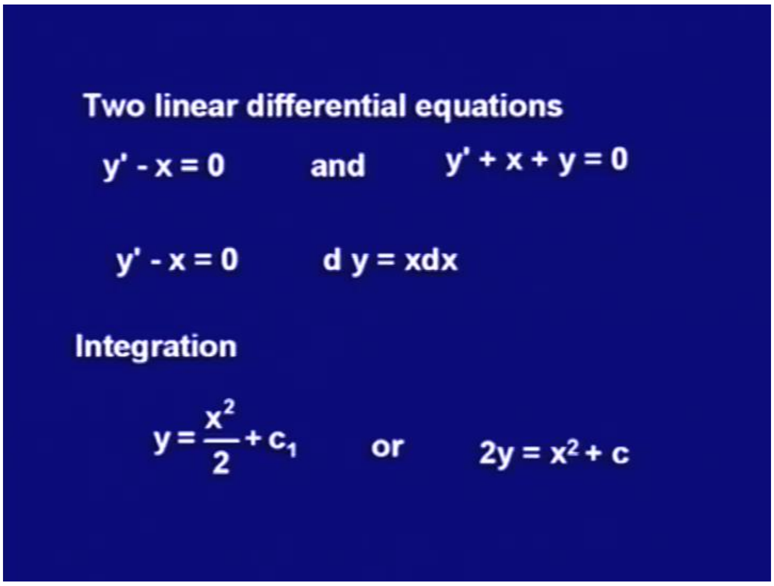

Namely, y dash minus x is equal to 0 and y dash plus x plus y is equal to 0 again we will solve these equations one by one, so let us take the first equation first y dash minus x is equal to 0. This is the linear equation we can also solve it directly, because this equation we do recognize very well, this is d y over d x is equal to x we can write it as d y is equal to x d x integrating on both the sides we do get, y as x square by 2 plus c 1 or we can write it as, y 2 y is equal to x square plus c, this is the solution of the first equation, now let us take the second equation.

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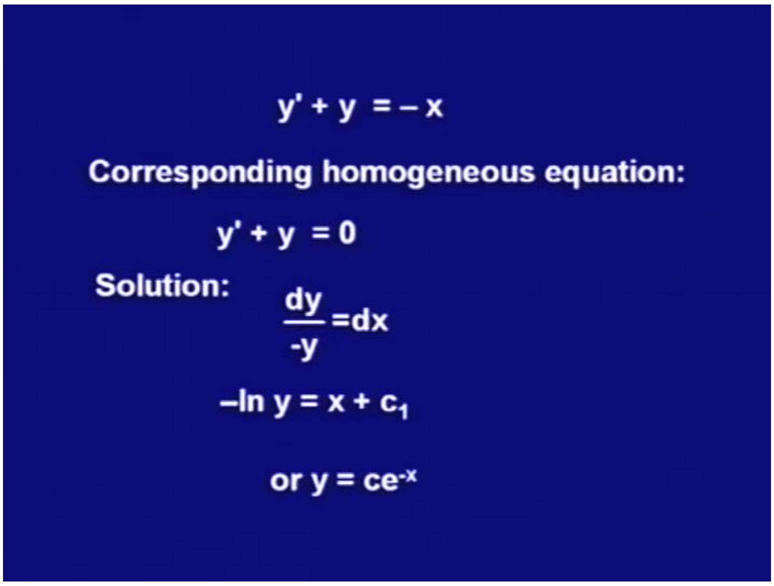

Y dash plus y is equal to minus x, now this is first order linear equation, but non homogeneous what will be the corresponding homogeneous equation that is y dash plus y is equal to 0. For solving it either we can use the method of solving linear equations or we can again use directly writing d y over d x and separating the variables, so here I have use the separating of the variable, I have got d y over minus y is equal to d x integrating on both the sides we do get, minus log y is equal to x plus c 1 or we could write y is equal to c e to the power minus x. Now, for getting the solution of non homogeneous equation we have to assume one solution.

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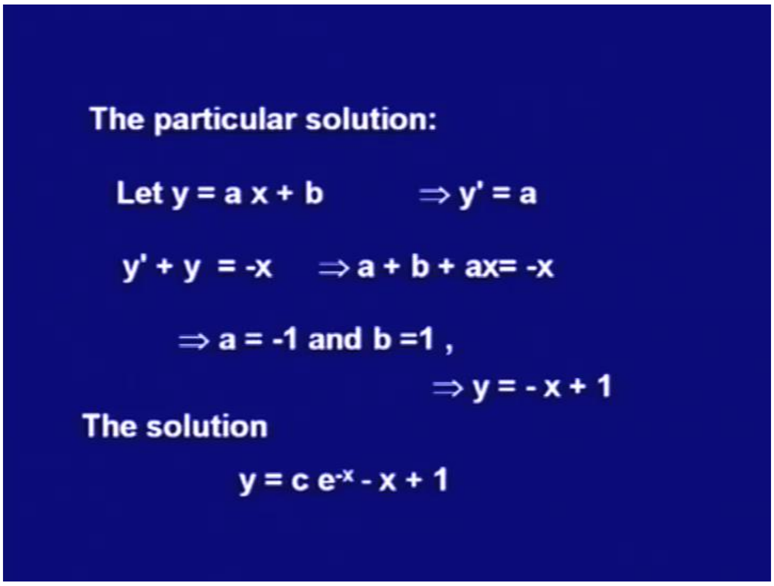

So let us say, the particular solution let y is equal to a x plus b what this gives that, y dash is a if this y and y dash we put in our given equation that y dash plus y is equal to minus x we would get that, a plus b plus a x is equal to minus x. Now, equate the coefficients of x and the constant we would get, a plus b as 0 and a as minus one thus we are getting the result that a a s minus 1 and b as 1 thus we would be getting the particular solution as, y is equal to minus x plus 1. So, what we have got the general solution of our second equation that solution is y is equal to c times e to the power minus x minus x plus 1.

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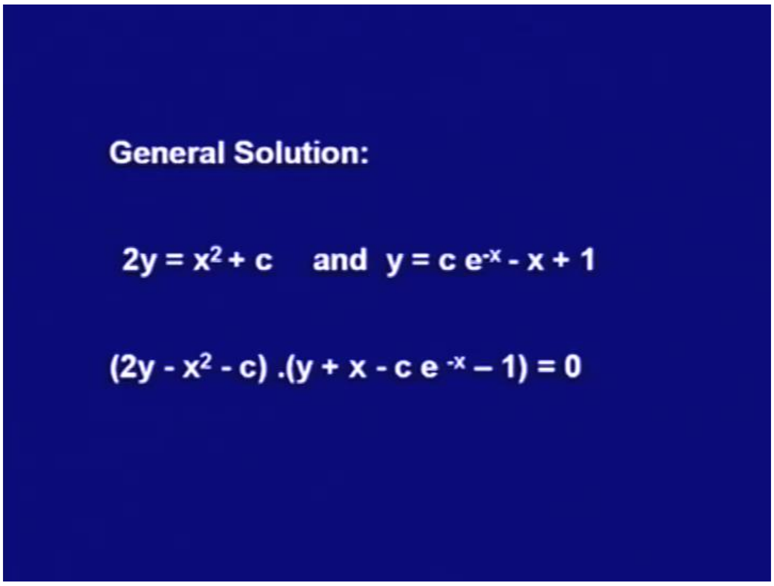

Now, what we have got we have got two solutions, one 2 y is equal to x square plus c another y is equal to c times e to the power minus x minus of x plus 1, these are constituting our general solution to get a final general solution we would write them as the multiplication of the two functions. So, what we are getting 2 y minus x square minus c multiplied by y plus x minus c e to the power minus x minus 1 is equal to 0 this is the solution of the given differential equation.

So, we had learnt of first method where, we are writing the equations in the form of degree of y dash as in we are factorizing it into n terms, which results each one as differential equation of first order and first degree. There could be of different kind of differential equations we can solve them, while using the methods which we have already learn and for general solution of that nth degree equation, what we do is we multiply all the solutions writing in the implicit form or writing the form y minus f of x y that kind of function equating them to be 0 thus we have learnt of first case

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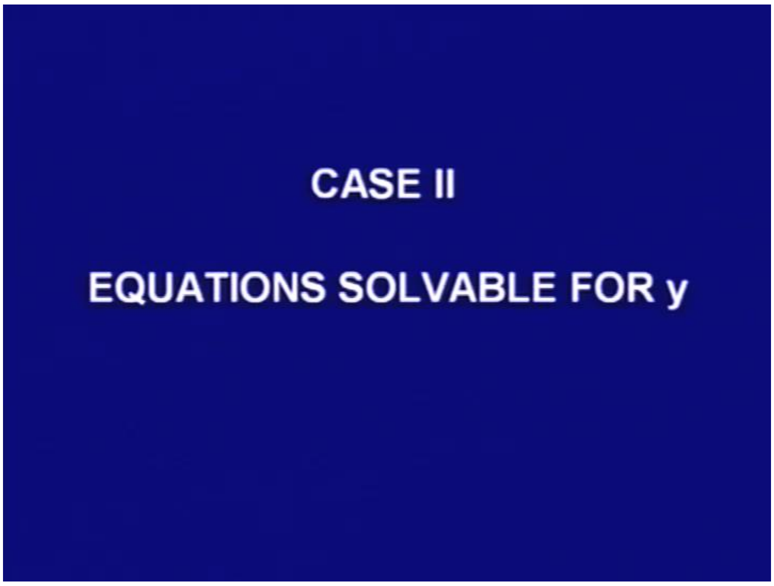

Now, go to the second case, where equations solvable for y, this is a typical case we see.

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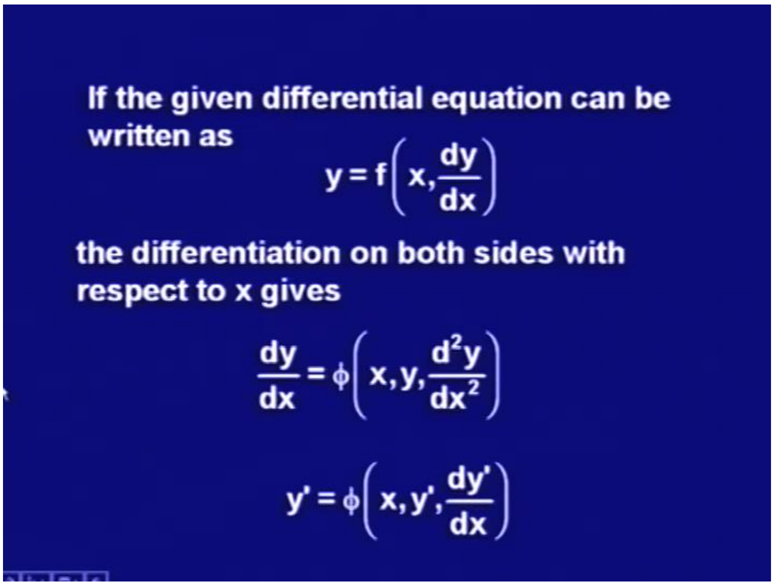

If our nth degree equation can be written as, y is equal to a function of x and d y over d x that is we are writing it as a function of x and y dash and that is equated to y that is why we are calling solvable for y that is we can write y is equal to something. Now, if I differentiate this on both the sides with respect to x what we would get, d y over d x is equal to, now let us say another function of phi of x y and d 2 y over d x 2, since what we are having is derivative of y would be d y over d x derivative of x may be there constant of whatever the function of x we may get x.

Then we may get y also as well and a derivative of d y over d x that is the second derivative of y d 2 y over d x 2, so we would get d y over d x as a function of x y and d 2 y over d x 2. Or another words what we are writing, y dash is equal to a function of x y dash and d y dash over d x that is now, I am writing this equation as y dash as the function of x y dash and derivative of y dash, now you see is that is what we have having something is here what we had y a function of x and y dash here we are having y dash as a function of x y dash and derivative of y dash.

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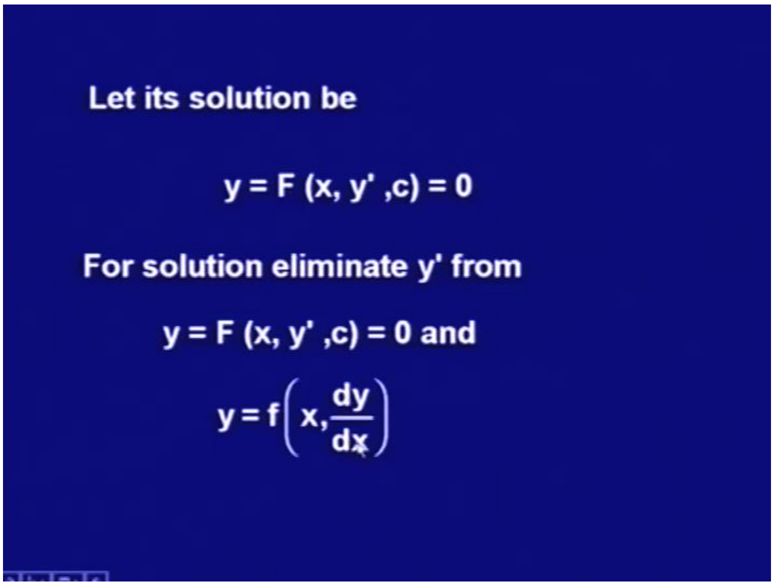

Let us say the solution of this kind of equation is a function of course, it will contain x y dash and c. So, for solution of the equation what we will do we will eliminate this y dash from this solution and though, original equation which we do have y is equal to f of x d y over d x and that would give me the solution of our equation y is equal to f x d y over d x, many times it may be not possible to eliminate.

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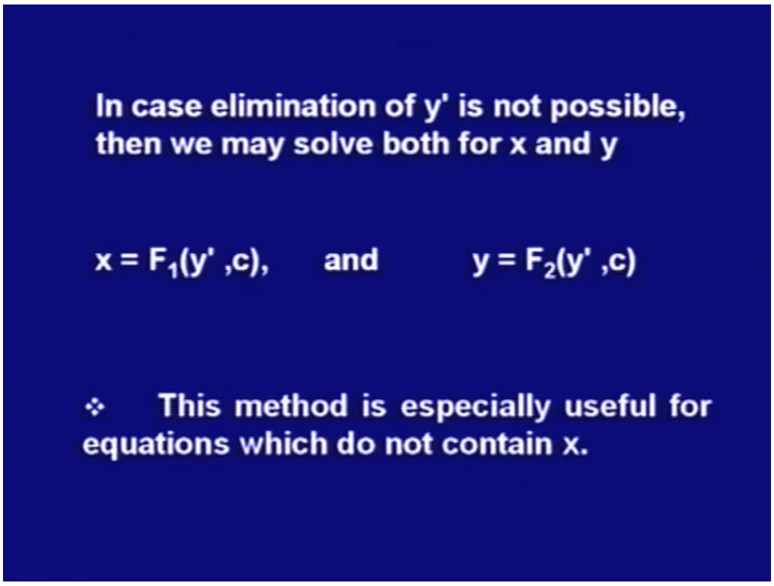

That is the equations we are getting they are not in simple form and we are not able to eliminate y dash then what we do is, we solve for both x and y and we write the solution as this form this is again a form of solution. Where we are saying is x is the function of y dash and c and y is also a function of y dash and c and of course, they are different functions here y dash is not being considered as d y over d x rather we are giving the solution as in the terms of y dash, so y dash we consider as a parameter. This method is especially useful for those equations which do not contain the terms of x that is it may contain the term of x square or something or x multiplied something, but not simply the terms of x.

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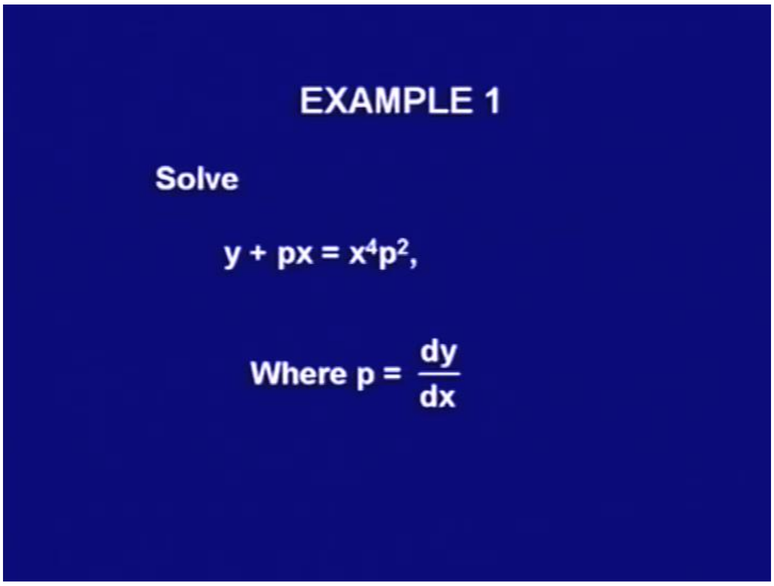

Let us do one example over here see, solve y plus p x is equal to x to the power 4 times p square, here p is being used to give the notation d y over d x if you do remember this method, we are doing is where we are getting the method equations involving y dash. So, we are using y dash as another parameter and we are taking it p s d y over d x, now see how we are going to solve using this technique.

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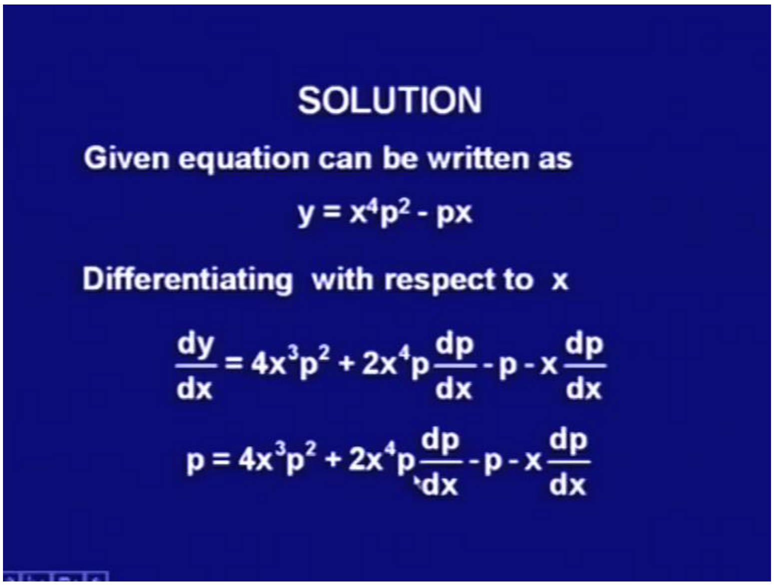

So, this given equation we can write as y is equal to x to the power 4 p square minus p x and now, we can write this equation as, y is equal to some terms of x and p here p is of course, y dash. Now, differentiate it with the respective to x what I will get first term d y over d x here the derivative of x to the power 4 p square this would be 4 x cube and p square as such, then we do have x to the power 4 as that what will be the derivative of p square 2 p and p is again a function of x, because it is d y over d x.

So, you I will write it as d p over d x similarly for the second term, minus p minus x times d p over d x, now d y over d x is nothing, but p, so we I will write it again and we do get that p is equal to 4 x cube p square plus 2 x to the power 4 p d p over d x minus p minus x d p over d x.

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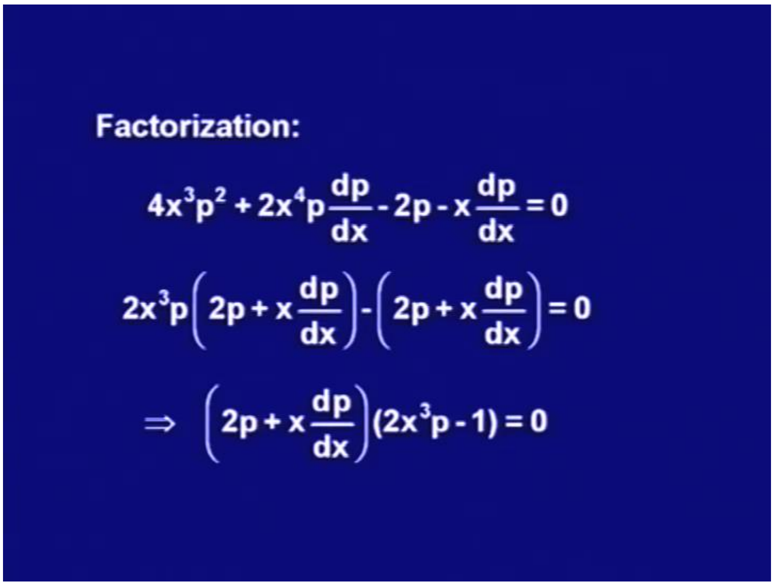

Now, again rewriting it and factorizing this one what we are getting is, now we are getting the equation in p and x and involving the term d p over d x that is we are having is again the first order equation, now we will factorize it you see how we are going to factorize it. From first two terms we are taking common to x cube p and we are getting the factor as 2 p plus x d p by d x minus from here we are just taking the common minus 1 and 2 p plus x upon d p over d x. So, we do get the two factors 2 p plus x d p over d x and 2 x cube p minus 1, now if I equate both these factors to 0 we would get two equations.

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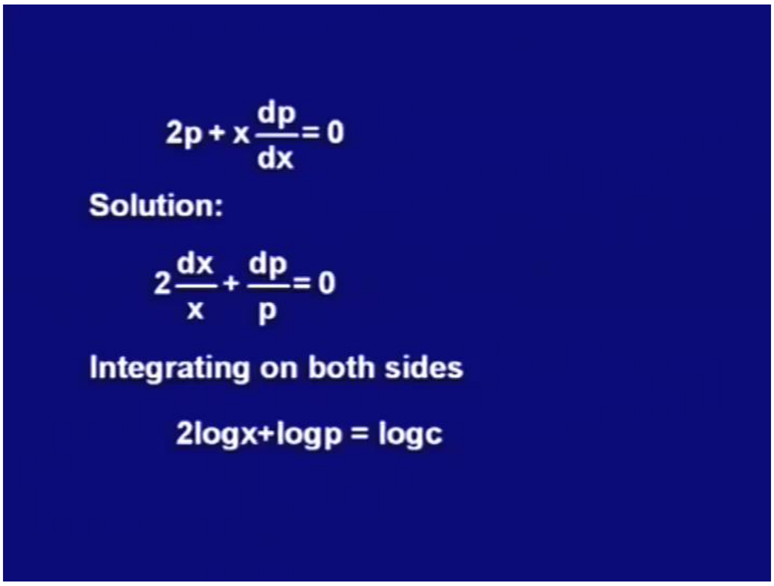

The first equation, 2 p plus x d p over d x is equal to 0 and other equation of course, we will talk later on this equation is actually a differential equation in p and x. So, we will treat it as a differential equation of first order in p and x where p is a function of x. So, we can solve it we do know the method to solve this first order equations linear, so we just choose again the variable separable method we do get, 2 d x over x plus d p over p is equal to 0. If I integrate it on both the sides I do get, 2 log x that is integration of 1 by x on integration of this as log p this constant again I have written here as a log c you do not understand that we write like this when we have to make the terms simpler.

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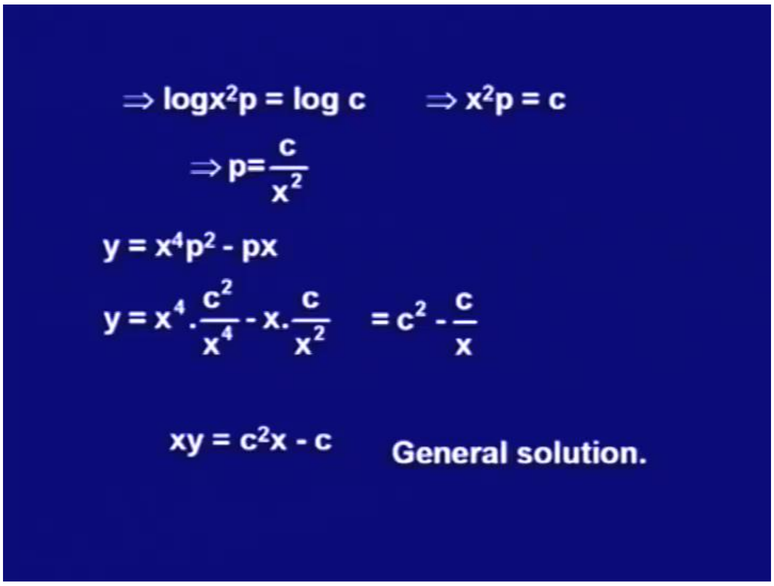

So, what we have got we have got log x square p is equal to log c or x square p is equal to c, so this is the solution of the first equation. Now, from here we do get p s c upon x square, now we have to solve the solution of this solution and the given equations, so that we can get the solution as y we have to eliminate p actually, so what we do is that here we are getting the value of the p as c by x square we will use this in number original equation, where we are having y is equal to x to the power 4 p square minus p x.

Now, if I put p over here what I would get y as x to the power 4 for p square we are writing c square upon x to the power 4 minus x and p we are writing c over x square simplification what it gives, c square times c by x or we are getting is x y is equal to c square x minus c. So, we have got this as the solution on this we would call the general solution.

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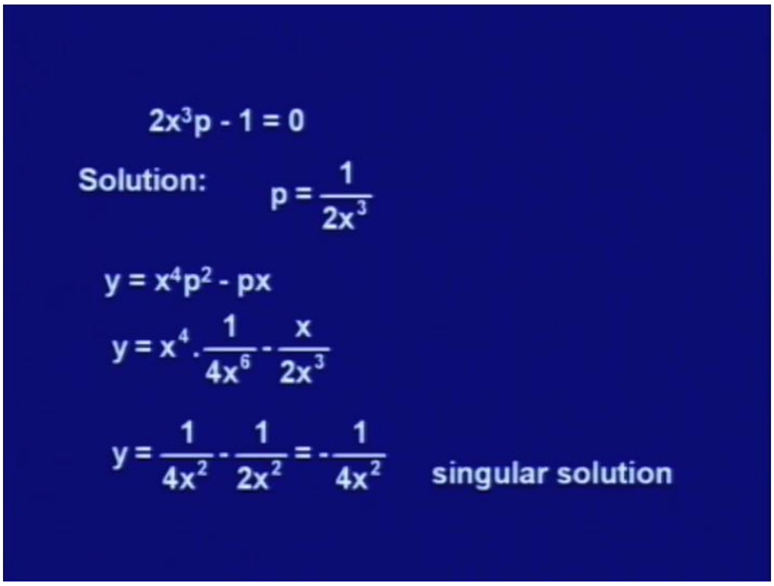

Now what will happen to the other equation which we have got in the factor let us see the other part we have got in the factor was 2 x cube p minus 1, if I see this as in x and p this is simple equation, so from here we do get the value of p, as 1 upon 2 x cube. Now, try to put this in the given equation that y is equal to x to the power 4 p square minus p x and what we would get we would get as, p square 1 upon 4 x to the power 6, because here the value of the p is 1 upon 2 x cube and here p is 1 upon 2 x cube into x. What it gives it gives me, 1 upon 4 x square minus x upon 1 upon 2 x square that is we would get, minus 1 by 4 x is square.

Now, this is called singular solution y if we see is that in the previous one what we have got that solution this solution cannot be obtained by putting the constant any value in the previous general solution. Now, we are not going to talk about the singular solutions many times, so we could make the this solution omit the solution in we can work only with the differential equation and get the general solution that is the first factor only to clarify more let us do one more example.

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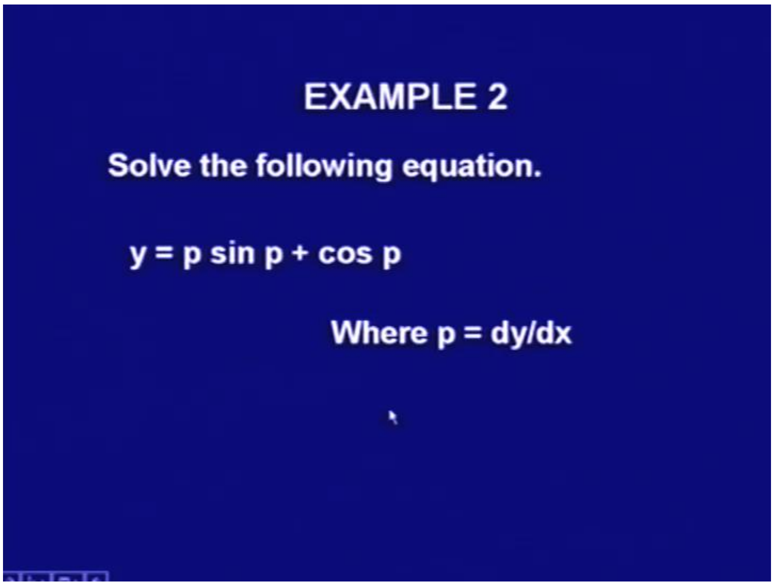

Solve the equation y is equal to p sin p plus cos p, again here p means d y over d x that is y dash see how you are going to solve it.

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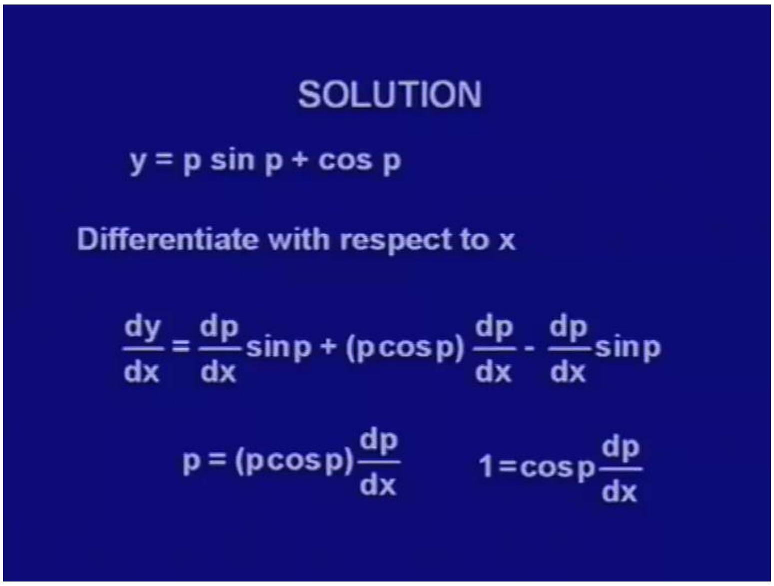

The Solution itself is given in the simple form that is, y is equal to d p sin p plus cos p we will differentiate it on both the sides with respective x what we would be getting, y d y over d x is equal to, now here d p by d x sin p plus p and the derivative of sin p is cosine p and then p is d p by d x similarly, for the second part it would be sin p and d p by d x. What we are getting is again we would write d y over d x as p.

So what we are getting p is equal to now here, d p by d x sin p and d p by that is why you see is that is this is been written in this form they are getting cancel at off we are getting is p sin p, sorry p is equal to p cos p d p over d x. Or 1 is equal to cos p times d p over d x again this is a equation, which we are getting in the first order equation.

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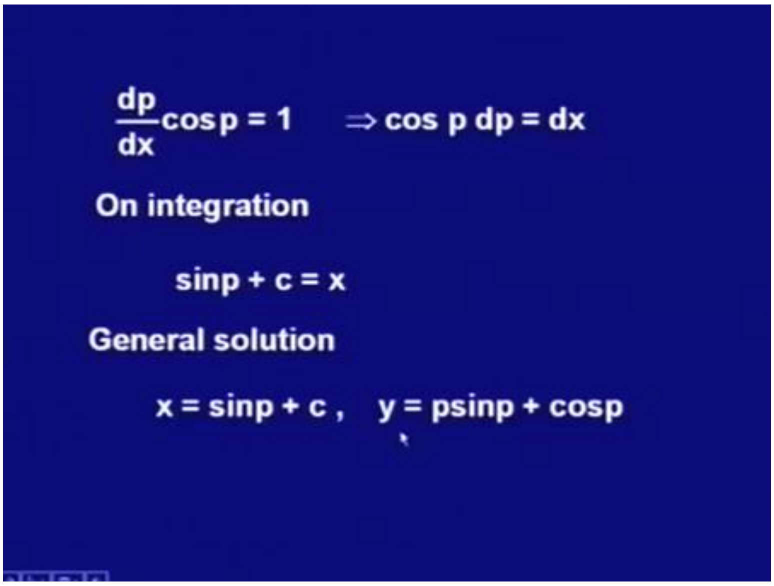

If I rewrite it I could write cos p d p is equal to d x differentiating on both the side integrating it on both the sides to get the solution, we get sin p plus c is equal to x you say is that is here, I have use this see just for the simplification. Now, we have got a equation sin p plus c is equal to x now, I cannot simplify it for p, because it will involve this which is involving this trigonometric term sin, so it will not give me simple solution or single solution.

Another equation what is the original equation we are having that was actually y is equal to p sin p plus cos p, so what you I will write the general solution here that x is equal to sin p plus c and y is equal to p sin p plus cos p. So, this we would write the solution in the form of x and y rather than giving y is equal to this, so we do write this is the solution of our giving differential equation.

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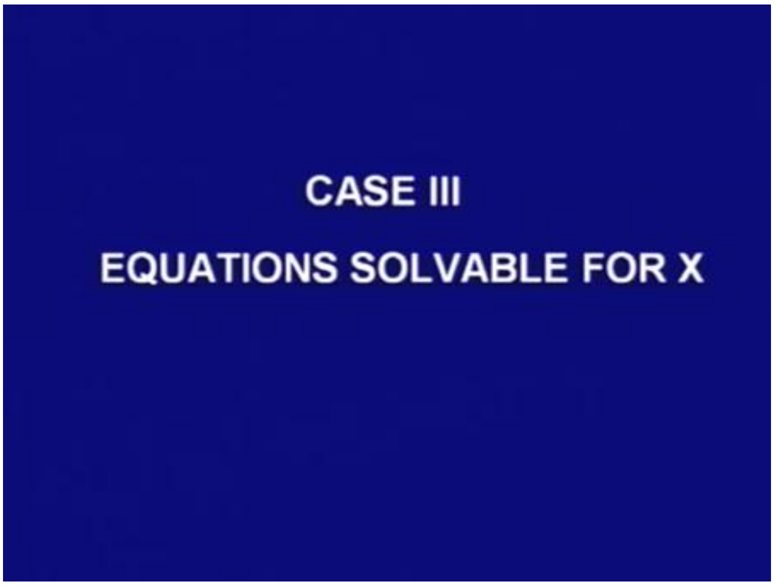

Now, let us move to the third case, equations solvable for x what are these equations?

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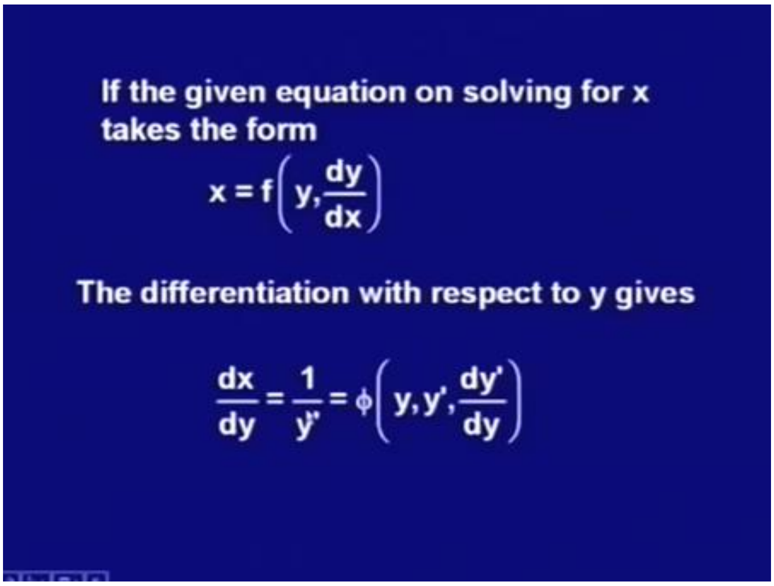

If the general equation of nth degree, if I could write in this form x is equal to a function of y and d y over d x you seen the previous case we had written y as a function of, x and d y over d x here we are writing it as x as a function of y and d y over d x. If this is possible what we will do, we will differentiate this with the respect to y what we would get, d x over d y this we can write as, 1 upon y dash as a function of y y dash d y dash over d y thus we are getting equation as, 1 upon y dash as a function of y y dash d y dash over d y this is again differential equation.

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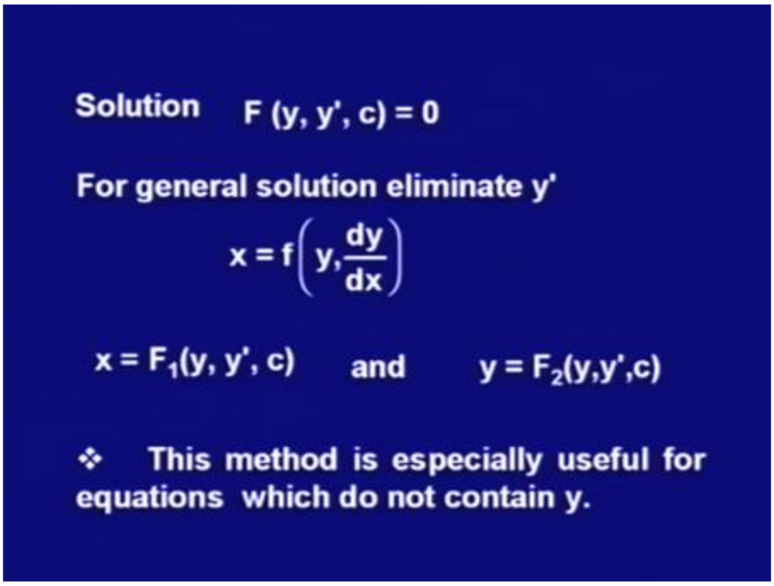

So, if the solution of this is of the form y y dash c is equal to 0, we could eliminate from the solution and the original equation the term of y dash and that will give me the general solution if it is not possible. So, let us say that is the solution we are getting is, we I will eliminate y dash from this equation and the solution x is equal to f 1 y dash y and c and this. If it is not possible to eliminate this y dash we can write this as a function one function of, y y dash and c in the x and another as, y is equal to f 2 y y dash as c as just as in the previous method that means, we are getting two values or 2 equations one is for x another for y.

Here we will treat this y dash as the parameter and this we will called the solution of the differential equation. This method is especially useful for the equations which do not contain the terms having only the term of y, so let us see the method to for how to solve it.

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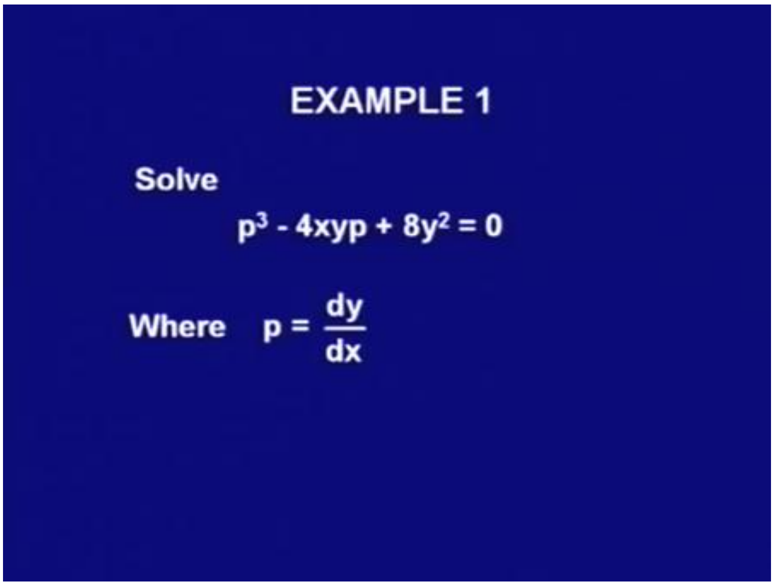

So, let us see one example question is solve p cube minus 4 x y times p plus 8 y square is equal to 0, here again p we are using for d y over d x.

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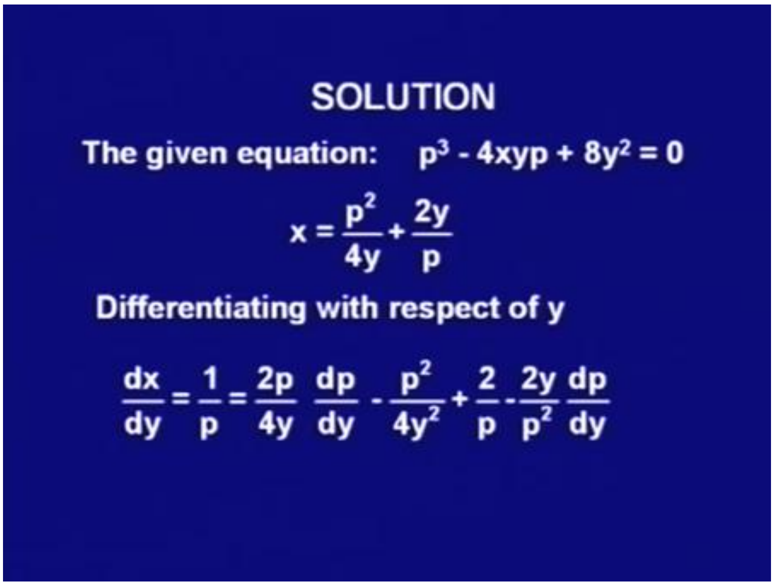

Now, the given equation we do write as p cube minus 4 x y p plus 8 y square is equal to 0, this we can write as x is equal to something. So, let us see that is how you could write, we could write x is equal to p square upon 4 y plus 2 y over p that is this is equation which we could write as a function of y and d y over d x that is p, now differentiate this both the side with respect to x.

What we are getting is d x over d y which we could write as one upon p derivative of p square upon 2 y. We are differentiating with the respect to y 2 p upon 4 y d p over d y minus p square as such derivative of y with the respect to y, that would be 1 upon 4 y square that is with the minus sign, similarly for the second term we are getting is 2 upon p minus 2 y upon p square d p over d y and simplifying it.

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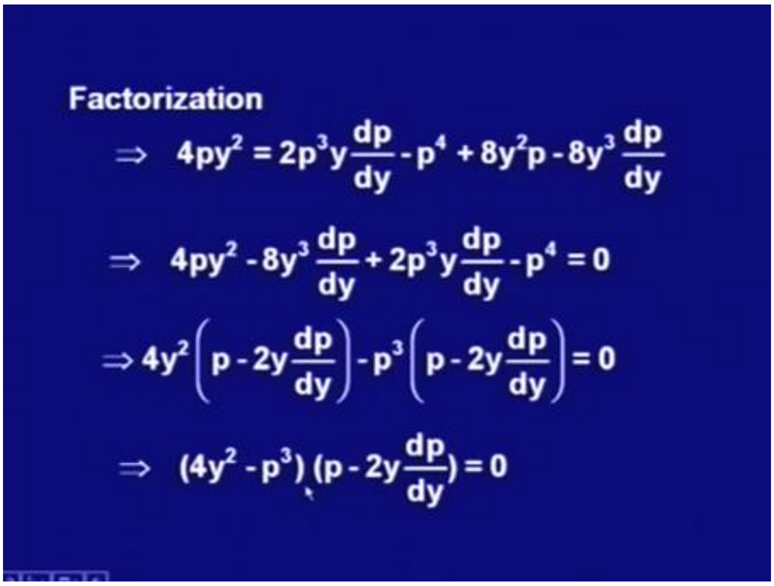

We could write it as, 4 p y square is equal to 2 p cube y d p over d y minus p to the power 4 plus 8 y square p minus 8 y cube d p over d y we are simplifying it, to get the factors. So, again more simplification we could write it as 4 p y square minus 8 y cube d p over d y plus 2 p cube y d p over d y minus p to the power 4 is equal to 0, that is how we have written it as for factorization, now you see that is from first two terms we can take 4 y square as common and will get the factor, p minus 2 y over times d p over d y.

From second two terms we are taking the factor common as, p cube and am writing the first term as second and second term as the first that is, here I am taking minus p cube outside and this term I am writing as first as p minus 2 y times d p over d y. Thus we are getting the two factors 4 y square minus p cube and p minus 2 y d p upon d y is equal to 0, now if I equate these two factors to 0 separately I would be getting two equations. The first equation is general equation in the p and y, while let us the second equation is differential equation in p and y, so let us see that is we are getting two equations.

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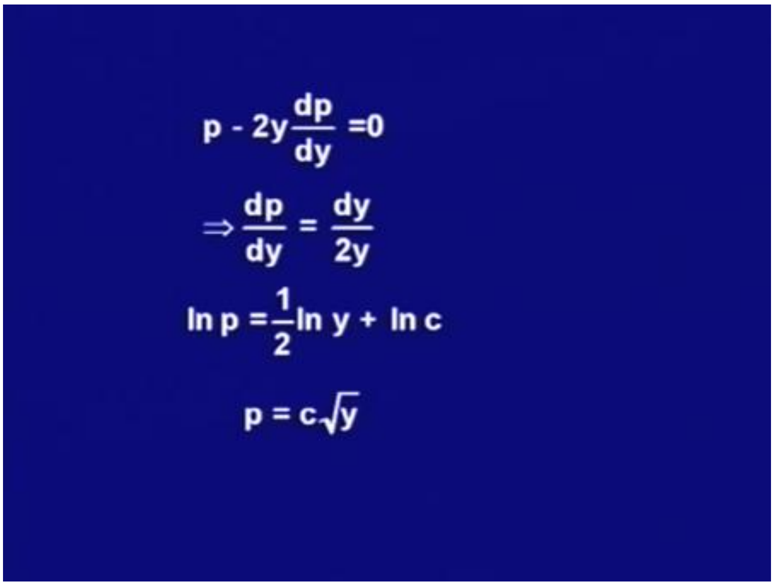

So, first we are talking about this differential equation as we had seen the previous method also, that is the equation simple equation may not give me the general solution. So, we are using the differential equation p minus 2 y times d p over d y is equal to 0 very simple, equation of first order and first degree in p and y to solve it, we do write it as d p over d y is equal to 2 y d y over 2 y integrating on both the sides we would get, log p is equal to 1 by 2 log y plus log c again the constant term we have written in the terms of logarithmic which gives me that p is equal to c square root y. So, this is we have got the solution of this differential equation, now use this solution into the original equation.

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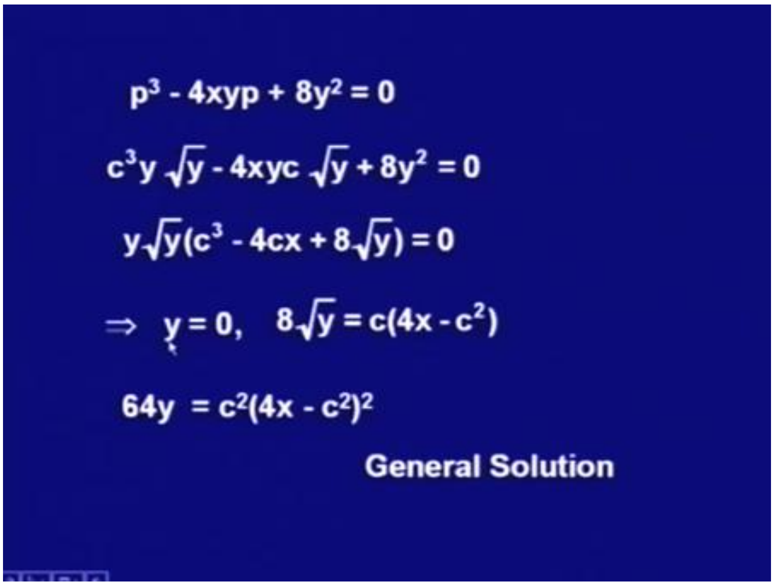

The original equation was p cube minus 4 x y p plus 8 y square is equal to 0, here we I will put p is equal to c root y in this one. So, what we would getting is c cube y root y minus 4 x y times c root y plus 8 y square is equal to 0. If we simplify it what we are getting is y root y c cube minus 4 c x plus 8 root y is equal to 0. Now, this we have got the two factors, one factor y root y and another factor is this equation, from this factor we are getting is that y is equal to 0 that says is, but this cannot be a solution, because y is equal to 0 is the solution in which we are not interested, so we will take this second factor.

So, second factor is giving as, 8 root y is equal to c times 4 x minus c square, now the second equation we square on both the sides give me 64 y is equal to c square 4 x minus c square whole square. So, this is what we are getting the solution of the differential equation, this we will call the general solution. You see this second here again in the first one we have got the two factors we have taken only the factor which involves the derivative for that, which is giving us the differential equation.

And when we are solving that differential equation, we put the value of p in the original equation and we have got again thus in the terms of factors this factor had given me the equation, solution as y is equal to 0. But this is not an acceptable, because it is solution y is equal to 0 will satisfy I need differential equations. So, this we are not interested we are interested in the second factor and from there we have got this as the general solution let us see one more example.

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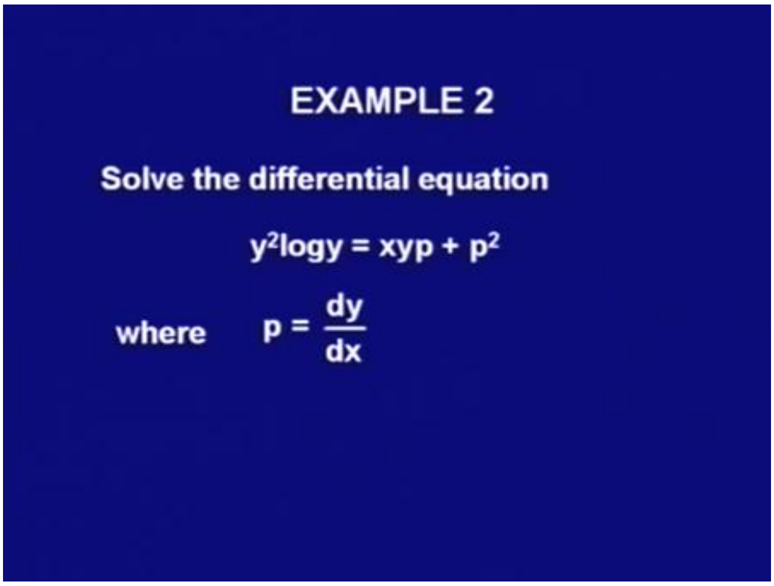

Solve the differential equation y square log y is equal to x y p plus p square, here again p is d y over d x.

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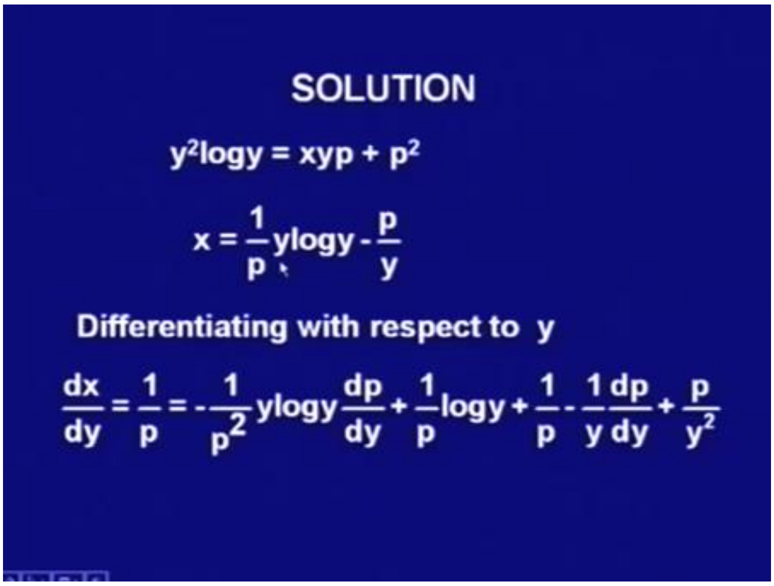

So, we are writing the equation y square log y is equal to x y p plus p square, again we see is that we can at we can solve it for x, but not for y, because y we are involving the term of y square as well as log y, so we can solve it for x, if we are solving it for x we can write it as one upon p y log y minus p upon y. Now, differentiate it with the respect to y what we do get, d x over d y which we can again write as one upon p derivative minus one upon p square then y log y terms.

And of course, with p d p over d y then one upon p as such derivative of y is 1, so it is log y only then one upon p again as such y as such derivative of log y is 1 by y, so y into 1 by y is giving me 1, so I have just got here 1 upon p. Similarly, the second term minus p upon y differentiating with the respect to y, so first differentiate p, so we do get minus 1 upon y d p over d y then minus p and what will be the derivative of 1 upon y that is minus 1 upon y square, so we are getting is plus p upon y square.

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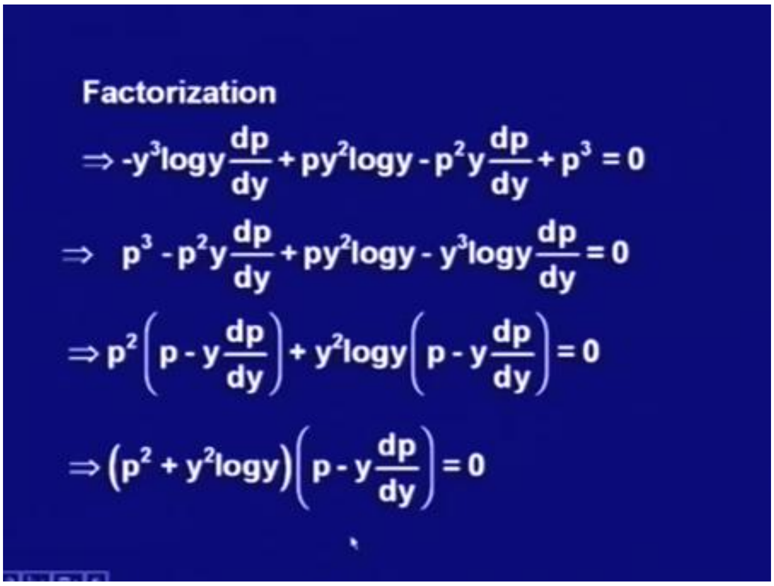

Now, simplify it now what we do get, minus y cube log y d p over d y plus p y square log y minus p square y d p over d y plus p cube is equal to 0. More simplification and factorization, so we are rewriting it as, p cube minus p square y d p over d y plus p square y log y minus y cube log y d p over d y is equal to 0. And just taking the common from first two terms p square gives me one factor p minus y d p over d y to get this same factor over here we take the common y square log y.

So, we are getting the two factors p square plus y square log y and p minus y d p over d y, again we are getting the two factors if I equate these factors after it lead to 0 from here we would be getting one equation in p and y, while here we are getting one equation in p y and d p over d y that is the differential equation in p and y. Again as that this is going to give us a particular solution this is going to give a general solution and we are more interested in general solution rather than a singular solution, so we I will take the second equation.

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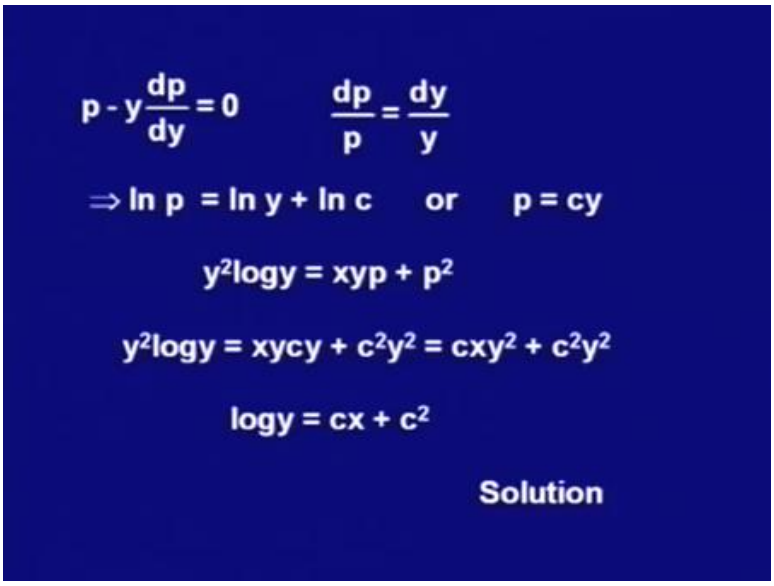

So, we are taking the second equation, p minus y d p over d y is equal to 0, which is a differential equation how to solve it you could rewrite it as, d p upon p is equal to d y over d y the simple equation integrating on both the sides we get log p is equal to log y plus log c or p is equal to c y. Now, use this solution in the given equation that is y square log y is equal to x y p plus p square y square log y is equal to x y p is cy and p square c square y square what we do get is, c x y square plus c square y square or we are getting in other word that is simplification log y is equal to c x plus c square.

If I take the anti logarithmic we can get the solution as, y is equal to e to the power c x plus c square or we could says that is, c times e to the power c x kind of solution and this is what is the general solution of our differential equation. Of course if I would have taken the other factor also that would give me a singular solution, but we are not interested in the singular solutions any more we are interested only in the general solution at this is stage.

So, now we had covered the methods, which are applicable to those equations which are of the first order, but have degree more than one there we have discussed three cases. The cases where, simply we can factorize the given equation into n factors and each equation is each factor results out to be a equation of first order and first degree, second method where we have seen that is if we could write the given equation in the form y is equal to a function x and y dash and y.

And the third one where we could try the give equation in the form of x is equal to a function of y and y dash. We had learn how to solve these three different kind of method they are most general methods which are going to solve the, particular for the problems which we are tackling in the fast course. So, we have learnt the first order differential equations with there of first degree or higher degree, so that is we have finished today.

Thank you.