Mathematics - III
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Lecture - 1
Introduction to Differential Equation

Welcome to lecture series on Differential Equations for under graduate students, today's topic is Introduction.

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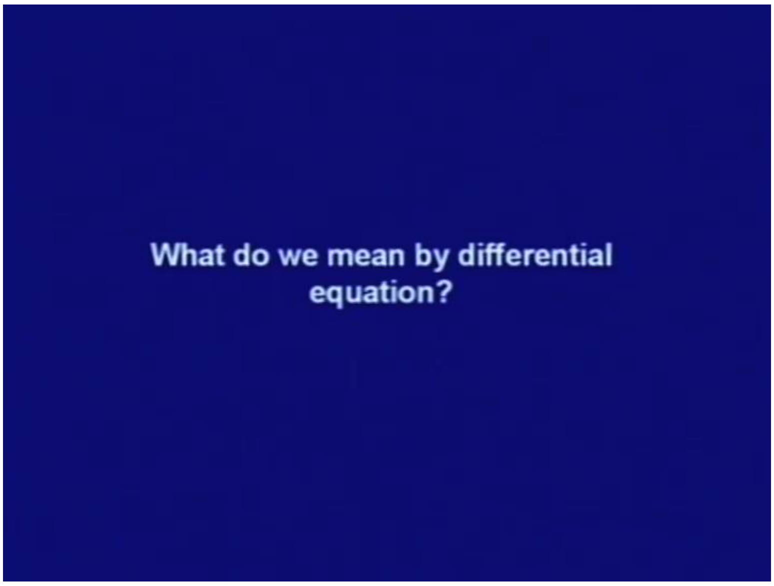

The first question arises, what we mean by differential equations before answering this question, let us try to remember one thing, which we have already written the exponential function.

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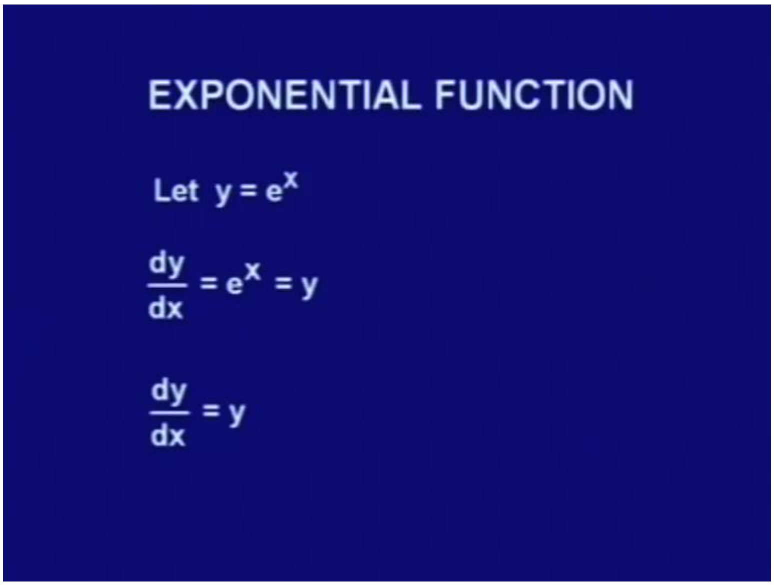

Yes, e to the power x, we know that this function has a property, that if we differentiate it with respect to x, we get the same function, that is d y over d x is e to the power x. Thus we got a relationship that d y over d x is equal to y, this relationship is called differential equation.

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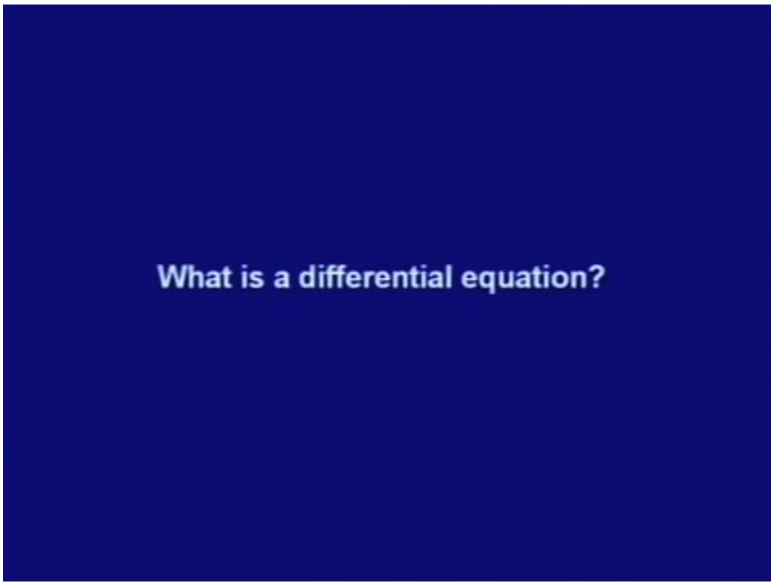

So now we are ready to answer, what do you is a differential equation.

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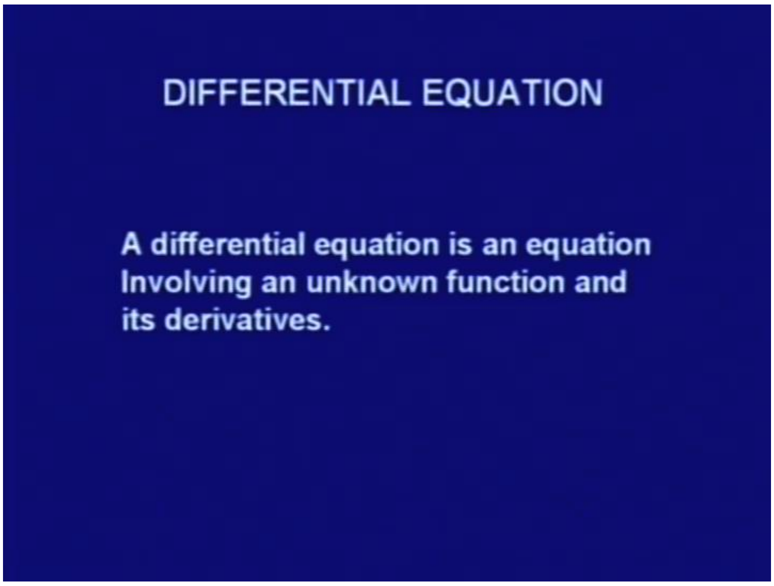

A differential is an equation involving an unknown function and it is derivatives.

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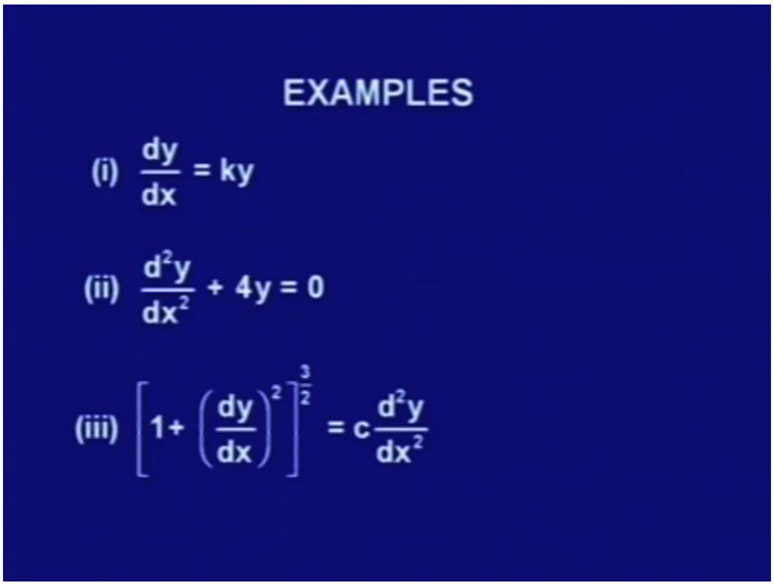

Thus d y over d x is equal to k y, d 2 y over d x square plus 4 y is equal to 0, 1 plus d y over d x square whole to the power 3 by 2 is equal to c times d 2 y over d x square.

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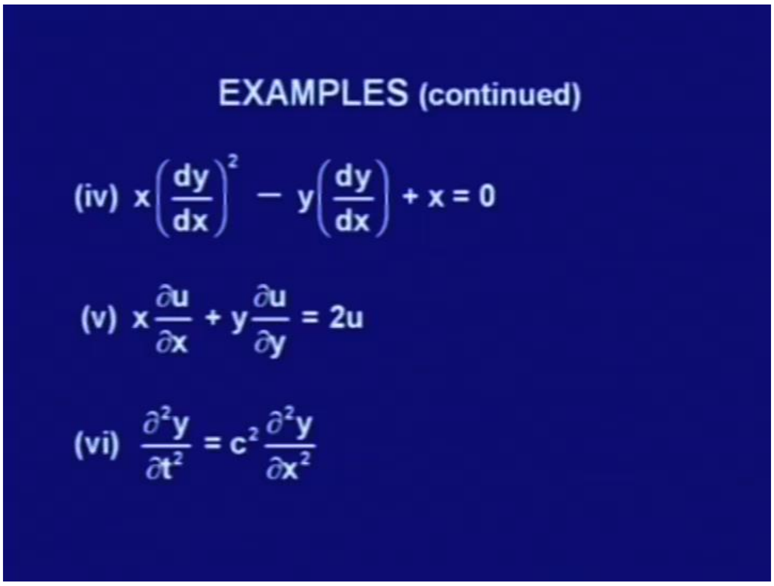

X times d y over d x whole square minus y times d y over d x plus x is equal to 0, x times del u over del x plus y times del u over del y is equal to 2 u, del 2 y over del t 2 is equal to c square times del 2 y over del x 2 are all examples of differential equations. In these examples we have seen, that some involves ordinary derivatives, while some examples involve partial derivatives. Thus, we categorize our differential equations into ordinary differential equations and partial differential equations.

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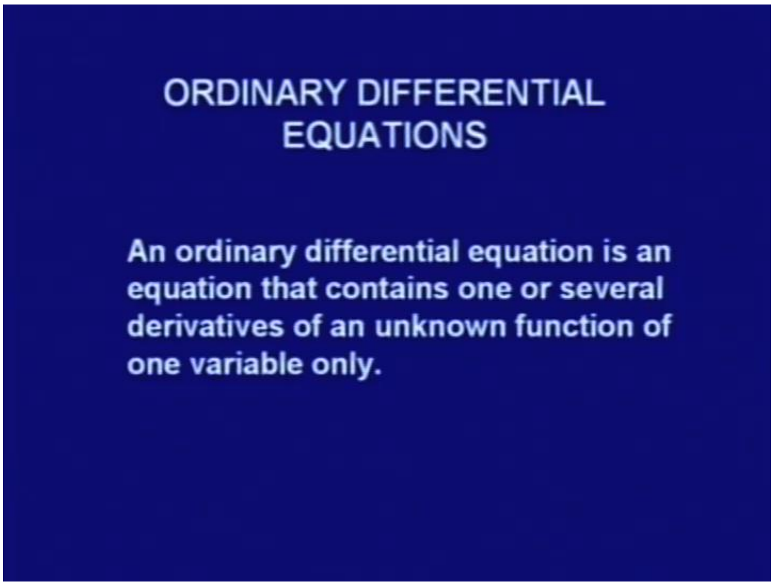

An ordinary differential equation is an equation that contains one or several derivatives of an unknown function of one variable only.

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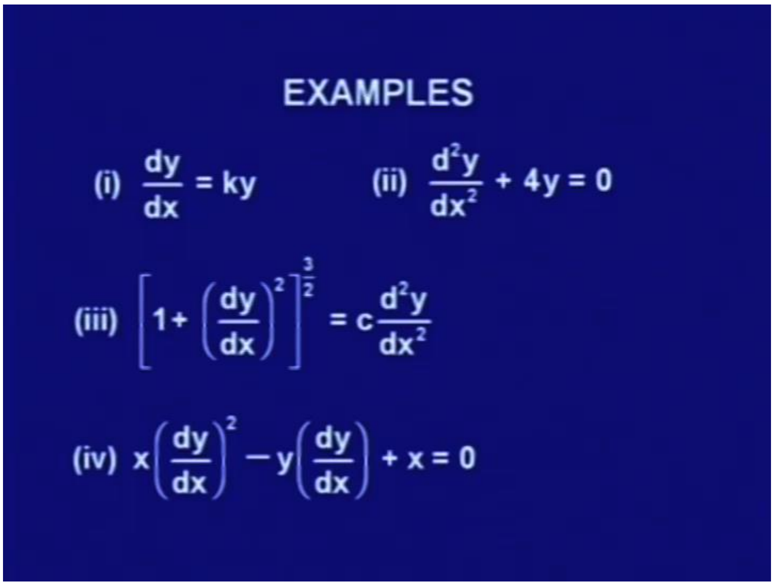

Thus, in our example the first example d y over d x is equal to k y, the second example d 2 y over d x 2 plus 4 y is equal to 0, 1 plus d y over d x whole square complete power 3 by 2 is equal to c times d 2 y over d x 2, x times d y over d x square minus y times d y over d x plus x is equal to 0 are all example of ordinary differential equations. Here we do have the unknown function y and it is derivatives and the function y was of only one variable and we had the ordinary derivative with respect to that variable only.

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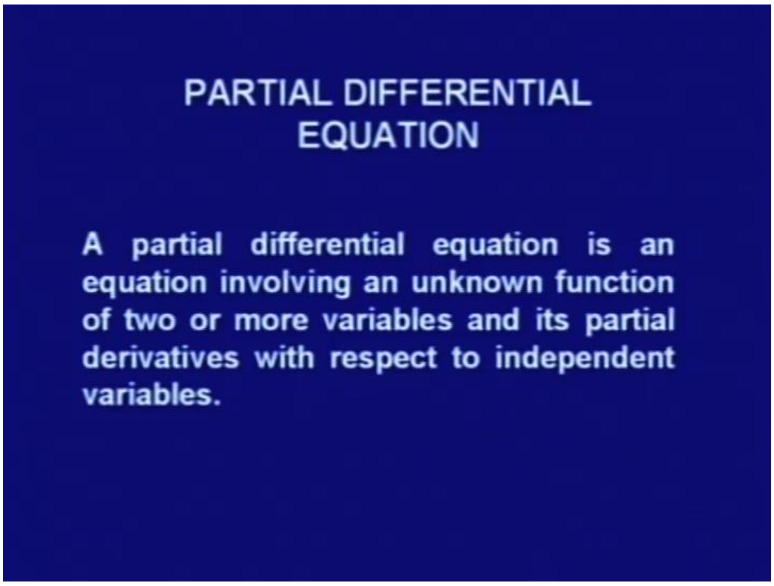

A partial differential equation is an equation involving an unknown function of two or more variables and it is derivatives with respect to independent variables.

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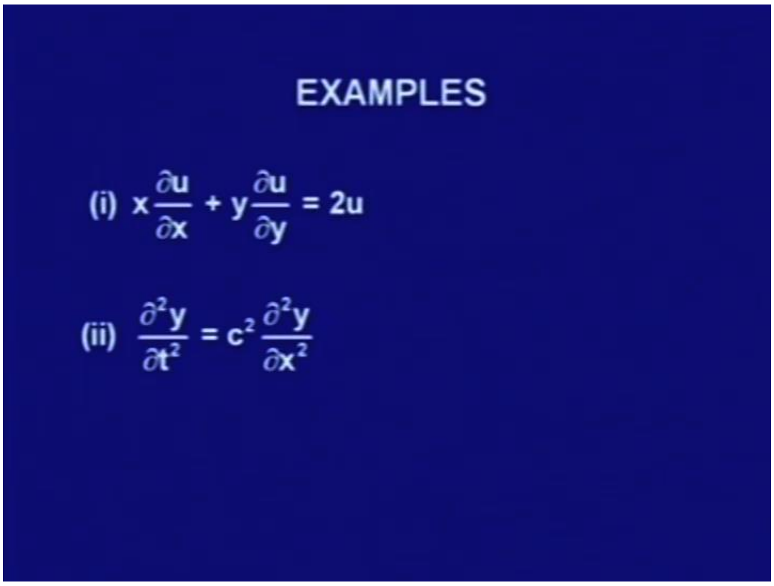

Thus in our examples, x times del u over del x plus y times del u over del y is equal to 2 u, del 2 y over del t 2 is equal to c square times del 2 y over del x 2 are examples of partial differential equations. Here in the first example, u is a function of two variables x and y and the equation involved is partial derivative with respect to x and y.

While, in the second equation, y is the unknown function and this function is of two variables t and x and it involves the second order partial derivative with respect to these independent variables. Thus we have seen that, all these equations are involving a function of one or more variables the derivatives ordinary or partial again the derivatives of first order or higher order, again the derivatives may be having the exponents.

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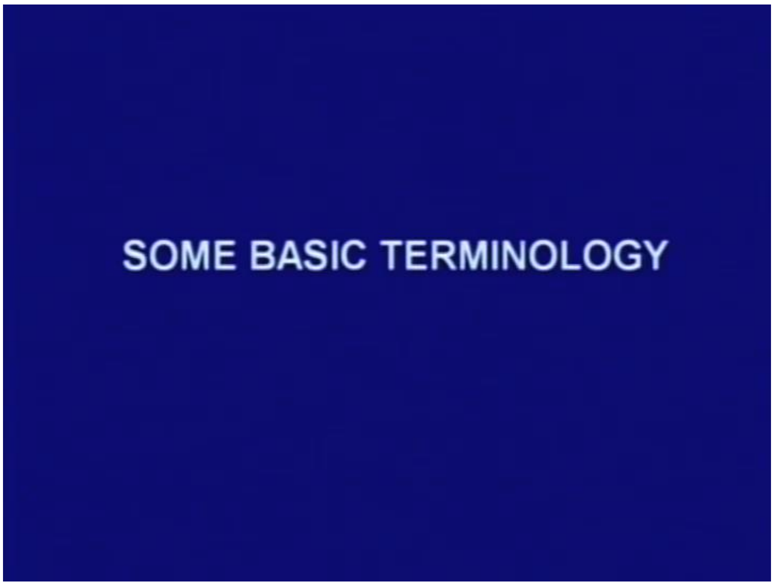
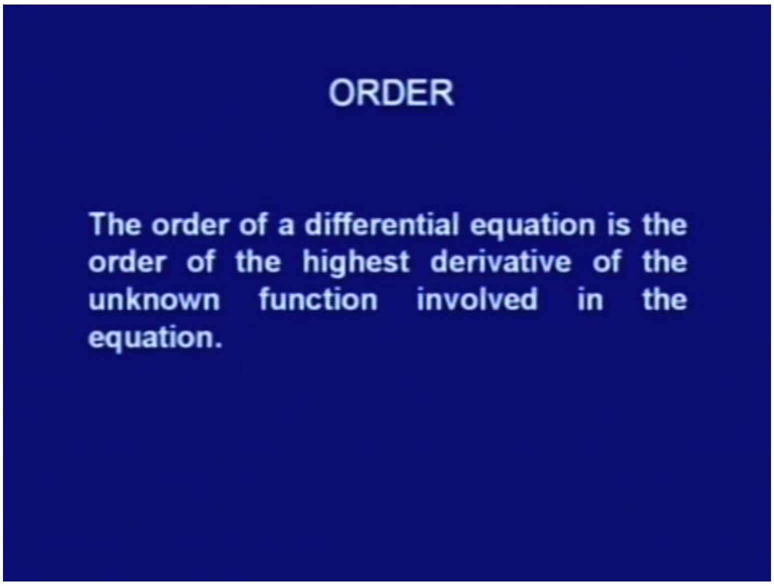


Thus, we are now ready to give some basic terminology of the differential equations. (Refer Slide Time: 05:05)



The first one is order; the order of a differential equation is the order of the highest derivative of the unknown function involved in the equation.

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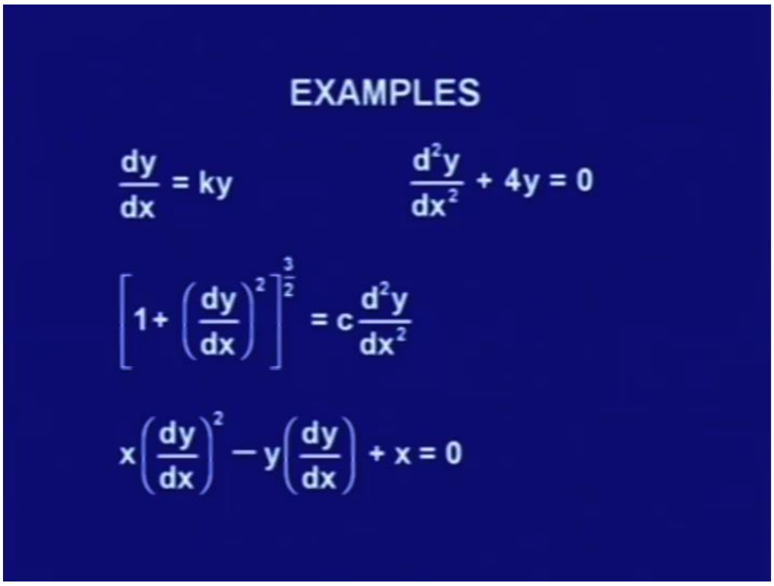

Thus in our example, let us see first example d y over d x is equal to k y, this involves the first order derivative only. Thus the order of this equation is 1, while as the equation d 2 y over d x 2 plus 4 y is equal to 0, this involves the second order derivative and that is the highest derivative involved in this equation, thus order of this equation is 2.

Similarly, in this equation 1 plus d y over d x square whole to the power 3 by 2 is equal to c times d 2 y over d x 2. In this equation, it involves the first order derivative as well as the second order derivative. So, the highest derivative involved is the second order derivative, thus this equation has added 2. Now, you see in the last example that x d y over d x square minus y times d y over d x plus x is equal to 0. We see the derivative involve are only of the first order, thus order of this equation is 1 only, this equation is also been called first order equation.

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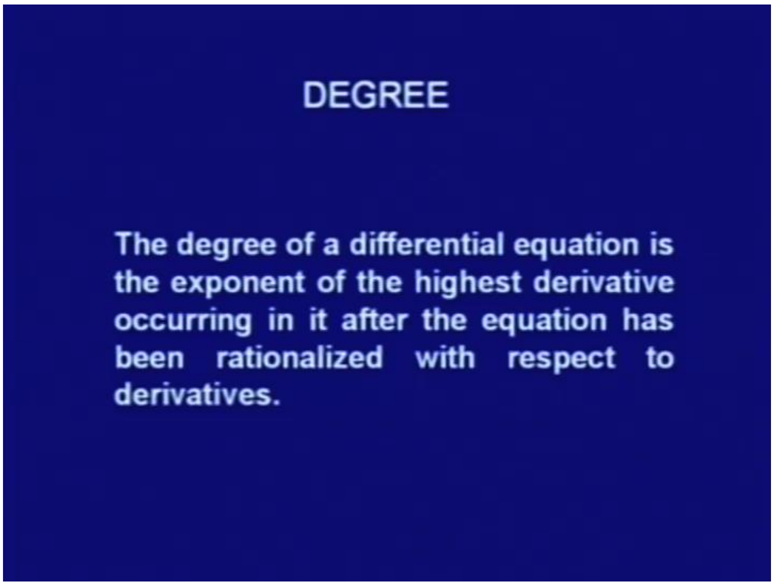

The second term is degree; the degree of a differential equation is the exponent of the highest derivative occurring in it. After, the equation has been rationalized with respect to derivatives, let us understand this more clearly with the help of examples.

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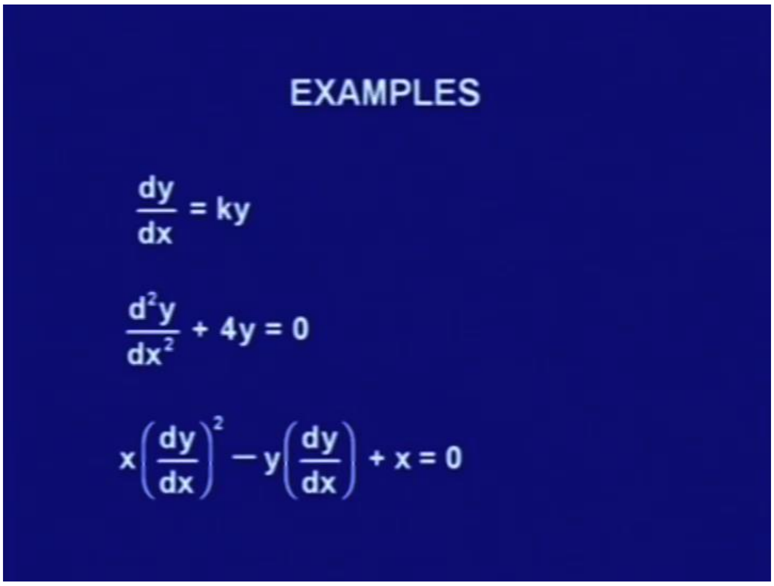

So, in our first example, we do have that the derivative involve is only that highest derivative is first order and it is exponent is 1, thus the degree of this equation is 1. In the second example the derivative which is involved that is second order derivative is the highest derivative and it is exponent is 1, thus this equation also has degree 1.

Now, see in this example we do have the derivative involve is of first order only, but we see that d y over d x has a exponent 2. So, the highest exponent is 2, thus degree of this equation will be 2.

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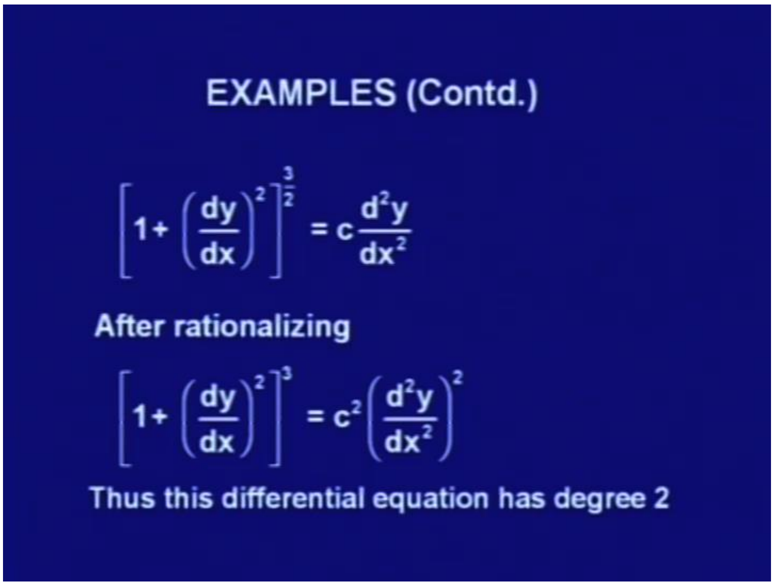

Now, let us see one more example we had in our examples, that is 1 plus d y over d x square whole to the power 3 by 2 is equal to c times d 2 y over d x 2. In this equation, we do have a power as 3 by 2 that is it is involving is square root.

Now, this is not called a rational equation because here the power of d y over d x is having 1 over 2 something. So, we will rationalize it, how will rationalize it, we will square this equation on both the sides and thus we would get 1 plus d y over d x square whole to the power 3 is equal to c square times d 2 y over d x 2 is equal to d x 2. Here, the highest derivative involved is the second order that is d 2 y over d x 2 and it is exponent is 2, thus degree of this equation has 2. Now, we can categorize the equations in different manners, so one more categorization of differential equations is coming as linear differential equation.

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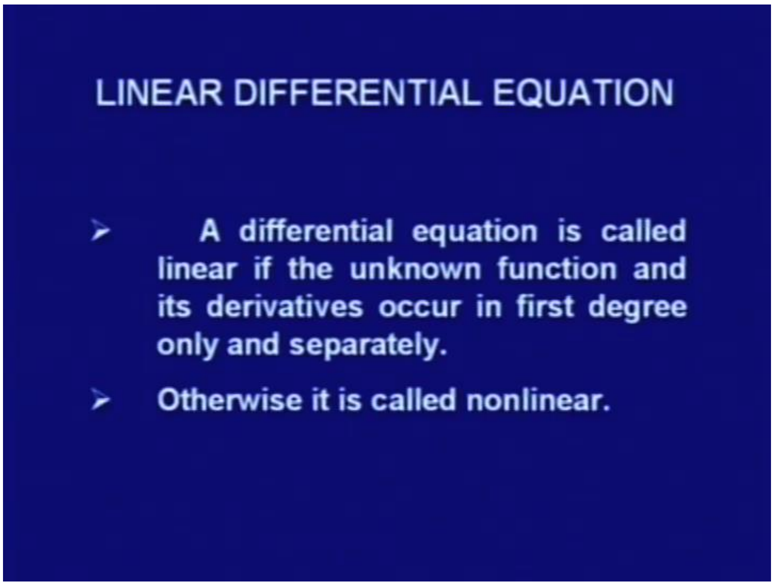

A differential equation is called linear, if the function and it is derivatives are appearing in the equation with degree 1 only and separately. If this is not happening then it is called non-linear, let us understand this more clearly with the help of examples.

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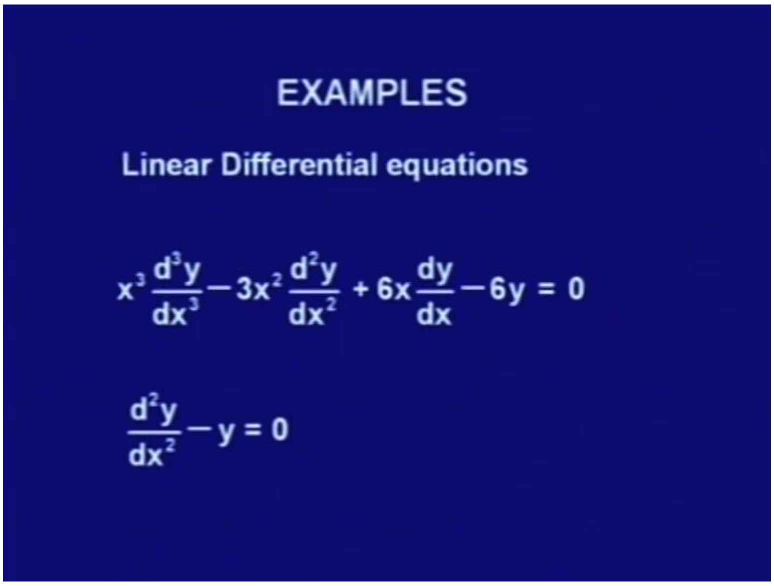

So, first let us see the example of linear differential equation, see this example x cube times d 3 y over d x 3 minus 3 x square times d 2 y over d x 2 plus 6 x times d y over d x minus 6 y is equal to 0. Here we see that the derivatives involved are first order, second order and third order. And all these derivatives and the function y are in degree 1 only, none of them have any exponent, more over none of the places, I too have that the function and derivatives are appearing simultaneously in any component.

Thus, this equation is linear in the unknown function y and it is derivative this is called a linear differential equation. Let us, see one more example d 2 y over d x 2 minus y is equal to 0, here we do see is that is the derivative is d 2 y over d x 2 and y both are occurring in degree 1 and separately thus this is also an example of linear differential equation.

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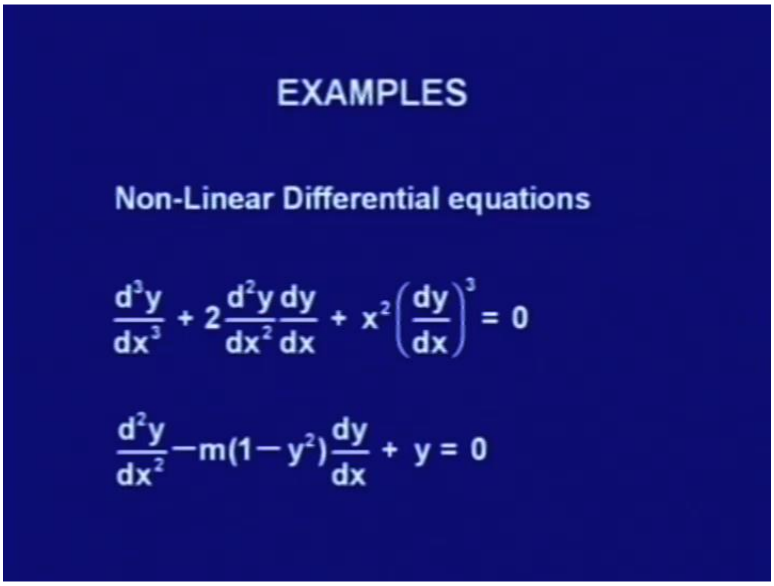

Let us see some example of non-linear equations, see this first example d 3 y over d x 3 plus 2 times d 2 y over d x 2 multiplied with d y over d x plus x square times d y over d x whole to the power 3 is equal to 0. Now, we see here that derivatives are coming with exponent as the first derivative d y over d x has degree 3, while as in the second component d 2 y over d x 2, that is the second order derivative and the first order derivative are occurring simultaneously, thus this is a non-linear equation.

The second example d 2 y over d x 2 minus m times 1 minus y square times d y over d x plus y is equal to 0. Here, we see that derivatives are occurring in the first degree only, but the function itself is occurring in the second degree, moreover in one component, we do have that the function and it is derivative are occurring simultaneously. Thus, this is also an example of non-linear differential equations.

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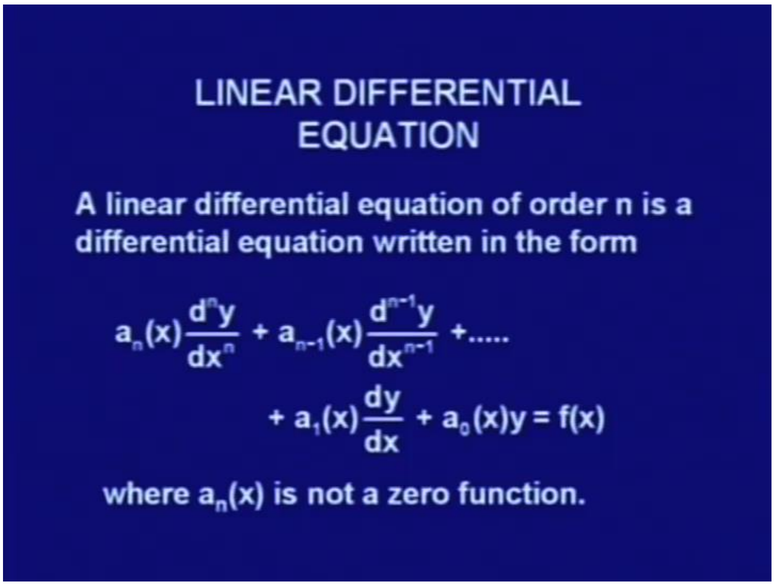

Let us have something more about linear differential equations, a linear differential equation of order n is a differential equation, which is written in the form a n x times d n y over d x n plus a n minus 1 x times d n minus 1 y over d x n minus 1 and so on, a 1 x times d y over d x plus a naught x times y is equal to f x. We see here that all the derivatives are occurring in first degree only and their coefficients or the function of x only, none of them involve the y or it is derivative, this is a linear equation.

Now, here the highest occurring derivative is d n y over d x n, that is nth order, thus order of this equation will be n only, if it is coefficient a n x is not a 0 function, thus this equation will be called a linear equation of order n. If a n x is not a 0 function, why, if a n x is a 0 function the first term will not be there and then the highest derivative would be only d n minus 1 over d x n minus 1, that is n minus 1th order, so this is a differential equation of order n.

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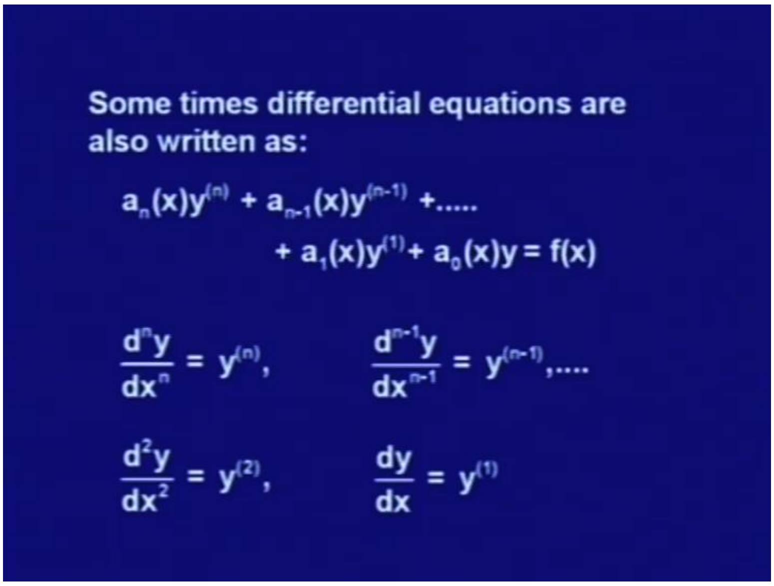

Many times this kind of equations are also being written as a n x times y n plus a n minus 1 times y n minus 1 and so on a 1 x times y 1 plus a naught x y is equal to f x, what the change we have done here, if you see is that we have replace this, y n with d n y over d x n that is the nth derivative, we have used another notation y n.

Similarly, for n minus 1th derivative, we have used the notation y n minus 1 and so on, for second derivative the y 2 and for first derivative y 1, these notations have been used. So, that it is convenient in writing, so this is also one way of writing the differential equations and these notations have been used for convenience. Now, we have seen that, these differential equations are functions, and their derivatives and their relationship.

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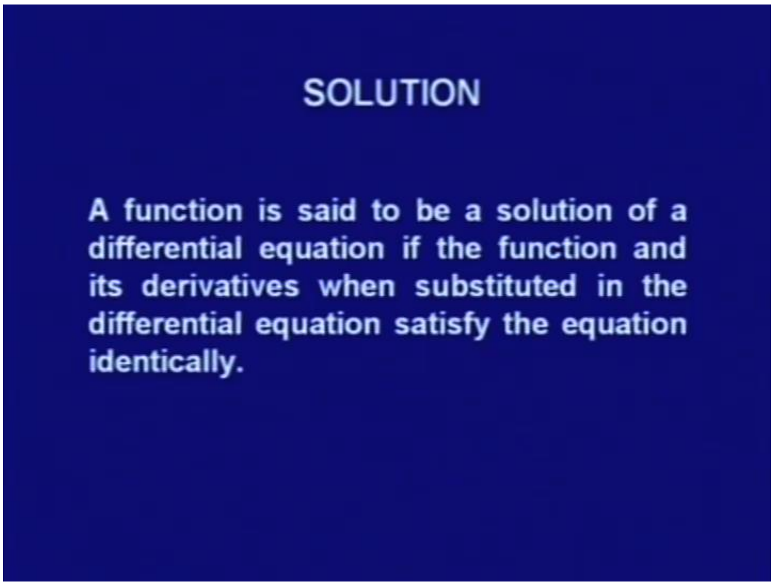

Thus a function will satisfy, so any function which is satisfying these equation, will call it is solution. So, now, we come to another terms solution, a function is said to be a solution of a differential equation, if the function and it is derivatives, when substituted in the differential equation satisfy the equation identically.

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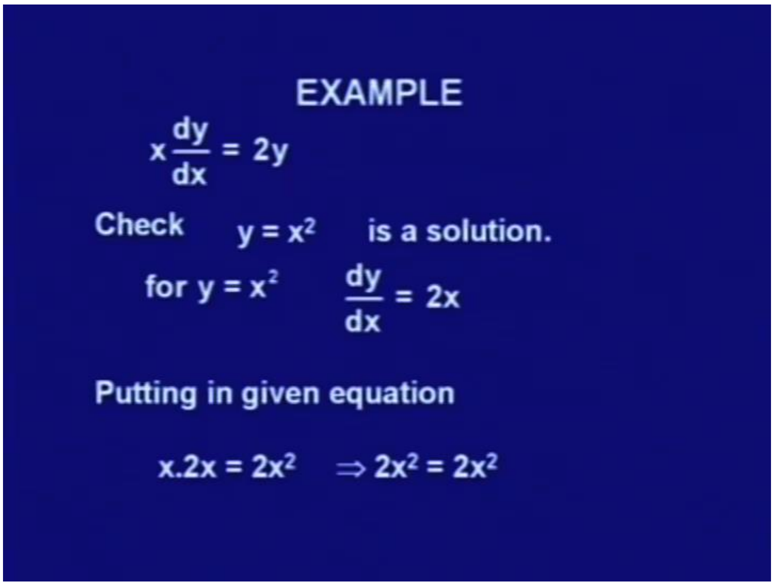

Let us, try to see it with the help of example, let us take this equation x times d y over d x is equal to 2 y, we can check that y is equal to x square; this is a solution, how can we check, let us see that is for y is equal to x square, what will be d y over d x that will be 2 times x.

Now, this value of the function that is x square and it is derivative 2 x, we can put into the given equation and thus what we will get x times 2 x is equal to 2 x square and right hand side is also 2 x square. Thus, we get that this equation is being satisfied identically thus, what we got that x square is a solution of the differential equation x times d y over d x is equal to 2 y, so we have got a solution.

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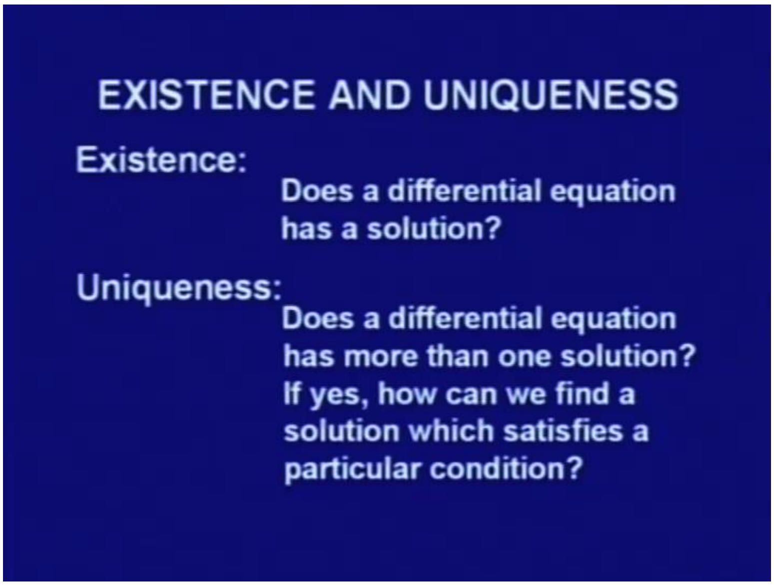

Now, let us see, what as soon as we talk about solution the question comes existence and uniqueness, what do we mean by existence, does a differential equation has a solution, the answer to this question is called existence. Then, it comes uniqueness does a differential equation has more than one solution, if yes can we find out a particular solution, that is if how can we find out a solution, which satisfies a particular condition. So, now we have seen that a differential equation may or may not have a solution, if it is having a solution, it may have more than one solution. So, now we are going to categorize the solutions also.

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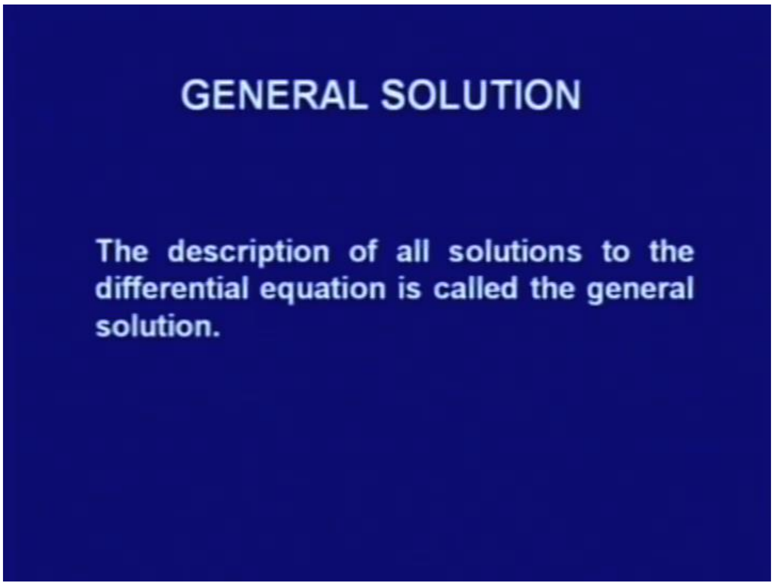

The first categorization is general solution the description of all solutions to differential equation is called the general solution, let us elaborate it little bit more.

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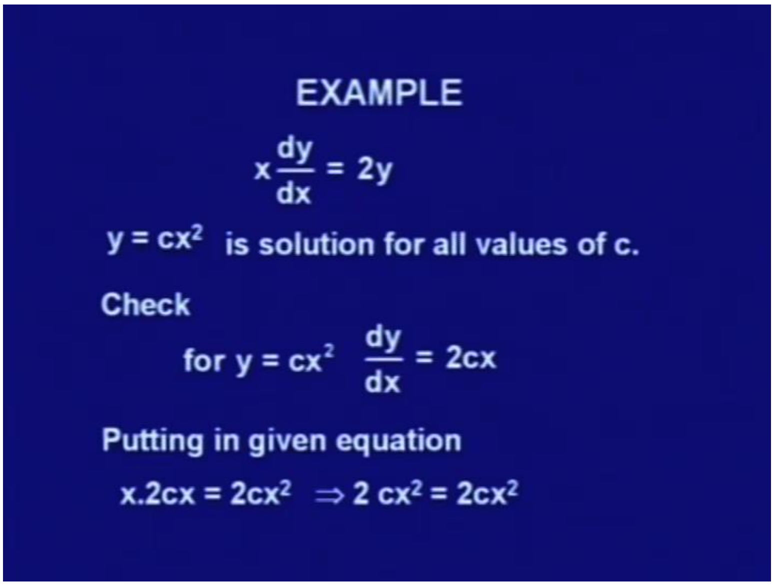

We have just seen this example that is x times d y over d x is equal to 2 y, now let us see little bit more, if I take this function c times x square, we can check that, this is also a solution of this equation for all values of c. We can check it c y is equal to c x square; the d y over d x will be two times c x.

Now, if I incorporate these things into the equation, what I would get is that x times 2 c times x is equal to 2 c x square, which is nothing but the right hand side. Thus c x square is also satisfying this equation for all values of c, what it says that is we are getting not only one solution, but many solutions that is for each value of c, I will get one solution.

So, we this c x square will be called the general solution of this differential equation, now the second question is uniqueness, before answering this let us try to define something more over here.

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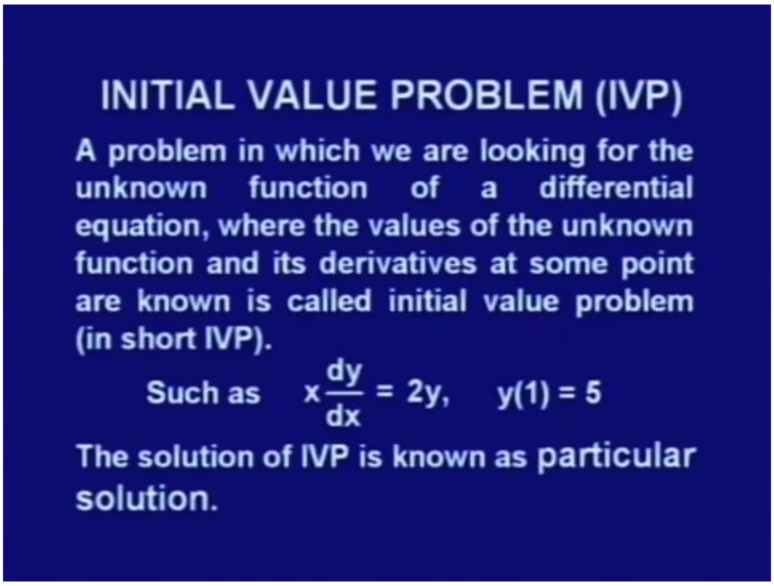

Initial value problem, a problem in which we are looking for the unknown function of a differential equation, where the equation of unknown function and it is derivatives at some points are known is called diff initial value problem; this is also referred as IVP in short. Say for example, the equation d y x times d y over d x is equal to 2 y, if I incorporate one extra condition that the function value is 5 at x is equal to 1, this is called an initial value problem, a solution to the initial value problem is known as particular solution.

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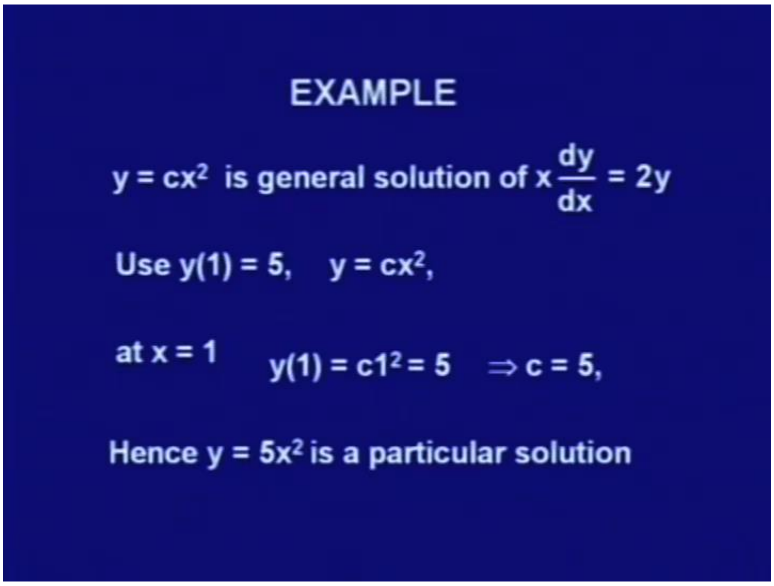

So, now let us see this with the help of example, just now we had seen that y is equal to c x square is a general solution of x times d y over d x is equal to 2 y.

Now, incorporate this condition that is y at 1 is 5; that means, in c x square at x is equal to 1, will put what we will get is that c is equal to 5. Hence, we got that y is equal to 5 x square is a particular solution, this solution we have got, when we have use this initial condition that is the function value is 5 at x is equal to 1, let us understand it little bit better with the help of graph.

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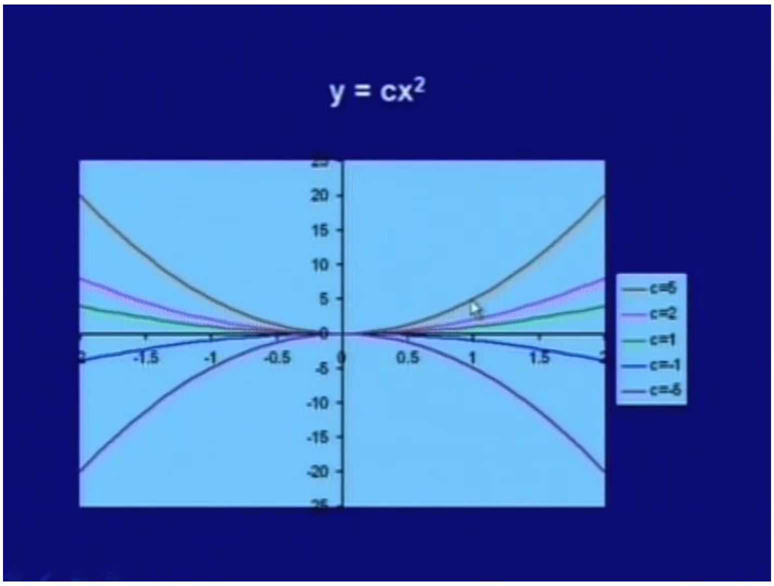

We see here, this is the graph of all the y is equal to c x square here these graphs, I have made for different values of c, that is all these functions are solution to the differential equation x times d y over d x is equal to y for different values of c, we are getting it. Now, in our condition what the initial condition was given that at x is equal to 1, the value of the function should be 5, we see is that is this is that is at 1, if we see here we are getting the value of 5. So, this is the graph which we will be or this is the function which is coming as a particular solution. We see is at 1 no other function has value 5 over here, so thus to arrive to a unique solution, we require a particular condition.

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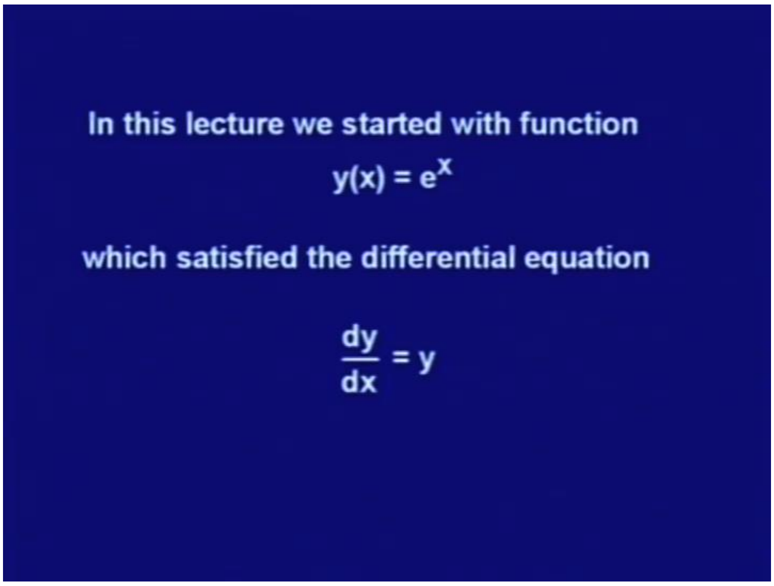

Now, we have started this lecture with this function, that is exponential function and we had seen that this is satisfying the equation d y over d x is equal to y.

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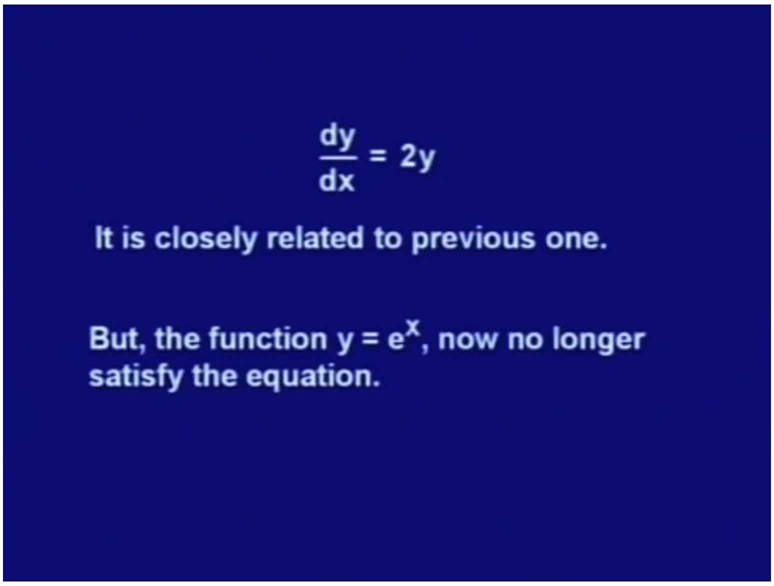

Now, let us take another differential equation d y over d x is equal to 2 y, this is closely related to the previous one, the only difference is of a constant 2. We have used at the y, but now this e to the power x, this does not satisfies this equation any more. Now, how to find out the answer to this equation; that is which function will satisfy this equation, let us take this assumption that the function should be of the same form that is exponential 1, we have to do little bit experiment with the same function.

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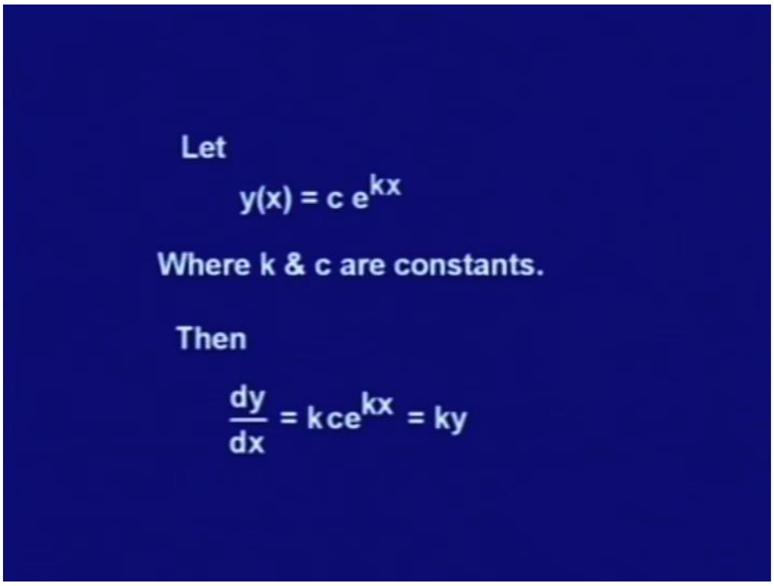

So, let us assume that we take a function c times e to the power k x, where both this c and k are constants. If I differentiate it, we get d y over d x is equal to k c times e to the power k x that is k times y.

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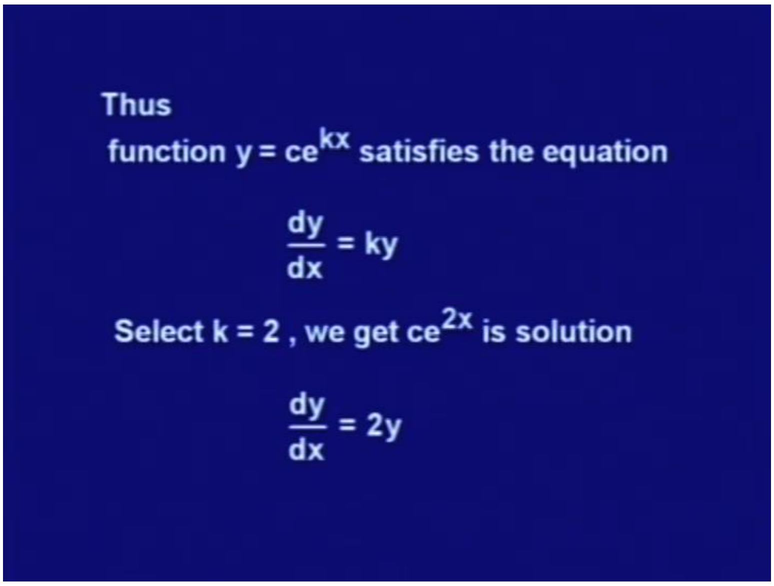

Thus we have got that function y is equal to c times e to the power k x satisfies the equation d y over d x is equal to k y. So, we had started with d y over d x is equal to 2 y, so now what I will do, I will replace this k by 2. So, we got that c times e to the power 2 x is solution to the differential equation d y over d x is equal to 2 y.

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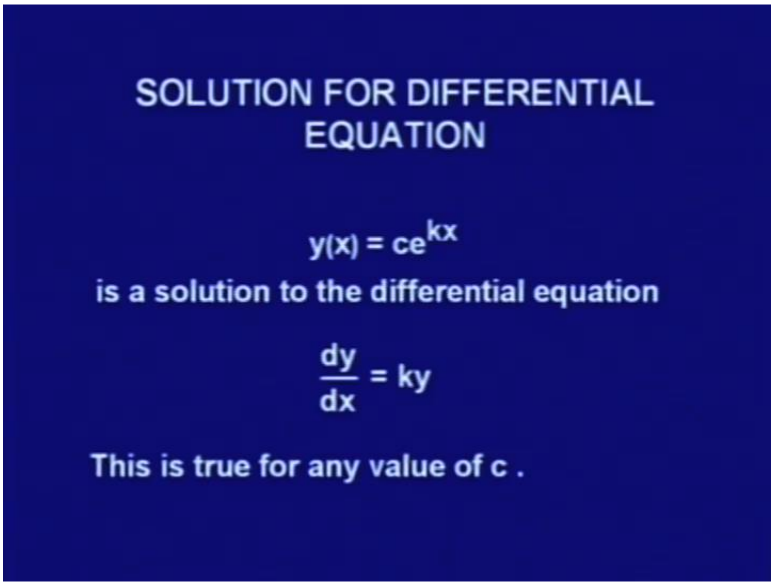

Now, what we have actually got, let summarize the result, we have actually got that c times e to the power k x is the solution to the differential equation d y over d x is equal to k y. This is true for every value of c, so thus what we have got actually we have got a general solution to the differential equation d y over d x is equal to k y.

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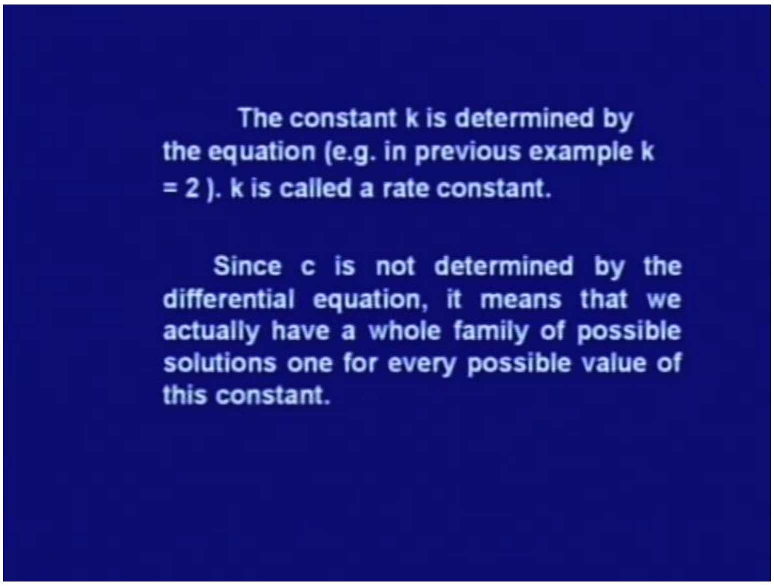

Where, this constant k is determined by the equation as in our example, what we have chosen, we have chosen k is equal to 2, this k is also referred as rate constant. But, this c is not determined by the differential equation, that is we have actually find out a whole family of possible solutions and one for every possible value of this constant is a solution.

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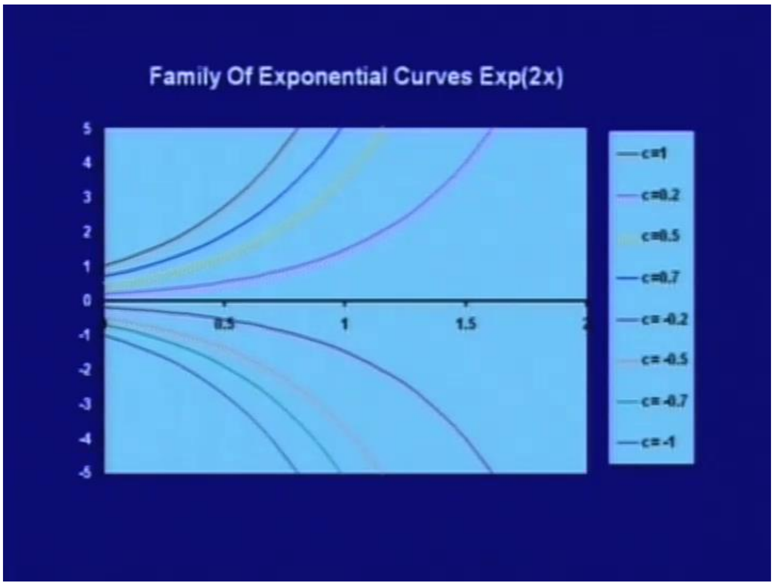

Let us see the graph of this function; this is the family of exponential curve. So, here I have made this, c is that is here we have different exponential curve all these are e to the power 2 x functions, but with different values of c. So, all these function satisfy the equation that d y over d x is equal to 2 y, how do we choose any particular solution, that says we require something more for that one, what that more, we say is that from where this function is starting, that is what is the initial value.

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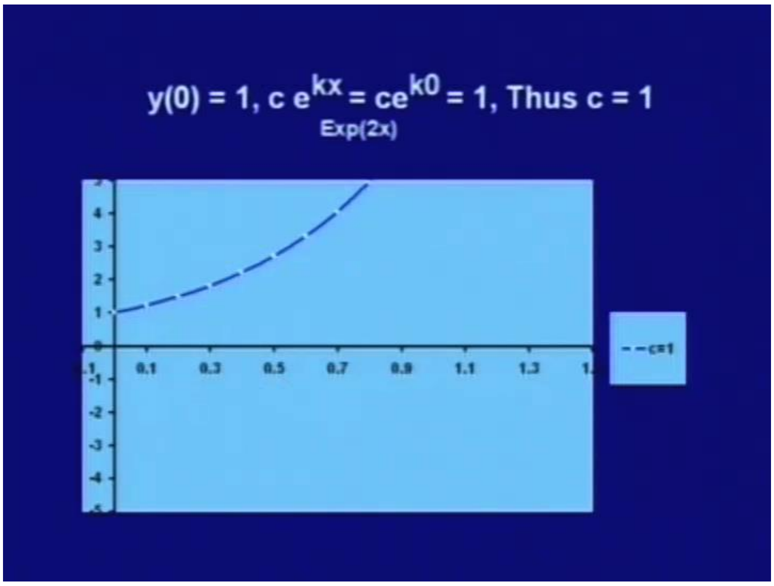

So, suppose I says that is my initial value that is y at 0 that is initially is 1, now if I get what is the solution that c times e to the power k x, that will be actually 1. Thus, what we get that c is equal to 1, if I replace c is equal to 1, I will get this function we see that this function we are initially at 0, we are starting at 1. Thus, we are having this as a function which is satisfying this particular condition.

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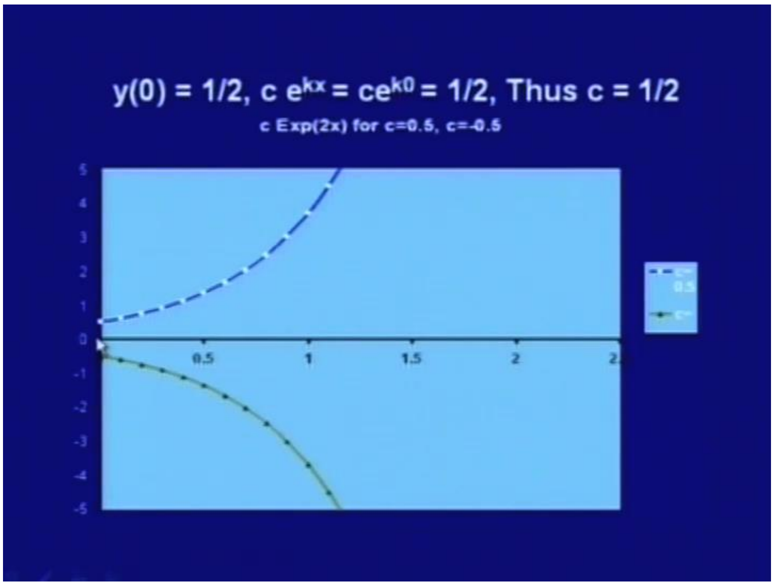

So, if we change our initial condition say the function which is starting at half, we will get that a different function, that is we will get here, y is equal to half times e to the power 2 x. This curve now, if I change my initial condition to minus half, that is the function is starting at minus half, we see that our function has solution has been changed. So, thus what we have got that we summarize the result, that we have got the initial condition is speaking up a particular solution. The differential equation gives me a whole lot of general solutions, so now I give you some question to think about.

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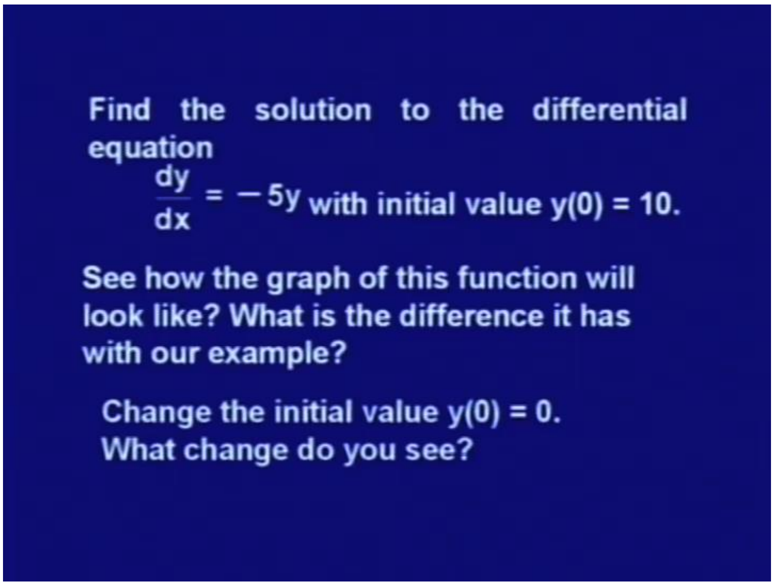

Find the solution to differential equation d y over d x is equal to minus 5 y, use the initial condition that at 0, the function which is starting with 10. See that is how the graph of this function will look like and what is the difference it has with our example, just now what I have done, more over you can experiment little bit more see that, if you change the initial value to 0 that is the function is starting to 0. Now, find it out what you are getting does it drastically change your solution think about it and then see.

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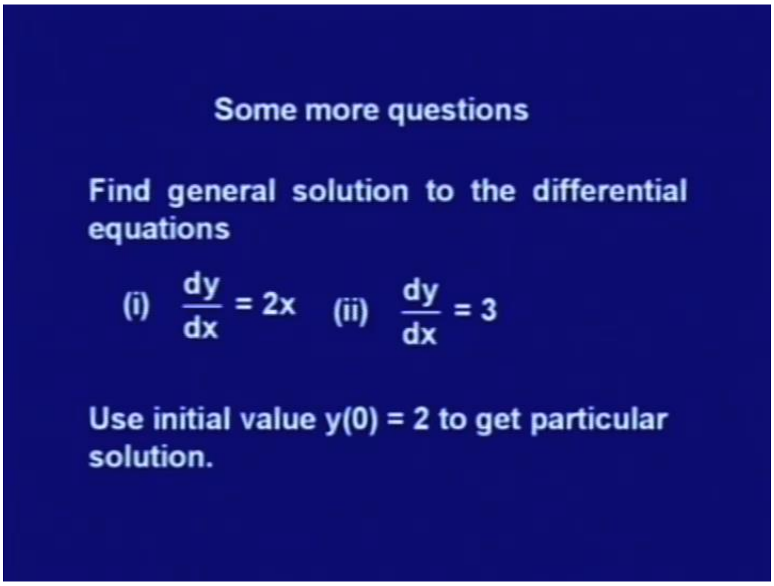

So, here are some more questions for you to think about find the general solution to the differential equation d y over d x is equal to 2 x, think about what should be the solution of this differential equation. Second question is d y over d x is equal to 3, what will be solution of this differential equation, more over use the initial condition that y 0 is equal to 2.

In both these questions and find out a particular solution, you can experiment with other initial conditions also and see what you are getting. These would give you little bit inside that is what we mean by differential equation and it is solution, so we have learn what are differential equations and what are solutions of differential equations.

Now, the question arise is why do we study differential equations or what we achieve by knowing the solution of differential equations. In other words, what is the application of these differential equation, where in real life, we will apply this differential equation, so that we study these differential equations.

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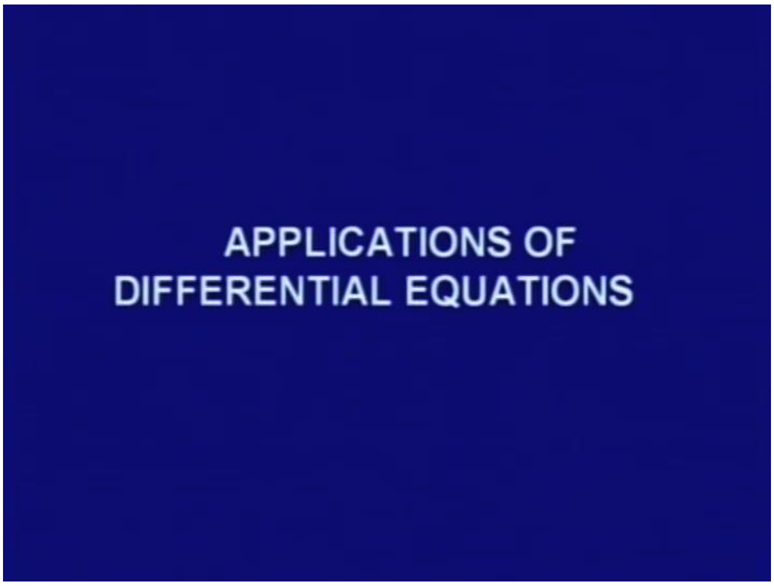

That says, what is the application of differential equation, the answer is modeling.

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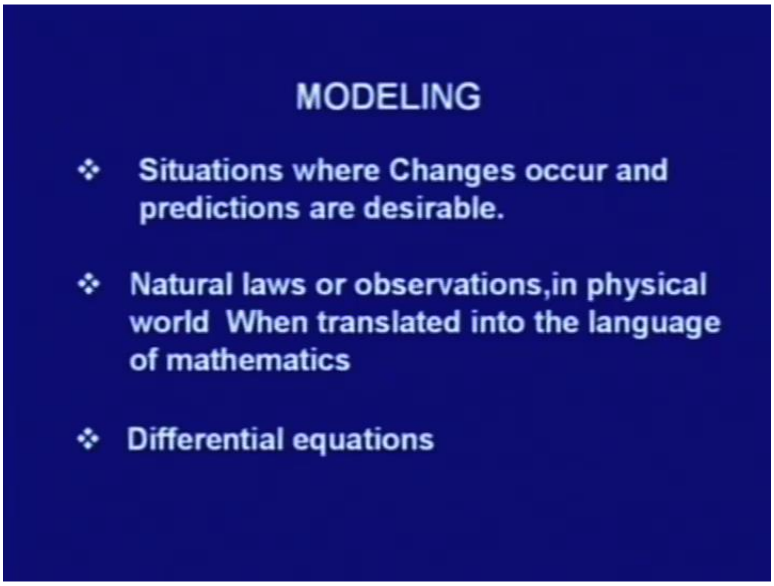

What is modeling? Modeling is that actually transforming the physical phenomenon into mathematical formulation by changing physical phenomenon into mathematical formulation. We do know more about that phenomenon, moreover we may predict the future behavior of that phenomenon and thus differential equations are very important topic in mathematics, science and engineering. In all those situations where changes occur and predictions are desirable.

The most of the systems, which comes understudy, they observe certain natural laws or observations, these physical changes when we transform into the language of mathematics may turn out to be differential equations. Thus, we try to see which kind of functions or which kind of phenomenon will translate into differential equation. A typical real what application of differential equations involve those phenomena's with changes with the time and the change is proportional to the rate of the change or the acceleration of the change. That is whenever these relationship, we can use as the differential equations.

Now, whenever a physical law involves a rate of change of a function, such as velocity or acceleration etcetera. We do get differential equations, thus in physics and engineering application; we do get many differential equations.

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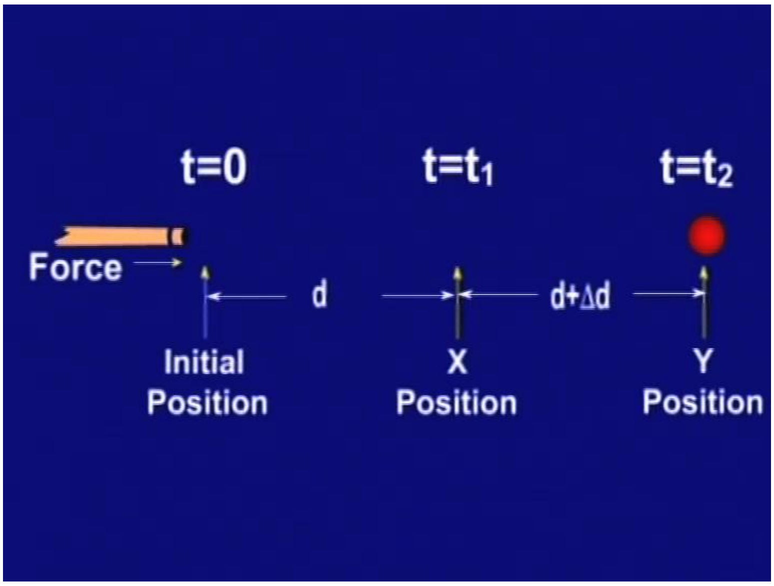

An example here, I have taken, here there is a stationary object which is initially at initial position, a force is been applied on it and this is stationary object starts moving. So, from where it is starts moving, that point I am taking initial position, after time t 1, it reaches to a position x. And it is still continues to move and at time t 2, it reaches to position y, we have observed here, that this object is not covering equal distance in equal time or in other words the equal distance has not been covered in equal time, that the movement of the object, that is the velocity of the object was changing and why it is happening.

Since, the force has been applied once only and object is moving on certain surface, that surface is also giving some force, so this velocity is changing. Now, this is an example where we can say is that we can use the differential equation, say for simplicity, if I use that the change, which is occurring is at constant rate. That is the distance; it is covering with respect to time that is changing with constant rate.

So, if I denote this distance by s, then d s by d t is a constant, say a and thus we have got that is the change in the distance is at constant rate, this is a differential equation and by solving this we can predict that at time t 3, where this object will be, this is what very simple application here we do have. So, now in this application, what we have done we have taken one assumption; that the change is at constant rate.

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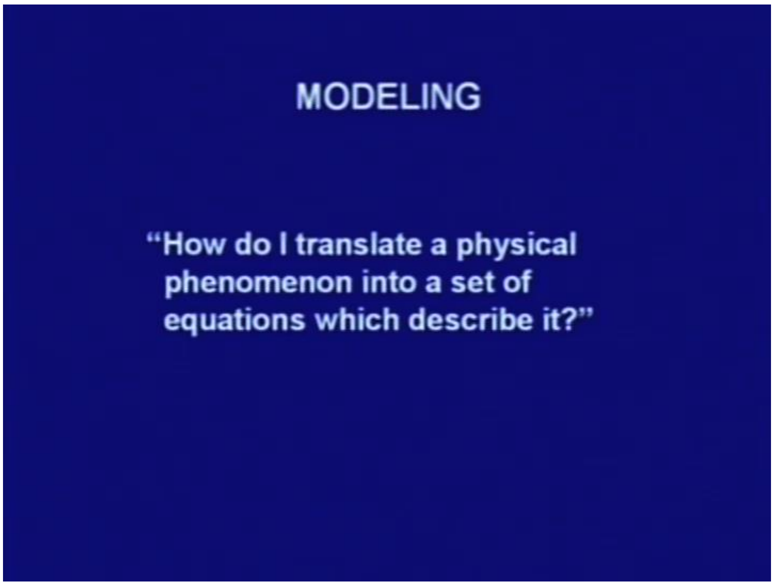

Thus, we have called that every day problem, what scientist is struggling is, how I translate a physical phenomenon into a set of equations, which can describe it.

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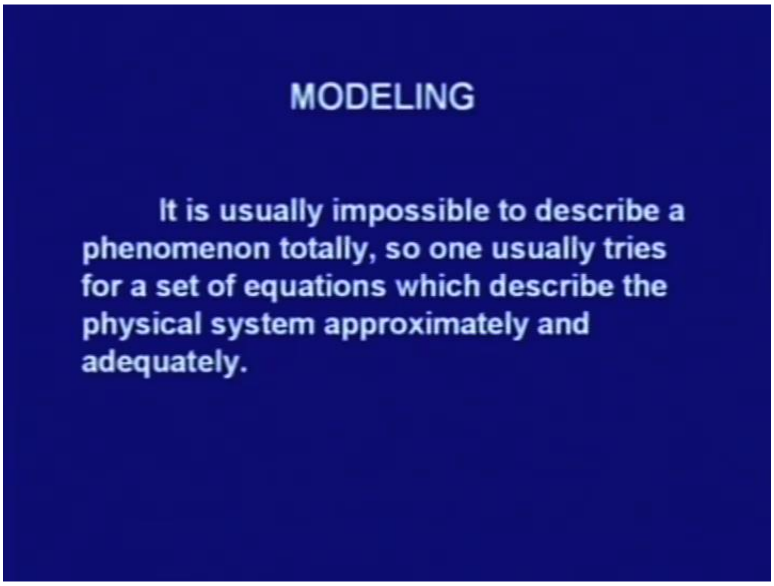

In real world, actually it is not possible to find out a system of equations or a set of equations, which describe this phenomenon completely and adequately.

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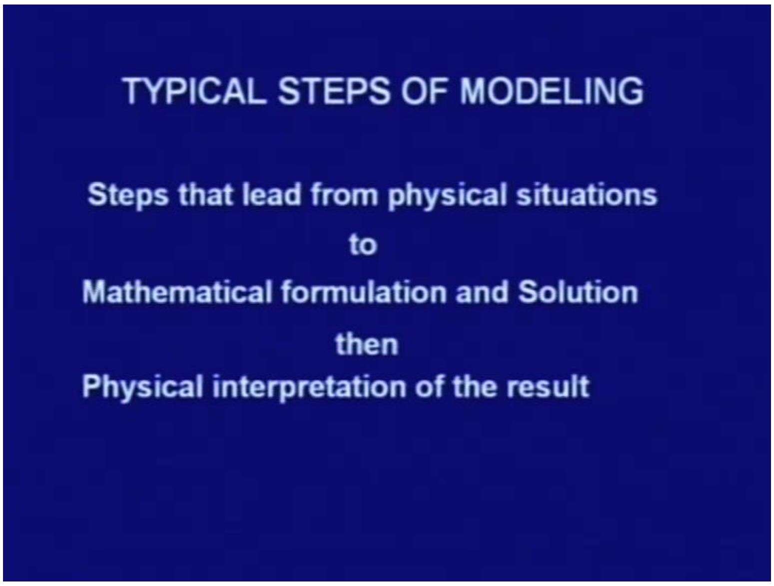

So, what we do is we find out a set of equations, which can describe this system approximately and when we says approximately of course, we are making certain assumptions of certain observations. And thus, we come up with some set of equations we try it we see that, how it is working, we compare it with real world research and then again we can improve up on our set of equations. This is what is the modeling we are having and in all those situation, where changes are occurring and we want to find out, how these changes are happening, all these are coming modeling through differential equations. Now, let us try to see this with a simple example.

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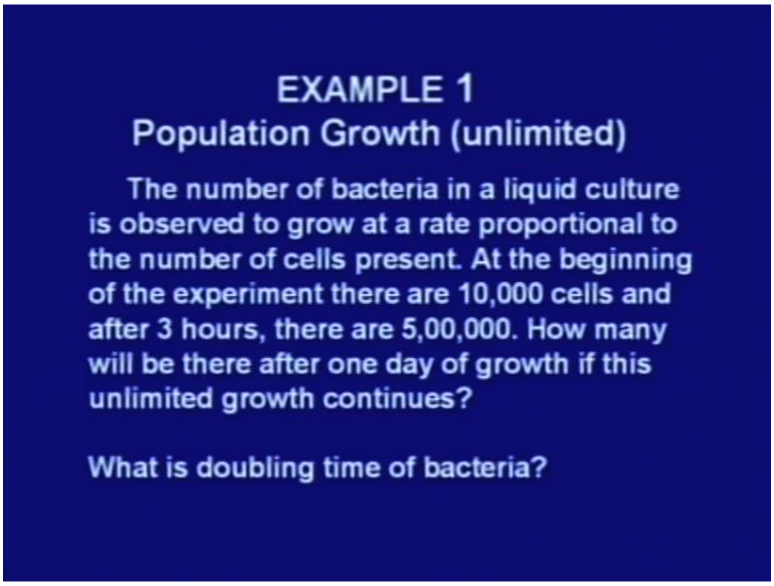

A first example is about the population growth, we have writing it as unlimited, because we had made an assumption that there is no condition or no control on the growth. The example is the number of bacteria in a liquid culture is observed to grow at a rate proportion to the number of cells present at the beginning of the experiment there are 10,000 cells and after 3 hours there are 5,00,000 cells.

Now, how many will be there after one day of growth if this unlimited growth continues, what is the doubling time of bacterial. Now, let us start with this example, what we have been given that is a liquid culture is having certain number of cells and it is growing with time. Now, how do I translate it, you see here first we will assume that is from the starting of the experiment, the time elapse that will denote by t and the time we will be counting here in the hours.

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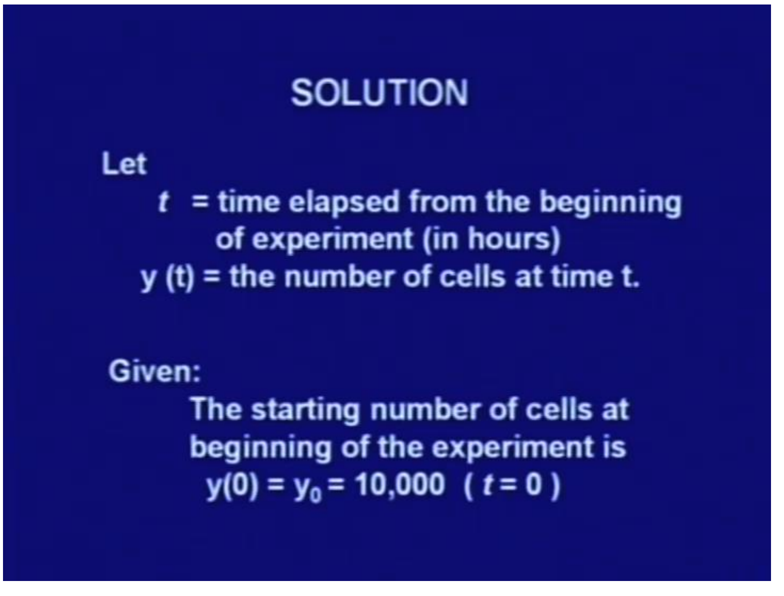

Second thing, we would be assuming is the unknown function y that is the number of cells at time t, why we are calling it a function, because it is changing with time t. Then, what we have been given, we have been given that the starting of the experiment, the number of cells are 10,000 that is y at 0 is 10,000. Here, I have used one another notation y naught will see that is this is also we are sometime referring that is the value of the function at that point.

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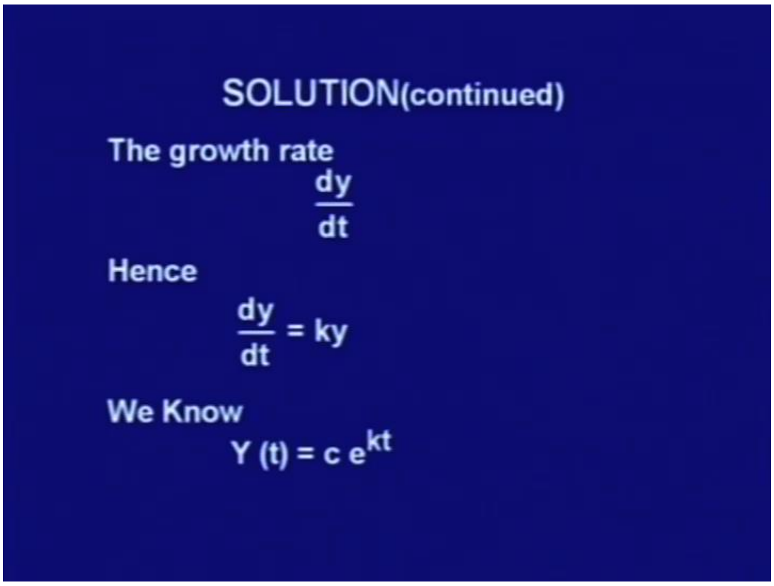

Now, what more we are been given in this one, that is the rate at which this growth is taking place or that is the number of cells are changing, that is called the growth rate. So, what will the growth rate, growth rate will be d y over d t, this is given that it is changing according to the size of the population at that time t. That means it is given that d y over d x is equal to k y, where k is the constant of proportion. Now, we do remember this equation we have same this equation in our first part of this lecture. We do know that this equation has a solution as y t is equal to c times e to the power k t.

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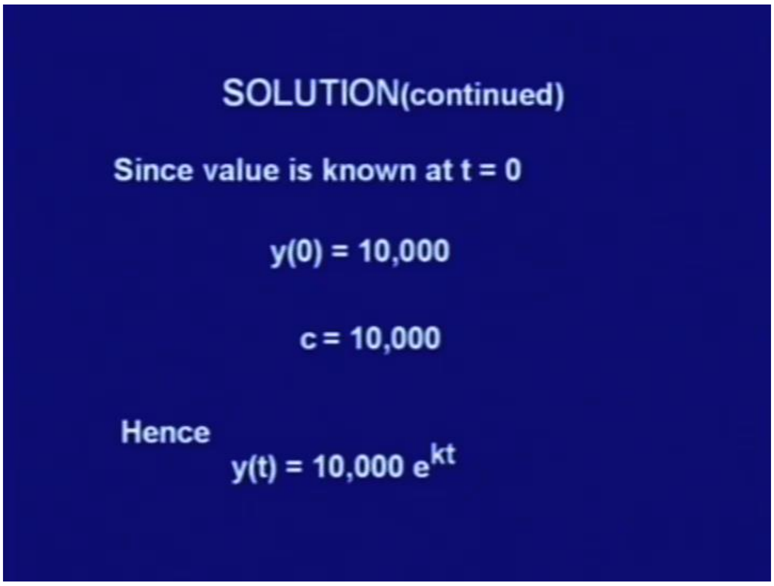

Now, we have been given initial value that the starting of the lay experiment, we do have y at 10,000 cells. Now, if I incorporate this initial value, we have seen in our first examples, this should be c, so c would be 10,000, thus what we have got that, this function would be 10,000 times e to the power k t. Now, the question is what will be this k, for that we have to see our experiment a little bit more.

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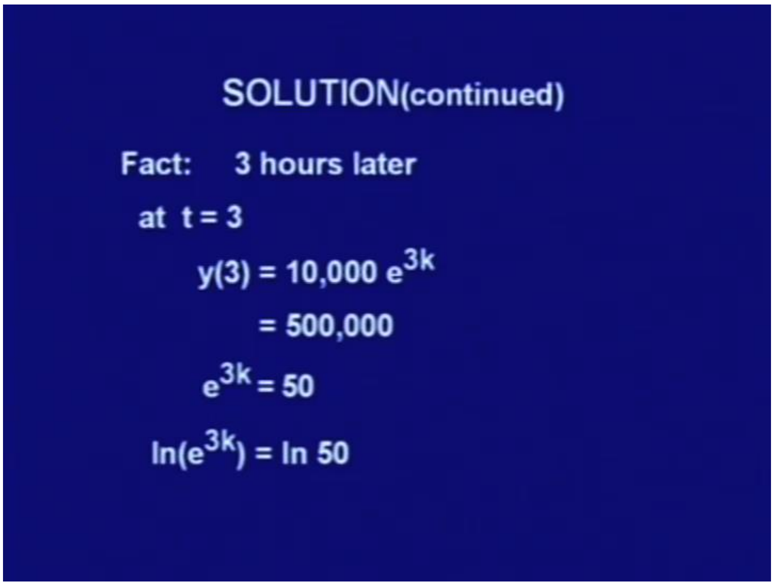

We are been given that after 3 hours, it has been observed that the number of cells are 5, 00,000. So, now translate this into the mathematical formulation, we are been given, so at t is equal to 3 my function would be 10,000 times e to the power 3 k, it is given that number of cells at that time are 5000. So, if I equate these two things, what I do get e to the power 3 k is equal to 50, now we can solve this to find out the value of k, how we will solve it, will just take the natural logarithmic on both the sides of this one.

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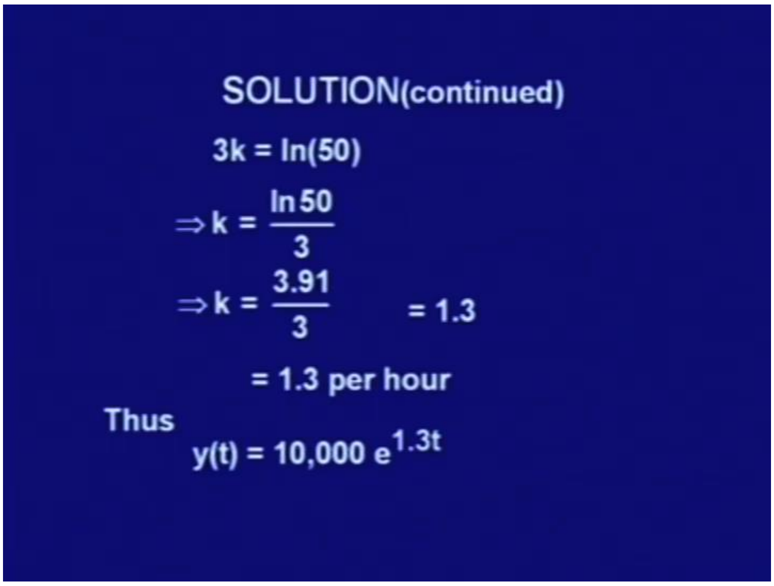

So, we do get log of e to the power 3 k is equal to log of 50, which is giving us that 3 k is equal to log 50, thus k is equal to log 50 by 3 and now we will incorporate the value of natural logarithmic of 50, which is approximately 3.91. Thus, we are getting the value of k as 3.19 by 3 is equal to 1.3, so what we have got that is the value of k is 1.3, now this is the rate constant and what rate we would have because, we have taken our t as the time in hour.

So, it would be that it is changing 1.3 per hour, that is rate of changes 1.3 per hour, so now, what is our solution, we have got that at the time t the number of cells will be 10,000 times e to the power 1.3 t. That is we have got this function as a solution or this is what we are giving as the number of cells at that time t, now let us see this by the graph.

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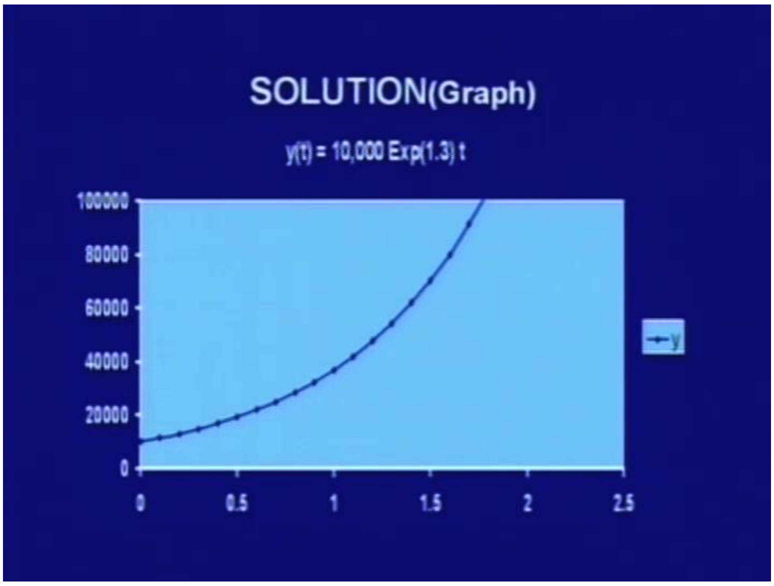

We see that is this is the graph of the function e to the power 1.3 t and since initially, we have started at the value 10,000, we see here in the graph, that it is 10,000, this is starting. So, this is the solution of this one, now we are ready to answer the question, that is what will be the growth, after one day.

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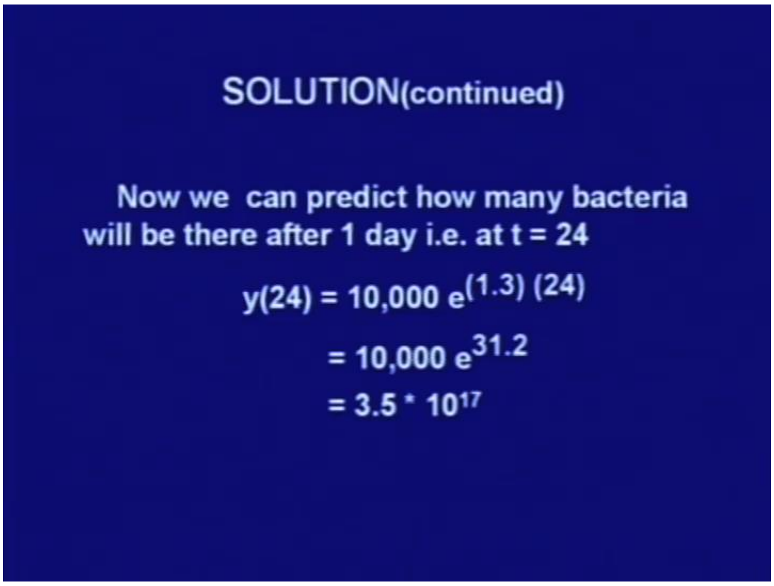

That is after 24 hours, that is we will incorporate t is equal to 24 in our solution, what it will be 10,000 times e to the power 1.3 and in place of t. I will put 24, which we will when solve will reach to 10,000 times e to the power 31.2, which will be giving me 3.5

into 10 to the power 17, that is these are the number of cells and the end of one day at the time of 24 hours.

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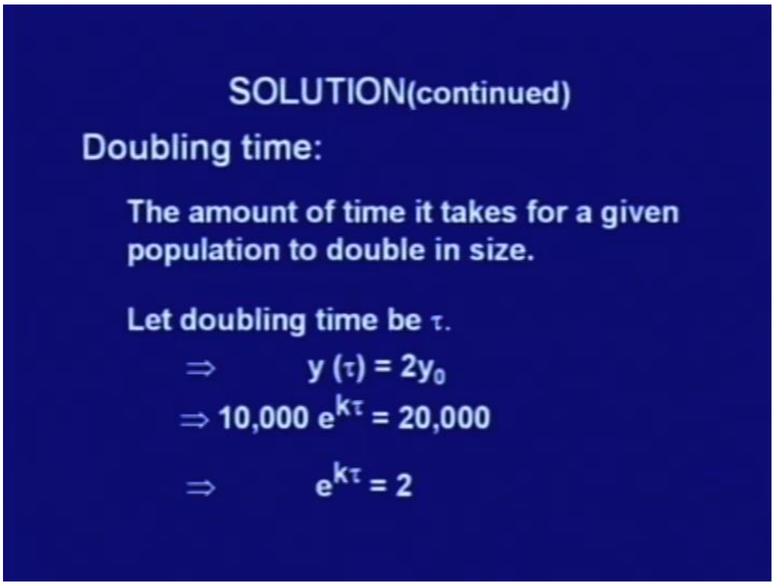

Now, let us try to solve the second part of the problem, what is the doubling time of the bacteria. So, first we should know, what is the doubling time, doubling time is actually the amount of time one takes for a given population to double in size. So, let us explain it let says that is tau is the doubling time, then what we have been said is that is the number of cells at time tau, that is y tau, should be twice the number of cells at initial position that is y naught.

So, now incorporate this information in our solution, because we do know, what is the y t, y tau is 10,000 times e to the power k tau and initially the value of the y naught, that is the number of cells at the beginning or 10,000. So, twice of this would be 20,000, thus what we got that e to the power k tau is equal to 2, now we do know that in this solution that k is 1.3.

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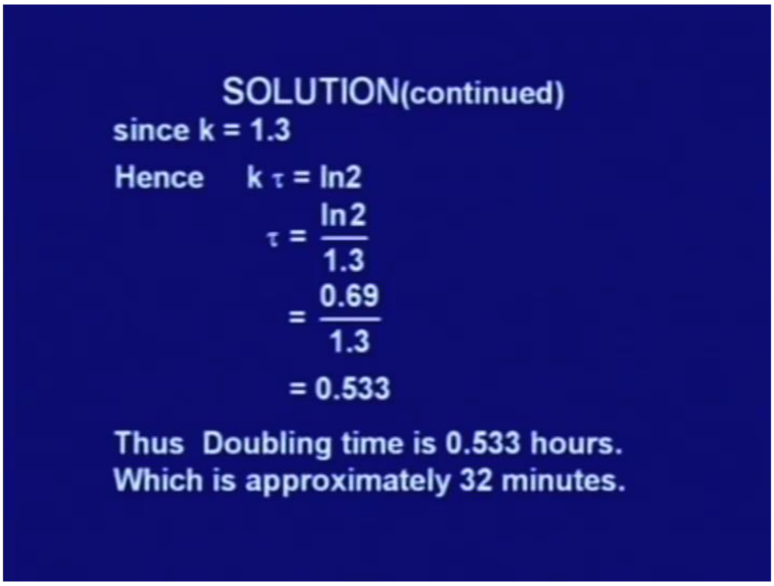

So, we incorporate this k tau is equal to log 2, after taking logarithmic and incorporing the value of k as 1.3, we do get that tau is equal to approximately 0.69 divided by 1.3, that is approximately 0.533, what is this is tau that is the time and how we are measuring in hours. So, this is doubling time is approximately 0.533 hours, which if I change into the minutes I do get approximately 32 minutes. So, we have here one problem, where we are in interested to know the growth of bacteria in a liquid culture.

We had observed that experiment for certain time, we had initially started our observations when there were 10,000 cells and after 3 hours, we have observed that there are 5,00,000 cells. And one more assumption we have made that, the growth is proportional to number of cells present in the culture at that time with the help of these three things, we have modeled our problem.

We said since the rate is changing according to the size of population, we got the equation d y over d k is equal to k y and we have solve that equation with the help of other observations, which we have made the first observation, that initially we have started at 10,000 cells. Then, the fact that after 3 hours, we have observed it as 5,00,000 from there, we have got the rate constant that is k.

And then we could predict it; that is after 24 hours how many bacteria will be in that culture, moreover we can also answer the question that is in how much time the population will get doubled and these answers we have made. Similarly, let us see one more example of the similar nature, but little bit different, that is radioactive decay.

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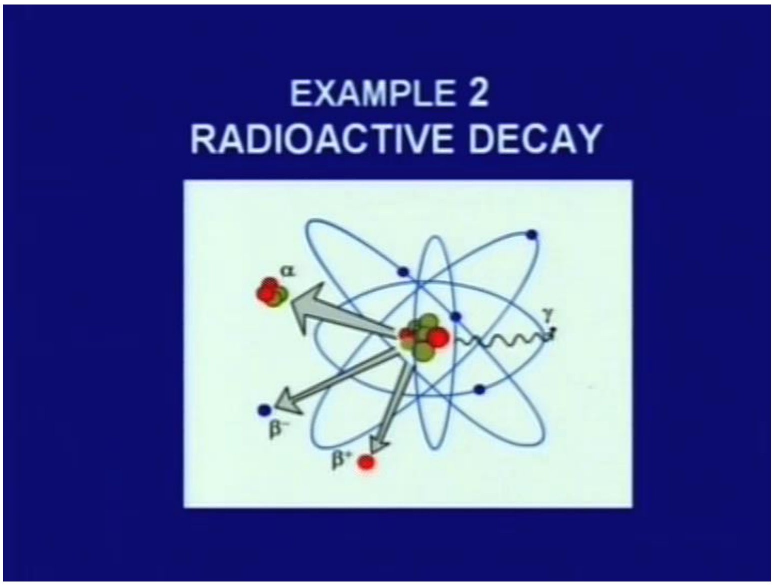

Radioactive decay is a property, which characterizes the substances, whose atoms undergo is spontaneous changes; these are being stored either in radioactive isotopes or in step that are unstable forms or in stable form and those materials which decays. These decays are normally at a constant rate, which when they burst are being countered on it is a counter.

This, these burst are being that is atoms are been burst at constant rate and this isotope, that is the radioactive material changes to it is stable material. That says is that this constant rate would be proportional to the number of atoms present at isotope at that time So, this decay is that is the change in the mass of radioactive material is proportional to the mass present at that time and this would be decreasing.

We can understand, this in another words as if example, if there is half of the material present at the isotope, then the number of burst in the atoms will be half of the times whatever it is previously. Thus, this change is in the decreasing order and this is called decay.

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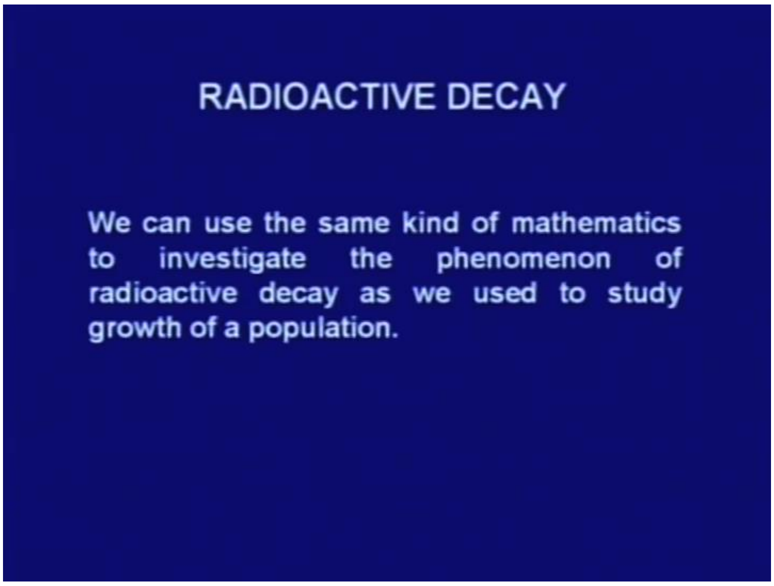

Now, this problem also we can study as, we have use this growth phenomenon, what is the difference between the growth phenomenon and this decay phenomenon, is the thing is that is their things, where changing to a they are growing up and here the population is decreasing or decaying. So, we can use the same kind of thing, what we are been given that is let us see.

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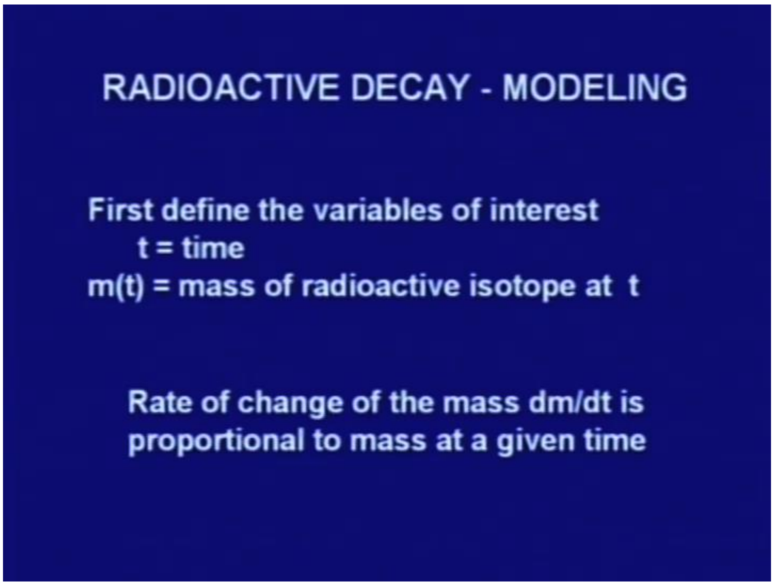

We have been defining the variable here first, that is t is the time and let us say m t is the variable, that is the mass of the ratio radioactive isotope at time t. It is given that, this mass is changing at a constant rate, which is proportional to the mass present at that time, that is the rate of change of the mass could be d m by d t, this is proportional to the mass given at a time.

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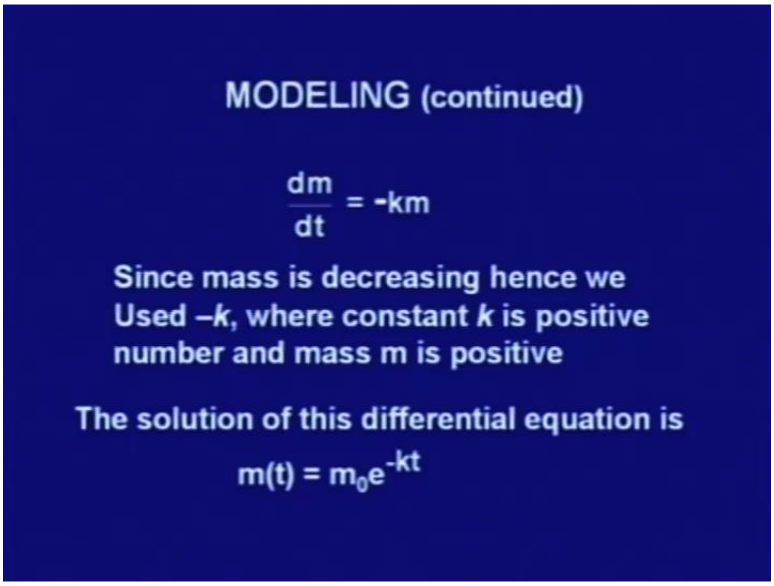

Thus, we can say that d m by d t is equal to minus k m, now see here I have used minus sign, why whatever be the mass present at the isotope; that will always be a positive number, that will be always a positive and it is decreasing, k I have used as a positive constants. So, I have to use the sign minus because as the time is changing, the mass is decreasing, so the rate of change should be negative, thus we have used here minus k and this solution of this differential equation.

Now, we have already seen this kind of differential equation, the difference is here, that is rather than having plus k, we are having here minus k. Of course, we have change the variable from y to m, so the solution would be that is in the solution itself, also we would change it to the minus k and the solution would be that m t is equal to m naught times e to the power minus k t.

Here, m naught I have used as the constant or that is the constant in the general solution instead of c, why m naught I have used because, there must be some initial value known for this experiment also. That is from, where we are starting observing this isotope to decay, that is that m naught is initial mass present on the isotope. Now, we are ready to solve a problem, let us see that is what is this problem, before going to the problem, let us see this function little bit more.

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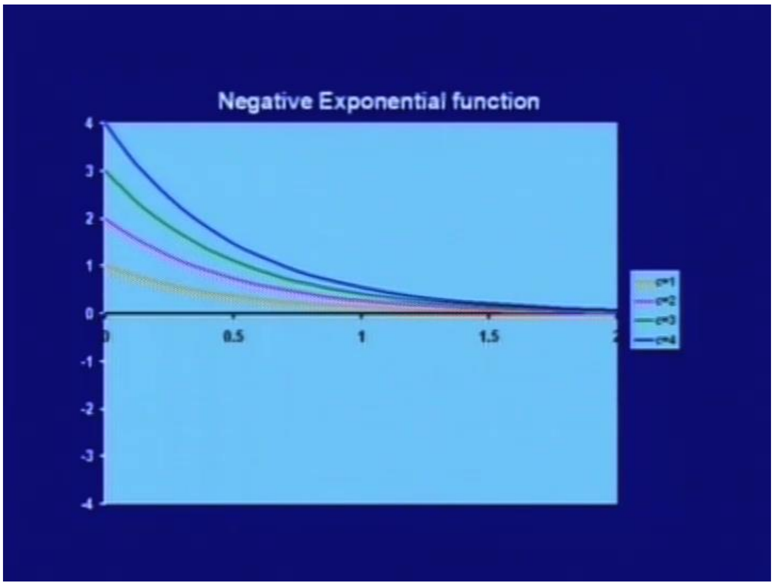

Since, this function is negative exponential, it is having c we have this function, which is decreasing, of course, this function should be decreasing. Because, what we are having is that solution must be a mass, which is a function which is decreasing and we see as initially we are starting at different values, we do have different solutions, now try to solve one problem.

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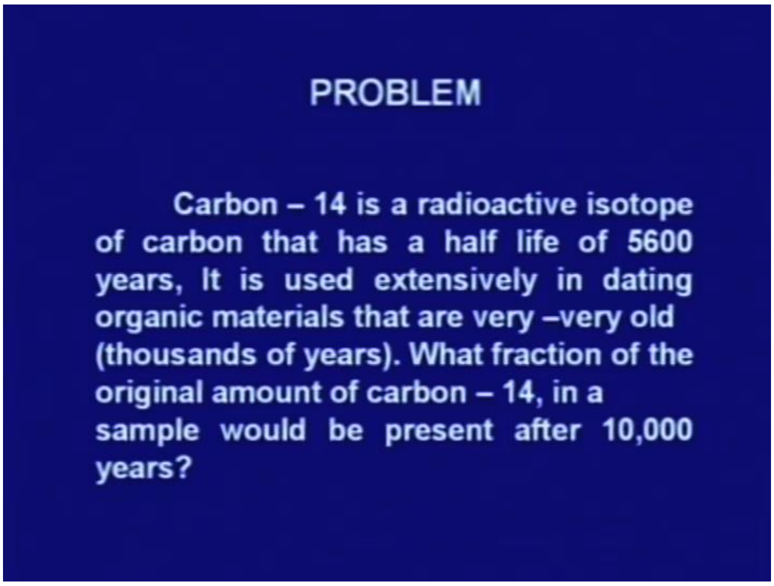

Let us see, what the problem we are going to solve here, carbon fourteen is a radioactive isotope of carbon; that has a half life of 5,600 years. It is used extensively in dating organic materials, that are very old sometimes 1000 of years. So, the question is what fraction of original amount of carbon 14, in a sample would be present after 10,000 of the years.

Now, let us see is that, how we are going to model this problem, carbon 14 is a radioactive isotope. So, it has a property that it will decay and the decay would be at a constant rate and that rate would be proportion to the amount of the radioactive material that is carbon 14, present at that time.

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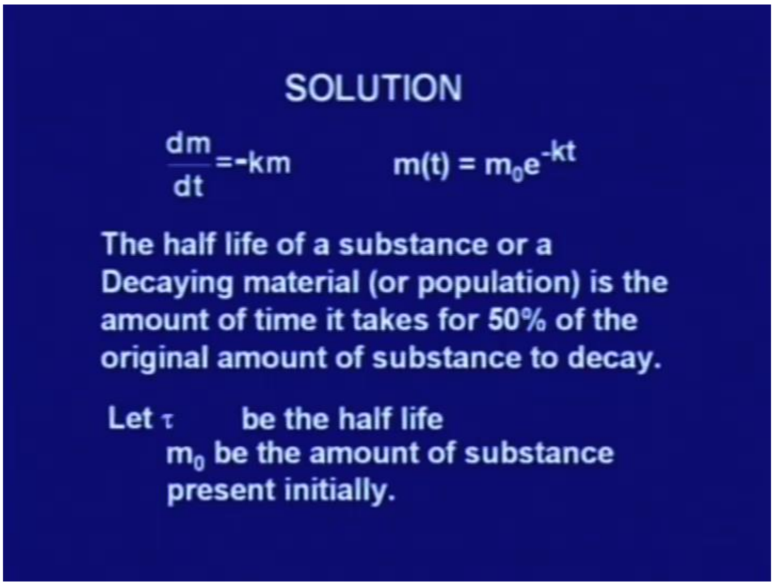

Thus, the equation will have is that d m by d t is minus k m, we have already seen that the solution of this equation is m naught times e to the power minus k t. Now, for this problem, we have to find out what is m naught and what is k or to answer this, we have to know is that is after 10,000 year, what will be the fraction of amount, which will be left that is we are not necessary want to know what is m naught, whatever would be m naught after 10,000 years, what fraction of that amount will be there.

So, we are more interested in finding first, what is k, how we are going to finding it out, we all see in our example, what other conditions are given to us, we are been given this information that half life. Now, what is the half life, half life of a substance or a decaying material is the amount of time it takes for 50 percent of the original amount of the substance to decay, we have been given here that half life of this is 5,600.

So, let here tau be the half life; that is the tau will be the time, it will take for material to come as the half of that percentage and let say that m naught is the amount of the substance present initially. Then, what it will give now because, we do know what is that a solution that is m t is equal to m naught times e to the power minus k t, we have been given that at tau is equal to 5000.

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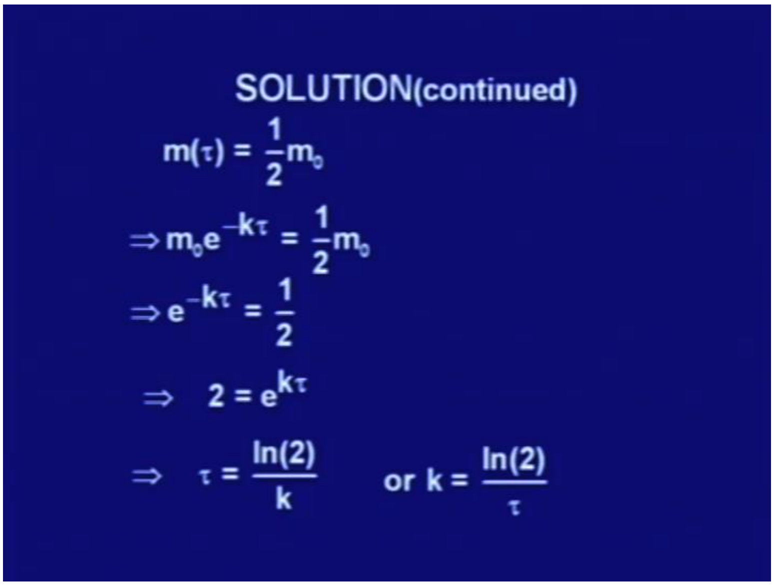

So, at tau I would get m tau is half times of m naught, now put tau as 5,600 in our solution and we do get that m naught times e power minus k tau as half m naught are e to the power minus k tau as half. Thus, what we are getting is that 2 is equal to e to the power k tau and thus tau is log 2 upon k. So, we are just solving this equation without actually incorporating the value of tau.

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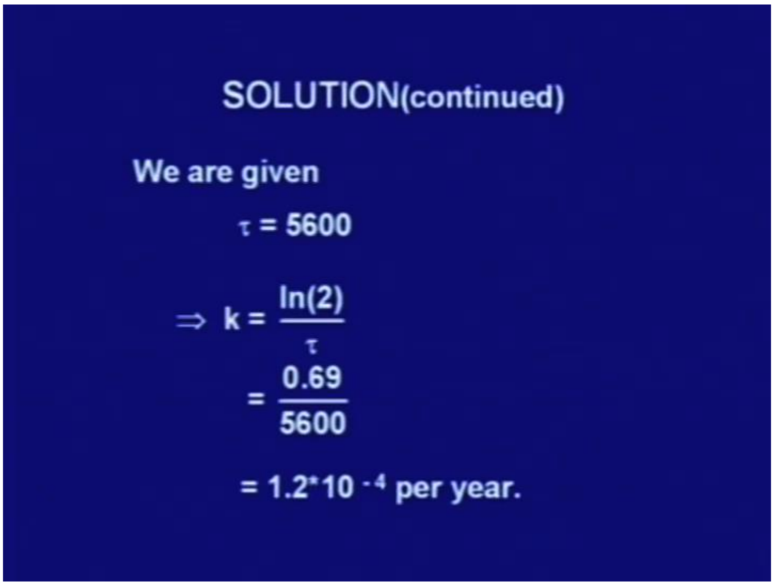

We are given that tau is equal to 5,600, so will incorporate this and we get k is equal to log tau log 2 upon tau, which is 0.69 divided by 5,600, which gives approximately 1.2 into 10 to the power minus 4 per year. That is the time; we have incorporated here in the terms of year, because it is takes a long time.

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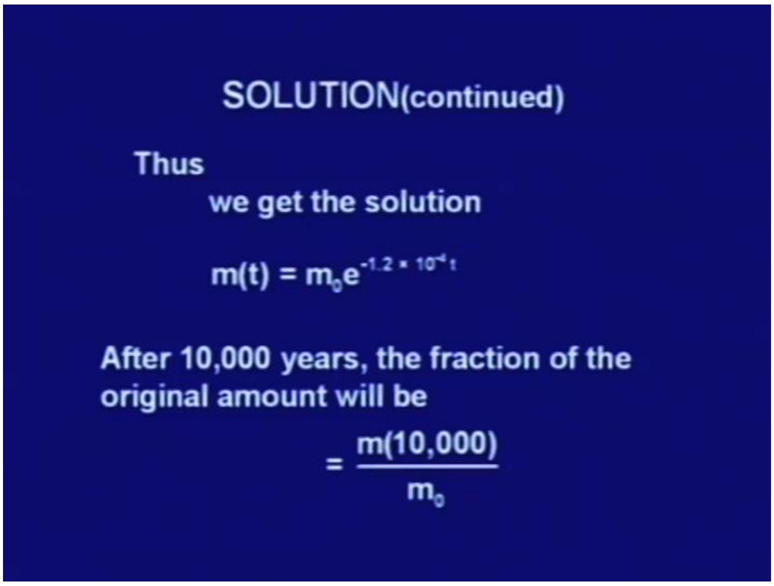

So, we have got that k is equal to this, thus what we have got the solution, we have got the solution as m t is equal to m naught times e to the power minus 1.2 times 10 to the power minus 4 t. Now, we are ready to answer our question, question is that is after

10,000 years, the fraction of the original amount will be m at 10,000 divided by m naught, because initially the amount is m naught.

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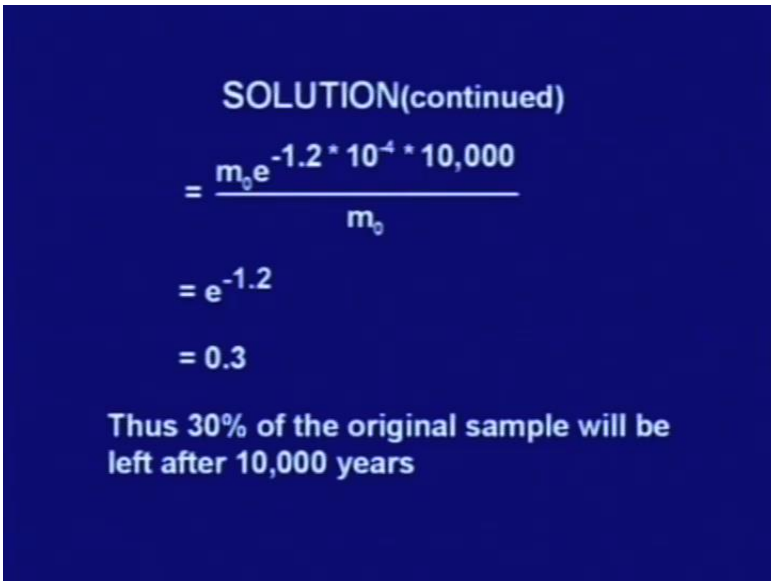

So, now, use this information in our solution m at 10,000 will be m naught times e to the power minus 1.2 into 10 to the power minus 4 and at the place of t, I will use 10,000 which gives me that it is e to the power minus 1.2. Now, we can calculate this value which is coming up approximately as 0.3, what does it says after 10,000 years, the 30 percent of the original sample will be left on the carbon 14 isotope.

That is again, we have seen here in this 1, that if we are talking about a decay which cannot be stopped by any outside source. So, we are having the decay is happening and that decay is again at a constant rate and that constant rate is proportional to the population size or the material present at that time. That, again we have used the same kind of differential equation and we have solve this problem also.

So, in both these examples, we have seen that, we have used the exponential function and it is differential equations, that is why I had use this example, just because I have started my lecture with exponential function. So, this is not the only application of differential equation, actually we can model many kind of physical phenomena's, all those phenomena's where changes are taking place and we are interested to know, what will be the future behavior of that phenomenon.

So, wherever this changing changes are occurring, we do have the differential equations, knowing the differential equations, knowing that how to solve those differential equations. We can know the function, that is we can know that phenomenon, what this phenomenon is the function says is that, if I say this is a function, it say is I do know completely about this thing.

So, if any phenomenon, I am saying is a function, I do completely know the phenomenon, I do not know the characteristic of that phenomenon, I can predict the future behavior of that phenomenon or I may actually calculate many things about many characteristics of that particular phenomenon. That is why this differential equations are very, very important, here I would conclude today's lecture.