Mathematics - III
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Lecture - 2
First Order Differential Equation and
Their Geometric Interpretation

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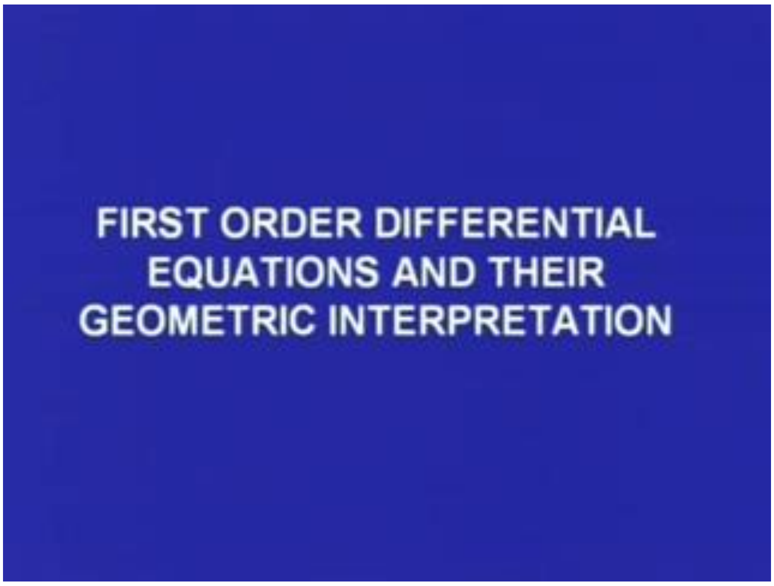

Differential equation for undergraduate students. Today's topic is First Order Differential Equations and their Geometric Interpretation, this is second lecture in this series. In first lecture we had learnt the differential equations their meaning and significance.

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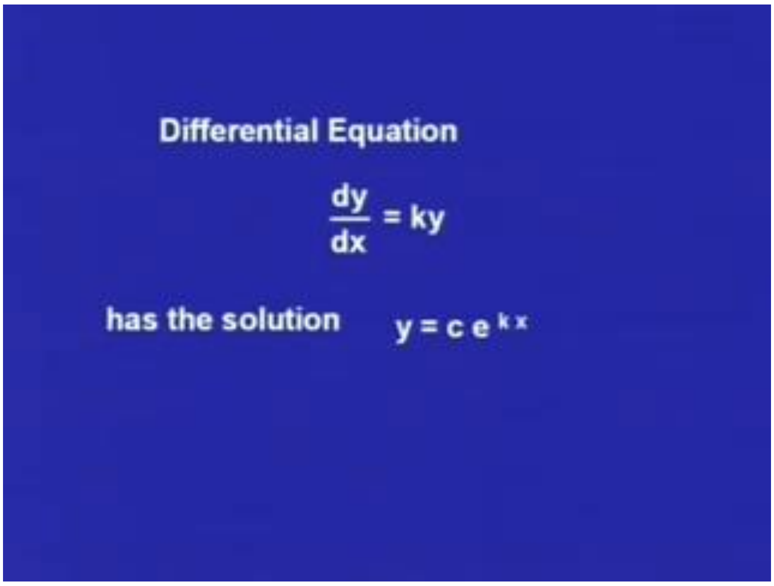

In first lecture we learn that the differential equation d y over d x is equal to k y has a solution y is equal to c times e to the power k x, this we had obtained by the relationship between the exponential function and it is derivative. If you do remember actually we first took the exponential function and then find out it is relationship with it is derivatives thus we obtained a differential equation and since this given function was satisfying this equation.

So, we said that this is solution of this differential equation, actually we had experimented with some other equations and some other functions as well, but in all those experimentation, what we have done we had actually said that this given function and its derivatives are satisfy this particular equation, so they are the solution of the differential equation. But, this may not be possible in many scientific applications, that is in those equations which are arising out of many scientific applications it is not that much easy to guess either from which it function these equations have been arisen. Or to guess which function will satisfy the given equation, they says is that we require to built some tools to solve the differential equations. So, we would start our topic formally, first we will look at ordinary differential equation.

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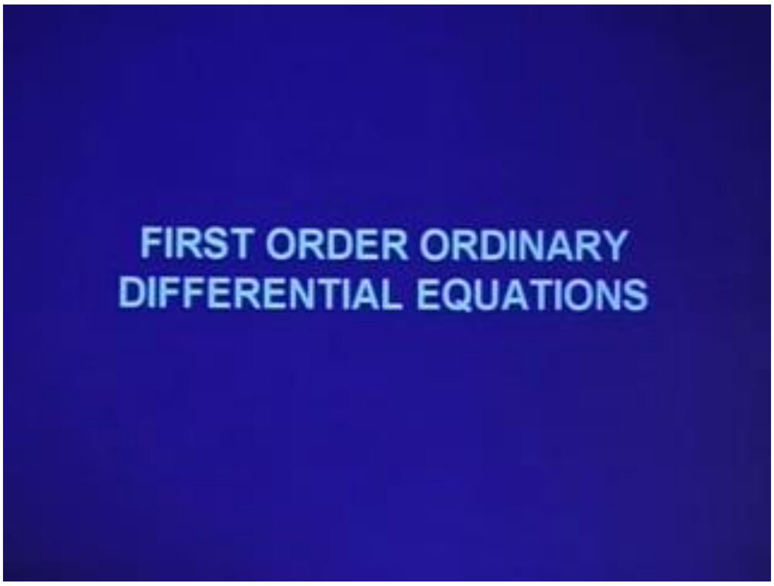

And the first topic we will cover first order ordinary differential equations, first we would look on ordinary differential equations, the first topic would be first order ordinary differential equation, what is first order ordinary differential equation?

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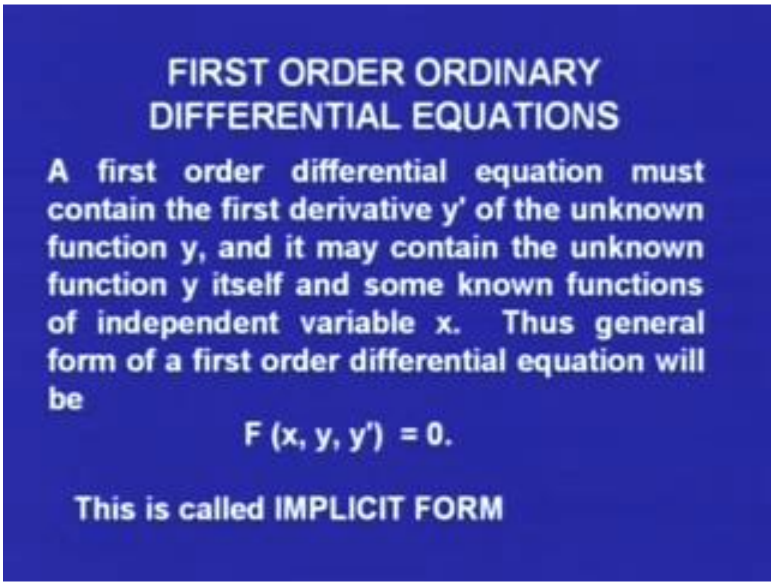

A first order differential equation must contain, the first derivative y dash of the unknown function y and it may contain the unknown function y itself and some known functions of independent variable x. Thus general form of a first order differential equation will be f of x y y dash is equal to 0, here we do have that this is a function of first derivative y dash, function y and it may contain some function of x, this is called implicit form.

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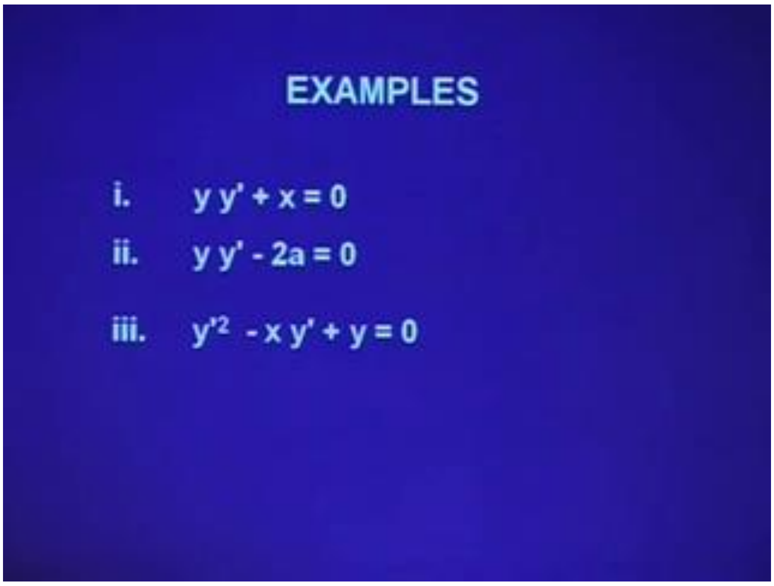

Some this kind of differential equations are say y y dash plus x is equal to 0, y y dash minus 2 a is equal to 0, y dash square minus x y dash plus y is equal to 0. We see that in all these equations, we are having the first derivative the unknown function, in this first equation I do have the function x. The second equation I do not have any term of x in the third equation we do have x y dash y dash square and so on, so these are implicit form, but in actual practice when we are actually solving we may require to write these equations in explicit form.

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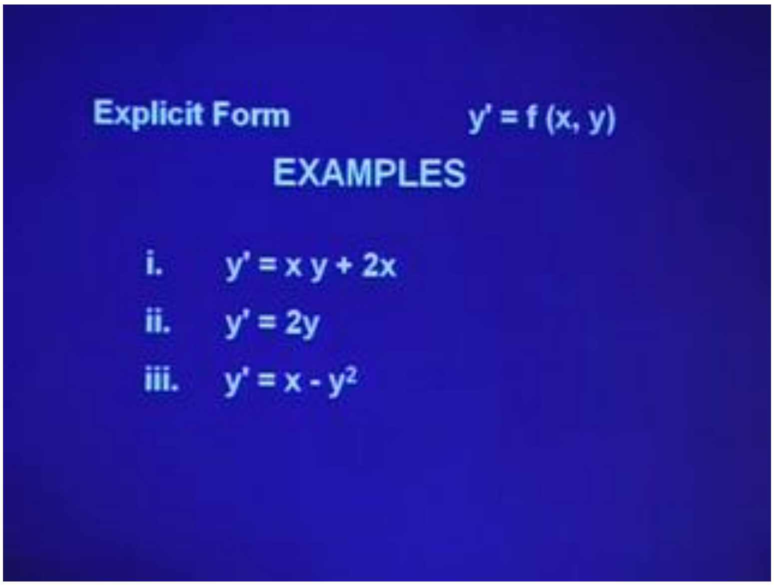

As y dash is equal to f of x comma y, some this kind of equations are as y dash is equal to x y plus 2 x y dash is equal to 2 y, y dash is equal to x minus y square. You see here that is we are explicitly writing that y dash that is the first derivative is the function of x and y. Now, before getting going for solving this finding out the method to solve these equations, let us first try to learn something geometrically, what these equations are saying and from that geometrical interpretation can be get something about the solution of this differential equation.

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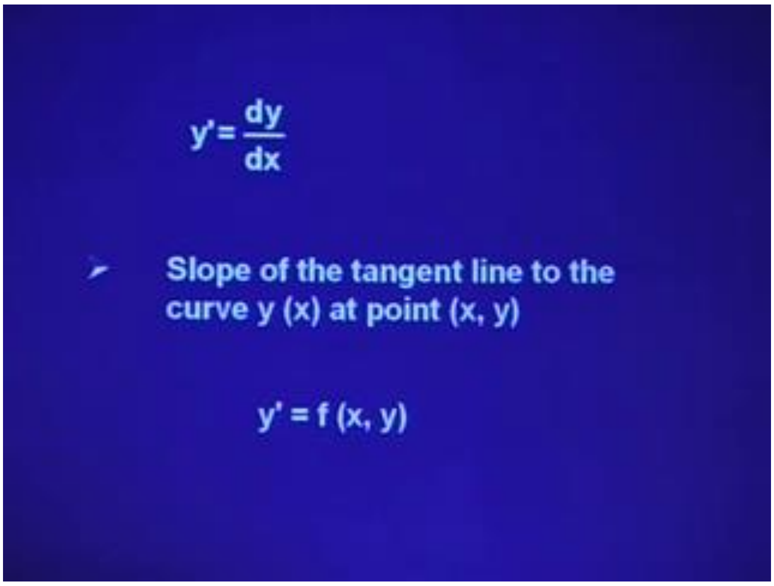

Let us look at we do know from the calculus that the first derivative d y over d x is actually presenting the slope of the tangent line to the curve y x at the point x y in x y plain. That is what this equation y dash is equal to f (x, y) is telling us, it is telling us about the slope of the curve y at the point x, y. Let me explain it, more clearly that how this idea is giving anything about the solution of this equation.

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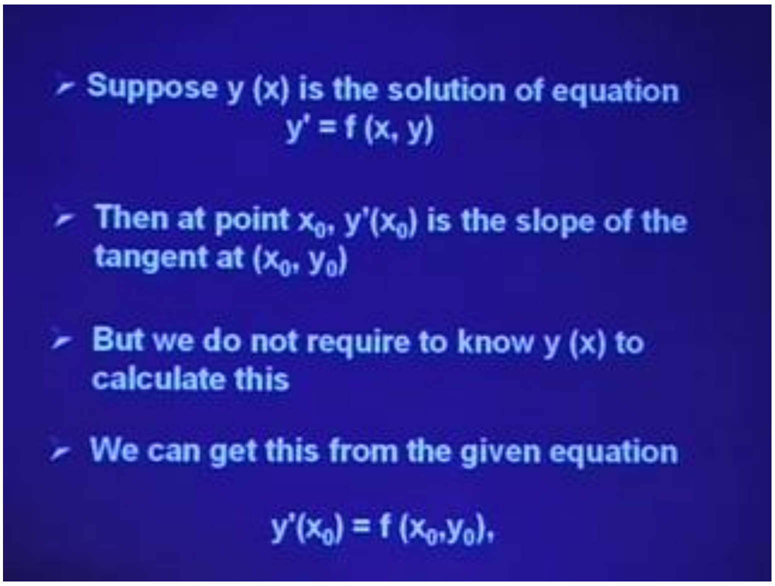

Suppose y x is the solution of differential equation y dash is equal to f (x, y), then what it says that at point x naught y dash x naught will give me the slope of the tangent at the point x naught y naught. But, for this we do not require to know the function y, what we do have is actually we do have that y dash is equal to f (x, y) that says is at point x naught y naught I can calculate y dash x naught from this as f of x naught y naught. That is we are not finding out what is y x this equation itself is telling us at the point x naught what is the slope at the point x naught y naught of this function.

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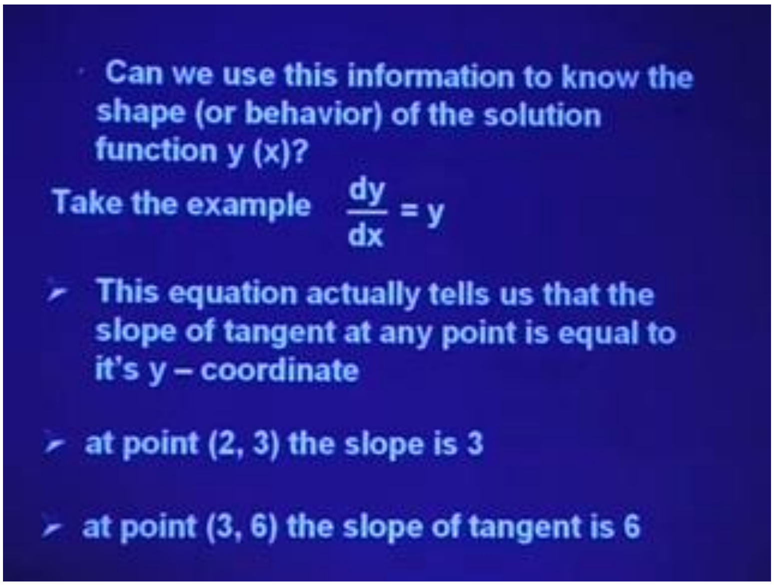

Can we use this information to know the behavior or the shape of the differential equation solution. Let us just try to say it with the help of some example, see let us take the example d y over d x is equal to y, why I had chosen this example we are very much familiar with this example that we had already done in our first lecture and we do know the solution of this example also. So, we would be able to more clearly understand the things, what does is telling us, this equation is actually telling us that d y over d x is equal to y; that means, the slope of the tangent at any point is equal to it is y coordinate. What does it mean, it says is that if I choose a point say 2, 3, then slope is 3, if I will choose a point say 3 6, then the slope is 6, now at different points if I am knowing this slope.

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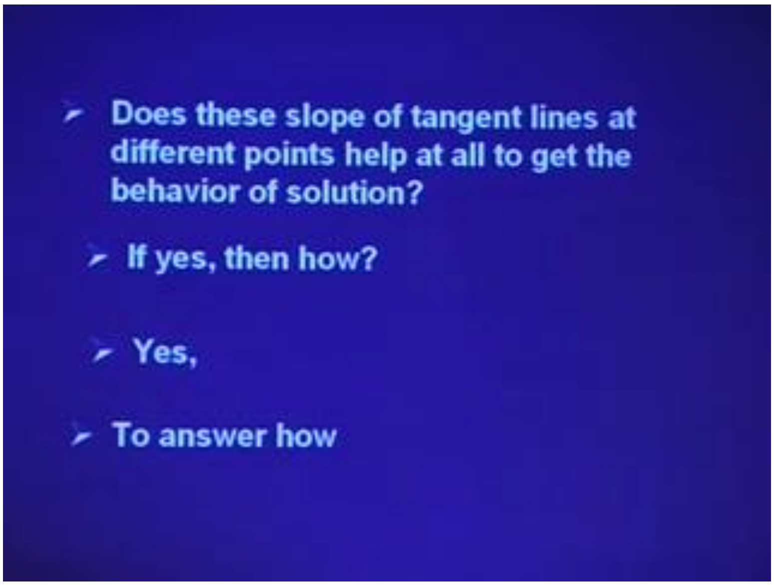

How this is going to help us to get any idea about the solution, so the question is does this slope of tangent line at different points help at all to get the behavior of the solution. And if yes then how they are getting it up, the answer here is for this question, off course, yes they can ask about the solution, to answer it how let us just say something more let me draw these slopes at different points and say some x, y plane.

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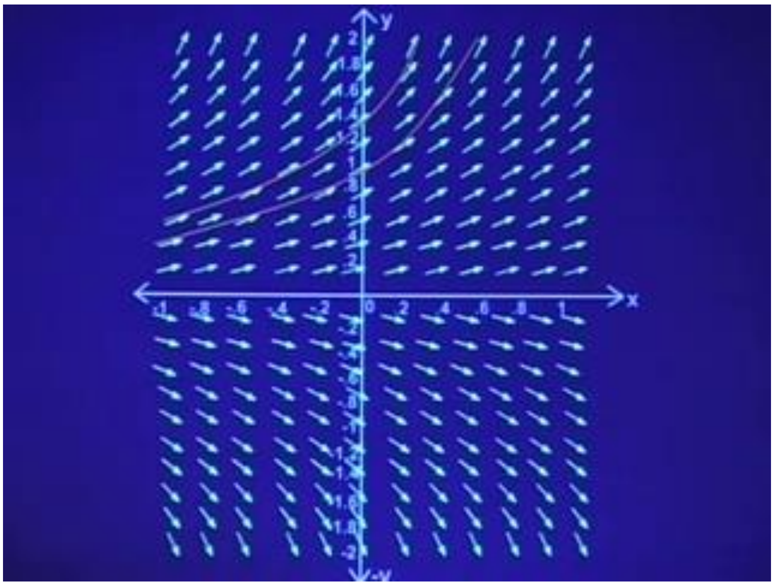

You see here, I have taken the same equation d y over d x is equal to y, so we are having here it says suppose that the 0.4 and 0.2 the slope would be 0.2, because the y coordinate is 0.2. So, in this line you see is that, all these y coordinate is point 2, so the slope is point 2, so if I take, so the point 1 and 1 the slope should be 1, so you see is that is the slope is 1. Similarly, if I see a point say this is point six and minus 1, so this slope should be minus 1.

Now, if I see in total it is this graph what it is telling it is giving us essence of direction, you see all these arrows are moving towards upwards, here the arrows are moving downwards. Why this downwards because we do know the second this y coordinate here is negative, so it is downwards slope is negative that says is that I do get an idea about the direction of the curves.

Now, for finding out an particular solution, we have to how a initial value problem; that means, suppose I say that at point x is equal to 0, the solution has the value 1, that is I am taking the initial condition y 0 is equal to 1. What it says that I have to include this point in the solution, the solution should pass through this point let us see if I do when a solution is it passing through this point. You see is that is see I can because, what I have can done is that is a just drawn the solution as that was going on this one. Now, if I change my initial condition says suppose I say that the at zero the function is 1.4.

Now, still my differential equation is same that d y over d x is equal to y, so this direction is not changing at all, this direction is same the only thing is it changing is that is. Now, I should have a function which should include this point over there; that means, I can draw another solution like this one, so what it says is that, this we are getting the solutions at different points. Now, this graph which we are having here this is called the direction field or slope field.

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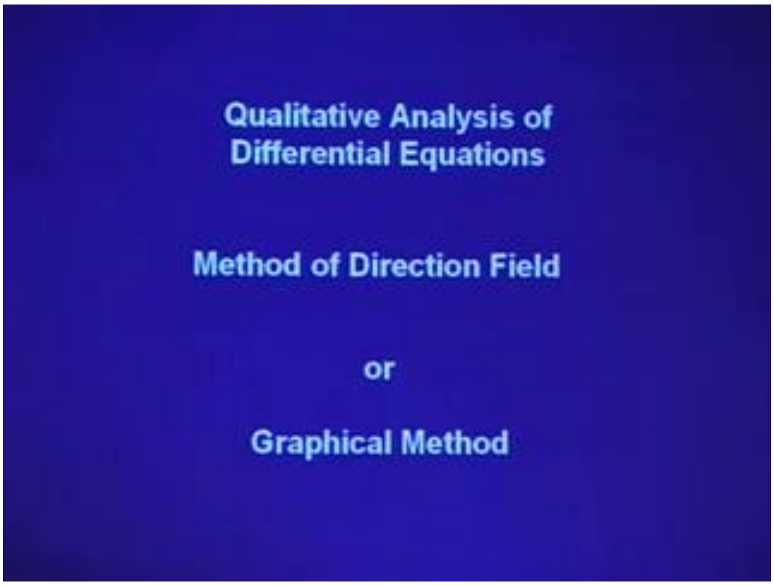

And this idea of getting a solution from this direction field is known as the qualitative analysis of differential equations or the method of direction field, sometimes this also referred as graphical method. So, what we had learn that is we had learn that is we have not solve the differential equation, what we have done actually we have find out the direction of the slope of the tangents at the points x naught and y naught and from their, we try to find out or try to approximate the solution. So, let us learn something more about this fun this method.

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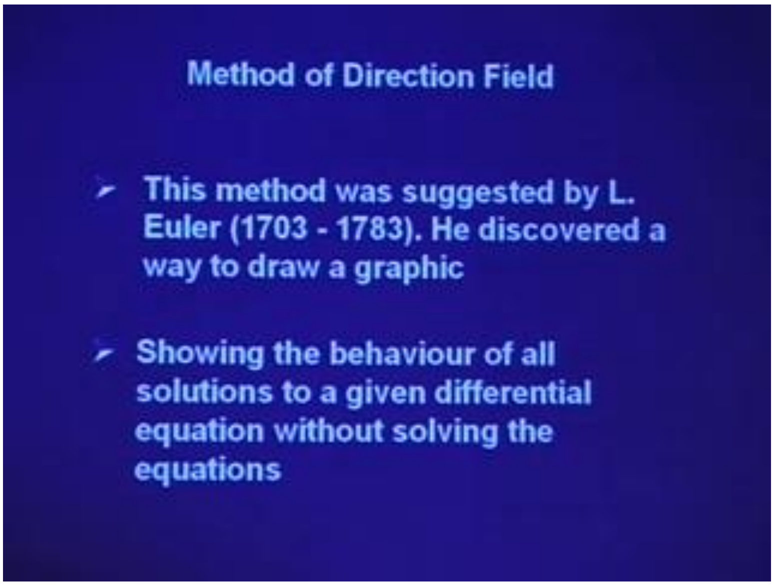

So, we will name this method as method of direction field, this method was actually first developed by Euler or in early you could says out 18 centaury, he discover a way to draw a graphic of those models, which we are modeling as the differential equations, he is modeled them as a graphic one. And then he try to says to build up or that showing that the behavior of all solutions to the given differential equation without actually solving them.

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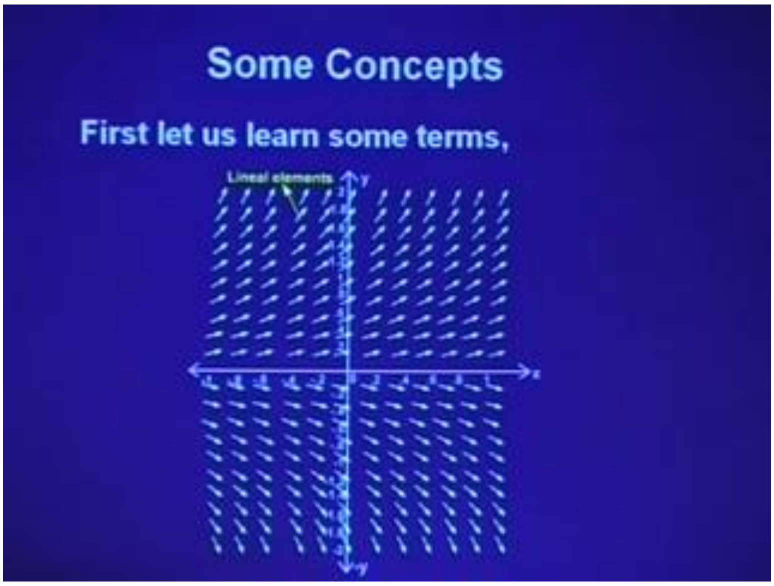

So, to learn something more about that we have to learn some concepts about this method, so first we will learn some chance. So, first is that is in this one this direction field which hours having, these small arrows which your hours having is the tangent lines at those points, these are called lineal elements.

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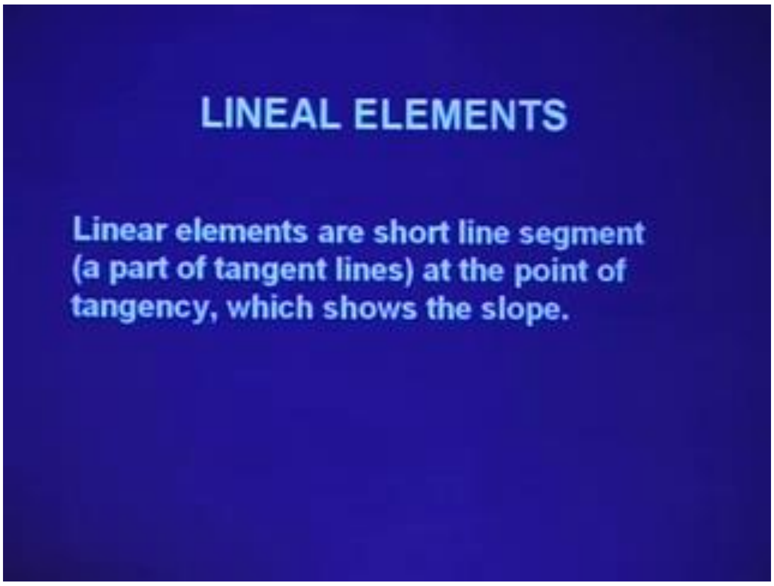

So, now, formally what will define the lineal element; lineal element are short line segments at or a part of the tangent line at the point of tangency, which show the slope of the tangent at that point.

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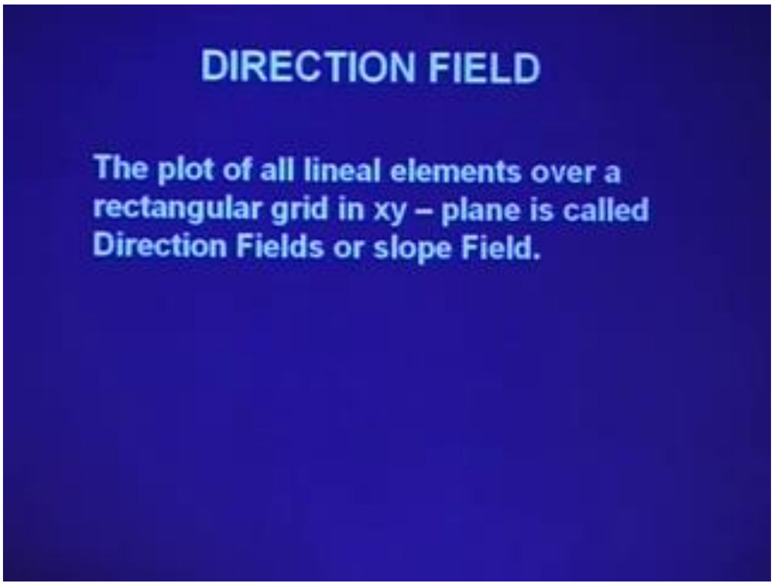

And the complete figure that is all the tangent lines at different points that is called the direction filed. So, formally what will be the direction filed the plot of all lineal elements over a rectangular grid in x y plane is called the direction field of or the slope field.

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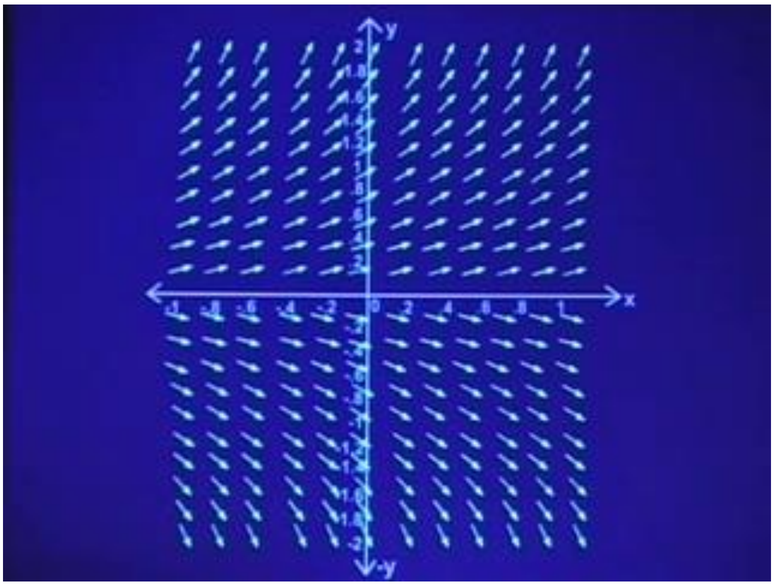

This direction field we are showing this direction in this, we are showing by the arrows how this arrows are being made, we just see the differential equation, because that we are having y dash is equal to f (x, y). And if at any point x y we are having that the function is giving me positive value; that means, the slope is increasing. So, we show upward direction and if at any point this function f (x, y) is giving as negative sign; that means, the function is decreasing and we show the slope downwards how we do get the solution from this direction field.

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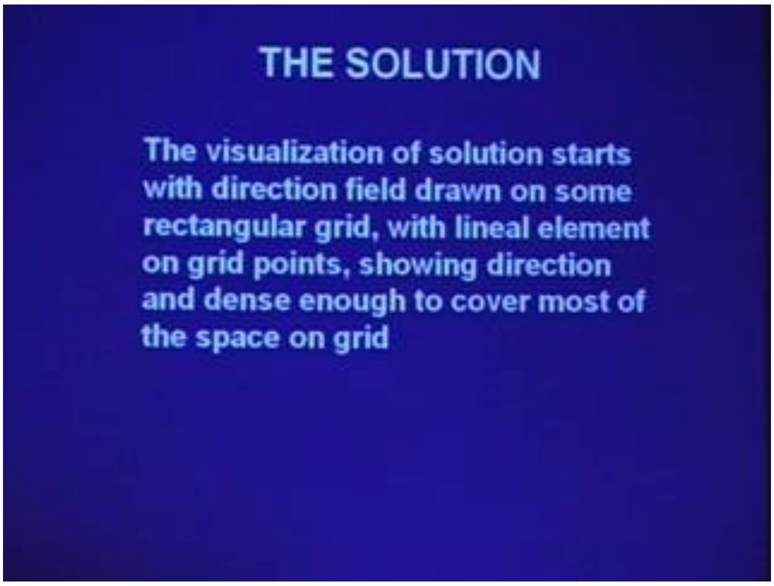

We see is that is we visualize this solution as soon as we get the complete direction field on the rectangular field with the lineal elements on the grid point, showing the direction and dense enough to the cover most of the space of the grid. How we are visualizing it, it is meeting that is we are actually basing our visualization on some facts.

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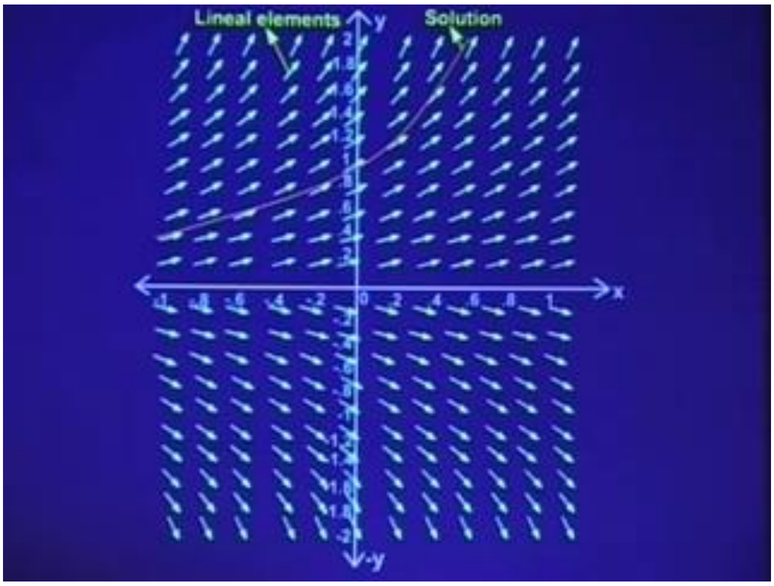

What are you see, that is was this direction is known we can visualize it that is it should the any curve it should move from here to upwards, if it is this side it should move the downwards So, this is based on some faults let me just explain those facts.

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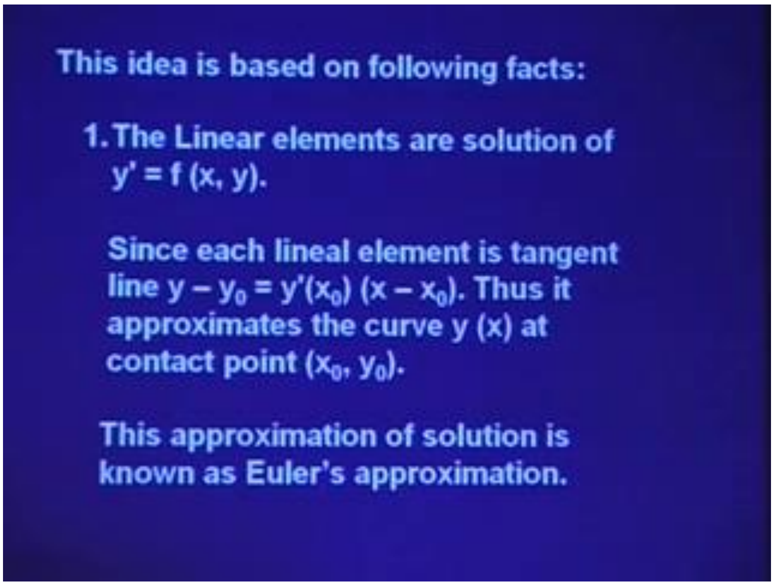

The first fact is the lineal elements are solution of y dash is equal to x y, why we say that y dash is actually the slope of the tangent at the point x y; that means, f (x, y) at the point x naught y naught is the tangent slope. So, what we do get is that, at point x naught y naught the tangent line would be y minus y naught is equal to y dash x naught times x minus x naught. So, at point x naught actually it will touch the curve, so that is at point x naught y naught the conduct point this lineal element that is the tangent and the curve they would be same.

So, it is a solution of this differential equation at the point x naught y naught this is the first fact, the second fact. So, from here actually what we are saying is that these at the point x naught y naught this is approximating the solution and this is also named as the Euler's approximation.

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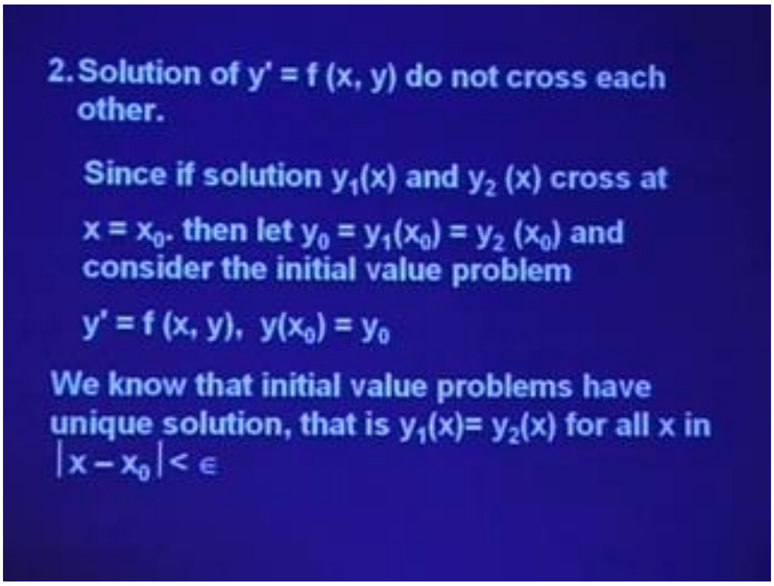

Second fact solutions of y dash is equal to f (x, y) do not cross each other, what does it mean, let us suppose that they are crossing, so suppose at point x naught they are 2 solutions y 1 x and y 2 x and they are crossing at point x naught. So, what we will do is then i would assume that at x naught, whatever the value of y 1 x naught and y 2 x naught that I will consider as y naught. And I will now, look at an initial value problem the initial value problem y dash is equal to f (x, y) y x naught is equal to y naught. We do know that the solution of initial value problems are unique, thus what it says is that y 1 x and y 2 x at x naught are same.

So, what we say for all x in a small is up silent neighborhood of x naught this y 1 x and y 2 x would match, because at x naught they are same that is at that point if they are same an initial value problem has unique solution, so they must be equal in that small number root. So, if in that small number root is equal like that we do say that says is that the solution, if they are two solutions they should not cross at all, they are that is the crossing is not possible from all this analysis actually we had learn something more also.

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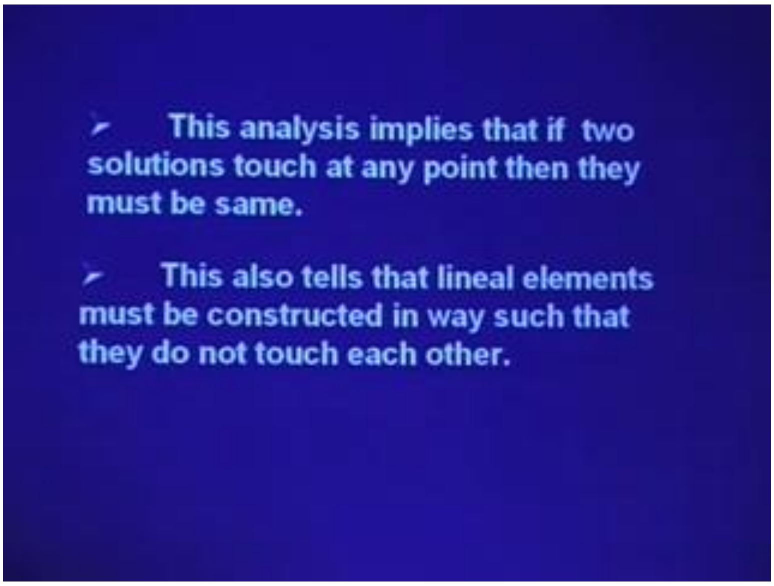

The first thing we had learn is that if the two solutions touch at any point, then they must be same and from this analysis, we had also learn that is when we have to find out solution by using this method we have to draw the lineal elements in such a manner that they should not touch each other. We want them to be thus enough in the given field, but we should construct in a manner such that they should not touch each other, they should be separated.

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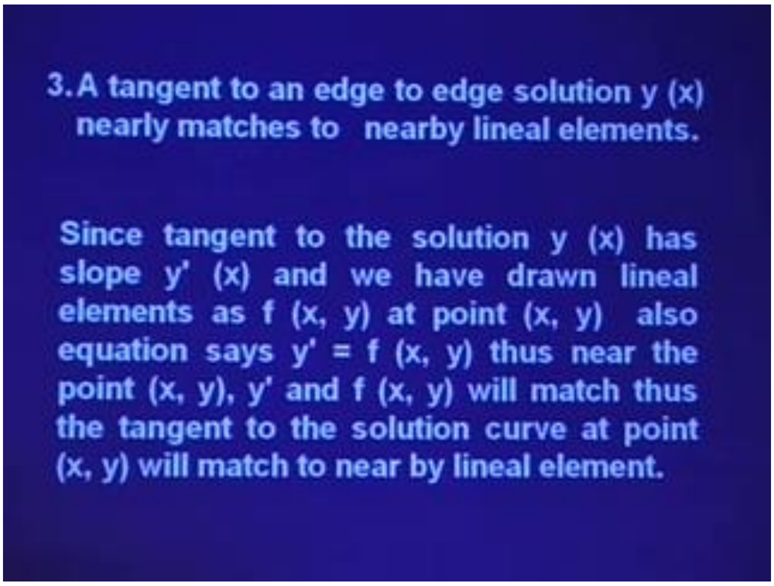

The third fact which we would be using is that is a tangent to an edge to edge solution y x nearly matches to the nearby linear element. Why again you see that is how we had got the solution, the tangent to the solution y x has slope y dash and the equation says that y dash is equal to f (x, y) and we have drawn it as equal to f (x, y) at point x y that says is that this nearby point at the near the point x y, this y dash and f (x, y) will match thus the tangent to the solution curve at point x y will match to nearby lineal elements. That says is actually that is if it is matching over where we keeping these facts in mind.

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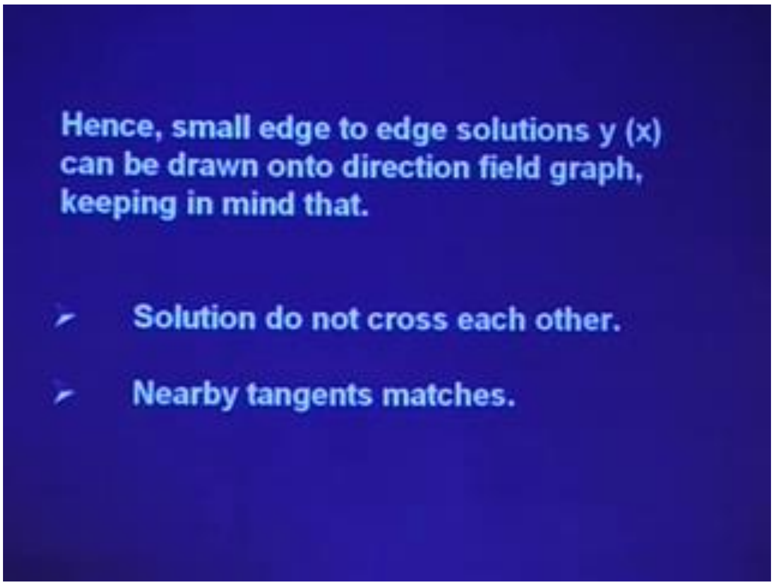

We are now, ready to draw edge by, what you mean by edge by edge solution will just go point by point. So, we can draw small edge by edge solutions y x on this direction field keeping in to mind two facts, one that the solutions do not cross and second that nearby tangent lines should match.

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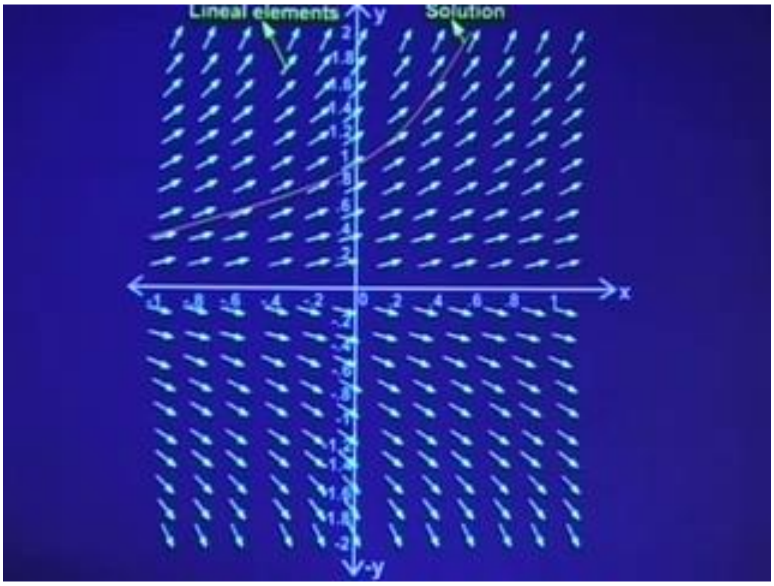

Thus we will get, see we just want at this point, so this nearby tangent this is matching here, here it is the point of tangency and we are saying is it is matching and we have drawn like this one. Now, if I says that is I want to draw at this point, so I would have the solution which would be moving from here and should not cross this first solution, that says is this line this lineal elements has to be drawn.

Off course it should be dense enough, so that I could get a one could get the idea of the direction, but they should not be in a dense in such a manner they that they touch each other or when I am drawing we are drawing the solution the solution should cross our solution should touch each other.

So this now, let us learn this method this direction fields are this graphical method, this direction field we can draw by an or now it is actually the computer software's are also available to draw these. But, before the invent of the computers thus we know that this method is been invented an early 18th century, this has been done by hand only and here we would learn this method to draw the direction field by hand only, this method when we are drawing it by hand is called isoclines method.

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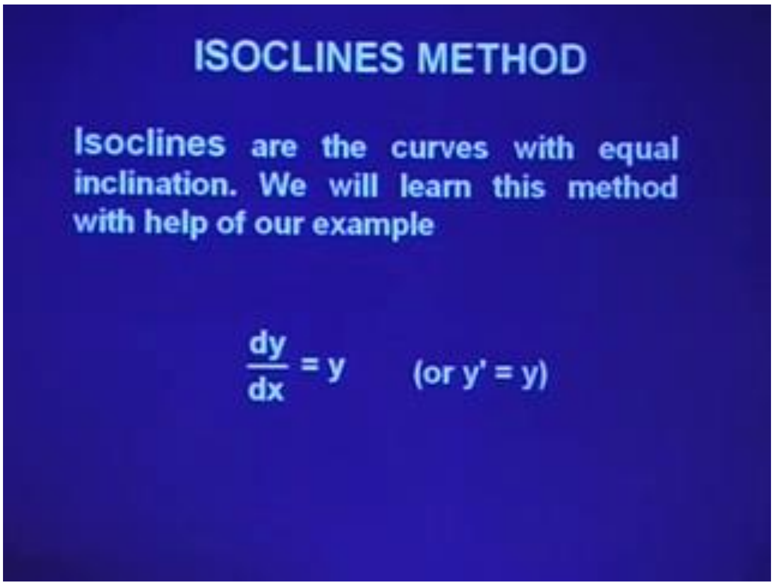

What is isoclines? Isoclines are the curves with equal inclination, that is one isocline curve means, here all the tangent points will have same slope. Now, we will learn again this method by the help of this our example that d y over d x is equal to y or we call it y dash is equal to y, so let us start a step by step.

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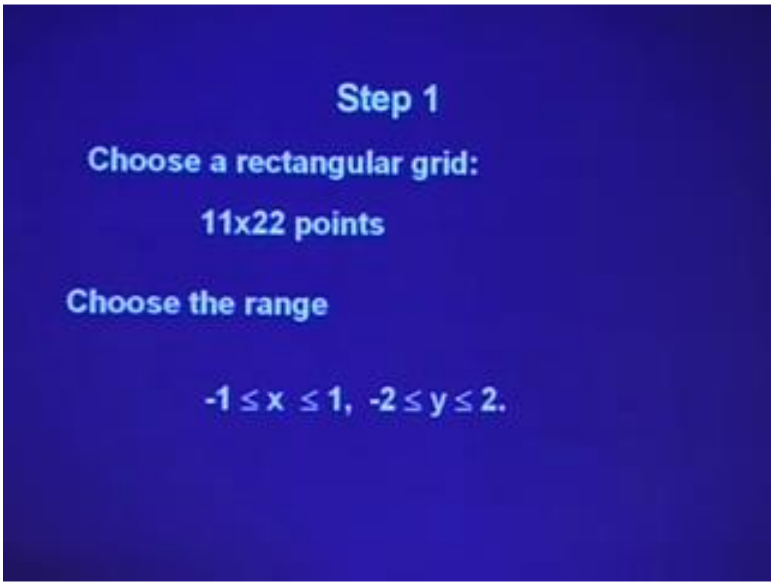

The first step would be choose a rectangular grid, so suppose here I choose a 11 cross 22 points, so that I feel is that it should be dense enough, but it will be dense enough if I choose the range also accordingly. So, will choose the range say for x minus 1 to plus 1 and for y minus 2 to plus 2.

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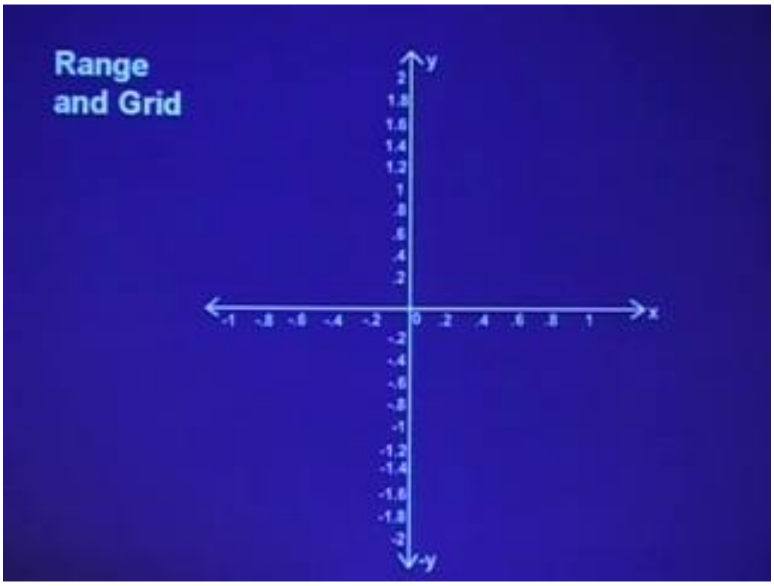

So we had chosen this range that is for x minus 1 to plus 1 for y minus 2 to plus 2, off course I have not plot it here the grid points that is I would choose all these points this 0.2, 0.2, 0.4, 0.2 and so on like that that we would have. So, once I have chosen this range and grid first thing we would draw isoclines.

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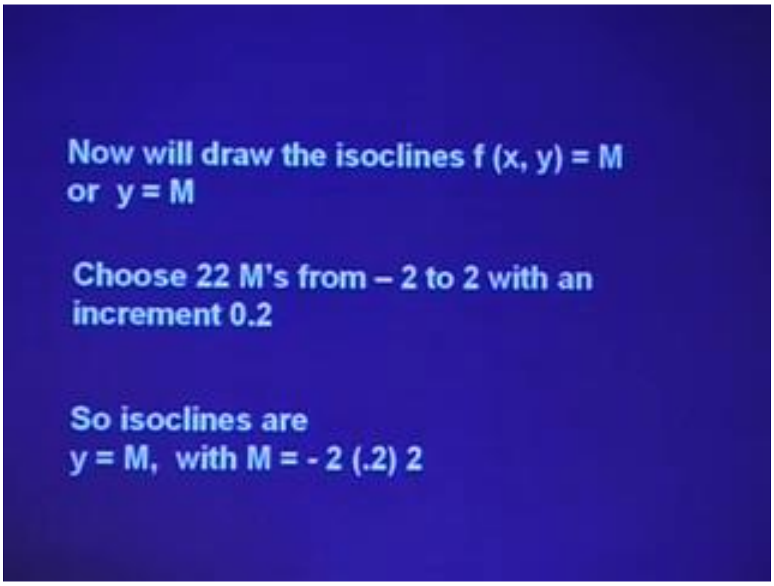

How do will draw the isoclines, first we find out what is the isoclines? Isocline is f (x, y) is equal to m, why because we are having the equation that y dash is equal to f (x, y) and y dash is telling us the slope of the tangent. So, the tangent would be f (x, y) is equal to m or we do call it that is in our example it would be y is equal to m, because f (x, y) was y. We have to choose 22m's that is will choose for different values of m, so that I could draw the many isoclines, so will choose from minus 2 to plus 2 with an increment of 0.2. So, isoclines are the line lines y is equal to m with m ranging from minus 2 to plus 2 with an increment of 0.2, let us see that is how we are going to draw it.

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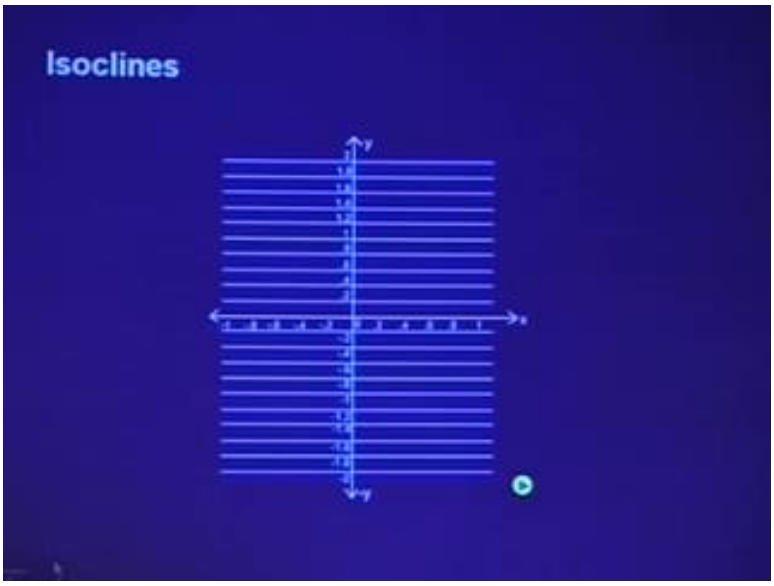

See we do have at 0.2 we have to draw the line y is equal to 0.2 at 0.4 we have to draw the line y is equal to 0.4 and so on at 0.6, 0.8 and so on. Similarly and the negative side at y is equal to minus 0.2 at y is equal to minus 0.4 and so on and all those points, thus we had covered the isoclines, we had drawn the isoclines, what should be the next step now the step two.

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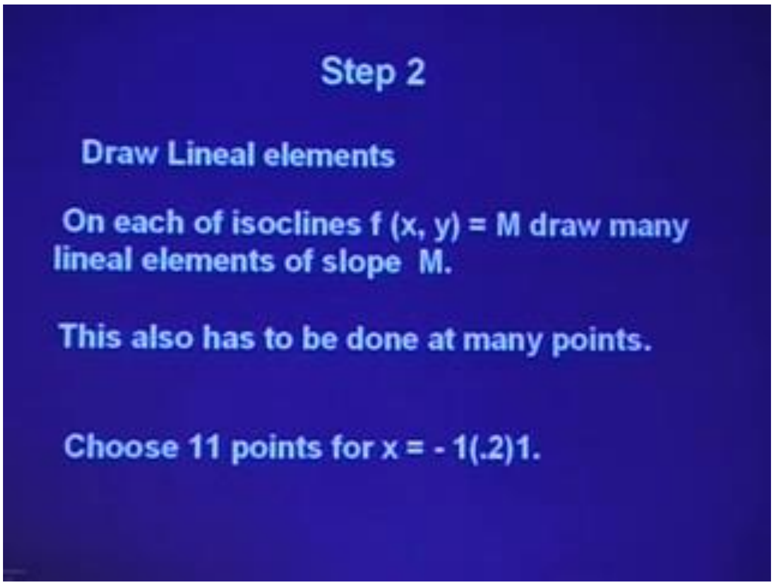

On these isoclines we have to draw the lineal elements, I have to draw this lineal element should be drawn on each isocline. So, on each isocline f (x, y) is equal to I am just is explaining here in method in general and showing with the help of our example also we have to draw many linear elements of the slope m. So, this also has to be done and many points, so again we will choose the grid points that is 11 points on the x axis because our isoclines are y is equal to m. So, we do just have to choose the grid points for x only that we have chosen from minus 1 to plus 1 with increment as 0.2, which says is that we would get at least 11 points and each a isocline.

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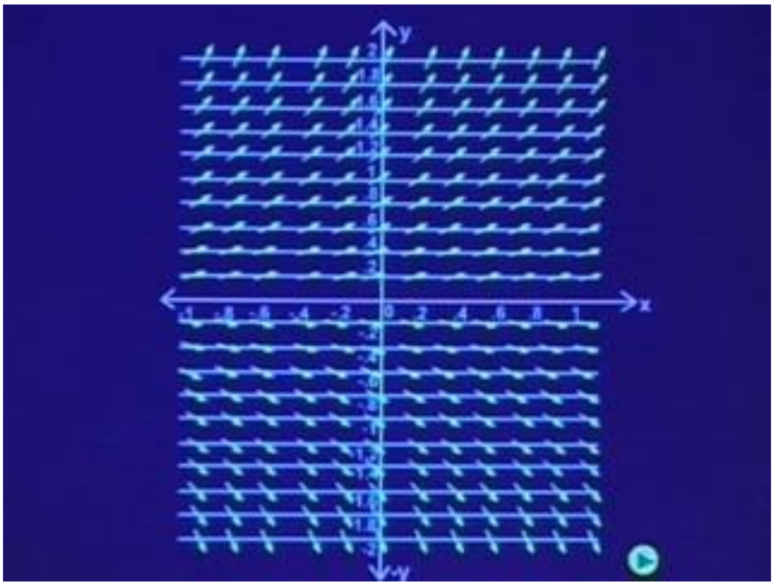

Now you see is that is how we are going to draw, let us choose first this isocline y is equal to 0.2; that means, here the slope is 0.2. Now, the grid points would be 0.1, 0.8 like this 1 from minus to plus 8. So, we would start the first point at minus 1 and 0.2, the slope should be 0.2 that is I have to draw a line, small line having slope as 0.2 at point minus 8 and 0.2 again, I have to draw a small line having the slope as 0.2. Actually this is isocline means is at all the points on this line I would be having the slope as 0.2, so thus at all points I will draw a small lines which has slope 0.2.

Now, choose this line y is equal to 0.4, this is isocline with our a slope as 0.4, so at first point minus 1 and 0.4 the slope will draw a small line having the slope as 0.4, you see is it little bit upwards to this one at the second point we do had it as 0.4 and so on and all the points I will draw it as 0.4. Similarly I would go for each of isocline and the positive side, you see is that is here the arrows are also drawn to the upwards, because these slopes are positive sides.

So, we do have and similarly we get in all the lines, we will draw on similar manner that is the slope is you see is that is how we are getting is this slope is increasing towards like this one at the last line I have got the slope should be the lines which has slope two. Now, come to the negative side, y is equal to minus 0.2 here the slope should be minus 0.2, that says is that is at this point minus 1 and minus 0.2 the slope should be minus 0.2, that is it should be magnitude is 0.2, but the direction is negative.

So, we see that is we have got direction negative, similarly the second point it should be again minus 0.2, so it would be going downwards and so on all the points on this line it should have the same magnitude, but direction as negative. Now, come to the y is equal to minus 0.4, now the magnitude should be 0.4 and the direction should be negative and at the second point as such, so in all the points over here. Now, similarly we can draw on all the lines, you see here is that is they are now giving the direction towards downwards only, once I had covered this drawing the lineal elements. Now, what will be the next step this step 3 is it is sketch the solution for sketching the solution.

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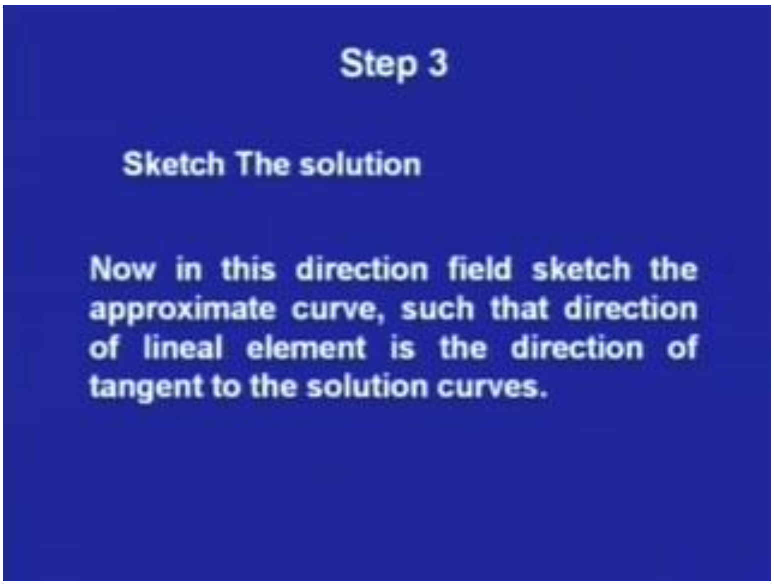

We see that first thing we have to see the direction of the lineal elements and we have to find out approximate curves, such that direction of lineal element is the direction of tangent to the solution curves. So, let see that is how we are going to do it over here this is our direction field you see here.

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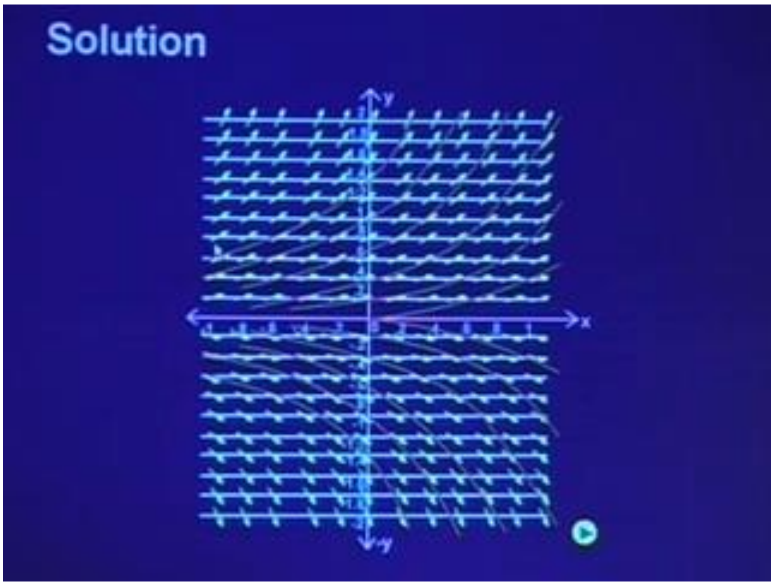

If you see at this point, the direction of the tangent is this way, so we would like to have a curve such that it is tangent should also be in this manner, so let us see is that is how we are doing it. So, let me just try it 1 or 2 solutions, so we see is that is here we are getting it say from this point or say it is from this point or you say it is at from this point or it is at from this point or this point, this is from this point or this point ((Refer Time: 31:40)) if you see is that is how we are getting these solutions at different points.

We do get it that is if I am having this point at with this point, we do see we do find out that the tangent should match over here, we see here the tangent should match at this point.

So, this is approximate solution, we are getting an idea that is how this curve should be going, so first we see the direction and then we try to draw solution approximately solution such that they are going. Now, do you remember that is in first lecture we have plotted the exponential function and there we if we do remember we are we had plotted like this one many exponential functions.

So, this is actually you are getting is it is approximately equal to the exponential function. So, we do have that this solution, we do know that the differential equation which we equation we had started d y over d x is equal to y and we do know the solution of this equation was y is equal to c times e to the power x and c we could find out if i choose any initial point.

So, you see here for y d y over d x we have got many solution that is general solution, we have got all kind of curves and you see is that, if you remember these are matching with the exponential curve in this is small field. And if I have to choose any particular solution, we just choose at that particular point, which value it is satisfying and that would be the solution.

So, this gives us how to find out the solution of a differential equation without solving, we just using the geometric meaning of a differential equation, that is the first order derivative and what the equation is saying about this first order derivative.

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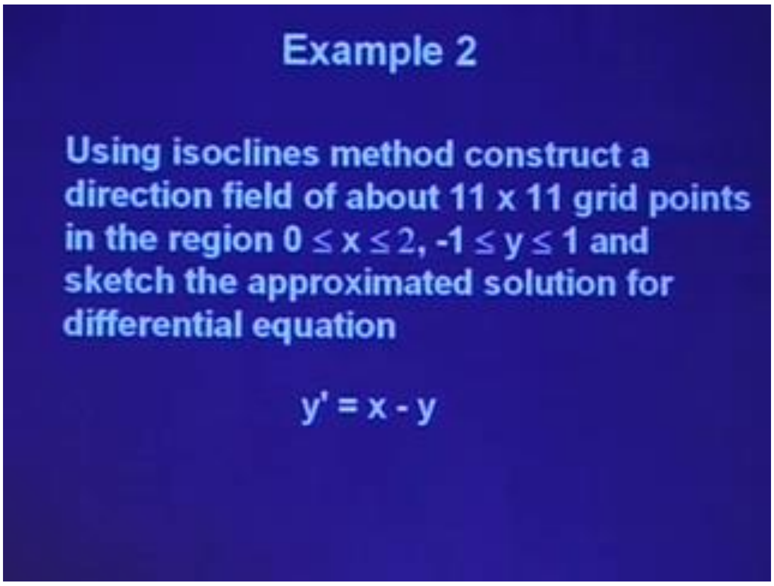

Second example using isocline method construct a direction field of about a 11 cross a 11 grid points in the reason x per 0 to 2 and y minus 1 to plus 1 and sketch the approximate solution for differential equation y dash is equal to x minus y. You see that differential equation here is written in explicit form y dash is a function of x and y, we will solve this equation by isocline method, so let us move to the solution.

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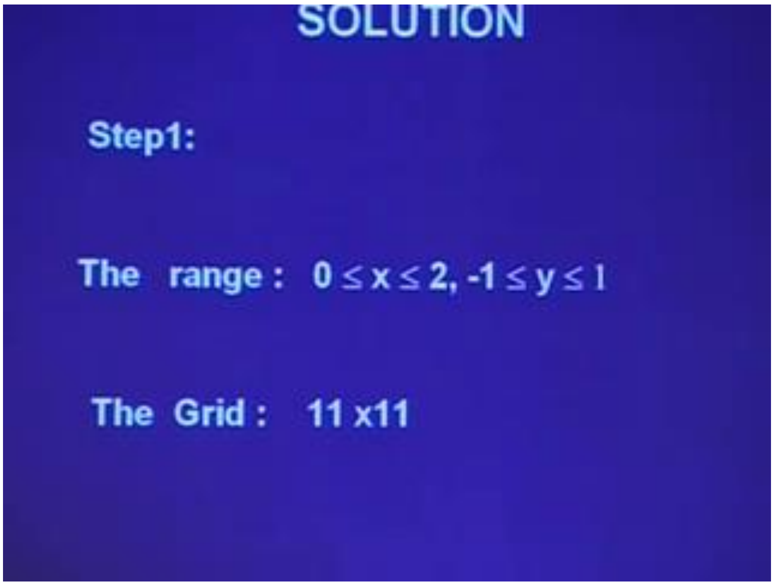

First step, first the range and grid range here, we have been given that we have to take the range for x from 0 to 2 and for y minus 1 to plus 1 and the grid we have been that we should have at least 11 cross 11 points, so first we will try that is what are the isoclines.

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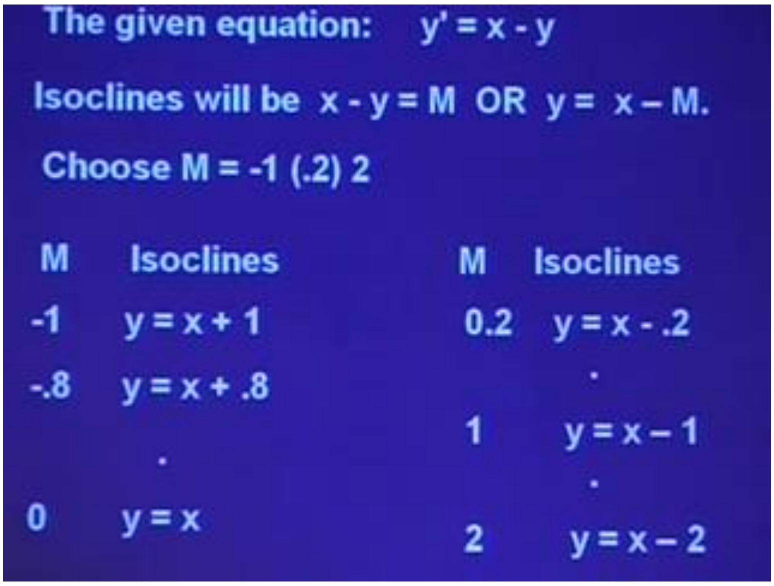

We see the given equation is y dash is equal to x minus y, so the isoclines would be f (x, y) is equal to m. So, here f (x, y) is x minus y that is x minus y is equal to m or we would write as y is equal to x minus m, so that we do have that they are a straight lines having slope one and they intercept as in that is the, as we are varying my m that is the slope of the tangent to the solution curve, we would have different lines with different intercepts only the slope of this isocline would be same.

That says is let us just try how many isoclines, we have to choose, we have do little bit experimentation to see that is whatever the range is given to us that is for x 0 to 2 and y is minus 1 to 2. We should choose all those m such that we could cover the complete field, that says is that is we would be choosing here after little bit experimentation, we had find it out that we will choose m as minus 1 to 2 with the increment of 0.2.

So, what will be our lines it see that is at minus 1 la isocline will be x plus 1 at minus 0.8 isocline will be x plus 0.8 and so on at 0 it would be y is equal to x, then at 0.2 it should be y is equal to x minus 0.2 you say that is our isocline is x minus m. So, thus we would have at 1 as y is equal to x minus 1 and at 2 y is equal to x minus 2. So, we are having this approximately a 15 lines which are we are having from minus 1 to minus 1 to plus 2 that is these are the my m's and we are having these isoclines now I will draw these isoclines on our, what is say, this let see how will do will do it.

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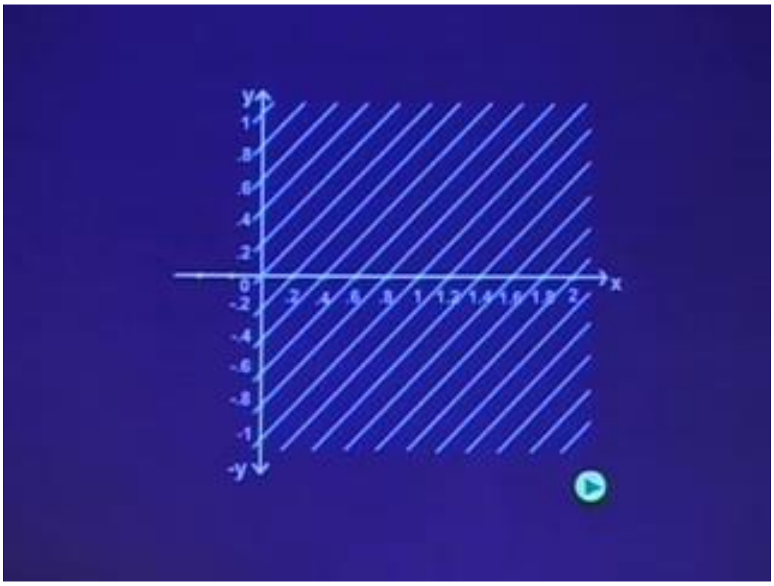

First will draw the range that is for x 0 to 2 y we have taken from minus 1 to plus 1, then we will try start drawing the isoclines at we do have the first isocline was y is equal to x plus 1. So, we do have is that would be you see at first 1 y is equal to x plus 1, here I have just chosen that is because it is going out of our range.

So, I am just taking a very small line here then at 0.8, it would be y is equal to x minus 0.8 and at 0.6 and so on. So, we are having all these isoclines for m ranging from minus 1 to 2 in this same this minus 1 to 2 that you see is that is why I have chosen this, because it is now completing our complete range is place, that is our complete range is been completed that is why. So, this little bit experimentation that is what would be covering our complete field, we could use this what m should be there, then what we do have the next step next step is that draw many lineal elements.

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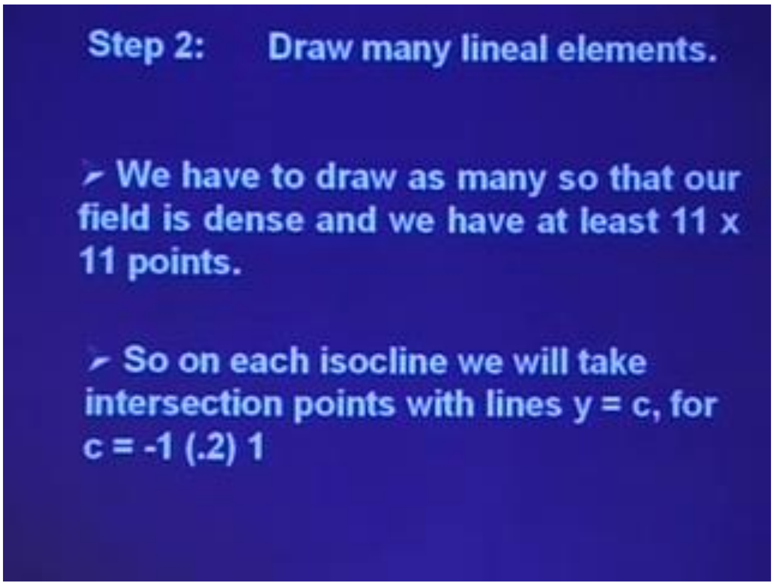

So, we have to draw these lineal elements as many as possible, so that our complete filed is dense enough and we should have at least a 11 cross 11 grid points that say is that is we have to choose grid points. So, what I have tried here is again these things you can learn by experimentation itself, here I am just showing this examples. So, that we do learn that is in the second example how the experimentation we are looking, what we are doing is on each isocline I have taken the intersection points with the line y is equal to c for the range c is equal to c minus 1 to plus 1 with 0.2, you see why I have taken it.

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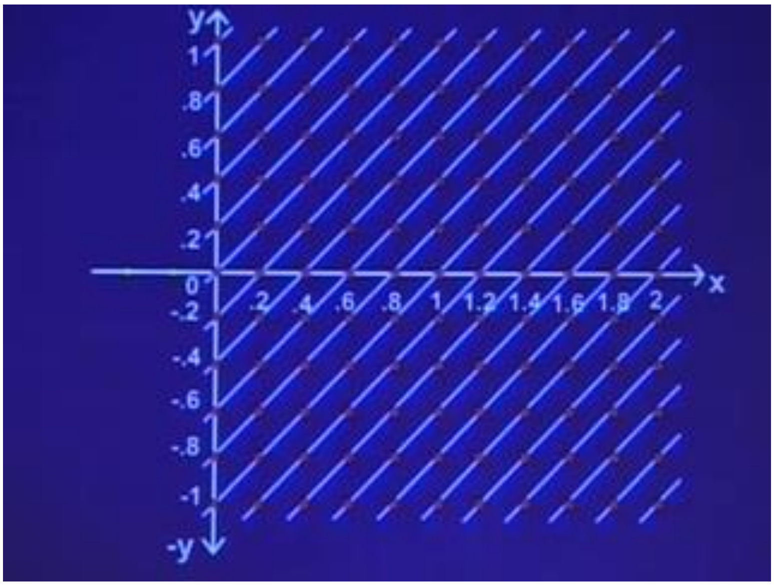

These were our isoclines, now how to choose those points we want 11 cross 11 points, so that say is that is here what I have done is that is you see I have chosen the lines y is equal to c like this one these lines. And I have taken all the points 0.2 difference with the 0.2 that say is that is I would be able to get some points on the isoclines itself. I had chosen first my grid and then isoclines, it may happen that I may not be having this grid point on the isocline.

So, it is better to choose grid points after I have drawn this isoclines, so like this when we are doing is you say is that is at least 11 cross 11 points I will be able to get in this grid.

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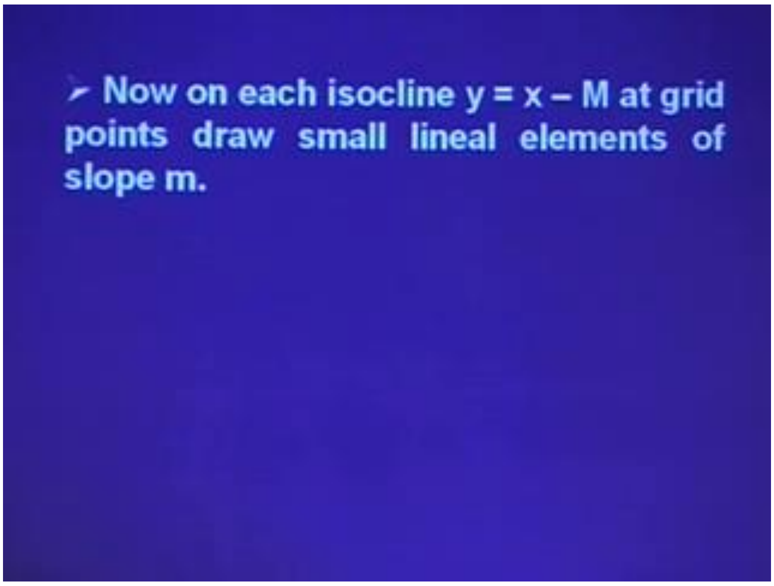

Now, what it says is that on these grid points, now we have to draw lineal elements that is on each isocline y is equal to x minus m at grid points draw a small lineal elements of the slope m.

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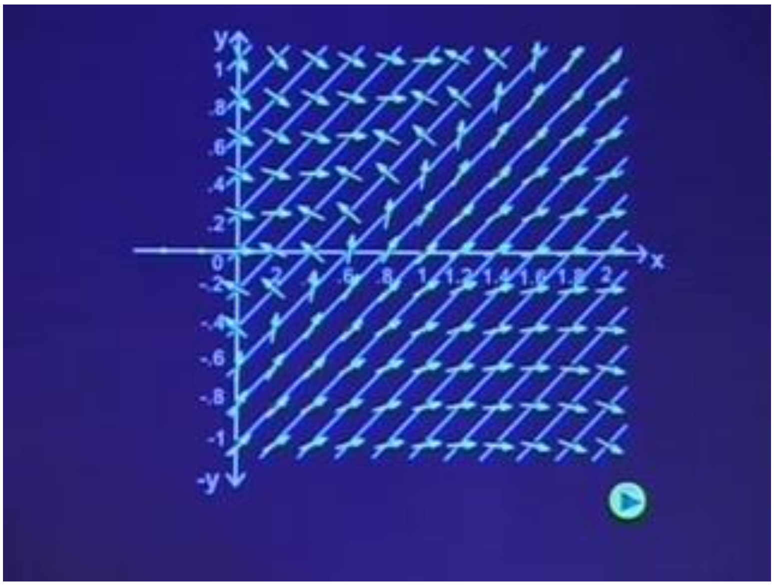

How we are going to do it again let us see that is how we are going to do, we have to do it by hand and we have to be very much accurate, because if we are not accurate we will not be able to get the accurate solution. So, you see we would start for the first isocline y is equal to x plus 1, what is the m here you do remember our isocline is y is equal to x minus m; that means, if y is equal to x plus 1 our m is minus 1.

That is here is this grid points of us let me just plot the grid points over here, so I have taken the first we have plotting the grid points over here you see that is all these grid points we are plotting over here on each isocline with the intersection y is equal to c. Now, on the first grid point isocline is y is equal to x plus 1 means m is minus 1, so I have to draw at lineal element that is a small line having slope minus 1 that says that is draw it.

Since the slope is minus, so I should have the arrow to downwards, now at the point this isocline is which one this is y is equal to x plus 0.8, that says m is minus 0.8; that means, I should have magnitude of the slope as 0.8 and direction downwards. See this one, so on the both the points it should be the same.

Now, this is isocline y is equal to x plus 0.6 and we do had three grid points here, so on all these three grid points we will draw a small lines that is the lineal elements with the slope as 0.6 and direction downwards. See like this and this way we would be going to do for all till here and you see is that is what is happening. At zero what we would have that direction is not being it important it should be just like that, but you see is as we are reaching to x minus 0.2; that means, my m is plus 0.2, so the direction should be upwards you see the direction has been changed. So, now, let me first draw all these lines and then will explain it little bit more, we do have it here, you see that what we are having is this is direction is slope is minus 1, here the slope is minus 0.8. So, 1 till 0 at 0 what will be that is this is the line y is equal to x, so m is 0, m 0 means is that it is horizontal one, so we do have that isocline should be parallel to the x axis.

And this direction is just been given at is not necessary that we should give the direction over here, because 0 does not have any direction positive or negative, then as we move y is equal to x minus 0.2. Now, that says is my isocline was y is equal to x minus m; that means, m is 0.2 that is the direction should be the slope should be have magnitude 0.2 and direction upwards you see all these points we do have direction upwards. And like that we are going, when we reach to the point that isocline y is equal to x minus 1 x minus 1 that says is that m is 1 and the line is y is equal to x minus 1.

This line also has the slope as one while as I have to draw lineal elements also having the slope 1. So, they are just parallel to that line that is they are matching with this line this has very important significance you see over here, then we are going upwards that is y is equal to x minus 1.2. So, the slope should be plus 1.2 and we are having like this we are just moving once like this, we had completed this direction field. Now, we had completed this direction field, what it says is that now what should be the next step, next step is to visualize the solution, how we are going to visualize the solution.

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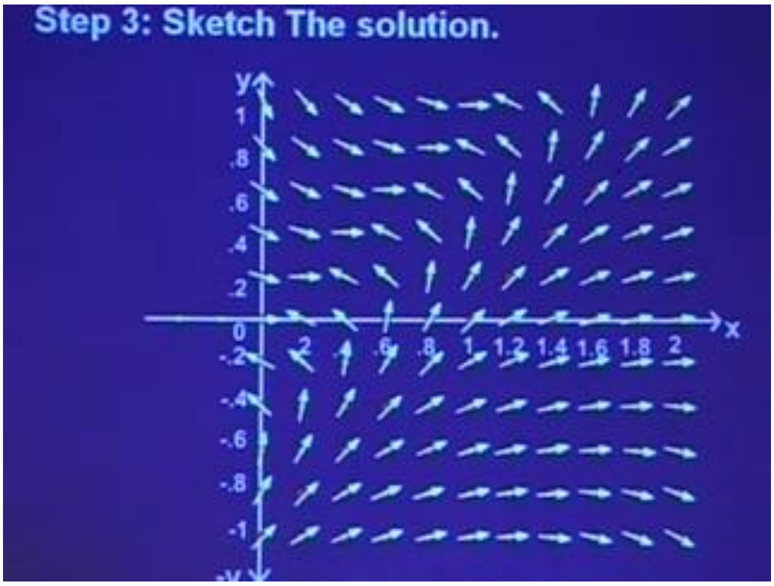

You see, we have got this direction field now I had remove the isoclines, so that we do not have any other idea of the direction we just have here our this lineal elements, which give us idea about the direction field. So, now let us see is that is what we have to do this third step is sketch a solution, for a sketching a solution the first thing is we have to find out the direction, that is how the this field what this is giving us about the curve, let us see one by one step by step.

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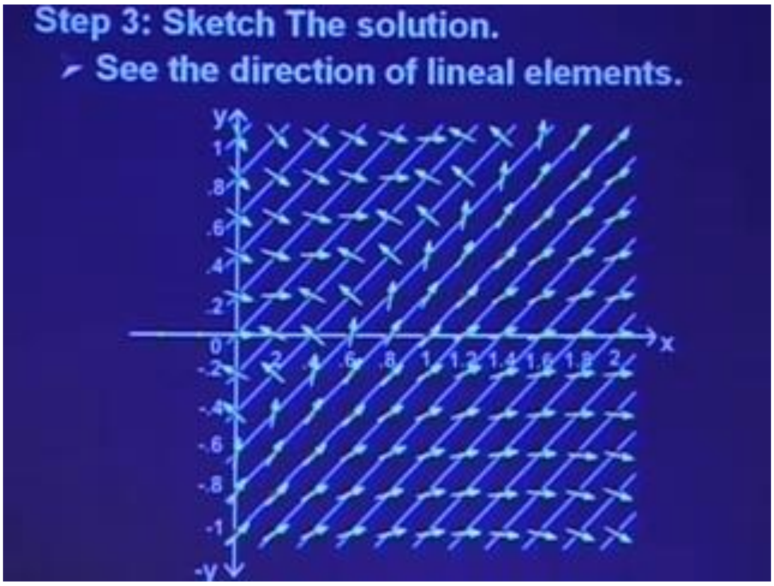

First is to see the direction of lineal elements, now I am again using my this isoclines, so that I could explain little bit more. We see is that from here, first we see is that is the direction is downwards over here let us see is that is one by one step.

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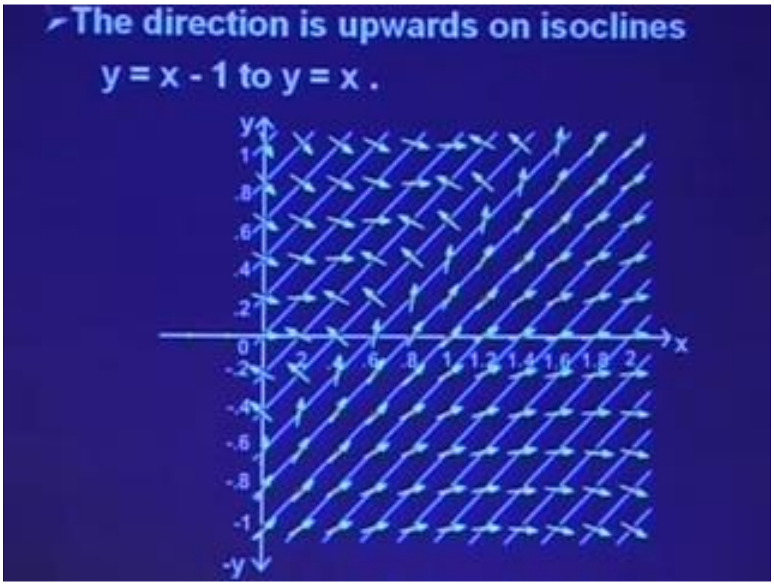

First we are seeing is that the line y is equal to x minus 1 this line, use this line and from this line if you see is that direction is moving here upwards something and then here also something downwards, then here it is downwards. So, we just make this first line as I said is this line as a special significance you see because it is matching with this one that says is actually says that this must be a solution to this differential equation. So, we say from this line we are having this is moving upwards, then it is coming up downwards that says the significant lines are y is equal to x plus 1, y is equal to x these are significant lines. So, we do have is that is y is equal to x minus 1 to y is equal to x the direction is upward, then the next point.

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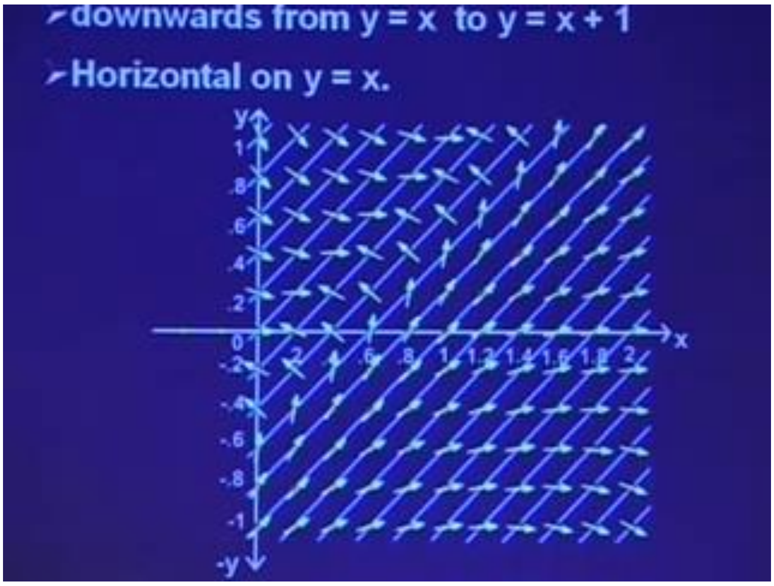

What we are observing that from y is equal to x minus for y is equal to x to the y is equal to x plus 1 this is the downward. And actually at y is equal to x we said is that to this is horizontal one, what it says is that the whatever be the curve it is changing it is direction at the point at the line y is equal to x that is changing the direction at y is equal to x minus 1.

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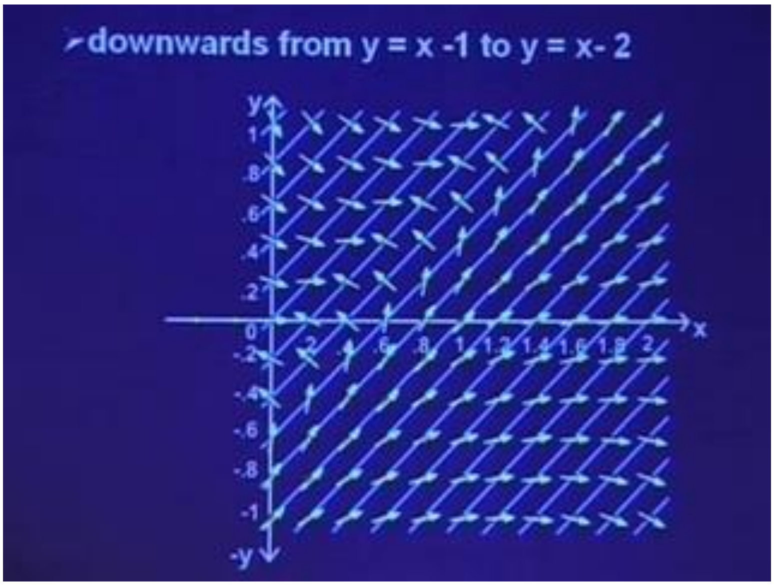

More over what we do have is that it is downwards from y is equal to x minus 1 to y is equal to x minus 2 all these things are downwards.

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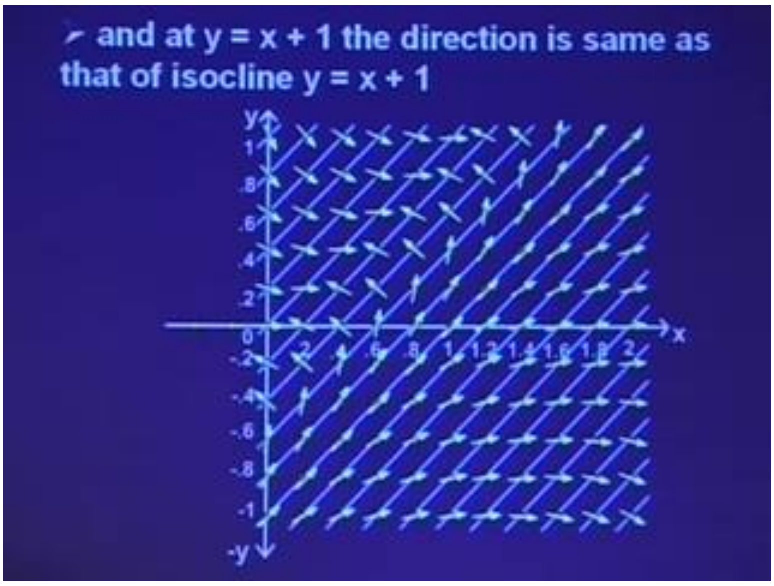

So, let me summarize what we had observed from here, we had observed that and again here I think is this one more point that y is equal to x minus 1, we have the direction is matching with the that of isoclines; now let me summarize the things which we had covered.

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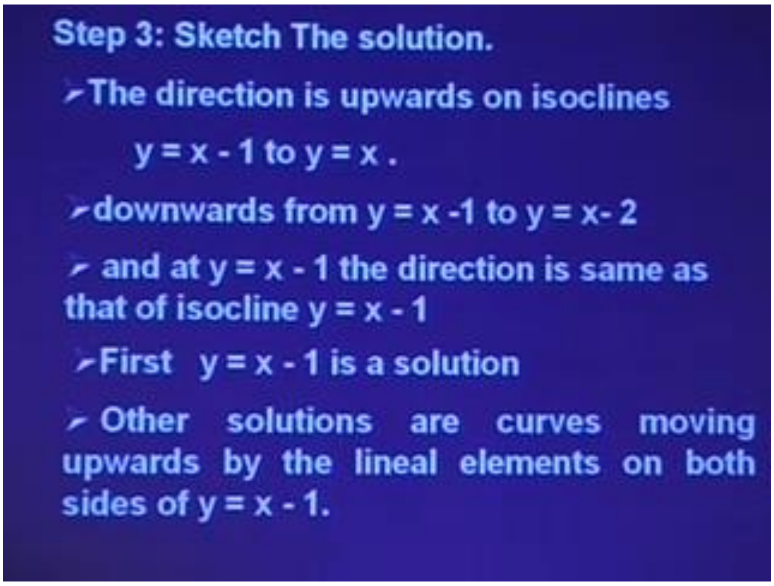

We had seen the direction is upward on isoclines y is equal to x minus 1 to y is equal to x it is horizontal at y is equal to x and it is downwards from y is equal to x minus 1 to y is equal to x minus 2. Here I think this is had left the point that y is equal to x to y is equal to x plus 1 it is again the downwards and at y is equal to x minus 1 the direction is same as that of that isocline y is equal to x minus 1.

So, first let us see this example, that says is that same as y is equal to x minus 1 what it says one thing it gives that this should be a solution. Second is that since it is moving downwards from x minus 1 to x minus 2, that says is my solution should be moving downwards, that solution curve should be moving downwards from this line and it is upwards from this line to this that says the solution curve should be moving upwards. So, that those points again that other solutions are curves moving upwards by the line lineal elements on both sides of the y is equal to x minus 1, that is upward downward like that we would be moving let see that is how we are going to is sketch it.

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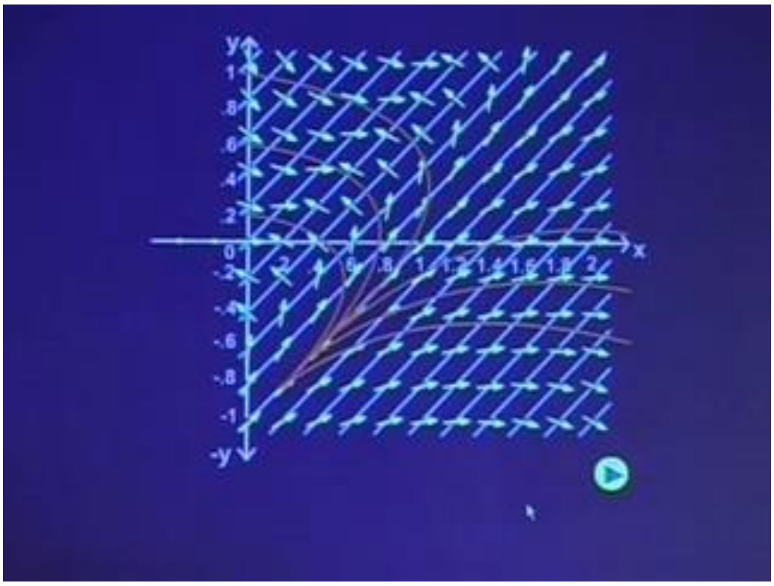

Here I am just giving you the solutions and then you see is that is how I had sketched it up we have got enough idea that is how we are doing the things, first on the upside and then on the downside of the line y is equal to x minus 1. So, first thing what we had is actually that this line y is equal to x minus 1 this is one solution, another is that the solutions are moving this way, because it is moving this way at the point y is equal to x in this isoclines.

You see is here it is little bit what it says is horizontal one that says is off course I think is I have not sketched any solution which is matching with any lineal element on this line. What it says is that from here it is downwards that says is the solution should that is the curve is moving downwards that is it is again going down from here that this line is an important line from where the direction is changing to the downwards while is here this direction is not changing in this field.

We are getting is the direction should be like this one that is getting a point we are going up like this one, that says is that is these are solutions of this differential equation. So, what we have got one particular solution we have got y is equal to x minus 1 this is a solution, then we are having these curves, what are these curves off course we cannot identify the curves directly from these graphs. But, we can check it whether this method is actually visualizing the solution or not because this method says is that is just visualizing the solution how the curves or that how this function y would be moving. So, we just go by this one, so let us just check it in the next one that is what we can how we can check whether we have visualized it correctly or not.

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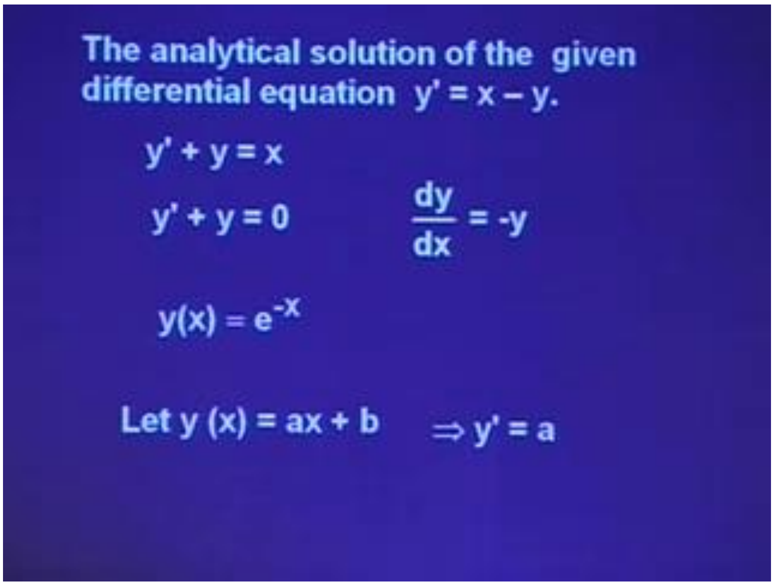

So, let see that is analytical solution and off course we have not learn till now, that is how to find out the solution, let us try again with the same method that is first assuming some function and then trying to get that is whether that function satisfies this equation, so that we could get an idea about the solution. So, let us write this differential equation as y dash plus y is equal to x. Now, first take this right hand side as zero that is called actually homogeneous equation, so first we will solve this homogeneous equation y dash plus y is equal to 0.

What we could write this I think you do remember this equation this is d y over d x is equal to minus y and we do know the solution of this equation is e to the power minus x. So, the general solution would be c times e to the power minus x, now we have to actually find out the solution of this equation not this equation. How we are going to do is, now we will assume one more function let us say that a x plus b if I assume this function let us try to see whether this function satisfies this equation or not, for that what we have to do, we have to find out its first derivative and then put this function and the derivative in to the equation and try to find it out.

So, that I could get what are this because I have taken here general form, we are I am not having any particular solution. So, here I am just giving you one way that is we can use in general that is a x plus b, why I have chosen this I can explain you, since I do have on the right on side x; that means, whatever be the function of the solution if I make it I should get something x over there here we are having y dash and y. So, they must be some term of x, but in y dash that term may should be disappearing that is why we should choose if it is right hand side is x, we choose it ax plus b. And now our aim is to find out this a and b such that this function may satisfy this particular equation, let us see it, so this says y dash is equal to a.

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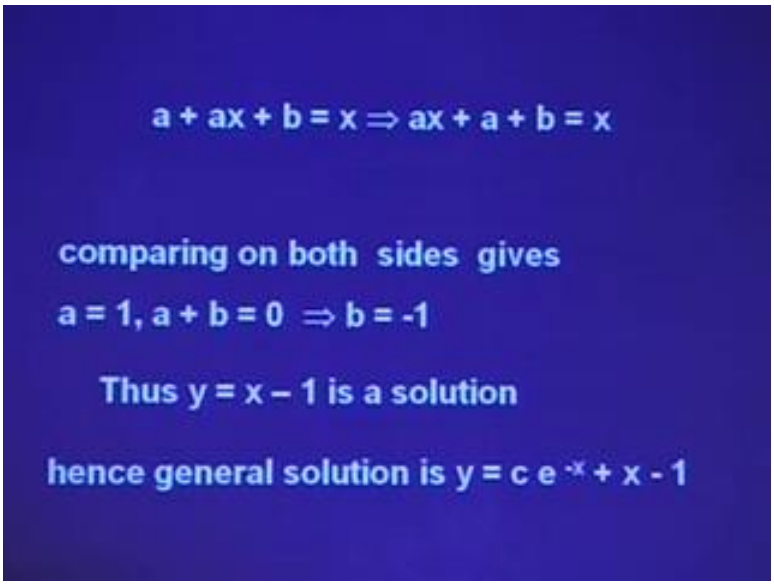

Now, if I am putting this on this given equation what I would get y dash plus y; that means, a plus ax plus b is equal to x what does it implies that a times x plus a plus b should be x. Now, equate the coefficients on both the sides what it gives me that a is equal to 1 and a plus b is equal to 0, now a plus b is equal to 0 what does it say that b is equal to minus a which is minus 1, thus what I have got the particular solution y is equal to x minus 1.

Now, we can check if I put what will be y dash y dash will be simply one say if I put y dash plus y what I would get 1 plus x minus 1 and the sum would be x; that means, this function is satisfying the differential equation. Now, what should be the general solution of this differential equation, because this is one particular solution here got I have got and before an I have got that homogeneous system, when I have taken that y dash is equal to minus y, I have got the solution y is equal to e to the power minus x.

Will learn actually the method of solving these equations later on, here I am just giving you one example just like that. So, what we do have that is, if we put this function general solution what we will be making y is equal to c times e to the power minus x plus x minus 1. Now, we can experiment if I find out what is y dash y dash would be minus c times e to the power minus x plus 1 and if I put y dash plus y is equal to what it will be I would be getting c times e to the power minus x plus x minus 1 and minus c times e to the power minus x plus 1; that will give me only x that says is that this function is satisfying the given differential equation y dash plus y is equal to x.

So, this is a solution and if now you can use any plotter or you can just find it out that is, just like that, we cannot handle that is how if this graph could be shown, we can just choose any plotter and plot this function and will find it out that this would be looking like.

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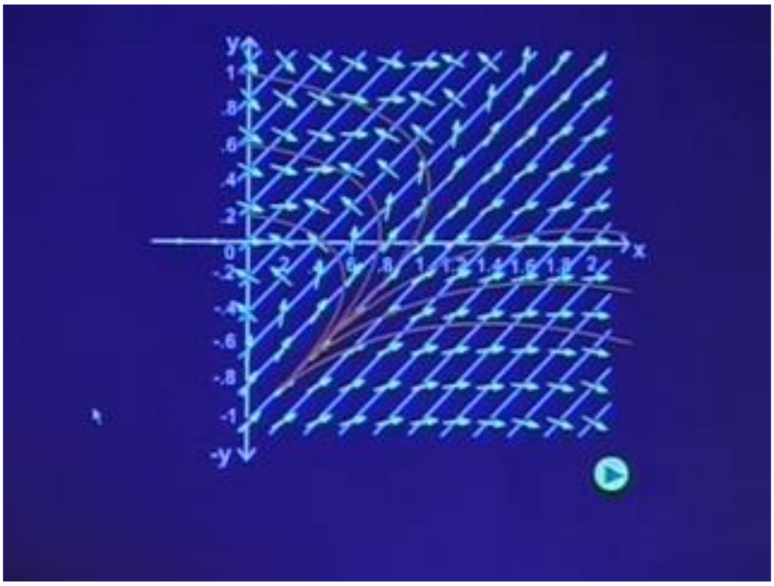

You just like this kind of functions you would be getting for different values of c you would be getting these different kind of plots that what would be the solution that says is that is by this we can visualize. So, this is not actually, if this method is not giving us exact analytical solution, we are visualizing the solution at that some different points that is we are just trying to get what would be at particular points rather you could says that at any particular point I could get the value of the function from here. We can just make the value of the function at any particular point I can just calculate the value of function that says is that is discretely or that we says is that is why we are calling it approximated solution.

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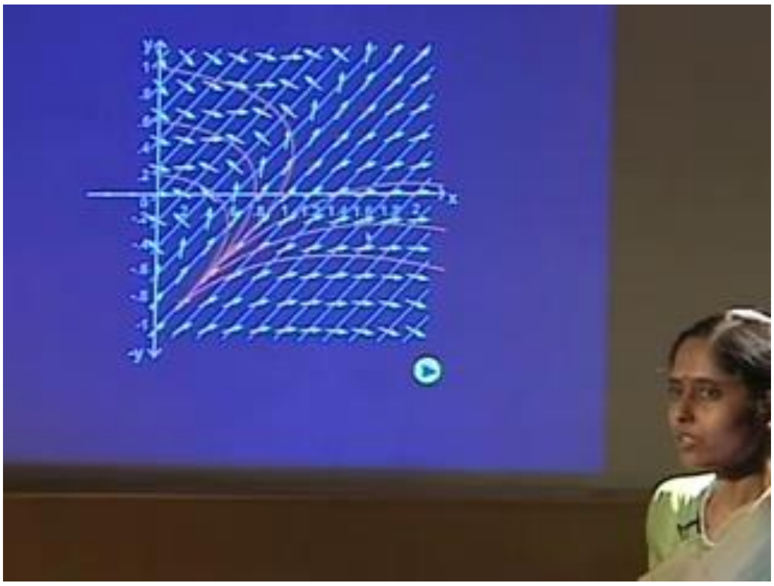

Let us summarize this method once again, first we have started with the I am just summarizing with the help of second example, first we had the range, then on this range first we had drawn the isoclines. Isoclines we have chosen for different values of m, m we had chosen from minus 1 to plus 2 with the difference as 0.2, then on these isoclines we made the grid points, how we had made the grid points we took the intersection of the isoclines with the lines y is equal to c.

And wherever they are intersecting those points we have taken as the grid points and what the range for c, we have taken we have taken for c minus 1 to plus 1 with again the difference as 0.2. Then on these grid points we have drawn the lineal elements, lineal elements means that is on we have to go isocline by isocline. So, on first isoclines, because the slope is minus 1 and there is only 1 grid point which is being showing in this 1.

So, this graph, so we have done only one lineal point showing the slope as minus 1 and second line slope is minus 0.8 and so on we have done till 0 at 0 the slope is horizontal. Then we had gone to the positive side and the slope is on the upward manner like that we have done and then what we have seen here one more thing, that at y is equal to x plus 1 the direction of the isocline as well as that of the slope is matching. Then what we have done we have seen the direction of this direction field, we have got that is, one thing we have got clear that this is one solution that is y is equal to x minus 1.

Then we have got that my solution should move upwards from here and so here if this is moving and when we whenever we are coming at 0, we see is ranging the direction; that means, it should move downwards again. And from this side it is moving to this side like this one, so thus we have drawn the solutions according to that first on the upper side of the y is equal to x minus 1 and then on the lower side of y is equal to x minus 1. So, we have got what we have got off course we cannot judge what is this function.

But, what we could do we could find out at different points we could find out the value of the solution at that point. If you remember our examples which we had in our first lecture, there wherever these differential equations were appearing, we were interested to find out the solution at those points, those points that is we are not interested very much what is actual function, we are interested only at certain points. So, this method actually gives me at those certain points I can find out that is what should be the solution.

Here we conclude today’s lecture, what we have learnt in today's lecture that is understanding first order ordinary differential equations, they are geometric interpretation that is geometrically what they are saying according to the calculus. And we try to learn something about the solution from that geometric interpretation in that process. We learn a graphical method or which is called a method of direction field or slope field to find out the solution of first order ordinary differential equations without solving them analytically.

We had also learn that this can give us the value of the function at a point and we cannot approximate complete function or complete analytical function, that says is that we can solve some application problems without actually knowing that from which function these are generated that is discretely we could find out the solution of those differential equations.

To sum up this one you can try this graphical method with the help of some exercises. So, here I am giving for you understanding some simple exercises, which you can do, first is that y dash is equal to 2 plus 3 y, another is y dash is equal to x plus y times 2 minus y. Another is that y dash is equal to y minus sin x, in all these exercises what you can do is you just first think about what should be a proper range in which you could draw it properly.

So, that you can understand that is the behavior of the solution, then choose proper slopes for drawing isoclines, try to see the isoclines that is what kind of equations you are getting whether they are simple lines or they are some curves, if they are curves then you have to draw those curves. As in the third exercise you have seen that isocline would be y is equal to sin x that is you have to draw the curve sin x for and what would be the isocline actually y is equal to sin x plus m that kind of curve. So, you have to draw the curves like that one and then on each isocline you have to draw the lineal elements of that one. So, I think that you would be able to do it by yourself and learn the method more properly and understand it more clearly that is all for today's lecture.