Mathematics II
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Module - 2
Lecture No - 1
Matrix Algebra Part - 1

Welcome, viewers. Today's topic is matrix algebra. We start matrix algebra with the definition of matrices.

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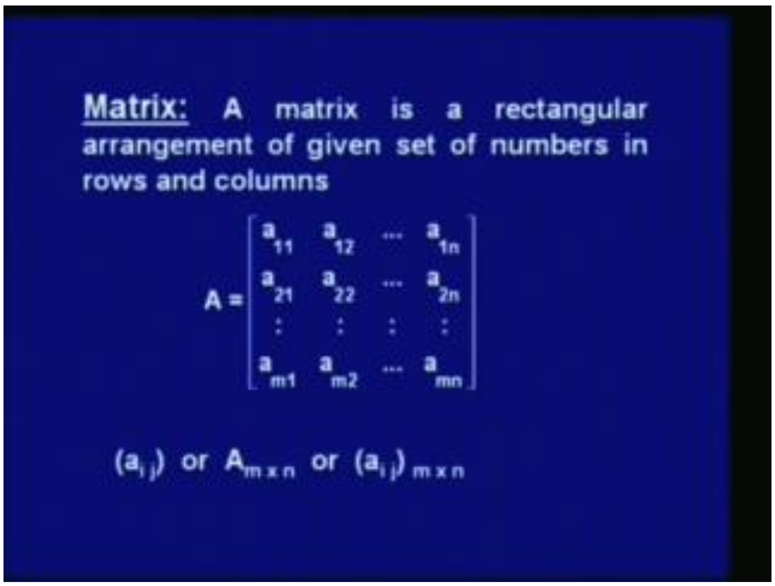

We define a matrix as a rectangular arrangement of a given set of numbers in rows and columns. Given a set consisting of m into n elements like a 1 1, a 1 2, a 1 n, a 2 1, a 2 2, a 2 n, am 1, am 2 am n, when they are arranged in rectangular matrix, we have rows and columns; then this arrangement is called a matrix. The matrix is enclosed in square bracket; we denote it by capital letter A. We give another notation as aij for the matrix. Sometimes we use it as am by n or we also write it as aij m into n.

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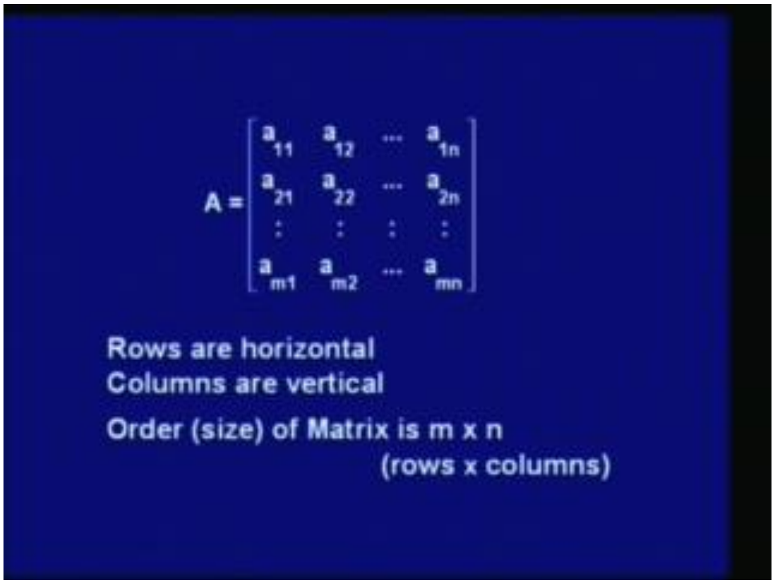

The rows are horizontal and are placed one below the other while columns are vertical and they are placed side by side. The size of matrix - or sometimes, we call it as order of the matrix - as m by n, meaning there by that there are m rows and n columns. That is why, the size or order is defined as rows into columns. So this matrix has m rows and n columns.

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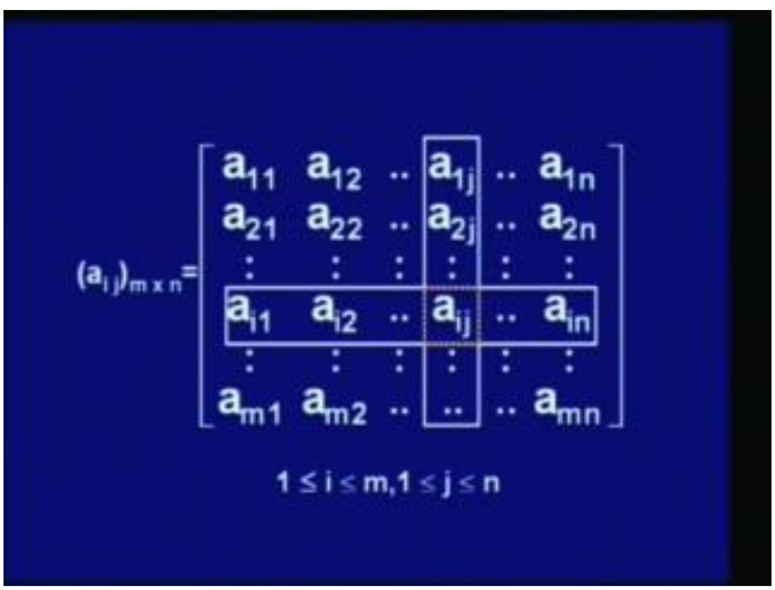

The individual item - or we call it an element - is identified by its position in the matrix. The matrix aij m into n - we have the ij th element as aij; that means, the element aij is in the i th row and j th column. So this is the i th row and this is the j th column and this element aji is a typical element of the matrix, and that is why this typical element is used in the notation. While this m by n means that the order of the matrix is m by n, in the matrix, the element has 2 indices - i and j. The index i represent its position in the row and j represents its column position. i varies from 1 to n, while the index j varies from 1 to n.

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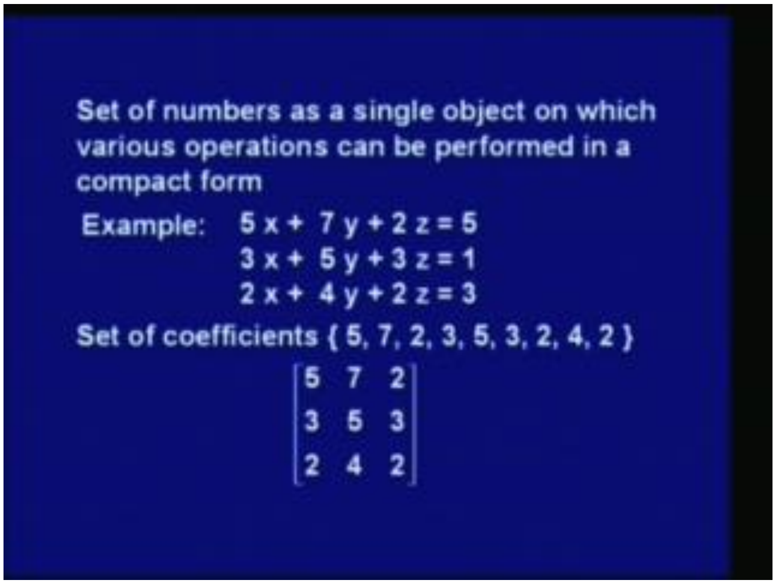

For example, if you have a set of numbers arranged in a set of equations like x plus y plus 2 z equal to 5, 2 x plus y plus 3 z equal to 1, 3 x plus 5 y plus z is equal to 3, then the set of numbers – 1, 1, 2, 2, 1, 3, 3, 5, 1 - these are the numbers which are actually the coefficients of these 3 equations. When these coefficients are arranged in this rectangular manner, then we say this matrix is the coefficient matrix; it has a specific meaning. The coefficients 1, 1, 2 they are the coefficients of the first equation - they appear in the first equation. While 2, 1, 3 are the coefficients of xyz in the second equation and 3, 5, 1 are the coefficients of xyz in the third equation. So row represents the 3 coefficients in the equations, while column represents the coefficients of x, the coefficients of y and the coefficients of z.

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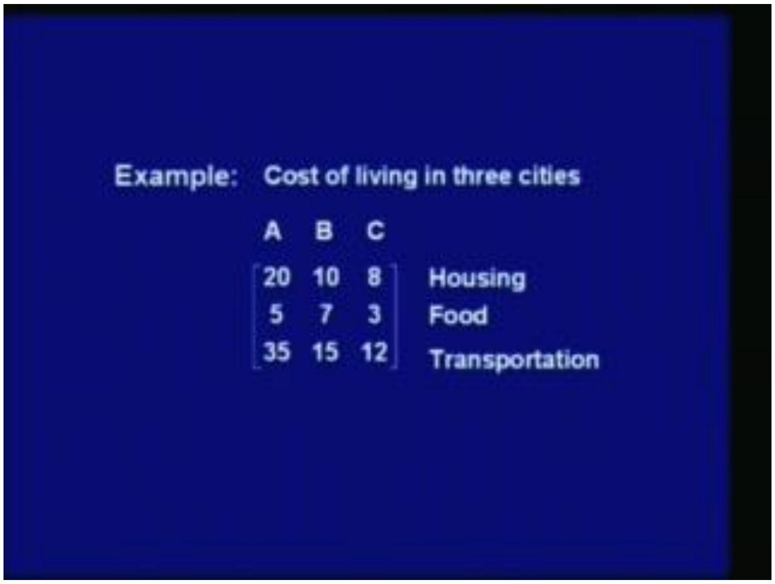

In another example, let us say we have a data about 3 cities and the living cost. So we have 3 cities – A, B, C - and the housing the cost, food cost and transportation cost. This data is arranged in this rectangular form. We say that 20 is the housing cost of city A, 10 is the housing cost in the city B and eight is the housing cost in city C. While 5 is the food cost in city A, 7 is the food cost of city B and 3 is the food cost in city C. Similarly, the third row represents the transportation cost in 3 different cities – A, B and C. 35 is the transportation cost in city A, 15 is the transportation cost in city B and 12 is the transportation cost in city C. So rows represent different cost - housing food and transportation - and columns represent cities. First column represents 3 costs – housing, food and transportation in city A; second column represents housing, food and transportation cost in city B and third column represents housing food and transportation cost in city C. Now we have types of matrices.

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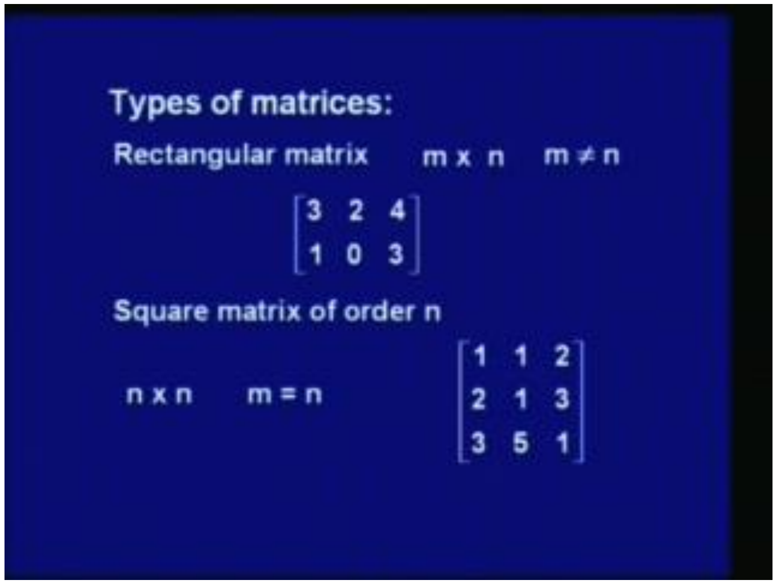

This classification is done on the bases of the shape a matrix is having, like a rectangular matrix is one where the number of rows and number of columns are different, that is m is not the same as n. For example in this matrix we have 2 rows and 3 columns; so m is not the same as n. However if number of rows and number of columns are equal, m is equal to n. Then we say we have a square matrix of order n. So m by n matrix is a square matrix of order n. In this example, we have a 3 by 3 square matrix of order 3. It consist of 3 rows and 3 columns; so it is square matrix of order 3.

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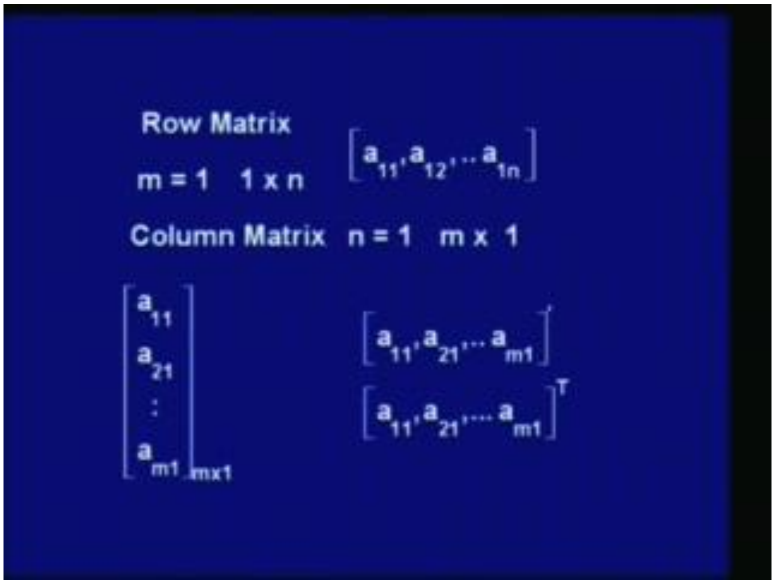

Row matrix is a special form of matrix where the number of rows is one; so m is equal to 1 and a typical row matrix is of order 1 by n. This is an example a row matrix. It has only one row here only Here the first index is fixed to 1 while the second index varies from 1, 2 to n. The column matrix has one column only. So n is equal to 1; the order of matrix is m by 1 and this is how a column matrix will look like. a 1 1, a 2 1, a m 1 and we say its of order m by 1. To save space, some author use a column matrix some authors represent the column matrix as this. Here, we have dash or sometimes we denote it by T; later on we will see more about this.

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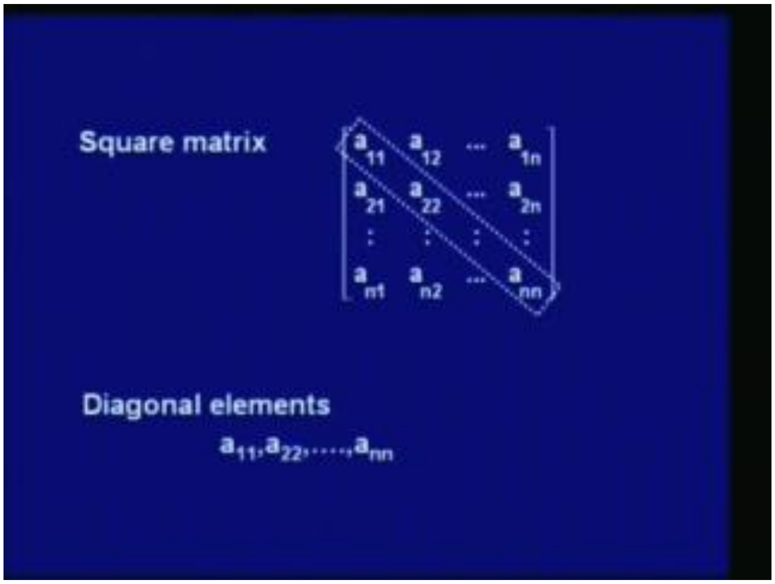

In the square matrix ,the number of rows and the number of columns are equal. So we have n rows and n columns. In all we have n square elements in a square matrix of order n. One can easily notice that a 1 1, a 2 2 a n n are the elements which lie on the diagonal of this square. Typically i and j are equal for the diagonal elements.

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So we define a diagonal matrix as a matrix in which ij th element is 0 if i is not j. However it is 0 whenever i is equal to j. Example is this matrix - I denote it by D 4 to indicate that it is a diagonal matrix of order 4 1. One can observe that only the diagonal elements are non 0 while rest of the elements are 0. So these non 0 elements appear in the diagonal matrix only.

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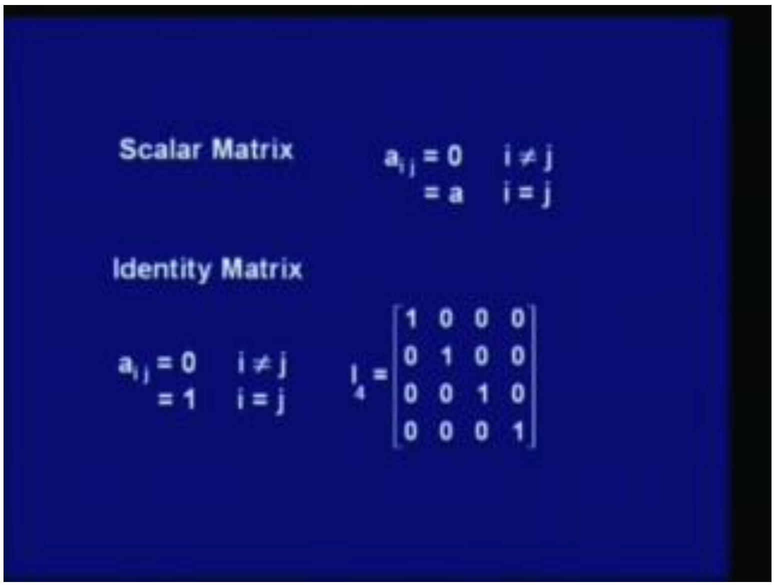

Scalar matrix is a very special form of diagonal matrix. It is a square matrix and it is defined as aij is 0 whenever i is not j. But the diagonal elements are all equal and they are a. So aij is equal to a whenever i is j. The identity matrix is also special form of a scalar matrix, where this a happens to be 1. So we define an identity matrix having aij is 0 whenever i is not j, but it becomes 1 when i is equal to j. So I 4: that is an identity matrix is denoted by I and 4 represents its order. So I 4 is a identity matrix of order 4. It has 1 as the diagonal elements and all elements are 0. So this is an identity matrix.

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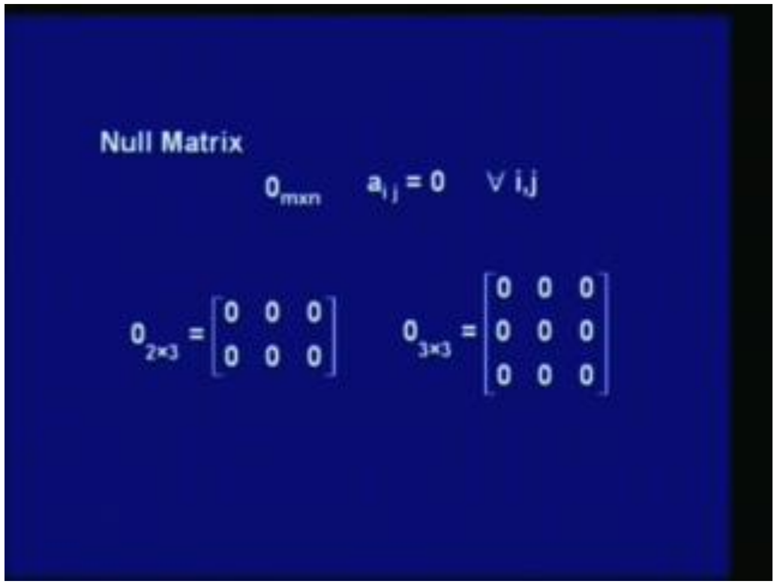

Null matrix is another useful matrix and we denote it by a symbol zero. m by n represents its order. So 0 mn is a null matrix of order m by n, and it is the matrix in which aij is 0 for all values of i and j. i takes values from 1 to m, while j takes values from 1 to n. Typically 0 2 by 3 -the null matrix of order 2 by 3 - is this matrix; it has 2 rows and 3 columns while 0 3 by 3 is a third order square matrix with all its entries as 0.

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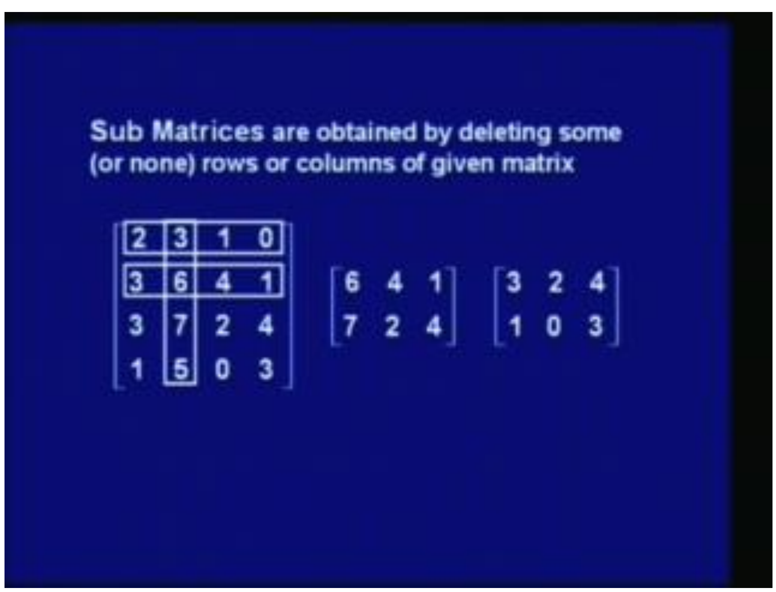

Sub matrices or obtained by deleting some rows or columns of given matrix. For example, we have a 4 by 4 square matrix consisting of elements 2, 3, 1, 0 in the first row; 3, 6, 4, 1 in the second row 3, 7, 2, 4 in the third row and 1, 5, 0, 3 in the last row. If I delete first row and last row together with first column, then what I have is this matrix. So 6 4 1 7 2 4 is a sub matrix of this matrix obtained by deleting first and last row and first column. This matrix is also sub matrix of this matrix, by deleting first and second rows and second column; so what I have is 3 2 4 and 1 0 3. In fact, one can obtain large number of sub matrices from a given set of given matrix having different order.

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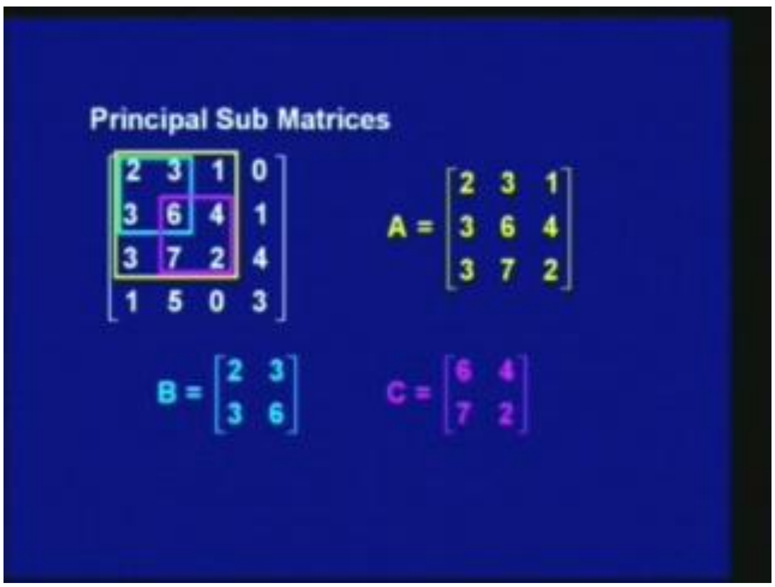

A principle sub matrix is also a sub matrix, but the diagonals are maintained like if I have this square matrix, then this is a principle sub matrix because the diagonal elements of this matrix and this matrix they are retained. In this case, the diagonal elements 2, 6 and 2 are retained. So this sub matrix is this matrix and it is a principle sub matrix. In a special case, a matrix itself is a sub matrix; so 2 3 1 0 3 6 4 1 3 7 2 4 1 5 0 3 is a 4 by 4 sub matrix of itself. Now we define operations on matrices.

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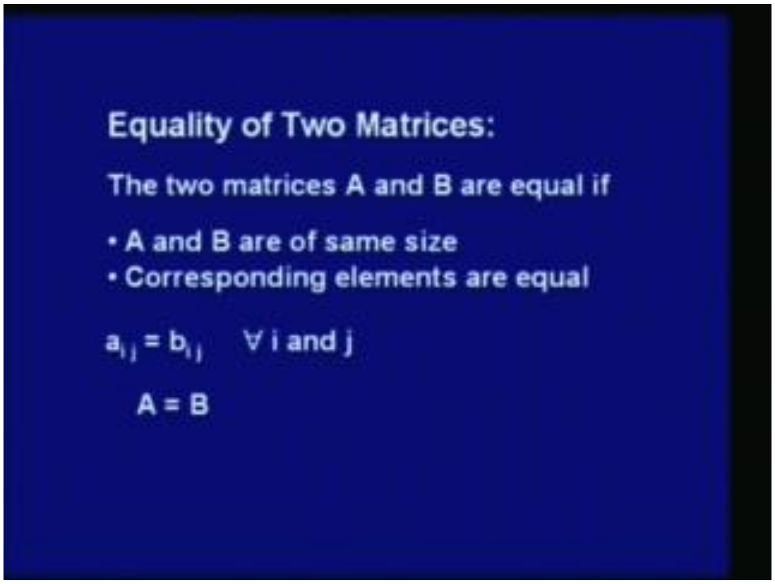

We say that the 2 matrix two matrices A and B are equal if some conditions are specified. The first condition is that the two matrices A and B are of same size; so if the 2 matrices are of same size, then they may be equal. If one more condition is satisfied and that condition is that the corresponding elements are equal, that is aij equal to bij for all values of i and j, in such case we say that the matrix A is equal to the matrix B. If the two matrices are not of same order, they cannot be equated.

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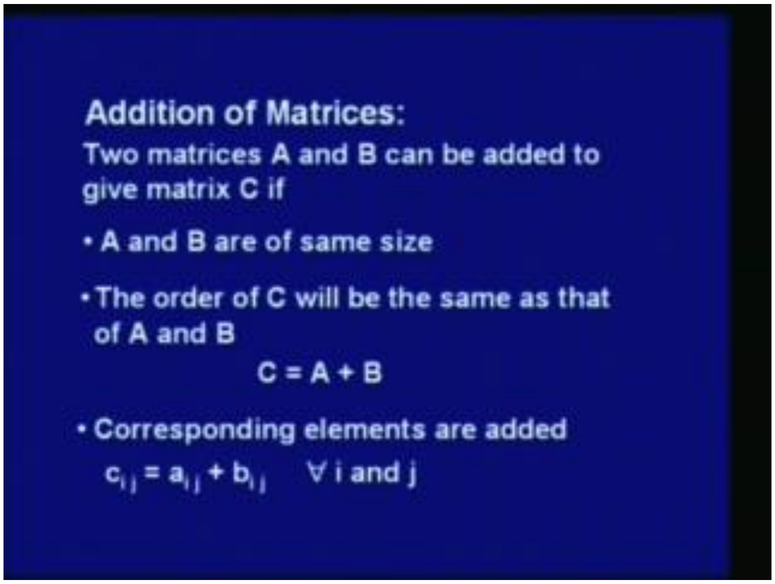

In the algebra of matrices, we define addition of matrices. If we are given two matrices A and B, then they can be added to give a matrix C - if A and B are of same size. So if the matrices A and B are of same size, then we say that they can be added and the result will be the third matrix C. This matrix C will be of the same order as that of A and B and we write C is equal to A plus B. Now to get the matrix C which is the sum of matrices A and B, one has to add the corresponding elements. That means, the typical element of the matrix C is cij is equal to aij - the element of A - plus ij - the element of B. So cij is equal aij plus bij for all values of i and j is the sum matrix. Now, this is illustrated in this example.

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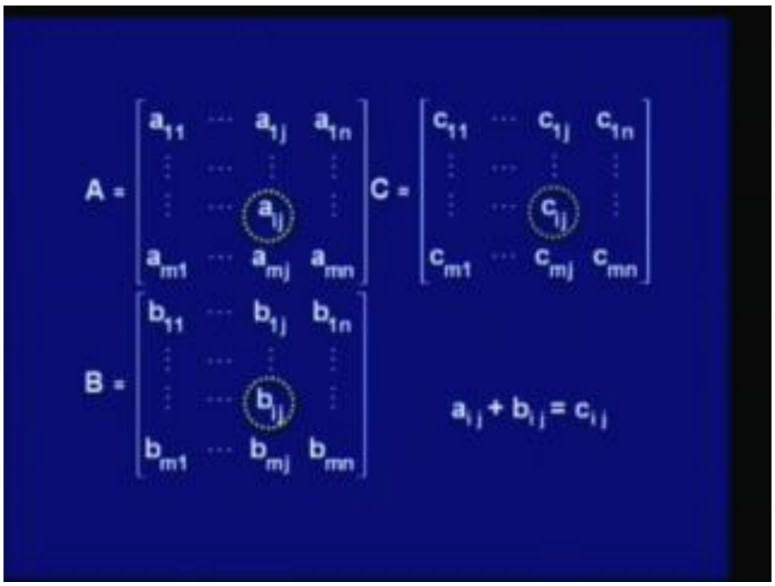

I have a matrix A consisting of m rows and n columns and B matrix also has m rows and n columns. So they are they can be added, and if they can be added we say they are confirmable to admission. So A and B are confirmable to admission and when they are added, I will get matrix C. The elements of matrix are obtained by adding corresponding elements; that is, the first element in the first row is obtained by adding first element of first row in first column of A and first row first column element of B. So a 1 1 plus b 1 1 gives the first element a 1 1 b 1 1 in the first row; the second element in the first row will be obtained by adding a 1 2 and b 1 2, which happens to be the second element of first row. So the elements are added, element by element, and the typical element aij of A matrix, which happens to be the i th row j th column element, is added in bij and what we get is the element aij plus bij. So C is the sum matrix of A matrix and B matrix and is given as this.

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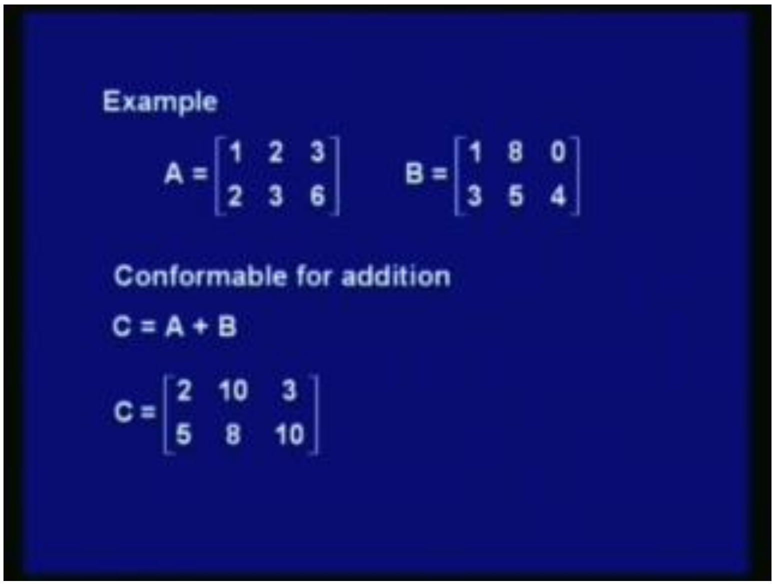

The example: one can take the 2 by 3 matrix A as 1 2 3 in the first row, 2 3 6 in the second row. The matrix B is also of order 2 by 3, consisting of first row as 1 8 0, the second row as 3 5 4 and they can be added because they are confirmable for addition and the result is the C matrix obtained as A plus B, as this matrix. One can notice that 1 plus 1 is 2, 2 plus 8 is 10, 3 plus 0 is 3, 2 plus 3 is 5, 3 plus 5 is 8, 6 plus 4 is 10. Next are properties of matrix addition.

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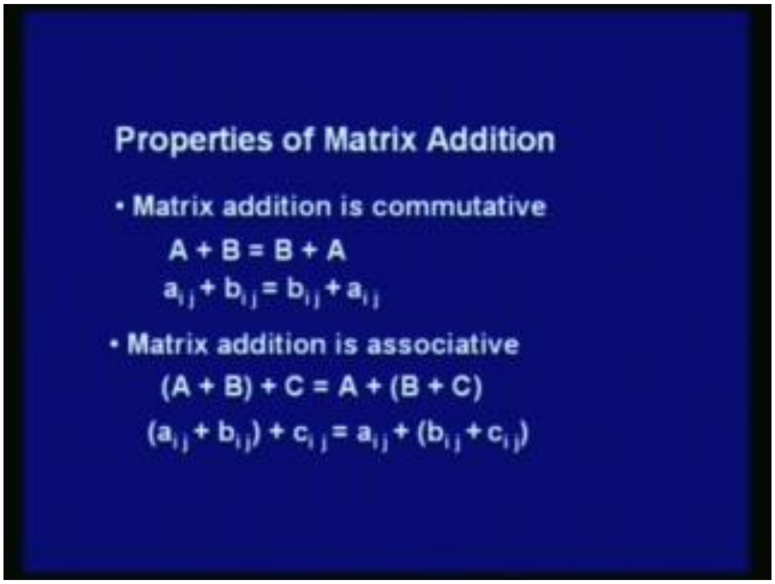

We say that the matrix addition is commutative, that means when A when B is added in A, we get the same result when B when A is added in B. That means the order is not important; whether you add B in A or you add A in B, the result is the same. And this can be proved easily because A plus B will have a typical element aij plus bij while B plus A will have bij plus aij as its ij th element and since aij and bij are numbers, so aij plus bij is the same as bij plus aij; so the matrix addition is commutative. Similarly, one can easily establish that matrix addition is associative. By this, I mean to say that you first add A and B, and in the sum, you add C; what you get is the same as you add B and C first, and then add A. So the order of addition is not important. We have three different matrices - you first add AB and then add C, or you first add BC and then add A; the result is the same. This is written as aij plus bij happens to be the ij th element of A plus B; cij happens to be the ij th element of C; and this is the same as this because the number are associatives, you can add them in any order.

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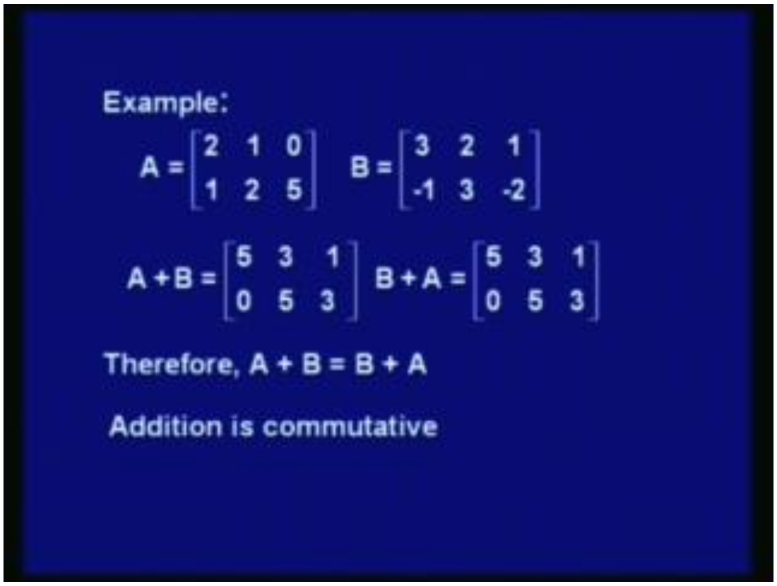

So the example: is you have A and B 2 by 3 matrix - B is also 2 by 3 matrix. They have to be of the same order; only then addition can be done. So given A and B - A plus B - is 5 3 and 1 in the first row, in the second row i have 0 5 and 3. If you add B plus A, then 3 plus 2 is also 5, 2 plus 1 is 3, 1 plus 0 is 1; minus 1 plus 1 is 0, 3 plus 2 is 5 and minus 2 plus 5 is 3. And One can note that A plus B is of 2 by 3, B plus A is of 2 by 3 and they are equal element by element, and that is why A plus B and B plus A are the same matrices; and this establishes that addition is commutative in this example. This example is shown to illustrate that the addition is associative

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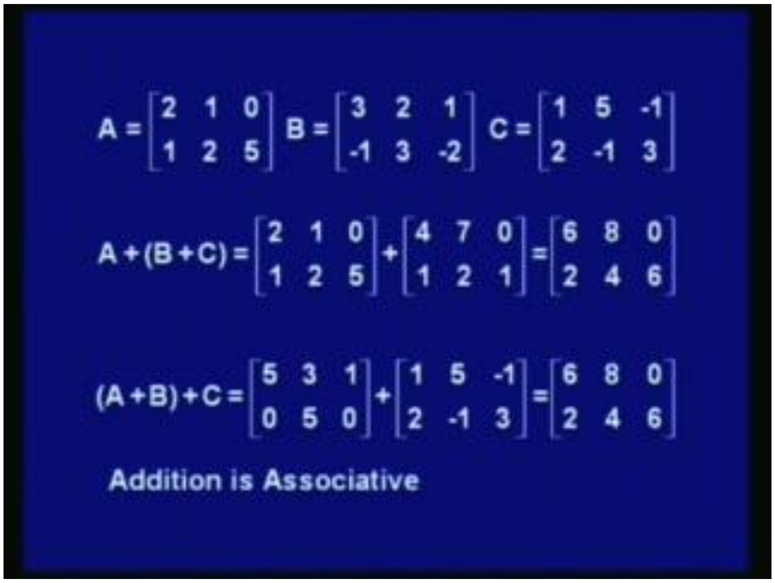

So I have taken A 2 by 3 matrix 2 1 0 1 2 5, B matrix as again 2 by 3 matrix consisting of 3 2 1 in the first row, minus 1, 3, minus 2 in the second row and C matrix as 1, 5, minus 1, 2, minus 1, 3. If I first add B plus C, then the result is 3 three plus 1 is 4, 2 plus 5 is 7, 1 plus minus 1 is 0 and then the second row is 1 2 1. is the sum matrix B and C In this, I add the matrix A. The result is this matrix. Then I add A and B first, and then add C to it. If i add A and B first, the result is 2 plus 3 is 5, 1 plus 2 is 3, 0 plus 1 is 1, 1 plus minus 1 is 0, 2 plus 3 is 5, 5 minus 2 is 0. In this, I add the C matrix and the result is this which is the same as the earlier matrix. So addition is associative. Further properties: matrix addition.

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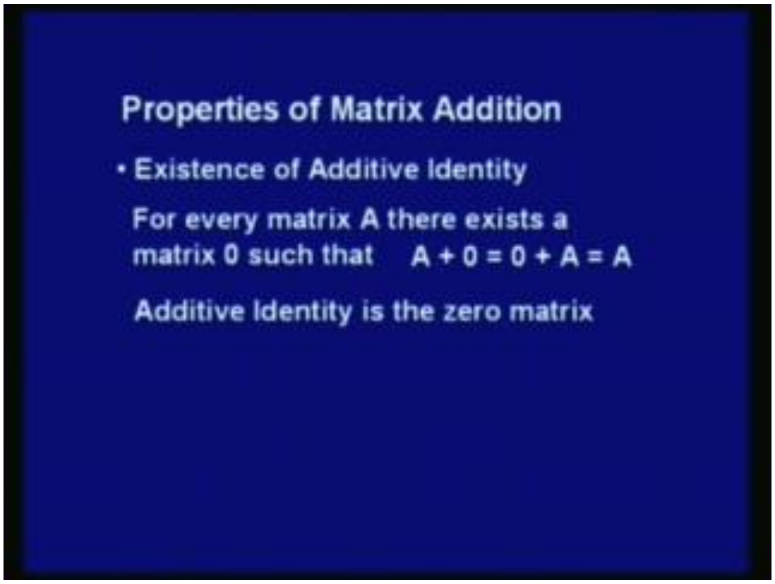

The existence of additive identity is an important property of matrix addition. By this, I mean to say that for every matrix A, there exists a matrix 0 or a null matrix such that A plus 0 is the same as 0 plus A, is the same as A. So whether you add 0 on the right side, or you add on the left side, the result is not affected at all and this matrix is called the additive identity, or the 0 matrix or we say additive identity is the 0 matrix. What is the order of this 0 - this null matrix? This must be taken so that this addition is confirmable. So this 0 matrix is of the same order as that of order of the matrix A.

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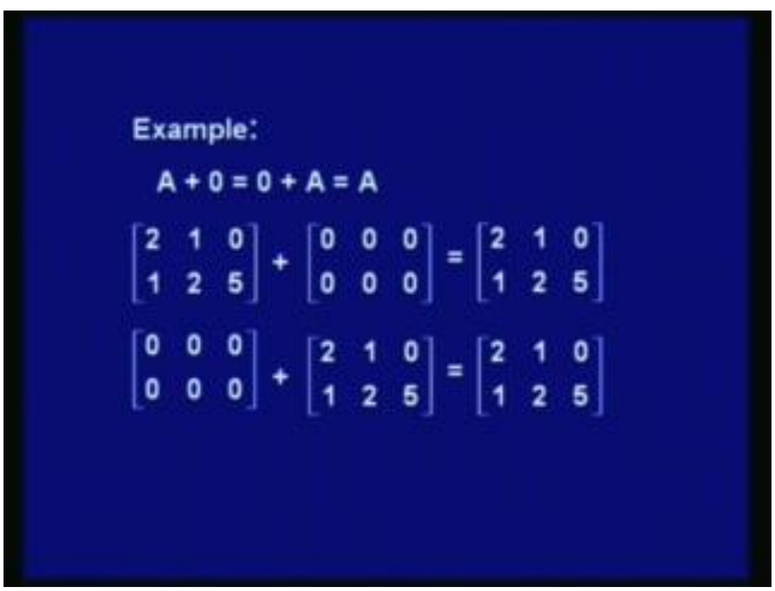

Like, if the matrix A is a 2 by 3 matrix, then 0 0 0 0 0 0 - a 2 by 3 null matrix - will give rise to 2 1 0 1 2 5, the same as the matrix A; and this can be done in any order, like 0 plus A is also A. So this is additive identity for this matrix A and this is true for all matrices. Further properties of addition!

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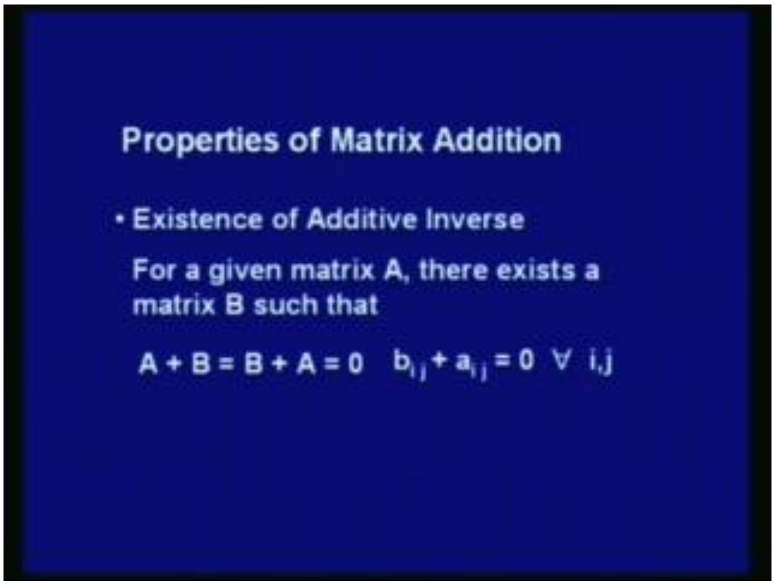

Another important property of matrix addition is that is the existence of additive inverse like for a given matrix A, there exists a matrix B such that A plus B is equal to B plus A equal null matrix; that means bij plus aij is 0 for all ai and j. For given matrix A, there must be some matrix B which when added in A gives rise to null matrix and that matrix B is the additive inverse of the matrix A.

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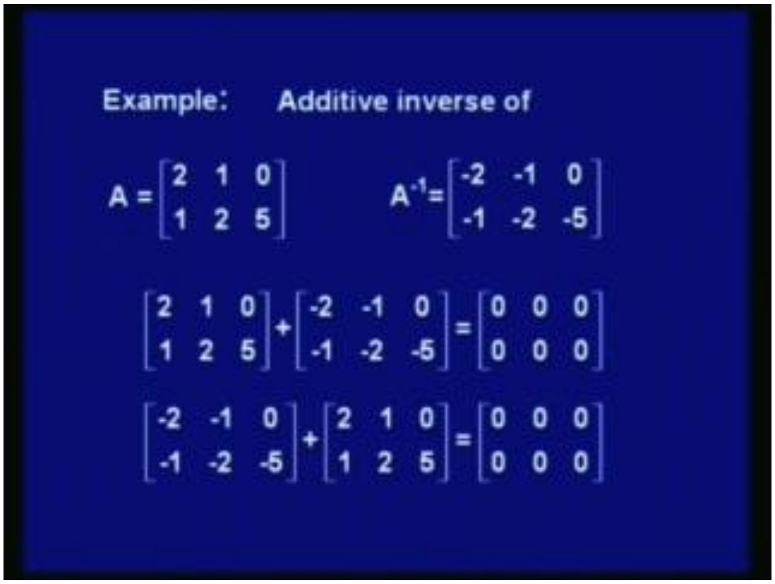

Example is if I have matrix A as 2 1 0 1 2 5 - a 2 by 3 matrix - then its additive inverse is minus 2 minus 1 0 minus 1 minus 2 minus 5 and this can be very easily verified as this. You can add 2 1 0 1 2 5 and minus 2 minus 1 0 minus 1 minus 2 minus 5 to give rise to the null matrix which is of order 2 by 3; and this can be done in any order. You first take this and then its inverse will get identity or you first take its inverse or then the matrix A will again get the identity matrix. Now we discuss another operation performed on matrices and that operation is multiplication by a scalar.

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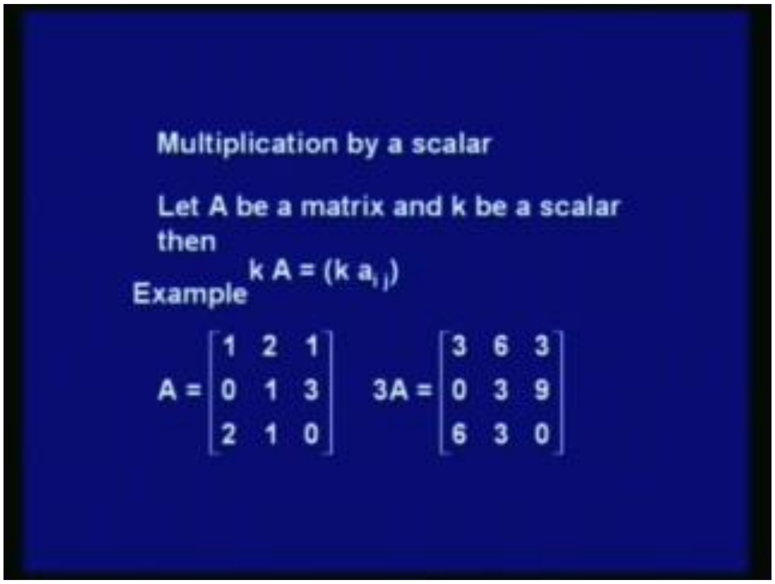

By this, I mean to say that if a matrix A is given and k is a scalar, then the matrix kA is obtained by multiplying each element of the matrix A each element aij of the matrix A by k. So this matrix k aij is the matrix obtained by multiplying by scalar k. Example: if i have a 3 by 3 matrix as this, then 3 times A is obtained by multiplying each element of this matrix A by 3. That is, 1 into 3 is 3, 2 into 3 is 6, 1 into 3 is 3; similarly 0 into 3 - 0 - 1 into 3 is 3, 3 into 3 is nine, 2 into 3 is 6, 1 into 3 is 3 and 0 into 3 is 0. So 3 A is this matrix obtained by multiplying A by the scalar 3.

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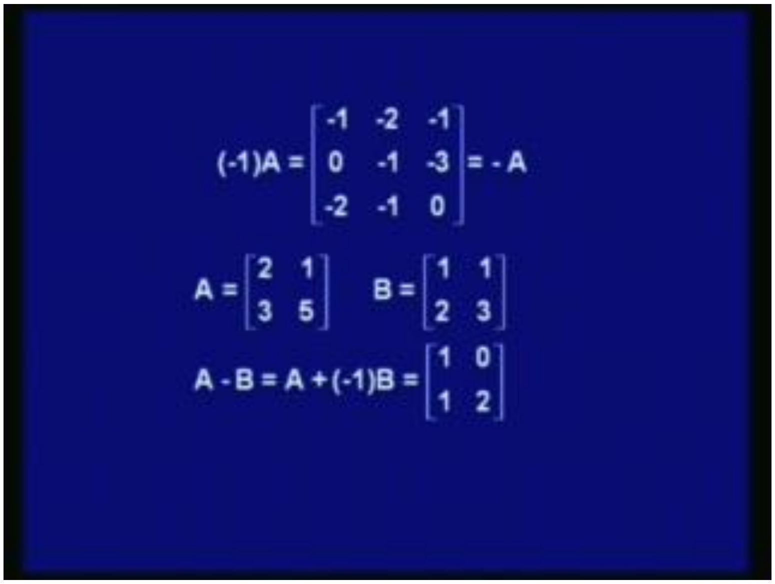

In another example, I multiply the matrix A by a scalar minus 1 and the result will be minus 1, minus 2, minus 1, 0, minus 1, minus 3, minus 2, minus 1, 0. I have taken the same matrix A as in the earlier example and this matrix is denoted by minus A. So minus A is a matrix which is obtained by multiplying A by scalar minus 1. So if i have A as 2 1 3 5 as a 2 by 2 matrix and B as another 2 by 2 matrix 1 1 2 3, then A minus B is the same as A plus minus 1 times B and the result is 1 0 1 and 2. So that is how we define subtraction of 2 matrices. So we have addition of 2 matrices and we have multiplication of 2 matrices.

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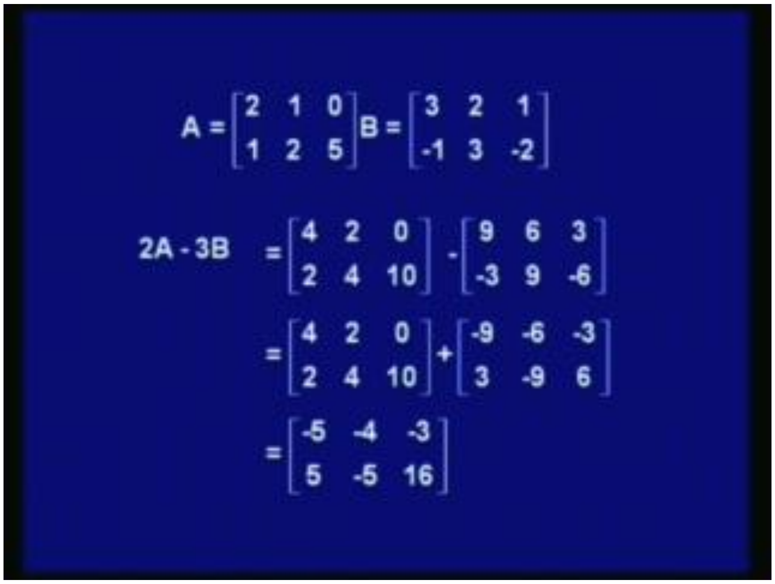

Now I take another example where I consider A matrix as 2 by 3 matrix, B matrix as this 2 by 3 matrix. Then 2 A minus 3 B: 2 A is the matrix obtained by multiplying every element of the matrix A by 2- that is 2 A is 4 2 0 2 4 and 10 - minus 3 B - means each element of B matrix is multiplied by 3, so it is 9 6 3 minus 3 nine and minus 6; and when we want to subtract them, then first element remains the same - the first matrix remains the same - but the second matrix is multiplied by minus 1. So we have first matrix plus - instead of this minus, now I have plus - but this matrix is modified by multiplying each and every element of this by minus 1.

So 9 becomes minus 9; here we have minus 6, minus 3, three, minus nine and 6. Now these 2 add matrices are of same order; they can be added to give rise to the result as minus 5 , minus 4, minus 3, 2 plus 3 5, 4 minus nine as minus 5, ten plus 6 as sixteen. So we can perform operations of this type on matrices. Also so we have started with simple addition, then I have introduced scalar multiplication and then I can perform operations of this type on matrices. So this are compact form of operations which can be easily performed on matrices. Next is multiplication of 2 matrices.

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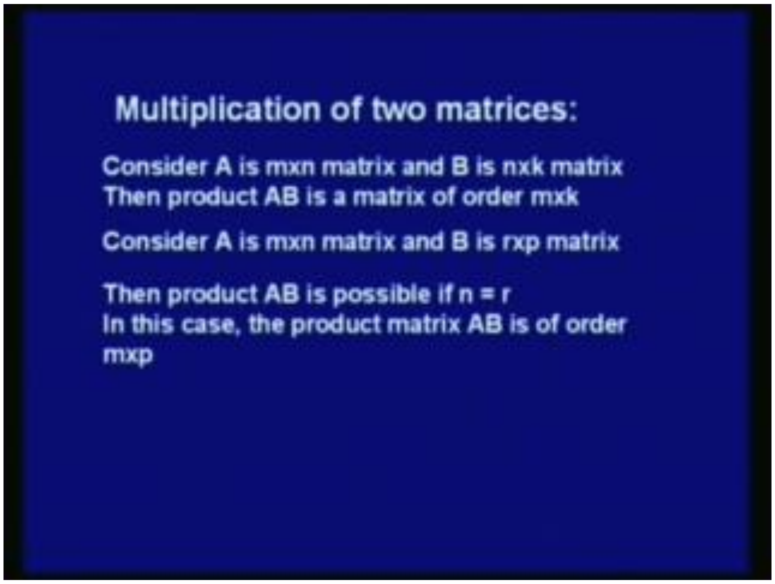

Consider a matrix A as m by n matrix and the matrix B as n by k matrix. Then the product of 2 matrices A and B is denoted by AB, is a matrix which is of order m by k. m by k - m the row of the first matrix, k the column of second matrix. Like Consider A is m by n matrix and B is r by p matrix; then product AB is possible if n is equal to r - this column size of A matrix must be the same as row size of B matrix. So this product is possible only when n is equal to r, and if this is the case, then product matrix AB is of order m by p.

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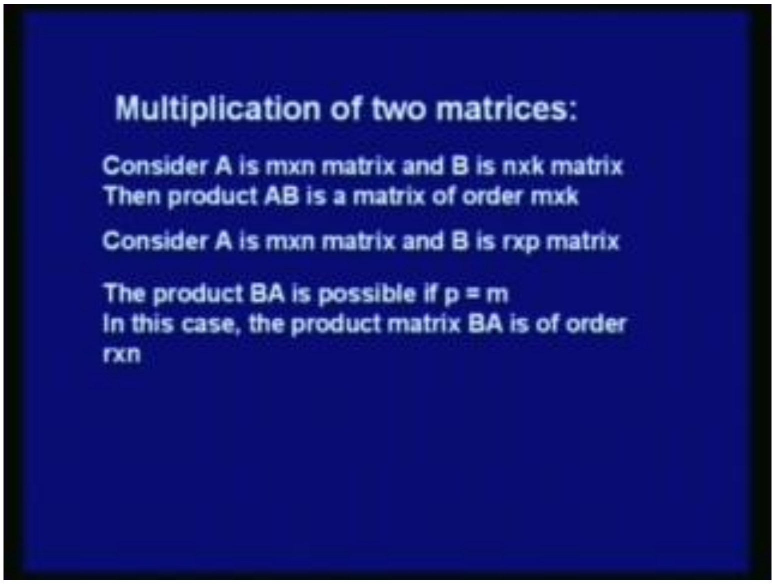

The product BA is possible - in this, the product BA is possible - if p is equal to m. In this case, the product matrix BA is of order r by n.

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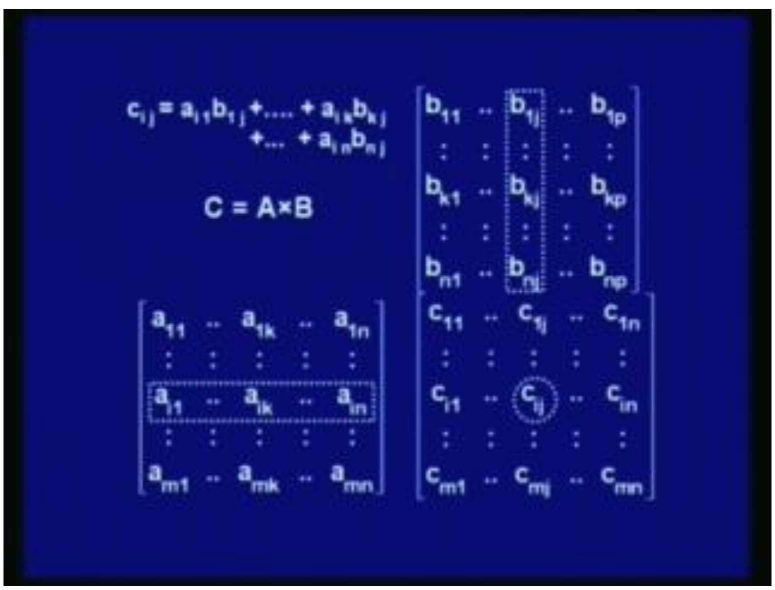

Now if I have matrix A as this matrix consisting of m rows and n columns and the matrix B as this matrix consisting of n rows and p columns, then they can be multiplied together because they have same number of columns. The product matrix C is actually obtained by multiplying the row of the first matrix - i th row of the first matrix - by the j th column of second matrix; and this multiplication is element wise -ai 1 multiplied by b 1 j, ai 2 multiplied by b 2 j, ajk multiplied by bkj, ain multiplied bnj. So we form this sum - this is the ij th element of the matrix C - and this way we say that matrix multiplication is row into column. So I have organized the calculation in this manner; this is i th row and j th column and they will give rise to ij th element of the product matrix C. Typically the cij element is obtained as the sum of this product.

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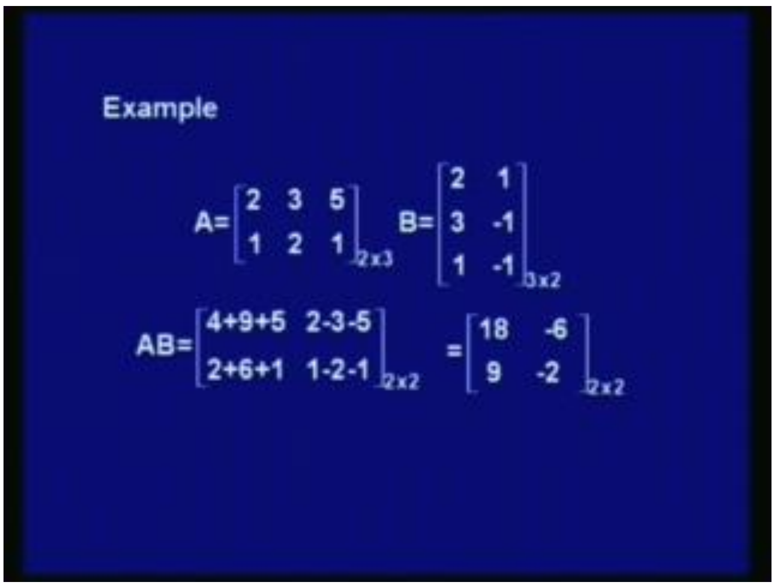

For example, if I have a 2 by 3 matrix A as 2 3 5 1 2 1 and the B matrix as 3 by 2 matrix consisting of 3 rows and 2 columns, then the product matrix AB will be of order 2 by 2. The number of columns in A is the same as number of rows in B; they have to be same if multiplication is to be possible. The first element here will be obtained by multiplying first row of A matrix with the first column of B matrix. So we have 2 plus 2 is 4, 3 plus 3 is nine, 5 plus 1 is 5 and the next element in the is the first row- second column element; so first row of A is to be multiplied by the second column of B term by term. So we have 2 multiplied by 1, 3 multiplied by minus 1, 5 multiplied by minus 1 and that gives me the element 2 minus 3 minus 5. The next element is this element which is the second row - first column element. So second row of A is to be multiplied with first column of B and the result will be 2 6 and 1.And the last element is second row - second column; so 1 multiplied by 1, 2 multiplied by minus 1, one multiplied by minus 1 gives me the result as this. So product AB is possible and the result is this 2 by 2 matrix.

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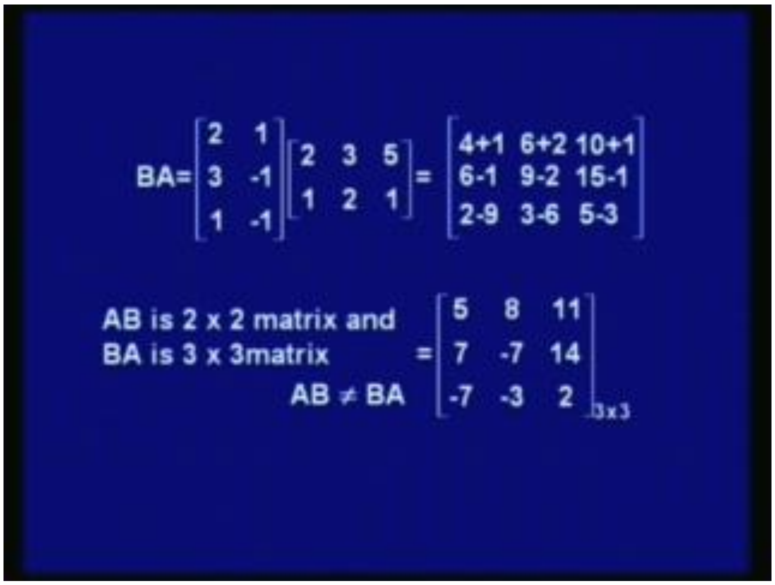

In this case, one can also perform the product BA. Why? Because B is of order 3 by 2, A is of order 2 by 3; so this 2 is the same as this 2. So this product is possible and this can be obtained as the matrix consisting of first row, first column - 4 plus 1. The next element will be this row and this column and this is 2 into 3 plus 1 into 2. this element The next element here will be obtained by multiplying this first row by third column and this element comes out to be this. Similarly, the second row multiplied by first column, second row multiplied by the second column, second row multiplied by the third column and third row is obtained by multiplying third row of this matrix by first column, third row by second column, third row by third column and we have a 3 by 3 matrix. That is clear from the order of B and A matrices. So BA is a 3 by 3 matrix. So my AB is 2 by 2 matrix; BA is of order 3 by 3 matrix. and Since the order of the AB and BA are not the same, so there is no question that AB is equal to BA; they cannot be equated. So although A and B are possible, B and A product BA is also possible; but they are not same. So AB is not the same as BA and this and this means that while addition of matrices is commutative, multiplication of matrices is not commutative.

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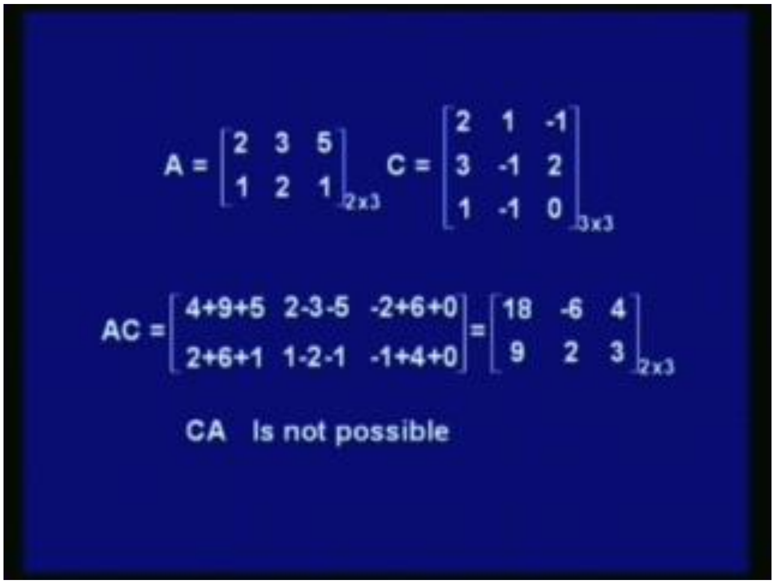

In another example, if I consider A as 2 by 3 matrix and C as 3 by 3 matrix, then the product AC will be a 3 by 2 by 3 matrix. the 2 by 3 matrix obtained as this But if you consider CA - CA is C is 3 by 3 and A is 2 by 3 - so this 3 is not the same as this 2; so CA is simply not possible. So there is in this example although AC is possible, but CA is not possible. In the earlier example, AB and BA both are possible but they are not equal. In this case, AC is possible but CA is not possible.

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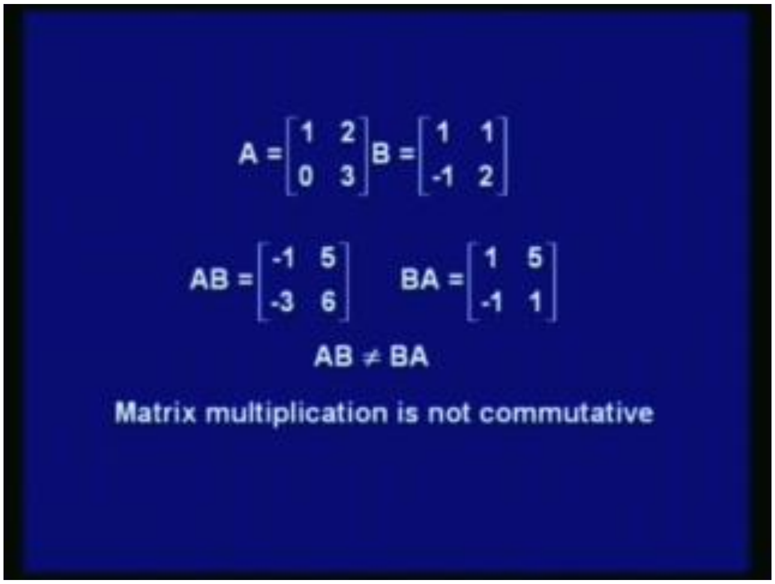

In another example, I consider the 2 by 2 matrix A and B also as 2 by 2 matrix. I multiply A and B to get this product AB as minus 1 5 minus 3 and 6, and the product BA is also a 2 by 2 matrix and it is 1 5 minus 1 and 1.and 1 can notice that although the 2 matrices are same, but their elements are different and we say that AB is not the same as BA; and this establishes that matrix multiplication is not commutative in this example. Now we discuss properties of matrix multiplication - to start with the associative property.

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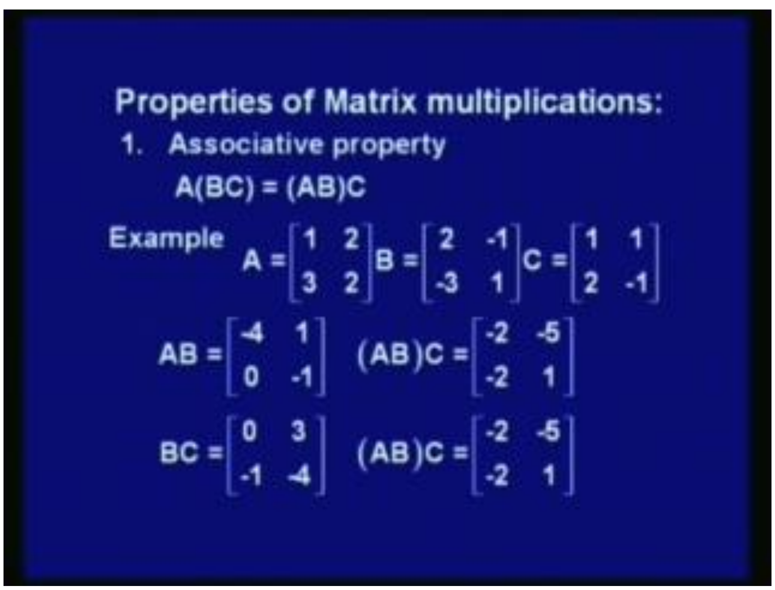

If we are given 3 matrices - A B and C - then they can be multiplied in any order. You can first multiply B and C and then multiply it by A, or you first multiplied A and B and then multiply it by C; the result will be the same. and I illustrate this with the example: if A is this matrix - A is 2 by 2 matrix - B is also a 2 by 2 matrix, C is this 2 by 2 matrix. Then AB will be 1 into 2 and 2 minus 3 is minus 4, 1 2 is multiplied by minus 1 1 to give me 1, this row is multiplied by this column to give me this and this row multiplied by this will give me minus 1. So this AB when multiplied by this C will give me : this row multiplied by the this column as minus 2, this row multiplied by this column as minus 5, this row multiplied by this column as minus 2, second row multiplied by second column of C gives me 1.

Now i compute BC also - this matrix BC. BC is this multiplied by this; so this multiplied by this gives me 0, this row multiplied by this column gives me 3. This second row multiplied by first column give me this element and this row multiplied by this column will give me this element. When this is this matrix BC is multiplied by the matrix A, I will get the product ABC which is the same as this AB into C. So whatever be the order, I will finally get the same result. So if there are 3 matrices, they can be applied in any order; they can be multiplied in any order and we will get the same result. Of course the important point is the multiplication should be possible in these 3 matrices. Next property of matrix multiplication is distributive property.

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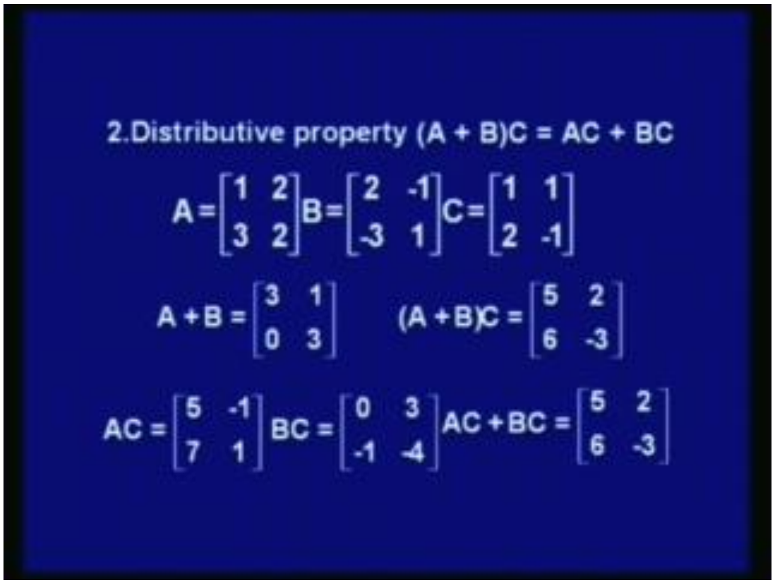

By this, I mean to say I have 3 matrices – A, B and C. A, B are added together and then multiplied by C. These is same as you first form the product AC, and then form the product BC, and then add together AC plus BC and the result is the same. So we say addition is distributive multiplication is distributive over addition. I can illustrate this with this example. I have three 2 by 2 matrices; I first formulate A, I first form A plus B as this 2 by 2 matrix, then I multiply it by C; I get the result as this. While when A and C are multiplied together, the result is the matrix 5 minus 1 7 1, when B and C are multiplied, I get the matrix 0 3 minus 1 and minus 4. When I add these 2 matrices, I get this and one can observe easily that this matrix is the same as this matrix, and this means distributive properties are satisfied for this set of 3 matrices.

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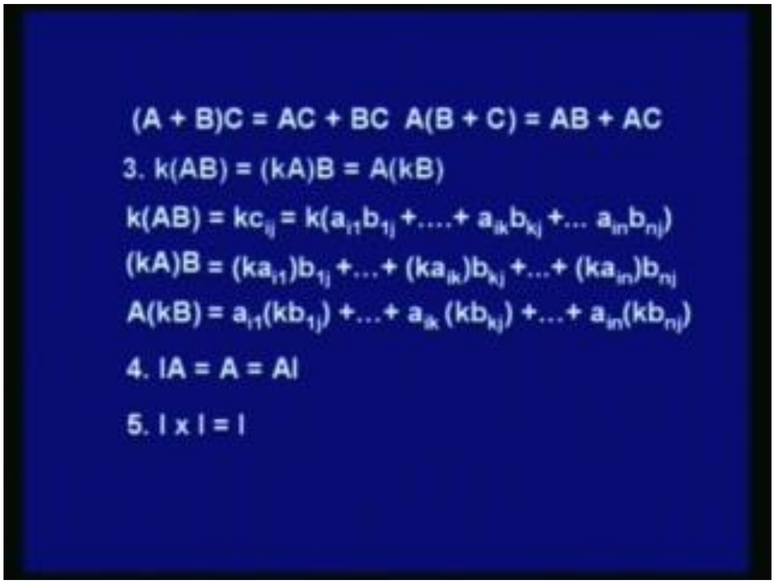

Now, I have A plus B multiplied by C - I get AC plus BC. I am multiplying I am first adding A and B, and multiplying it on the other side; this is important because matrix multiplication is not commutative. So C is post multiplied - will have AC plus BC; here in this case, A is pre-multiplied - I add B and C and multiply it by A and here we have AB plus AC. That means left or right distributive properties - both -are applicable in the case of matrix multiplication. Now this matrix multiplication - when this product is multiplied by a scalar k - then k AB is same as k times A when multiplied by B or A multiplied by kB. So this can be proved here.

What is kAB? AB is the product matrix and cij is typical element of A. So cij, we have proved that this is this elementl; ai 1 b 1 j plus ai 2 b 2 j, ij th row and column are multiplied, that will give me cij. When we multiply it by k, then the typical element of kAB, will be this element and this can be rewritten by ki 1 multiplied by b 1 j and then kai 2 multiplied by b 2 j, kaik multiplied by bkj plus this. And This is the same as if you write this terms and in this form. This is the same; so this establishes this property. Then the next property says that identity matrix multiplied by A will give me the matrix A itself, and whether A is multiplied by identity matrix or identity matrix is multiplied by A, the result is a matrix A itself; but this product has to be possible. So if A is a square matrix of order n, then i has to be square matrix of order n. So this i is a square matrix of order n, A is square matrix of order n, then ia is equal to A and this is equal to ai. Further if i multiply 2 identity matrices, then result will be an identity matrix.

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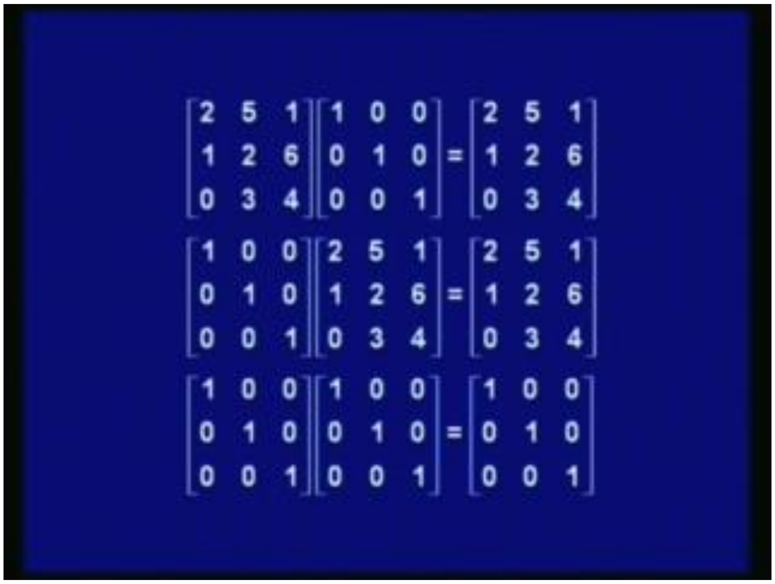

So, if I take the matrix A as this 3 by 3 matrix multiplied by an i 3 matrix - identity matrix of order 3 - then the result will be the same. This can be established very easily because when I multiply this row by this column, only this element will be there; rest of the elements will be gone so we will have only two in the result. When i multiply this by this, then only this will be there; rest will not contribute, so I have only 5. When i multiplied this row by third column, then only this element will contribute; rest of the elements will not contribute and the final result remains the elements 1 itself.

The same thing happens with the second row multiplied by this; so this will not contribute, only this will contribute and that’s what I am having here. When this is multiplied by this, then I will get 2, when this is multiplied by this I will be getting 6; so it is 1 2 6. Similarly this row multiplied by first column will be give me 0; this row multiplied by second column will give me 3; this row multiplied by this will give me 4. So ia is the same as A; same thing can be done a times i; I still get the same matrix A. So

2 into 1, 5 into 0, 1 into 0 will give me 2 and same thing can be done for each and every element, and one can establish that ai is the same as a. In the last row, I have multiplied 2 identity matrices; they have to be of the same order. So when they are multiplied, the final result is also an identity matrix.

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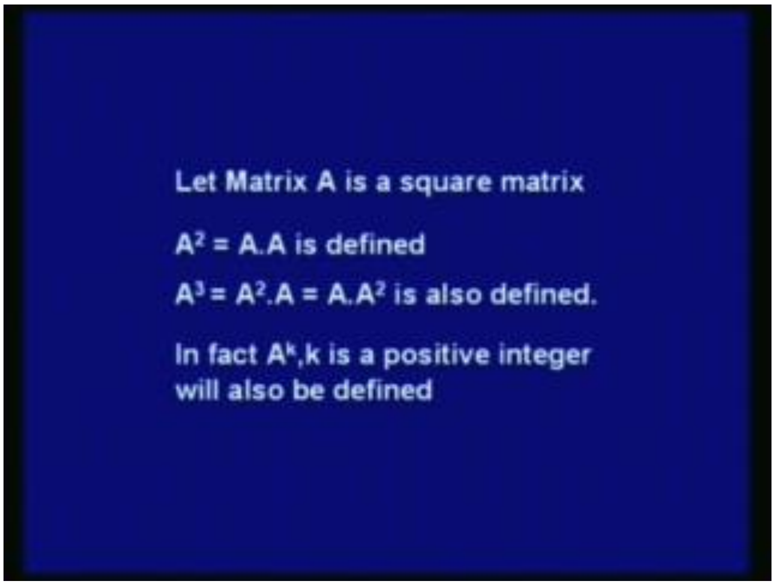

Now if i have a square matrix A, then A square is defined; A cube is also defined as A square A square into A and it is also same as A multiplied by A square. They both are defined and in general Ak, the k th power of A - k happens to be any positive integer will also be defined. But this is true only for square matrices; for other matrices this is not possible.We have finally defined the product Ak times as Ak matrix, which is a square matrix, multiplied k times. So viewers, today we have started our discussion with the basic concepts on matrices. Starting with the definition, I have introduced types of matrices. I have then defined addition of matrices, then its properties, then matrix multiplication by a scalar and finally matrix multiplication and then we have seen properties of matrix multiplication. In my next lecture, we will discuss more about matrices. Thank you.