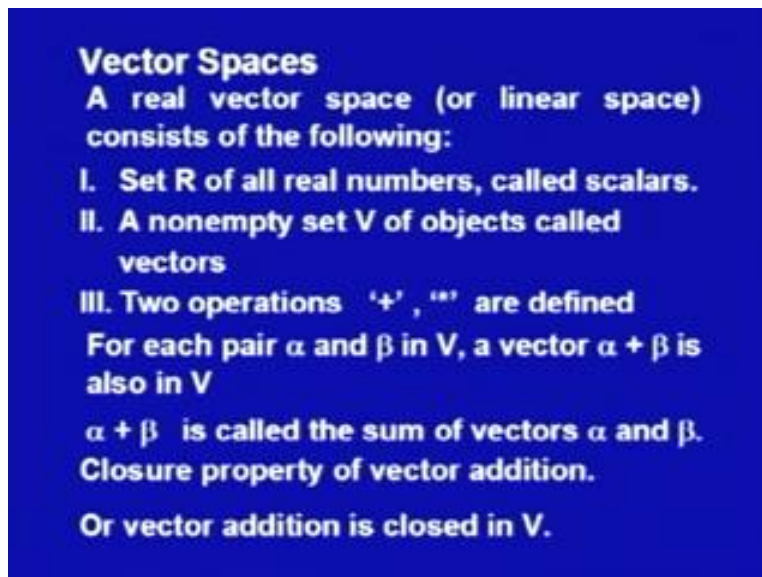


Mathematics-II
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Module-2
Lecture - 6
Linear Algebra Part-1

Welcome, viewers. We are starting series of lectures on linear algebra. This is the first lecture in this series. In this, we will be discussing vector spaces. Viewers, we are performing operations of additions and multiplication on numbers, but they are performed on vectors as well. We have seen how addition and scalar multiplications performed on matrices **they** can be performed on polynomials also. Similarly, the linear combination of objects appears in many different contexts, like we can have linear combination of vectors solution of differential equations. **Now** A branch of mathematics dealing with the common properties of sets and liner combination of its elements is linear algebra. We start with the definition of vector spaces.

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Vector Spaces
A real vector space (or linear space) consists of the following:

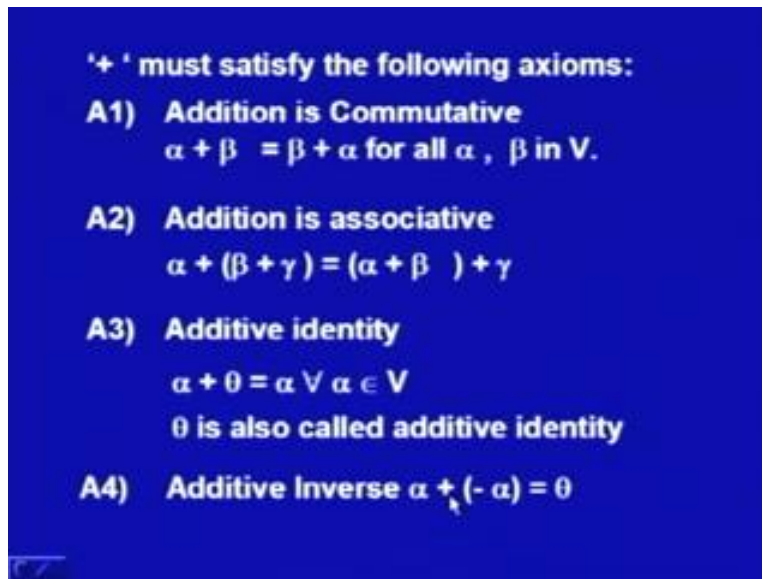
- I. Set R of all real numbers, called scalars.
- II. A nonempty set V of objects called vectors
- III. Two operations $+$, \cdot are defined

For each pair α and β in V , a vector $\alpha + \beta$ is also in V
 $\alpha + \beta$ is called the sum of vectors α and β .
Closure property of vector addition.
Or vector addition is closed in V .

Now **vectors** vector spaces - sometimes we call it linear spaces - consist of the following: the first is the set of real numbers -we call them scalars, a n 1 mpty set V of objects or

vectors and 2 operations defined on them - plus and star; they are defined as plus is defined on set of vectors, so if we have a pair of alpha and beta in V, then another vector alpha plus beta is also in V. We say alpha beta is the sum of 2 vectors alpha and beta and if this is also in V, the we say vector addition is satisfying the closure property or we say vector addition is closed in V.

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Now this vector addition must satisfy following axioms. The first axiom is that addition is commutative; that means, if we add beta in alpha, that is alpha plus beta, or we add alpha and beta, that is beta plus alpha, it is same for all alpha beta in V; that is alpha plus beta is equal to beta plus alpha for all alpha beta in V; if this property satisfies then we say addition is commutative. The second axiom is that addition is associative; by this, we mean to say that if there are 3 vectors - alpha, beta and gamma - then alpha plus beta plus gamma is equal to alpha plus beta added and then we add gamma into it; so we will have alpha plus beta plus gamma is equal to alpha plus beta and then we add gamma to it; that means it doesn't make difference whether we add beta plus gamma first and then addition alpha into it, or we add alpha plus beta first and then add gamma. In fact this means these brackets are meaningless; we can we can ignore them. So if this property of addition is satisfied, then we say addition is associative. The third axiom is that there exists an

additive identity in the set V . By this, I mean to say that there exists a member - call it 0 - in V such that $\alpha + 0$ is equal to α ; that means whenever we add 0 , whenever we add anything on 0 , **the** that thing remains the same; that is, $\alpha + 0$ is equal to α for all α belonging to V . Such a vector is called additive identity in V . Then the next axiom is that additive inverse exists for any number - any vector α - in V . By this, I mean to say that for given α there exists another vector - I call it minus α - so that $\alpha + (-\alpha)$ is equal to additive identity 0 . So $\alpha + (-\alpha)$ is equal to 0 or we say minus α is the additive inverse of α .

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Scalar Multiplication ""
 For each c in R and vector α in V , the vector $c * \alpha$ is called the scalar product of c and α . When the vector $c * \alpha$ is also in V . (Closure property)

"" satisfy the following Axioms:

A5) $c * (\alpha + \beta) = c * \alpha + c * \beta$
 $\forall \alpha, \beta$ in V and c in R

Multiplication by scalars is distributive with respect to vector addition

Similarly, scalar multiplication **is** also **is** has to satisfy certain properties. First thing is the closure property; by this, I mean to say that if there is a scalar c in set of real numbers R and a vector α in V then the vector $c * \alpha$ is called the scalar product of c and α ; and if $c * \alpha$ is a vector and it is also in V , **then** we say it satisfies the closure property. So scalar multiplication satisfies the closure property means that $c * \alpha$ is also in V for any given scalar c . It has to satisfy other axioms. Call it A5 - it means $c * (\alpha + \beta)$ is equal to $c * \alpha + c * \beta$ for given α, β in V and c in R . By this, I mean to say **is** that you add $\alpha + \beta$ - 2 vectors - and then multiply it by the scalar c . This amounts to be the same thing as if we have multiplied α with c

and beta with c. So this is a scalar multiplication; we add the 2 vectors and what we have is another vector which is the same as c star alpha plus beta. Now this property is called that multiplication by scalar is distributive; so this scalar multiplication and this is addition. So this addition this scalar multiplication distributes the addition of vectors; that is why it is a distributive property of scalar multiplication.

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A6) $(c_1 + c_2) \cdot \alpha = c_1 \cdot \alpha + c_2 \cdot \alpha$
 $\forall \alpha \text{ in } V \text{ and } c_1, c_2 \text{ in } R$

Multiplication by vectors is distributive with respect to scalar addition

A7) $(c_1 \cdot c_2) \cdot \alpha = c_1 \cdot (c_2 \cdot \alpha)$

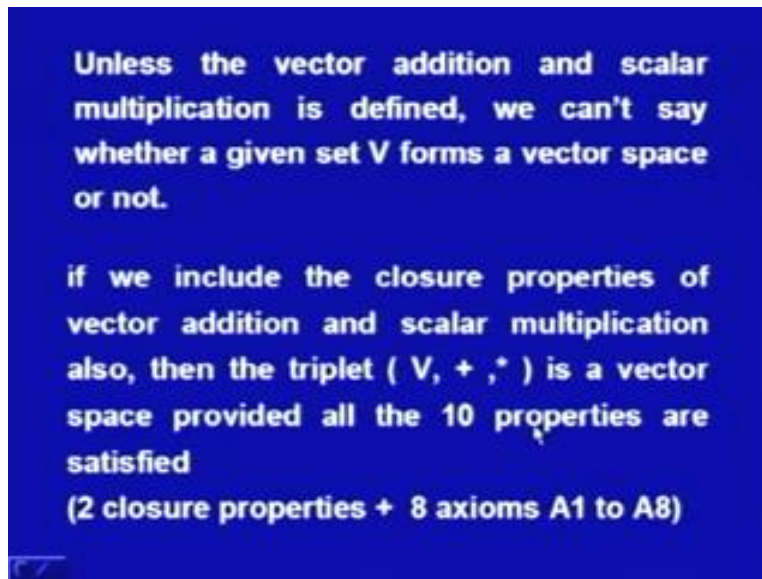
Multiplication by scalars is associative

A8) **Multiplicative Identity**
 There exists 1 in R such that
 $1 \cdot \alpha = \alpha \quad \forall \alpha \text{ in } V$

Next is c 1 plus c 2 - 2 scalars; then they are added and then multiplied by a vector alpha; then this amounts to c 1 star alpha plus c 2 star alpha for all alpha in V and c 1 c 2 in R. We say that multiplication by vectors is distributive with respect to scalar addition. So this alpha when multiplied by these 2 - addition of 2 scalars - then it is distributed as c 1 star alpha plus c 2 star alpha. The next is the associative property. Here we will have c 1 dot c 2, that is, multiplication of 2 scalars multiplied by a vector alpha is equal to c 1 star c 2 star alpha. Now here, you may notice that this dot simply means multiplication of 2 scalars - simple multiplication of 2 scalars. So When the 2 scalars are multiplied, we will be getting another scalar. Now when this scalar is multiplied by vector alpha, then what we have is c 1 star c 2 star alpha; that means, this is equal to c 2 is multiplied by c 2 scalar is multiplied by vector alpha, this becomes a vector and then this vector is multiplied by the scalar c 1; then we will have c 1 star c 2 star alpha. So this is such sort

of associative property; so multiplication by scalars is associative. Last property is multiplicative identity: that if there exists 1 in R such that $1 \star \alpha$ is equal to α for all α in V ; so there has to be a scalar 1 in R , which when multiplied by α , α remains unchanged. So this property has to be there and when all these properties are satisfied from a 1 to a 8, then we say that we have a vector space.

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Now we say that unless the vector addition and scalar multiplication is defined, we can't say that whether a given set V forms a vector space or not. That means, we define a vector space; we have to have a set V of vectors and 2 operations: the vector addition and scalar multiplication. Now with this, if we include the closure property of vector addition and scalar multiplication also, then this triplet V plus and star is a vector space, provided all the ten properties are satisfied. These ten properties are 2 closure properties - 1 with respect to addition, another with respect to scalar multiplication - and eight axioms which I have just now discuss from a 1 to a 8.

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Example 1: Let us consider the set of vectors in two dimensional plane \mathbb{R}^2
 $V = \{\alpha \in (x_1, x_2): x_1, x_2 \in \mathbb{R}\}$.

The sum of two vectors $\alpha = (x_1, y_1)$ and $\beta = (x_2, y_2)$ is defined as
 $\gamma = (x_1 + x_2, y_1 + y_2)$

Vector addition is closed

The scalar multiplication of the vector (x_1, y_1) with scalar c is defined as
 $c * (x_1, x_2) = (c x_1, c x_2)$.

Scalar multiplication is also closed

Now, let me explain this with the help of an example. **So** Let us consider the set of vectors in 2 dimensional plane \mathbb{R}^2 . **So** V is a set of vectors where α is a vector in \mathbb{R}^2 , so it has to be in order pair x_1 comma x_2 where x_1 and x_2 are real numbers. **So** If we consider this set of vectors and we define the addition of 2 vectors, α x_1 comma y_1 and β x_2 comma y_2 as another vector γ which is x_1 plus x_2 and y_1 plus y_2 , - **so the** if we define this way - then vector addition is closed and we also define the scalar multiplication of vectors x_1 y_1 with scalar c as c star x_1 comma x_2 as c x_1 comma c x_2 . Both the components of the vector are multiplied by c . Now this is a real number, this is a real number and so again a pair. So this also belongs to V and that means scalar multiplication, if we define this particular way, is also closed or closure property is satisfied by scalar multiplication.

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A1: Addition is commutative
 $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

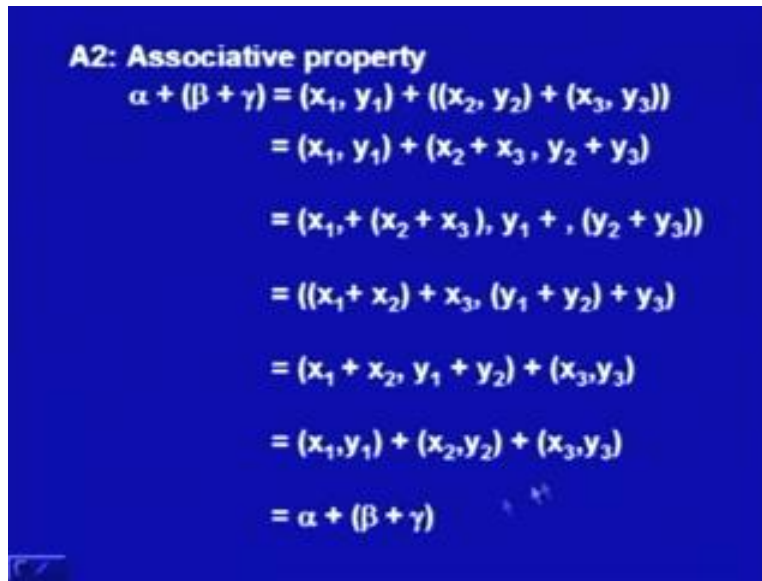
+ appearing on two sides of expression have different meanings

Or $(x_2 + x_1, y_2 + y_1) = (x_2, y_2) + (x_1, y_1)$

Or $(x_1, y_1) + (x_2, y_2) = (x_2, y_2) + (x_1, y_1)$

Now we have to check whether all the other axioms A 1 to A 8 are satisfied or not. So let
To start with, we first consider A 1 which means addition is commutative; so to check
this, we consider 2 vectors first: x_1 comma y_1 and the second is x_2 comma y_2 . By
definition, the way we have defined vector addition it is x_1 plus x_2 comma y_1 plus y_2 .
But x_1 , x_2 , y_1 and y_2 - being real numbers - their sum is also real number. So x_1
plus x_2 and y_1 plus y_2 can be written as x_2 plus x_1 and y_2 plus y_1 . Now here, one
may notice that this plus here and this plus here - that means, plus on the left hand side
and plus on the right hand side - have different meanings. Here the plus means we are
adding 2 vectors; so it is between 2 vectors; so this plus means vector addition. While this
plus means simply we are adding 2 numbers; so this plus refers to addition of 2 real
numbers; so this is a real number, this is a real number, that is why we can write it as x_2
plus x_1 and y_2 plus y_1 and by definition, this is equal to x_2 comma y_2 plus x_1 comma
 y_1 and that simply means x_1 y_1 plus x_2 y_2 is the same as x_2 y_2 plus x_1 y_1 . That
means order of addition of vectors doesn't make much difference; whether we add x_2 on
in x_1 , the whether we make we add this vector in this vector or this in this, it is 1 in the
same. So we say that addition is commutative.

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A2: Associative property

$$\begin{aligned}\alpha + (\beta + \gamma) &= (x_1, y_1) + ((x_2, y_2) + (x_3, y_3)) \\ &= (x_1, y_1) + (x_2 + x_3, y_2 + y_3) \\ &= (x_1 + (x_2 + x_3), y_1 + (y_2 + y_3)) \\ &= ((x_1 + x_2) + x_3, (y_1 + y_2) + y_3) \\ &= (x_1 + x_2, y_1 + y_2) + (x_3, y_3) \\ &= (x_1, y_1) + (x_2, y_2) + (x_3, y_3) \\ &= \alpha + (\beta + \gamma)\end{aligned}$$

So the next property is associative property. **So** If we consider 3 vectors now – alpha, beta, gamma as x_1, y_1 , x_2, y_2 and x_3, y_3 respectively - then this sum is given by this pair and this can be simplified further; this second bracket can be simplified to x_2 plus x_3 and y_2 plus y_3 , and then you can add this vector here. So it is x_1 plus x_2 plus x_3 comma y_1 plus y_2 plus y_3 and since x_1, x_2, x_3 are simply numbers, **so** they can be associative; so you can change the order of bracket. **so** That simply means x_1 plus x_2 and then add x_3 ; in the same way y_1 plus y_2 and then add y_3 , and further simplification will give us x_1 plus y_1 plus x_2 plus y_2 plus x_3 y_3 ; so, order is not important and what we have finally is alpha plus beta plus gamma. So, associative property is satisfied by vector addition.

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A3: Additive identity

$\theta = (0, 0) \in V$ is additive identity

$$\alpha + \theta = (x_1, x_2) + (0, 0) = (x_1 + 0, x_2 + 0)$$
$$= (x_1, x_2) = \alpha$$

A4: Additive Inverse

For gives vector $\alpha = (x_1, x_2) \exists (-x_1, -x_2)$ in V such that

$$(x_1, x_2) + (-x_1, -x_2) = (0, 0) = \theta$$

If $\alpha = (x_1, x_2)$ in V then its additive inverse $-x = (-x_1, -x_2)$ also exists in V

So, first two properties being satisfied, let us check the third property, that is additive identity. **So** In this set of vectors, we say theta is equal to 0 comma 0, where both the components are 0; they belong to V, because they are real numbers; so V is additive identity and it belongs to V. Let us check there is additive identity or not for this. Consider any vector alpha. Add theta into it and what we have is x 1 comma x 2 plus 0 comma 0 and it amounts to be the same vector as alpha. So we can say that there exists additive identity theta. The next property is additive inverse. **So** For given vectors alpha x 1 and x 2, there exist minus x 1 and minus x 2 in V, such that x 1 plus x 2 plus minus x 1 comma minus x 2 is equal to 0 comma 0 and this is nothing but theta. **So x 1 plus x 2** For given x 1 x 2, we have minus x 1 minus x 2 in V **they are it is in V** because x 1 and x 2 are numbers and if x 1 x 2 belongs to R, minus x 1 minus x 2 also belongs to R. So inverse of x 1 x 2 is minus x 1 comma minus x 2. So, for given vector alpha, there exists its inverse. **So** In this way, additive inverse exists in V for all alpha given in V.

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Scalar Multiplication
A5: Distributive property
Consider $c * (\alpha + \beta) = c * ((x_1, y_1) + (x_2, y_2))$
 $= c * (x_1 + x_2, y_1 + y_2)$
 $= (c(x_1 + x_2), c(y_1 + y_2))$
 $= (cx_1 + cx_2, cy_1 + cy_2)$
 $= (cx_1, cy_1) + (cx_2, cy_2)$
 $= c * (x_1, y_1) + c * (x_2, y_2)$
 $= c * \alpha + c * \beta$

Now comes this scalar multiplication. We first check the distributive property; so c star α plus β is equal to c star x_1 comma y_1 - that is, α - plus x_2 comma y_2 . We can add the two: it is x_1 plus x_2 comma y_1 plus y_2 multiplied by c and then by the definition of scalar multiplication, c can be taken inside; so it is c x_1 plus x_2 comma c by 1 plus y_2 . Then you can simplify this because c is a scalar and x_1, x_2 are real numbers; so we can write it as cx_1 plus cx_2 comma cy_1 plus cy_2 . Then this is nothing but cx_1 comma cy_2 plus cx_2 comma cy_2 and by definition of scalar multiplication, we can say it is c star x_1 y_1 plus c star x_2 y_2 . Finally, it is nothing but c star α plus c star β and **this is** we have started with c star α plus β and finally we arrived that this result, and this proves that scalar multiplication is distributive over vector addition.

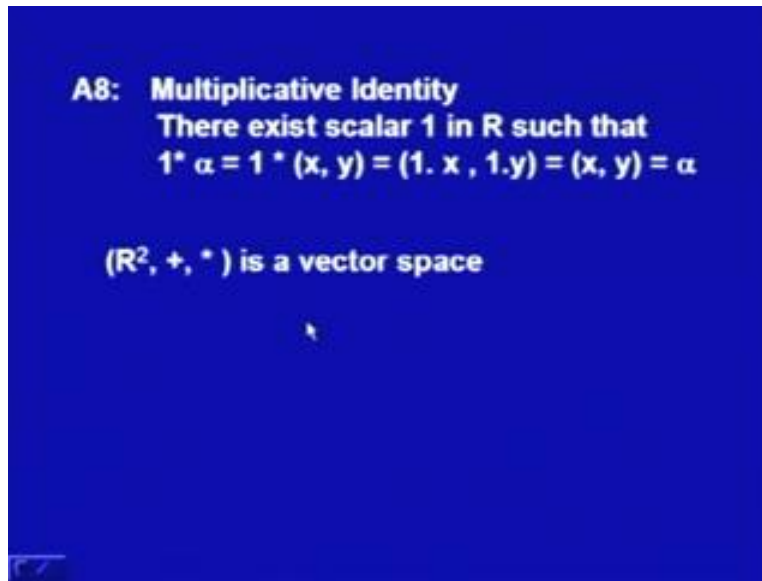
so This is the next axiom.

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$$\begin{aligned} \text{A6: } (c_1 + c_2) * (x, y) &= ((c_1 + c_2)x, (c_1 + c_2)y) \\ &= (c_1x + c_2x, c_1y + c_2y) \\ &= (c_1x, c_1y) + (c_2x, c_2y) \\ &= c_1 * (x, y) + c_2 * (x, y) \end{aligned}$$
$$\begin{aligned} \text{A7: } (c_1 \cdot c_2) * \alpha &= (c_1 \cdot c_2) * (x, y) \\ &= ((c_1 \cdot c_2)x, (c_1 \cdot c_2)y) \\ &= (c_1 \cdot (c_2x), c_1 \cdot (c_2y)) \\ &= c_1 * (c_2x, c_2y) \\ &= c_1 * c_2 * (x, y) \\ &= c_1 * (c_2 * \alpha) \end{aligned}$$

Then A 6 means scalar addition star the vector xy . So it is c_1 plus c_2 x comma c_1 plus c_2 y, by the definition of scalar multiplication, and then of course these are simply numbers and we are doing again and again; so they can be this they can be simply multiplied and then we finally have c_1 star x comma y plus c_2 star x comma y . So the property A 6 is also verified for this plus. Then Next is A 7, which is the associative property; so it is $c_1 c_2$ multiplied by α ; so we have $c_1 c_2$ star - α is x comma y ; so we say c_1 dot c_2 multiplied by x comma $c_1 c_2$ multiplied by y - so this is a vector. And then we again apply the the associative property for real numbers and it is c_1 dot c_2 x; so we can change the order here, the bracket principle changes. Same thing will be d 1 here and then c_1 can be taken outside and what we have is c_2 x comma c_2 y. This is nothing but c_1 star c_2 star x comma y and finally, what we have is c_1 star c_2 star α . So we have proved that c_1 dot c_2 star α is the same as c_1 star c_2 star α .

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So The last property is multiplicative identity. So We have to prove that there exist scalar 1 in R, which when operated upon the vector alpha, it remains the same. So To check this, we say 1 is that multiplicative identity; so 1 star alpha is equal to 1 star x comma y - we apply the multiplicative definition of multiply ah definition of scalar multiplication - it is 1 dot x comma 1 dot y which is nothing but x comma y; So this is the same as alpha. So Finally what we have is 1 star alpha is equal to alpha; so this 1 is such a multiplicative identity. So there exists some multiplicative identity 1 in R. So All the properties are satisfied; so one can say that \mathbb{R}^2 , that is a set of vectors in \mathbb{R}^2 , plus the vector addition, star - the scalar multiplication - is a vector space. So to check that a given set and the 2 operations defined on them on this particular set is a vector space, we have proved all the properties and in this particular case, we have shown that all these properties are satisfied; so \mathbb{R}^2 plus star is a vector space. Now this is one simple example. I will extend this simple example to \mathbb{R}^n ; that means, we have earlier taken 2 dimensions vector 2 dimensional vector space. Now we are taking n dimensional vector.

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Example 2:
The set $V = \{(x_1, x_2, \dots, x_n) : x_1, x_2, \dots, x_n \in \mathbb{R}\}$
is a vector space in \mathbb{R}^n with respect to the
vector addition

$$(x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n) = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$$

Scalar multiplication :

$$c \cdot (x_1, x_2, \dots, x_n) = (cx_1, cx_2, \dots, cx_n)$$

Solution: Closure, commutative, associative
Distributive

The zero vector in \mathbb{R}^n is $0 = (0, 0, \dots, 0)$.

So we will have V . The set V is consisting of pair ah of n real numbers - so it is $x_1 \times x_2 \times \dots \times x_n$. All these belong; to \mathbb{R} then this is a vector space in \mathbb{R}^n with respect to vector addition. So how do you define vector addition in this? This is a vector addition. We will have $x_1 \times x_2 \times \dots \times x_n$ plus $y_1 \times y_2 \times \dots \times y_n$ is equal to x_1 plus y_1 , x_2 added into the second component, x_3 added into the third component, x_n added into last component y_n . So that is if that is how we define vector addition. Then how do we define scalar multiplication on this set? So If we have a scalar c when c is multiplied with this vector, then every component is scale by this c . So it is $c \times x_1 \times x_2 \times \dots \times x_n$ is cx_1 comma cx_2 comma cx_n . So If we define scalar addition in this way and scalar multiplication in this way then one can very easily check that this is also a vector in V ; this is also a vector in V . So addition is closed and scalar multiplication is closed. One can very easily check all other properties A 1 to A 8 for this set and one can prove that this is also a vector space. So closure - we have checked. Commutative property - we have to see; this will always be satisfied because these numbers $x_1 \times x_2 \times \dots \times x_n$ etc these are real numbers. So and as we have proved in earlier example, this will also be proved in the same very manner. So closure property, commutative property, associative property, all the distributive properties are satisfied; only thing is you have to show that there exists a 0 vector in \mathbb{R}^n ; so we say that 0 vector is nothing but all the components are 0. So If this is the 0 vector, then one can also find

additive inverse for all $x \in V$. So inverse also exists. Similarly scalar one is also there, which when multiplied by the given vector $x \in V$ will remain unchanged; so this is also a vector space. So V - define in this manner; addition, in this particular manner; scalar multiplication in this manner; this also defines a vector space.

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Example 3:
The set of all $m \times n$ matrices forms a vector space over addition and scalar multiplication of matrices.

Solution:
Addition of matrices is closed since sum of two $m \times n$ matrices is also an $m \times n$ matrix
Scalar multiplication of $m \times n$ matrices is also closed .

A1 : Matrix addition is commutative
A2 : Matrix addition is associative
A3 : θ = null matrix of order $m \times n$
A4 : $-A_{mn}$ is additive inverse of given A_{mn}

The third definition **is third example** is the set of m by n matrices; they also form a vector space over matrix addition and scalar multiplication of matrices. Now to prove this, one can say that addition of matrices is closed because if we are given two m by n matrices, **then** the result is also the sum of 2 m by n **matrix is also an m by n matrix**; so it is closed. Similarly when you multiply a scalar c to a matrix, which is for m by n , the result is also an m by n matrix; so this is also closed. So the two operations are closed in the set of matrices. Then comes other properties; **like** but these have already been established - **that** matrix addition is commutative, matrix addition is associative, one can say that θ or the null matrix of order m by n is the negative identity; **now the** the next is the additive inverse. So for a given m by n matrix A_{mn} , the additive inverse is minus A_{mn} ; this always belongs to the given set of m by n matrices. So this is also defined.

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$$\begin{aligned} \text{A5: Consider } c * (A + B) &= c\{(a_{ij})_{m \times n} + (b_{ij})_{m \times n}\} \\ &= c(a_{ij} + b_{ij})_{m \times n} \\ &= (ca_{ij} + cb_{ij})_{m \times n} \\ &= c(a_{ij})_{m \times n} + c(b_{ij})_{m \times n} \\ &= cA + cB \end{aligned}$$

The next is A 5, that is scalar multiplication, distributed over vector addition. So we consider ij th element of m by n matrix - a - and b_{ij} is the ij th element of m by n matrix b . **So** When c is multiplied with this, let us see what will happen. c multiplied by a_{ij} plus b_{ij} is - when we add 2 matrices, **then** they are being done element wise; so ij th element of a and ij th element of b matrix will be added together giving us a_{ij} plus b_{ij} of the m by n matrix. **So** This is scalar multiplied by c ; **so** this simply means c times a_{ij} plus c times b_{ij} of m by n matrix and then one can split the two - it is $c a_{ij}$ m by n plus $c b_{ij}$ m by n and that simply means cA plus cB . So what we have is c star A plus c star B is equal to cA plus cB ; **so** this property is also valid for m by n matrices.

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$$\begin{aligned} \text{A6: } (c_1 + c_2) A &= (c_1 + c_2)(a_{ij})_{m \times n} \\ &= ((c_1 + c_2)a_{ij})_{m \times n} \\ &= (c_1 a_{ij} + c_2 a_{ij})_{m \times n} \\ &= c_1 (a_{ij})_{m \times n} + c_2 (a_{ij})_{m \times n} \\ &= c_1 A + c_2 A \end{aligned}$$

Now comes the next distributive property. **So** We have 2 scalars - c_1 and c_2 ; they were multiplied by an m by n matrix. Let us say a_{ij} is a typical element of A . **So** c_1 plus c_2 multiplied by a_{ij} of m by n order matrix **this** comes out to be c_1 plus c_2 multiplied by a_{ij} ; **so** it is multiplied with each and every element of the matrix. So it is $c_1 a_{ij}$ plus $c_2 a_{ij}$ because these are simply numbers. So they are distributed and then you can again collect the elements and we will have $c_1 a_{ij}$ m by n plus $c_2 a_{ij}$ m by n and this simply means $c_1 A$ plus $c_2 A$. So what we have is c_1 plus c_2 A is equal to $c_1 A$ plus $c_2 A$. That means A 6 also holds for this set.

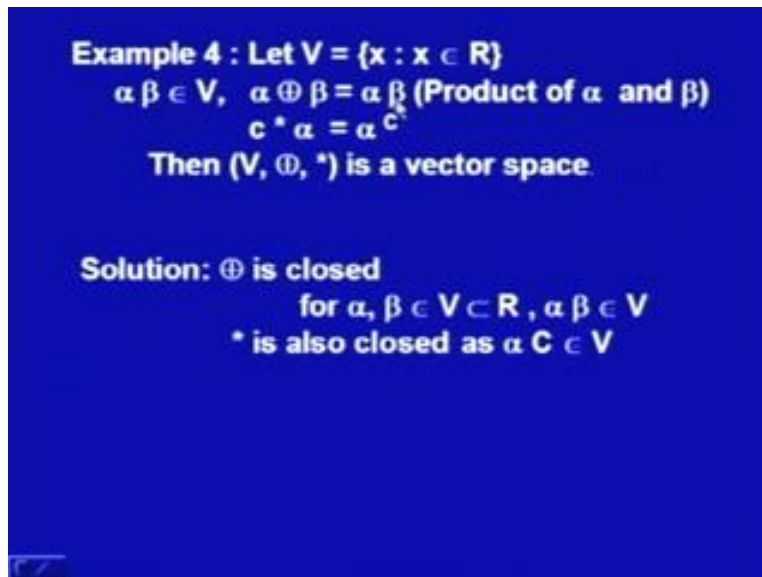
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$$\begin{aligned} \text{A7:} \quad & (c_1 \cdot c_2) A = c_1 \cdot c_2 (a_{ij}) \\ & = (c_1 c_2 a_{ij}) \\ & = (c_1 (c_2 a_{ij})) \\ & = c_1 (c_2 a_{ij}) = c_1 (c_2 A) \end{aligned}$$
$$\begin{aligned} \text{A8:} \quad & 1(A) = 1(a_{ij}) = a_{ij} \\ & = A \end{aligned}$$

Hence the set of all $m \times n$ matrices forms a vector space.

The **next is** next property is distributive property. **So** Let us see: $c_1 c_2$ two scalars multiplied by A . So the $c_1 c_2$ multiplied by typical element of the matrix - I am not writing m by n here again and again; **so** but this simply means the a_{ij} th element of the m by n matrix. **So** It is $c_1 c_2 a_{ij}$; it is multiplied with each and every element of the matrix and then this product; this is simply a number, so they can be distributed. You can **put the ba you can** interchange the brackets and finally what we have is $c_1 c_2$ into A . **So** This property also holds for scalars and matrices of order m by n . Next and the last is the existence of identity; **so** 1 belongs to set of real numbers, so 1 is a scalar which when multiplied by an m by n matrix A , **will** will not affect the matrix and what we have finally is A . So 1 multiplied by each and every element of the m by n th order matrix will not change any element here. So what we have is simply A . **So** All the properties are satisfied and one can say that and set of all m by n matrices form a vector space.

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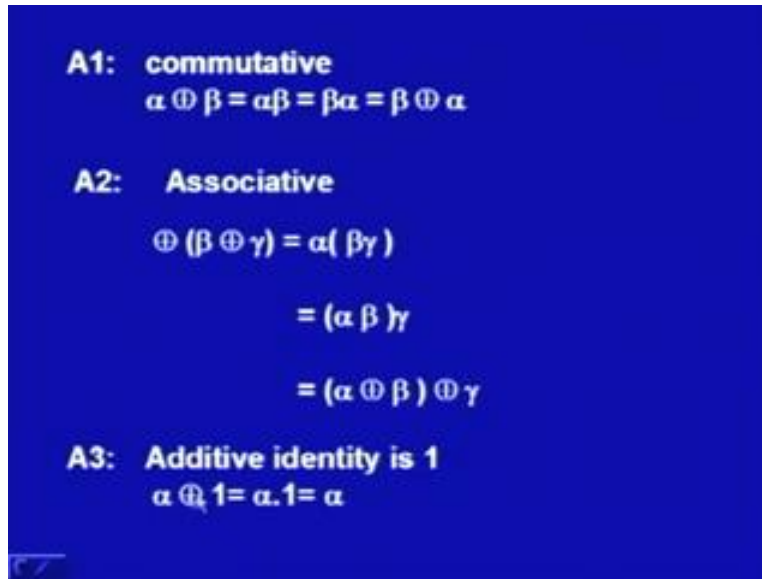
Example 4 : Let $V = \{x : x \in \mathbb{R}\}$
 $\alpha, \beta \in V, \alpha \oplus \beta = \alpha \beta$ (Product of α and β)
 $c * \alpha = \alpha^c$
Then $(V, \oplus, *)$ is a vector space.

Solution: \oplus is closed
for $\alpha, \beta \in V \subset \mathbb{R}, \alpha \beta \in V$
 $*$ is also closed as $\alpha^c \in V$

Now ah this ah These are some of the examples which we are the the simple examples we are aware of them with . Let us take one example which is slightly different from what we had so far. So now we define of vector V consisting of all real numbers belonging to \mathbb{R} and here, I am defining the addition of 2 vectors in a different manner. So I am not using plus sign here, because plus may be little bit confusing here because here i mean to say the addition ah by addition I mean to say actually the product of 2 vectors alpha and beta - alpha being defined as a real number. So alpha plus beta is equal to alpha beta product of alpha and beta and c star alpha - by this, I mean to say alpha raised to power the scalar c . So alpha is a real number and c star alpha means alpha raised to power c . Now my claim is then V plus, that the operation of addition which I have defined in this particular manner and scalar multiplication, if I define in this particular manner, then it is also a vector space. Now To prove this, I have to first check that this operation is closed or not. So for this, I consider 2 vectors - alpha and beta; they belong to V , they belong to V . But what is V ? V is actually a subset of real numbers. The way I have defined this - V is a subset of \mathbb{R} and $\alpha \beta$ belongs to V . Now When $\alpha \beta$ belongs to V , then this product means $\alpha \beta$ - that also belongs to V - because multiplication of 2 real numbers is also a real number; so this is also closed. Star is also closed as alpha raised to

power c - α is real number raised to power, a real number c - also belongs to V . So plus is closed as well as star is closed.

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A1: commutative
 $\alpha \oplus \beta = \alpha\beta = \beta\alpha = \beta \oplus \alpha$

A2: Associative
 $\oplus (\beta \oplus \gamma) = \alpha(\beta\gamma)$
 $= (\alpha \beta) \gamma$
 $= (\alpha \oplus \beta) \oplus \gamma$

A3: Additive identity is 1
 $\alpha \oplus 1 = \alpha. 1 = \alpha$

Next is the commutative property. So α plus β is equal to $\alpha\beta$ which is equal to $\beta\alpha$ because these are numbers; they can be commutative and this is nothing but β plus α . The next property is associative. **so** We will say α plus β plus γ is equal to $\alpha\beta\gamma$ is equal to $\alpha\beta\gamma$; these are our numbers, **so** these can they can be associative and they can be written in this form. So this becomes associative property. **So** Next is additive identity; it is 1. So α plus 1 is equal to α multiplied by 1 because the way I have define this operation - it is the product of the 2. So this α into 1 - real number multiplied by 1- is the same real number α ; so this is additive identity.

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A4: Additive Inverse
 $\forall \alpha \in V, \exists 1/\alpha \text{ in } V \text{ such that}$
 $\alpha \cdot 1/\alpha = 1$
additive Inverse for α is $1/\alpha$

A5: Distributive
 $c * (\alpha \oplus \beta) = (\alpha\beta)^c = \alpha^c \beta^c$
 $= \alpha^c \oplus \beta^c$
 $= (c * \alpha) \oplus (c * \beta)$

Additive inverse in this case: if I have alpha belonging to V, then there exists 1 by alpha in V such that when they are multiplied together, what I have is additive identity. In this case, additive identity is 1. So alpha multiplied by 1 by alpha is equal to 1. So we can say additive inverse of alpha is nothing but 1 by alpha. So that is how we define additive inverse.

Next thing is the distributive property. Now To check the distributive property, we will have c star alpha plus beta is equal to - this operation is alpha beta - c star; this means alpha beta raised to power c and this these are simply numbers. So I can write down this number as alpha raised to power c beta raised to power c or we can say this is the product; then I can say alpha raised to power c added - this multi ah operation - beta c. So this is the product of the 2 and alpha is nothing but c star alpha plus c star beta. So c star alpha plus beta is nothing but c star alpha plus c star beta. So we have proved the distributive property.

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$$\begin{aligned} \text{A6: } & (c_1 c_2) * \alpha = \alpha^{c_1 c_2} = \alpha^{c_2 c_1} \\ & c_1 * (c_2 * \alpha) = c_1 * (\alpha^{c_2}) \\ & (\alpha^{c_2})^{c_1} = \alpha^{c_2 c_1} \\ & \Rightarrow (c_1 c_2) * \alpha = c_1 * (c_2 * \alpha) \\ \text{A7: } & (c_1 + c_2) * \alpha = \alpha^{c_1 + c_2} = \alpha^{c_1} \alpha^{c_2} \\ & = \alpha^{c_1} \oplus \alpha^{c_2} \\ & = c_1 * \alpha \oplus c_2 * \alpha \\ & \Rightarrow (c_1 + c_2) * \alpha = c_1 * \alpha \oplus c_2 * \alpha \\ \text{A8: } & 1 \in V \quad \text{s.t.} \quad 1 * \alpha = \alpha * 1 = \alpha \end{aligned}$$

Thus with the given operations V is a vector space.

alpha c 2 into c 1 can be written as alpha c 2 c 1 and this simply means that c 1 c 2 star alpha is the same as c 1 star c 2 star alpha. So that proves the A 6 -the associative property. Finally the A seven is c 1 plus c 2 star alpha is equal to alpha times alpha raised to power c 1 plus c 2 - this is the scalar - so it is c 1 plus c 2 and this can be written as alpha c 1 multiplied by alpha c 2 - this is the property of real numbers. So we can say alpha c 1 plus alpha c 2; **so we can** this is the product of 2 numbers alpha c 1 into alpha c 2; so we can say alpha c 1 plus alpha c 2. So this is equal to c 1 star alpha; this is nothing but c 1 star alpha and this is nothing but c c 2 star alpha and that proves the result that c 1 plus c 2 star alpha is equal to c 1 star alpha plus c 2 star alpha and with this we come to the last property. **That** There exists 1 belonging to V **such that** - 1 belonging to the set of real numbers - such that 1 multiplied by alpha is equal to alpha times 1 or it is alpha. So there exists a scalar 1 such that 1 alpha is equal to alpha 1 is equal to alpha. Thus with the given operations, V is a vector space. So we have defined operations in slightly different manner but still we could prove that this given set V is a vector space with respect to defined operations. Now so far we have taken examples which are actually vector spaces, but in this example we are defining a set V of all vectors alpha such that alpha belongs to R and alpha is a positive number. Then we say that it is not a vector space with respect to the operations which we have just defined.

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Example 5:
The set $V = (\alpha : \alpha \in \mathbf{R}, \alpha > 0)$ is not a vector space with the operations

$$\alpha \oplus \beta = \alpha + \beta$$
$$c * \alpha = c \alpha$$

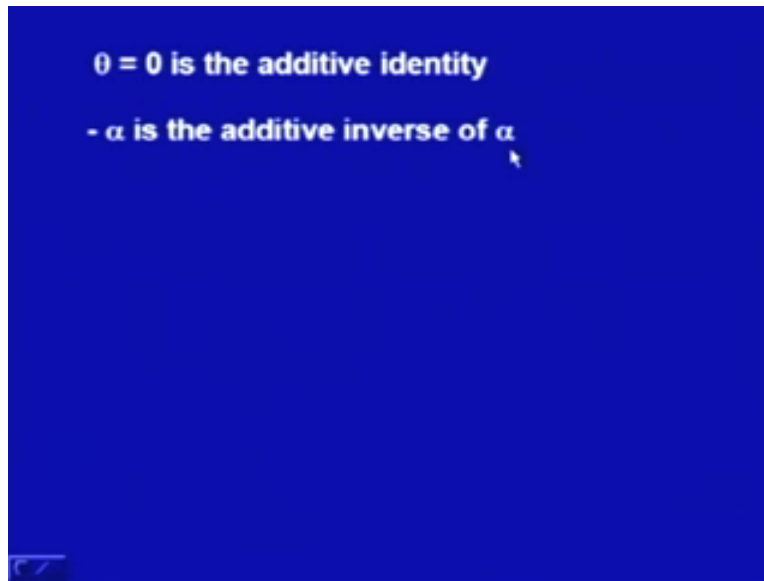
Solution:
At least the one of axioms fails

- Addition is Closed**
- Commutative**
- Associative**

Also $r < 0, r \alpha < 0$ does not belong to V
Scalar multiplication is not closed.

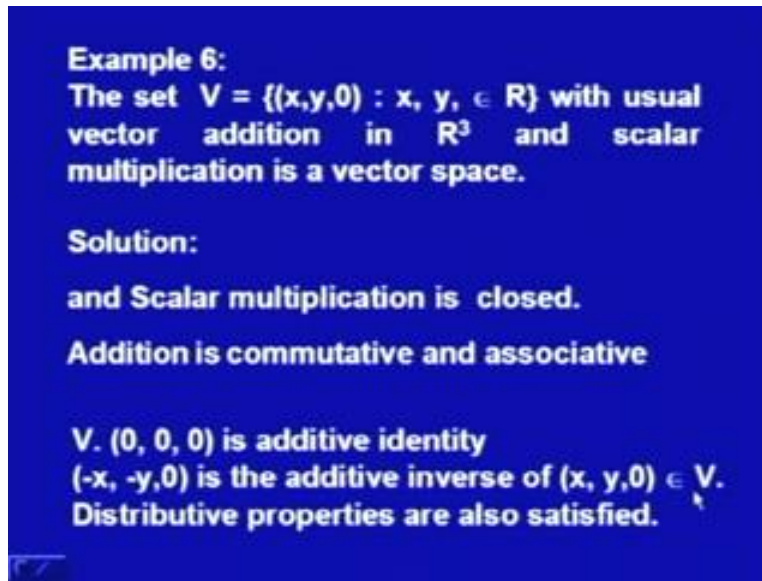
alpha plus beta is equal to alpha plus beta and c star alpha is equal to c into alpha. Now, **proof is** to prove this what we have to do is we have to show that at least 1 of the axioms fails in this case. So we have to check all the axioms and if there exists an axiom which is not satisfied **then** we can say that this set is not a vector space. So we start with addition; **so** addition is closed because if the alpha belongs to R then alpha plus beta **will** also belong to R. alpha is a positive number, beta is a positive number; add the two. It is a positive number; so it is closed. It is commutative because alpha is a real number; so alpha plus beta is also a real number; so it is commutative. So **really** to prove that **it is an associative this as a** addition is associative - in this case that is also satisfied; but when it comes to scalar multiplication: so if we take r -a scalar which is negative - then r alpha is also negative. **So** If alpha is a vector, r is a negative scalar; then r into alpha by this definition is a negative number and this negative number because of this property will not belong to V; so we can say that this multiplication is not closed. **So** When this one property fails here, **so** it is not a vector space. So we can say the scalar multiplication is not closed. **So** This will not form a vector space.

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In fact, if you can in fact the proof is done. But still we can say that theta is equal to 0 is the additive identity - absolutely no problem; but minus alpha is the additive inverse of alpha - that's true - but minus alpha will not belong to V . So for given alpha, additive inverse will not exist. So This is another property which fails in this case or we say that in this particular example V is not a vector space. You can check other properties but the solution is complete because we have proved it is not a vector space. Then another example:

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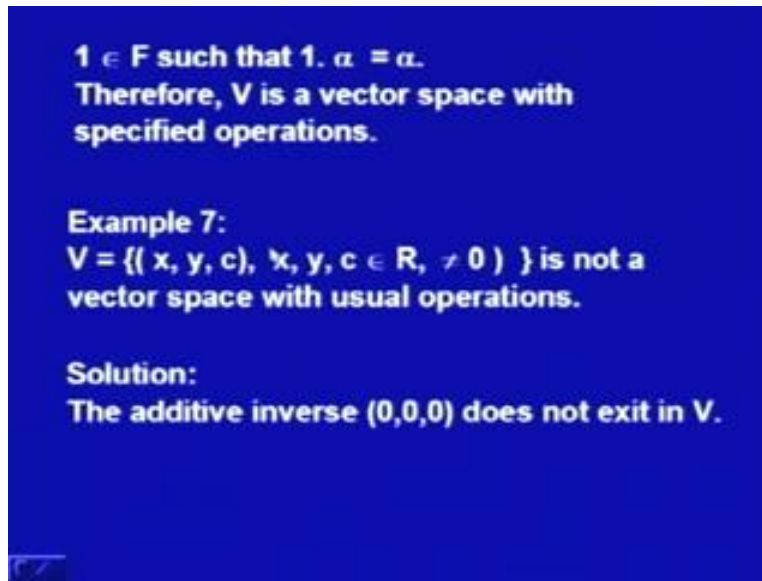
Example 6:
The set $V = \{(x,y,0) : x, y, \in \mathbb{R}\}$ with usual vector addition in \mathbb{R}^3 and scalar multiplication is a vector space.

Solution:
and Scalar multiplication is closed.
Addition is commutative and associative

V . $(0, 0, 0)$ is additive identity
 $(-x, -y, 0)$ is the additive inverse of $(x, y, 0) \in V$.
Distributive properties are also satisfied.

We have a set V consisting of \mathbb{R}^3 - vectors in \mathbb{R}^3 : x , y and z is equal to 0. x and y belongs to \mathbb{R} with usual vector addition in \mathbb{R}^3 and scalar multiplication is a vector space. So this set with usual vector addition and scalar multiplication is a vector space. Now we have proved this. One can check that addition and scalar multiplication - they are closed. Addition is commutative and associative; this one can be very easily proved. I leave this in exercise for the viewers. Then we can say $(0, 0, 0)$ is additive identity for the set V ; further $(-x, -y, 0)$ is the additive inverse of $(x, y, 0)$ for any vector belonging to V in this particular form. One can very easily check that when we add the two, **it is** the additive identity is 0. Distributive property is also satisfied.

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$1 \in F$ such that $1 \cdot \alpha = \alpha$.
Therefore, V is a vector space with specified operations.

Example 7:
 $V = \{ (x, y, c), x, y, c \in \mathbb{R}, c \neq 0 \}$ is not a vector space with usual operations.

Solution:
The additive inverse $(0,0,0)$ does not exist in V .

Then we can talk about the scalar 1 such that 1 into alpha is equal to alpha and all these properties proved that V is a vector space with specified operations. But if we slightly modify this set V as xy comma c , where c belongs to \mathbb{R} but - it is not 0 - c is not 0, then it is not a vector space with usual operations. So slightly you make a change in this and it will not be a vector space like the additive inverse 0 comma 0 comma 0 comma does not exist in V . So this is one property which says that V is not a vector space. This does not belong to V because if it has to belong to V , then this must be c and c is not 0; so c is not a 0 so 0 0 0 cannot belong to V . So this is Because of this reason, we can say that x by c is not a vector space with usual operations.

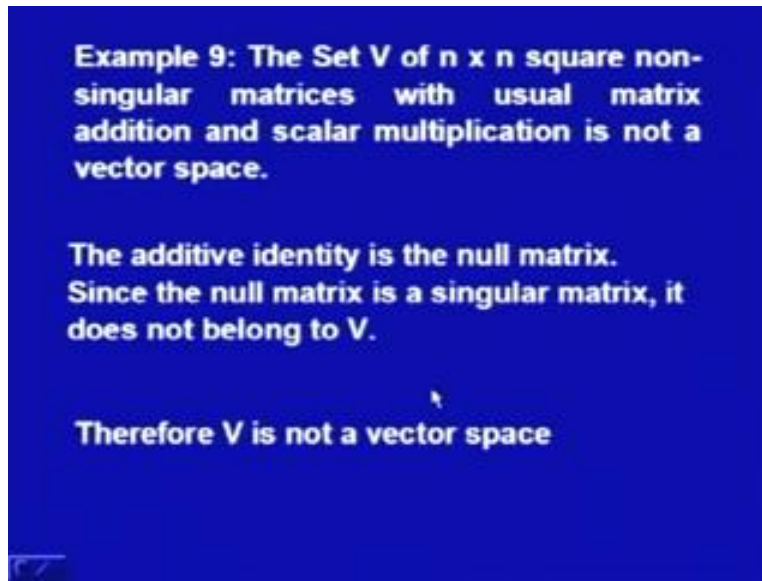
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Example 8:
Let $V = \{x, y, x, y \in \mathbb{R}\}$ with the usual vector addition '+' and scalar multiplication defined as
 $c * (x, y) = (c x, y)$
Then $(V, +, *)$ is not a vector space.

$$\begin{aligned}(c_1 + c_2) * (x, y) &= ((c_1 + c_2) x, y) \\ &= (c_1 x + c_2 x, y) \\ c_1 * (x, y) + c_2 * (x, y) &= (c_1 x, y) + (c_2 x, y) \\ &= (c_1 x + c_2 x, 2y) \\ (c_1 + c_2) * (x, y) &\neq c_1 * (x, y) + c_2 * (x, y)\end{aligned}$$

Another example which is not a vector space: **now** we take an example where we have x, y - if they belong to \mathbb{R} . So this vector x, y belongs to \mathbb{R}^2 be the usual vector addition plus and scalar multiplication defined as $c * (x, y) = (cx, y)$. Now **the now** we are defining the scalar multiplication in a slightly different manner. See, earlier we took $c * (x, y) = (cx, cy)$. Now I have changed the meaning of scalar multiplication here. It is $c * (x, y) = (cx, y)$ and that means then $(V, +, *)$ is not a vector space. One can prove it very easily. You can check $(c_1 + c_2) * (x, y)$ is equal to $(c_1 + c_2)x, y$. So **this is** only first component is being multiplied by this; so it becomes $c_1 x + c_2 x, y$ and now we can see $c_1 x + c_2 x, y$ plus $c_2 x, y$; if I take the right hand side for this I will have $c_1 x, y$ plus $c_2 x, y$ and that simply means - the vector addition means $c_1 x + c_2 x, y + y$ that is $c_1 x + c_2 x, 2y$. That means this is not equal to this; so it is not distributive. **So** With this scalar multiplication, the same set which we have taken earlier will not form a vector space. So it is important how **the way** you **are** define the 2 operations. **Its not** It depends upon the set V ; it also depends upon the operations which you define on them. So with this operation the same set V will not form a vector space.

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Again I will take an example of matrices. So if we have a set of n by n square matrices - but these square matrix are non singular matrices - with usual matrix addition and scalar multiplication is not a vector space. We have proved that set V of n by n square matrices with usual matrix addition scalar multiplication is a vector space but when it comes to square matrices with usual matrix addition scalar multiplication, then it is not a vector space. **And** The reason is the additive identity is the null matrix. We have proved that **additive** null matrix is the additive identity. With this additive identity is the null matrix which is non -singular matrix. Null matrix has determinant 0 and it becomes nonsingular it becomes a zero matrix. So null matrix does not belong to this set of n by n square non -singular matrices and that is the reason **that** this will not form a vector space. **So** While n by n matrices **they** form a vector space, **but** when it comes to n by n square non singular matrices, they do not form a vector space. So V is not a vector space.

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Result 1: There exist only one additive identity in a vector space V

Proof: (i) Proof by contradiction.
Assume that there are two distinct additive identities θ_1 and θ_2

$\theta_1 + \theta_2 = \theta_1$	θ_2 being additive identity
$\theta_2 + \theta_1 = \theta_2$	θ_1 being additive identity
$\theta_1 + \theta_2 = \theta_2 + \theta_1$	Additive commutative
$\theta_1 = \theta_2$	contradiction.

Hence there exist a unique additive identity in the vector space

Now we talk about some simple results which will hold for all the vector spaces irrespective of the set V , irrespective of the operations which we define. The first result is that there exists only one additive identity in a vector space. By this, I mean to say that if we have a vector space, then there exist only one additive identity. You cannot have 2 additive identities in a given vector space whatever be the vector space - whatever be the operations. Now, **to prove this** we prove it by contradiction. By this, I mean to say we assume that there are 2 distinct additive identities: θ_1 and θ_2 . So by this, I mean to say that $\theta_1 + \theta_2$ is equal to θ_1 because θ_2 is additive identity. So **when** additive identities added into θ_1 will have θ_1 ; but θ_1 is also an additive identity; so $\theta_2 + \theta_1$ - θ_1 being additive identity - then added into θ_2 - it will not change θ_2 ; so $\theta_2 + \theta_1$ is equal to θ_2 . **When** θ_1 **being** considered as additive identity **and by this** I means **to say** $\theta_1 + \theta_2$ is equal to $\theta_2 + \theta_1$ and this means θ_1 is equal to θ_2 . So this means θ_1 is equal to θ_2 . $\theta_1 + \theta_2$ is equal to $\theta_2 + \theta_1$. Why? Because it is commutative. So they are equal - that means this is equal to this; θ_1 is equal to θ_2 . So **what** we have started with **we have started with** two different additive identities θ_1 and θ_2 . They are not equal; but finally we have proved that they are equal. So it is not possible that a vector space has 2 distinct additive identities; even if we

start with different, we finally arrive at the same values. So by contradiction, we have proved that there exists only one additive identity in a vector space.

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Result 2: There exist only one additive inverse for given $\alpha \in V$,

Proof: Proof by contradiction

Let α and α are two distinct inverses of α in V

$$\alpha + \alpha_1 = 0 \quad \text{definition of inverse}$$

$$\alpha + \alpha_2 = 0 \quad \text{definition of inverse}$$

$$(\alpha + \alpha_1) + \alpha_2 = 0 + \alpha_2 = \alpha_2$$

$$(\alpha + \alpha_2) + \alpha_1 = 0 + \alpha_1 = \alpha_1$$

The next result says that there exist only 1 additive inverse for given alpha and V. So any member alpha and V cannot have two different inverses. **So** Again we prove this result by contradiction. **So** Let us say alpha 1 and alpha 2 are 2 distinct inverses of alpha in V; so they are 2 different inverses alpha 1 and alpha 2. **So** Since alpha 1 is a inverse of alpha, alpha plus alpha 1 is equal to theta by definition of inverse. Similarly alpha plus alpha 2 is also theta because of definition of inverse. **Now to prove we will actually prove that** Although we have started with different values of alpha 1 and alpha 2, **but** finally we will prove that alpha 1 is equal to alpha 2. Now to prove this, what we have done is in the first, I have added alpha 2. So alpha 1 plus alpha 1 plus alpha 2 is equal to theta plus alpha 2; I have added alpha 2 in this, but what is theta plus alpha 2? It is nothing but alpha 2 because theta is the additive identity. Now in this, I add alpha 1. So alpha plus alpha 2 plus alpha 1 is equal to theta plus alpha 1; theta plus alpha 1 is alpha 1. But what is this? alpha plus alpha 1 plus alpha 2 is equal to alpha 2.

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$$\begin{aligned}\alpha_2 &= (\alpha + \alpha_1) + \alpha_2 \\ &= \alpha + (\alpha_1 + \alpha_2) \text{ associative Law} \\ &= \alpha + (\alpha_2 + \alpha_1) \text{ Commutative Law} \\ &= (\alpha + \alpha_2) + \alpha_1 = \alpha_1 \text{ associative} \\ \alpha_2 &= \alpha_1\end{aligned}$$

Contradiction

Hence inverse of a given α is unique in a vector space

So if I start with this, alpha 2 is equal to alpha plus alpha 1 plus alpha 2. But since this addition is associative so I can write down this as alpha plus alpha 1 plus alpha 2; that means I can change the brackets like this. But then this addition is commutative. So I can change the order; so it is alpha plus alpha 2 alpha 1 and again, I can change this bracket because of associative property. So what I have is alpha plus alpha 2 plus alpha 1; but I have already proved that this is nothing but alpha 1. So I have started with the alpha 2 and I have arrived at alpha 1. What does it mean? It means alpha 2 is equal to alpha 1. But this is the contradiction; that simply means it is not possible to have 2 different additive inverses for a given element in V and that means inverse of a given alpha in V is unique. So for a vector space, we will have a unique inverse for a given vector.

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Result 3: Let V is a vector space then
 $0 \cdot \alpha = 0$ for every α in V and scalar 0
 $c \cdot 0 = 0$ for any scalar c and 0 in V

Proof:
 $0 \cdot \alpha = (0 + 0) \cdot \alpha = 0 \cdot \alpha + 0 \cdot \alpha$

Adding both sides $-0 \cdot \alpha$
 $0 \cdot \alpha + (-0 \cdot \alpha) = (0 \cdot \alpha + 0 \cdot \alpha) + (-0 \cdot \alpha)$

$0 \cdot (\alpha + (-\alpha)) = 0 \cdot \alpha + (0 \cdot \alpha + (-0 \cdot \alpha))$

$0 = 0 \cdot \alpha + 0 = 0 \cdot \alpha$

The next result says that if V is a vector space, then 0 multiplied by α is equal to 0 for every α in V and scalar 0 and c multiplied by 0 is equal to 0 for any scalar c and 0 in V . Now one thing to be observed here is that this 0 and this 0 **they** are different. This 0 is a scalar; when multiplied by an α vector, the result is a vector. So this 0 is a 0 vector and in this case this is a 0 vector and this is also a 0 vector. Now to prove this we start with the first one - 0 multiplied by zero. **Now** I write this as **0 as 0** plus 0 multiplied by α . It is absolutely no problem because 0 is a scalar quantity; so 0 plus 0 is a scalar - still 0 . So 0 plus 0α is equal to 0α plus 0α . Why? **Because** This is distributive because it is a vector space; so distributive property is satisfied. Now **adding** both sides minus 0 times α . So if I add this, 0α plus minus 0α is equal to 0α plus 0α - this is what I had - plus minus 0α - I have added this in both the sides. Now the left hand side is simplified as this, because 0 is a scalar which you can take outside; so 0 multiplied by α plus minus α is equal to 0α ; and then I **have so distributive property so I** apply distributive property here and what I have is 0 on the left hand side. On the right hand side, I will have 0α . So I have proved that 0 is equal to 0α ; so that first part has been proved. This is true for any scalar 0 in any vector α .

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Proof:

$$c \cdot 0 = c \cdot (0 + 0)$$
$$c \cdot 0 = c \cdot 0 + c \cdot 0$$

Adding both sides $-c \cdot 0$

$$c \cdot 0 + (-c \cdot 0) = (c \cdot 0 + c \cdot 0) + (-c \cdot 0)$$
$$(c - c) \cdot 0 = c \cdot 0 + (c \cdot 0 - c \cdot 0)$$
$$0 = c \cdot 0 + 0$$
$$0 = c \cdot 0$$

Now **this for the second part** for the second part, the proof goes like this. I start with $c \cdot 0 - 0$ - zeroes are vector now - is equal to c multiplied by 0 plus $0 - 0$ being the additive identity; so 0 plus 0 is also 0 . So $c \cdot 0$ is $c \cdot 0$ plus 0 . This dot represents scalar multiplication. So this can be written as $c \cdot 0$ plus $c \cdot 0$ - distributive properties applied. Now adding both sides minus c times 0 , again the same way this can be taken together in the left hand side, then it is 0 and then you can again apply the associative property on the right hand side. So this proves that 0 is equal to $c \cdot 0$ and that proves the second part of the result.

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Result 4: Let V is a vector space and $c \cdot \alpha = 0$ then $c = 0$ or $\alpha = 0$

Proof: Consider $c \neq 0$ $c\alpha = 0$

$$\frac{1}{c}(c\alpha) = \frac{1}{c}0 = 0$$
$$1 \cdot \alpha = 0$$
$$\alpha = 0$$

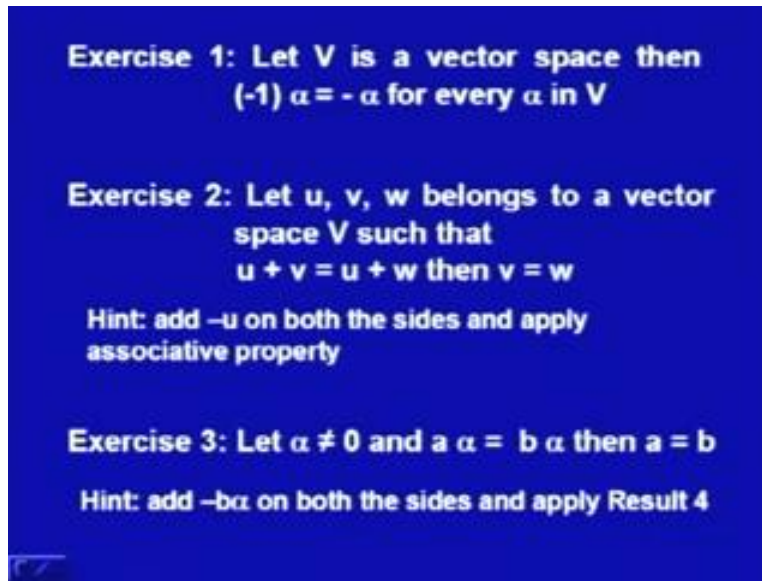
Consider $\alpha \neq 0$ $c\alpha = 0$

Adding both sides $-c \cdot \alpha$

$$c\alpha + (-c\alpha) = 0 + (-c\alpha)$$
$$0 = (-c\alpha)$$
$$\Rightarrow -c = 0 \text{ or } c = 0$$

Now the fourth result says that if V is a vector space and c dot α is equal to 0, then c is equal to 0 or α is equal to 0. So whenever c α is 0, then one of the two things must happen - either the scalar is 0 or the vector is the null vector. To prove this, we first consider c as not zero. With this, we will prove that α is equal to 0. So c is not 0 but c α is 0. Then I can multiply this quantity by 1 by c . So 1 by c into c α is **1 into** - I am multiplying 1 by c on both the sides; so 1 by c into 0 is **because** - 1 by c is possible because c is not zero, that is why I have taken c as not 0; so I am multiplying 1 by c on both the sides. 1 by c into c α is equal to 1 by c into 0 . So right hand side is 0 but this side is 1 into α ; but we know 1 is a vector space, so 1 dot α is equal to α ; so this proves α is equal to 0. Now to prove the other part, we consider α not 0 and by adding minus c α on both sides and proceeding in this particular manner, we can prove that c is equal to 0; that establishes this result.

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**Exercise 1: Let V is a vector space then
 $(-1) \alpha = -\alpha$ for every α in V**

**Exercise 2: Let u, v, w belongs to a vector
space V such that
 $u + v = u + w$ then $v = w$**

**Hint: add $-u$ on both the sides and apply
associative property**

Exercise 3: Let $\alpha \neq 0$ and $a \alpha = b \alpha$ then $a = b$

Hint: add $-b\alpha$ on both the sides and apply Result 4

Towards the end of the lecture, I will give few exercises. The viewers are advised to go through these exercises. They actually give some important results related to vector spaces. Exercise 1 says that if V is a vector space, then minus 1 into alpha is equal to minus alpha for every alpha in V . While the second exercise says that if u, v, w belong to vector space V such that $u + v = u + w$, then $v = w$; so it is actually a cancellation law. u and w can cancel out and we can have $v = w$. So to prove this, you simply you have to add minus u on both sides and apply associative property. The third exercise is that if $\alpha \neq 0$ and $a \alpha = b \alpha$, then $a = b$. So this is again a cancellation property where we can add the 2 where you can cancel the two scalars. Now to prove this, you have to simply add minus $b \alpha$ on both the sides and apply the results which we have already discussed.

So viewers, today we have discussed vector spaces. I have started with differential of vector spaces. I have given some examples where we have define the vector space vector V . We defined operations addition and multiplication and we have seen whether a given V plus star is a vector space or not. We have taken some examples where it forms a vector space and we have illustrated in few examples where the given set with given operations do not form a vector space. Thank you.