Engineering Physics 1

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Module-02
Lecture-02

Stationary Waves, Reflection, Refraction & Diffraction

This is the second lecture of the four lecture series on Acoustics. In the last lecture which was the first one, we considered sound generation and propagation.

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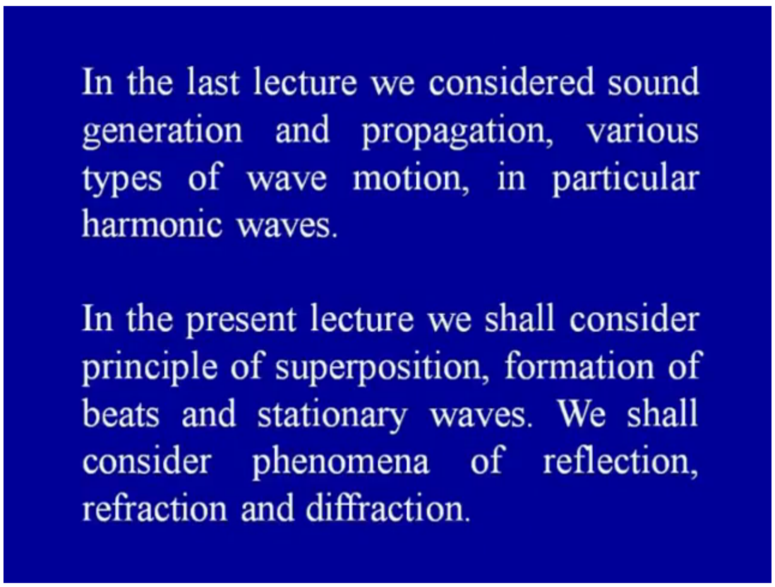

Various types of wave motion, in particular, harmonic waves. It was pointed out that the elasticity of the medium is a very important aspect. Sound waves are mechanical waves; the particles of the medium vibrate. And therefore elastic properties determine how the wave will propagate in the medium. We also considered an equation, a very general equation for wave motion for propagation along the x axis.

This equation was a partial differential equation in x and t. We considered if the general solution which were in the form of the solution of the combination x - vt or x + vt giving rise to waves moving along the positive direction of x axis or the negative direction of x axis. No other form of the solution is admissible for this equation. Then, we considered waves of different types the transverse waves, where the particle motion is perpendicular to the direction of propagation.

It is in the transverse plane can have any direction but always remaining in the transverse plane. A tyPical example was pointed out the vibrations of a stretched string, a Taurus spring for example. And the longitudinal waves, we the particle motion is along the direction of propagation. This is a very common thing because the sound waves in air are longitudinal.

Then, we considered, in particular, harmonic waves because the particles would vibrate the most of the time they vibrate simple harmonically, because the force restoring force due to elasticity comes out to be proportional to the displacement. We considered various properties of the harmonic waves their main characteristics. Now, in the present lectures this one, we shall consider principle of superposition, formation of beads and stationary waves.

We shall consider phenomena of reflection, refraction and diffraction of sound, principle of superposition.

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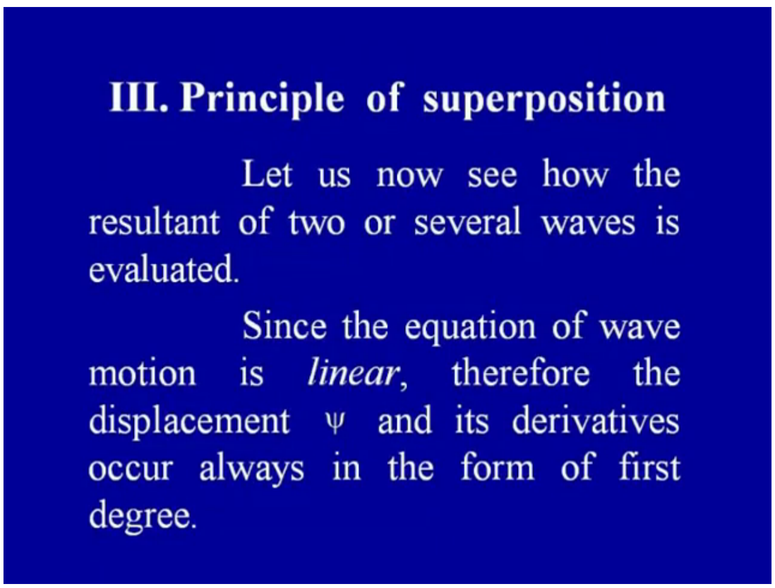

Let us now see, how the resultant of two or several waves is evaluated. See the equation of a motion is linear and is homogeneous; therefore the displacement side and its derivatives occur always in the form of first degree.

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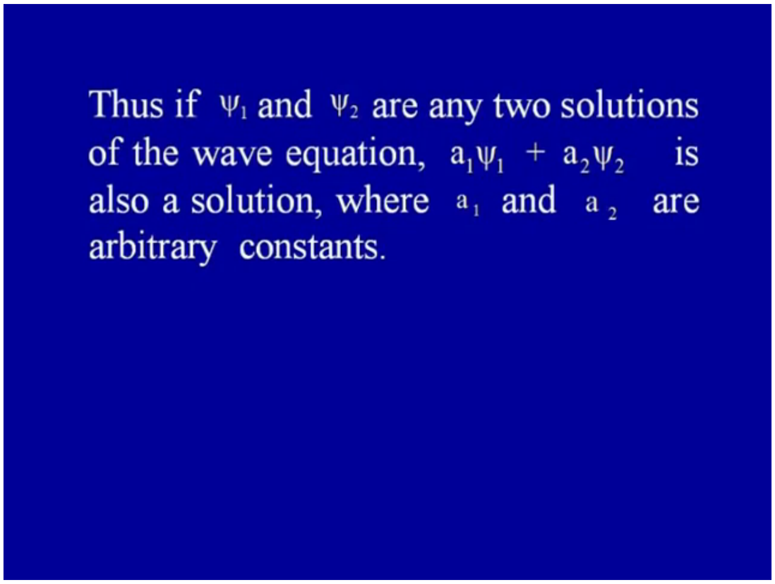

Thus if Psi 1 and Psi 2 are any two solutions of the wave equation, any combination like a1 Psi 1 + a2 Psi 2 is also a solution where a1 and a2 are arbitrary constants.

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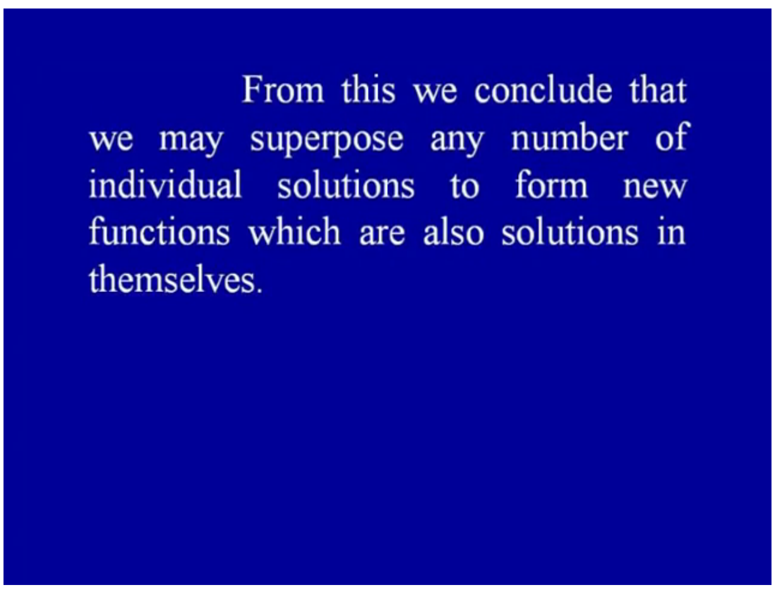

From this, we conclude that we may superpose any number of individual solutions to form new functions which are also solutions in themselves.

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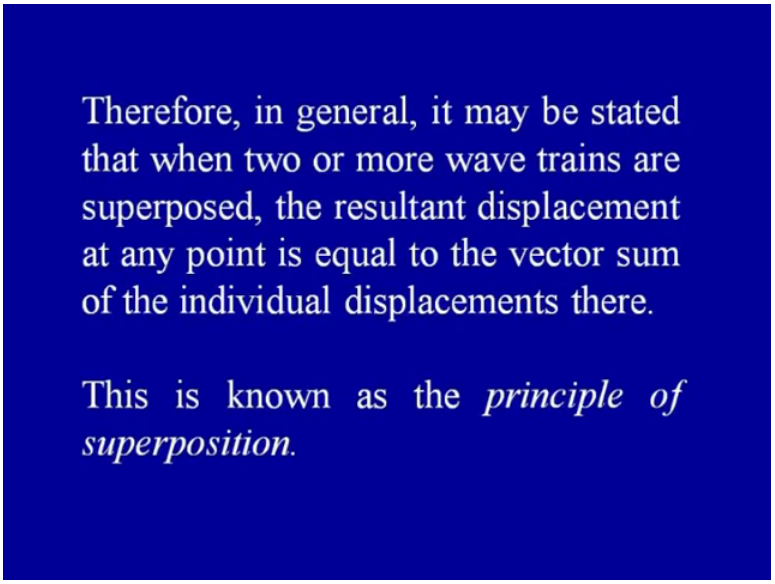

Therefore, in general, it may be stated that when two or more wave, wave trains are superposed the resultant displacement at any point is = the vector sum of the individual displacements there. This is known as the principle of superposition.

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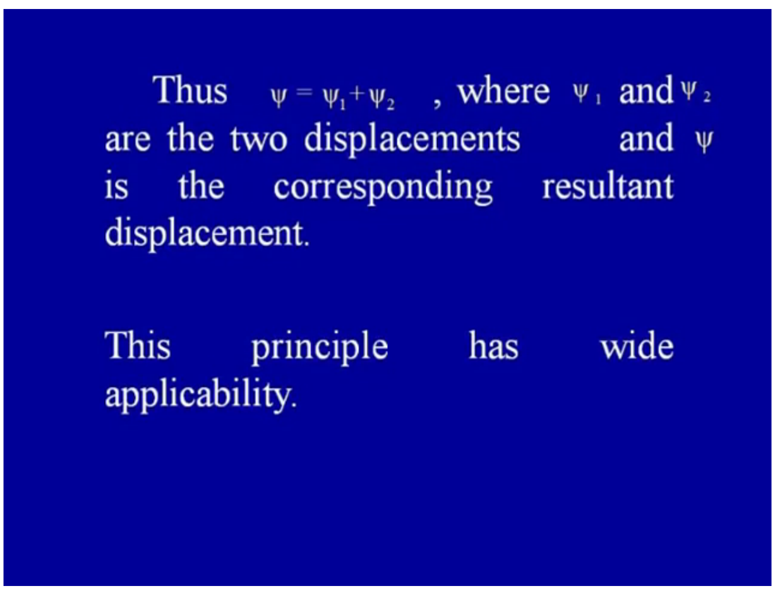

Thus Psi will be = Psi 1 + Psi 2. If Psi 1 and Psi 2 are two displacements and Psi is the corresponding resultant displacement. The displacement has wide applicability. Let us consider superposition of two harmonic waves moving in the same direction. Let Psi 1 is = a cos Omega 1 t - k1x + alpha 1.

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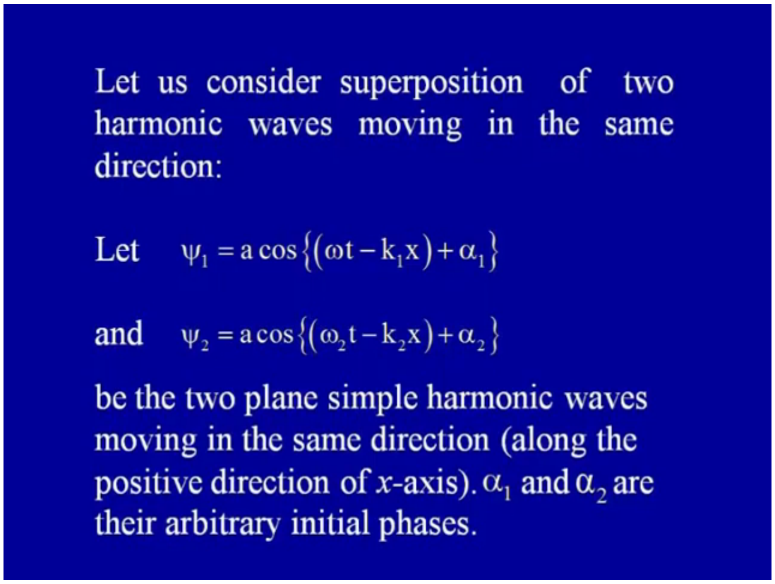

For the first wave traveling along the positive direction of x axis and the other one Psi 2 = a times cos of Omega 2t - k2x + alpha 2. This is the second one. So, these two are there; alpha 1 and alpha 2 are arbitrary initial phases.

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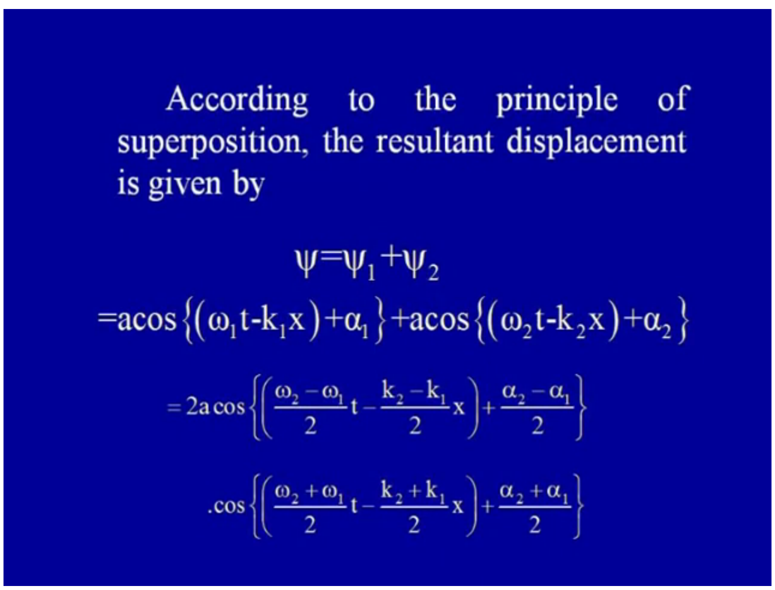

Now, according to the principle position the resultant displacement is given by Psi is = Psi 1 + Psi 2, just vector sum. This gives a times cos of Omega 1t - k1 x + alpha 1 + a times cos of Omega 2 t - k2 x + x Phi 2. This gives 2a times cos of Omega 2 - Omega 1 by 2 times t - k2 - k1 by 2 times x + alpha 2 - alpha 1 by 2 multiplied by cos of Omega 2 + Omega 1 by 2 times t - k2 + k1 by 2 times x + alpha 2 + alpha 1 by 2.

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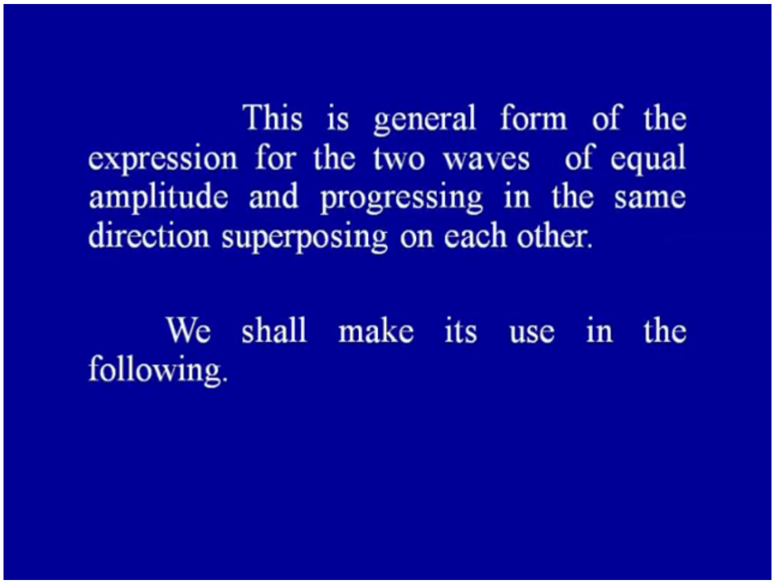

This is general form of the expression for the two waves of equal amplitude, both having amplitude a, progressing in the same direction. Both were progressing along the direction of positive x axis superposing on each other. We shall make use in the following.

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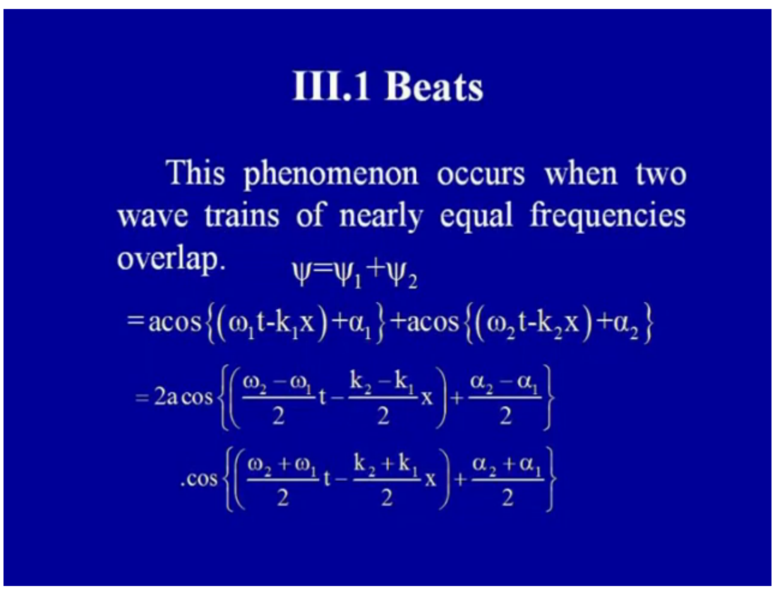

Let us consider the phenomena of beats. This phenomena occurs, when two wavelengths of nearly equal frequencies Omega 1 is almost = Omega 2, k1 is almost = k2, when such waves, they overlap. Again using principle of superposition Psi is = Psi 1 + Psi 2.This is the same expression as I had a little while ago.

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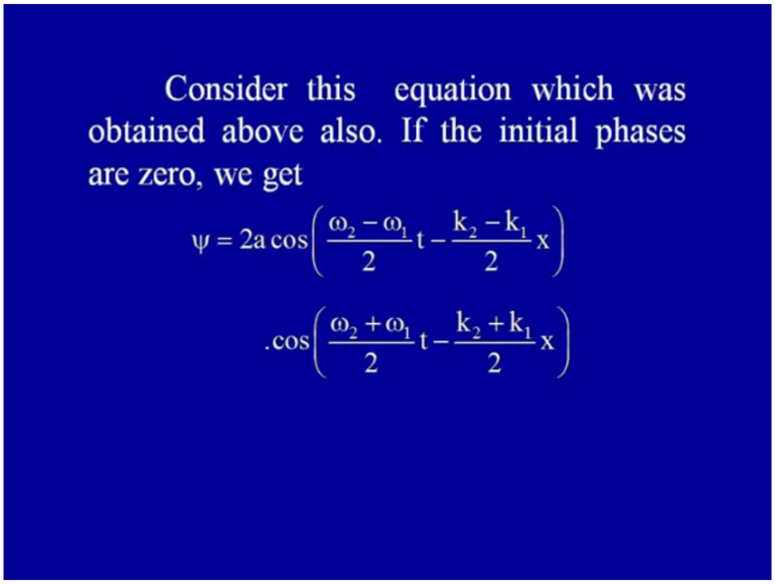

But now, consider this equation, with initial phases are set to zero just for simplicity. If you put alpha 1 and alpha 2 = 0, we get for Psi which is = Psi 1 + Psi 2 2a times cos Omega 2 - Omega 1 by 2 times t - k2 - k1 by 2 times x multiplied by cos of Omega t + Omega 1 by 2 times t - k2 + k1 by 2 times x.

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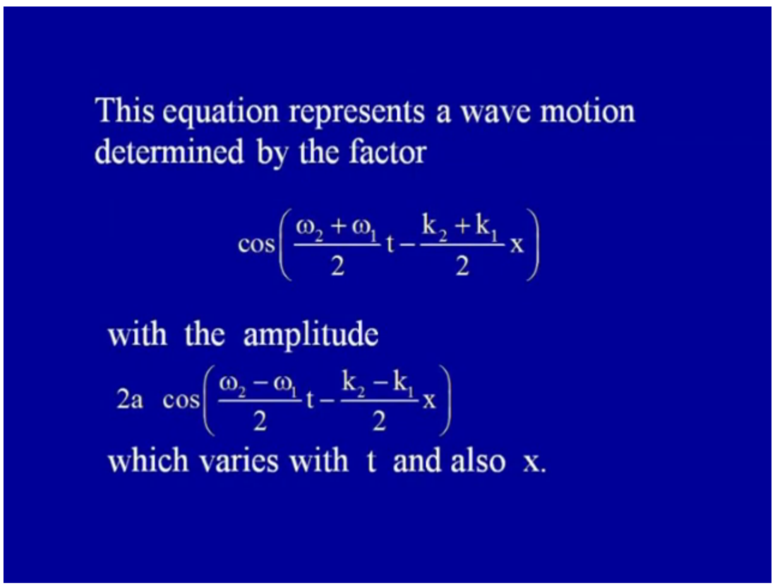

This equation represents a wave motion determined by the factor cos of Omega 2 + Omega 1 by 2 times t - k2 + k1 by 2 times x. Remember, the frequencies are nearly equal. Omega 1 is almost = Omega 2; k1 is almost = k2. Essentially, this factor is cos of Omega t - kx with the amplitude. Usually, the amplitude discussed but here in this case the amplitude is 2 a times cos of Omega 2 -

Omega 1 by 2 times t - k2 - k1 by 2 times x. If we put Omega 2 = Omega 1 and we put k2 = k1, this will be cos of 0 which is 1.

And the amplitude will be just = a constant value 2a. But here, we find this is the expression which varies with t and also x.

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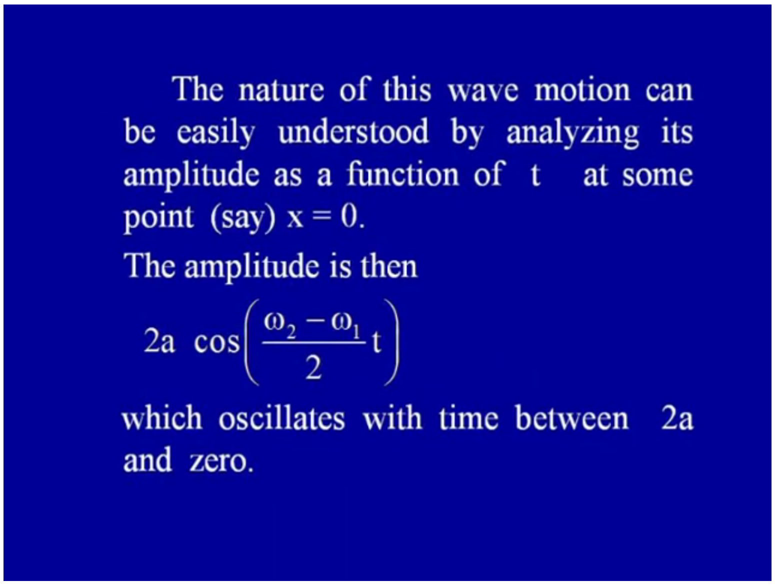

The nature of this wave motion can be easily understood by analyzing the amplitude term, as a function of t as a function of time, at some fixed point. Let us take x is = 0 just for simplicity. It does not affect any physics; the amplitude is then 2a times cos of Omega 2 - Omega 1 by 2 times t. This oscillates with time between the maximum value 2a and the minimum value 0.

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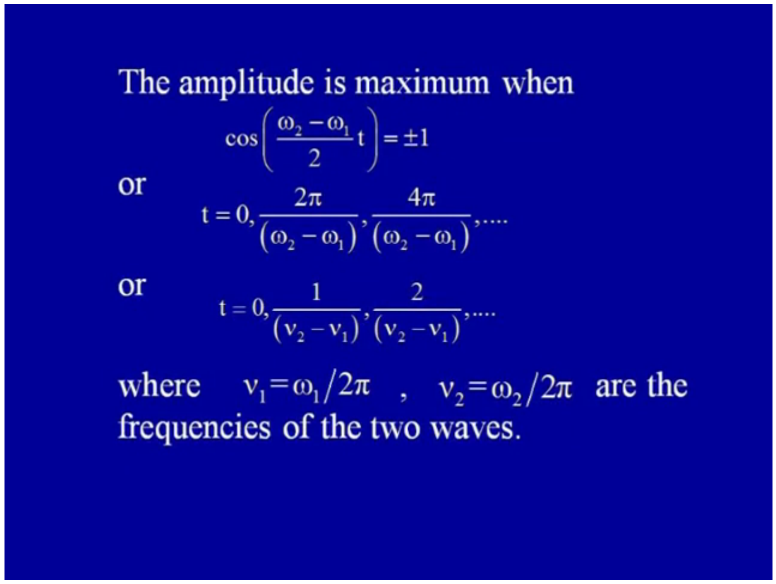

The amplitude is maximum when the cos factor is = + - 1 this means at time t = 0 at 2 Pi upon Omega 2 - Omega 1 at 4 PI upon Omega 2 - Omega 1 or 6 PI upon Omega 2 - Omega 1 or in terms of frequency at time t = 0, 1 upon Mu to - Mu 1 2 upon Mu 2 - Mu 1, 3 upon Mu 2 - Mu 1 where Mu one is Omega 1 by 2 Pi Mu 2 is Omega 2 by 2 Pi.

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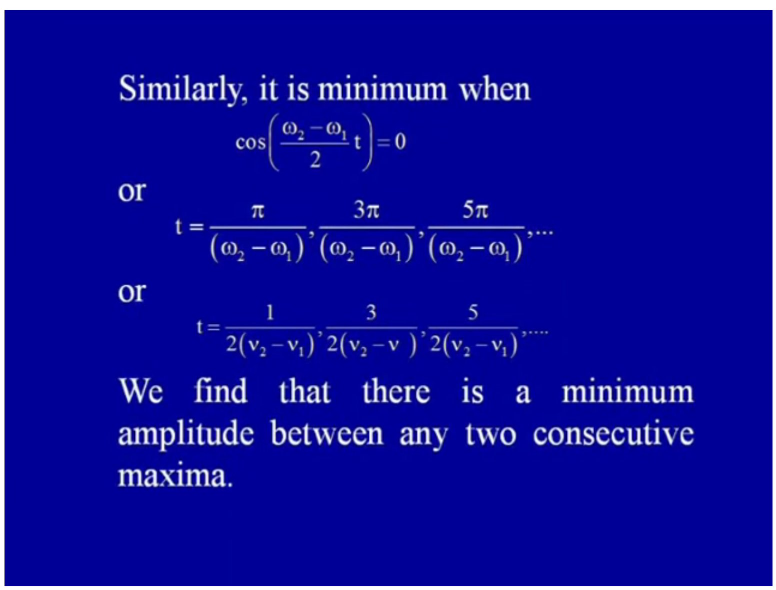

Similarly, the amplitude is minimum when the cos vector is 0; this means the argument is like PI by 2 or 3 Pi by 2 or Pi Pi by 2 that is at t = Pi upon Omega 2 - Omega 1 or 3Pi upon Omega 2 - Omega 1 or 5Pi upon Omega 2 - Omega 1, like this; Or in terms of frequency at times t = 1 upon twice of Omega 2 - Omega 1at 3 upon twice 2 Omega 2 - mega 1 or 5 upon twice the omega 2 -

Omega 1, these times. We find that there is minimum amplitude between any two consecutive maximum.

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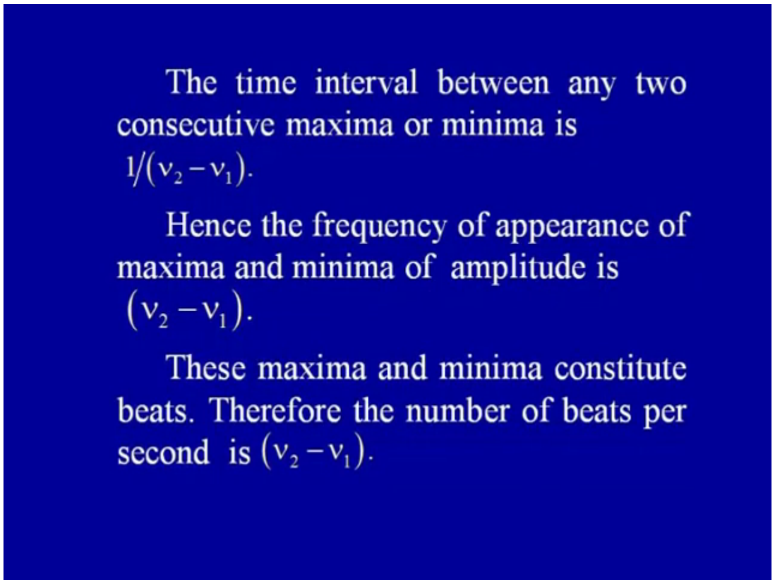

You see the time interval between any two consecutive Maxima or minima 1 upon Omega 2 - Omega 1 with an interval of this Maxima repeats or minima repeats. Hence the frequency of appearance of Maxima or minima of this amplitude is Mu 2 - Mu 1. These phenomena, maxima minima constitute Beats. And therefore, we say, that the number of beats per second is Mu 2 - Mu 1 just a difference of the two frequencies.

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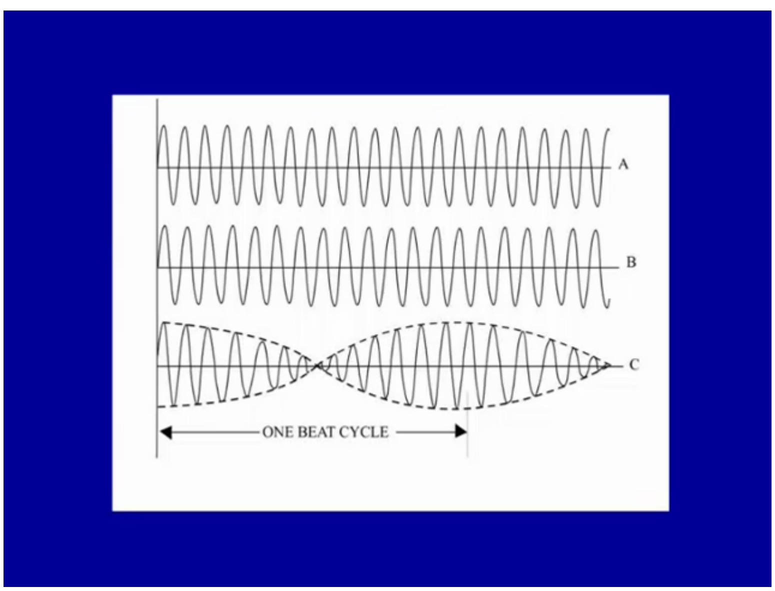

Consider this figure. This figure is an envelope of this transient amplitude modulation, resulting from the superposition of the waves A and B of slightly different frequencies.

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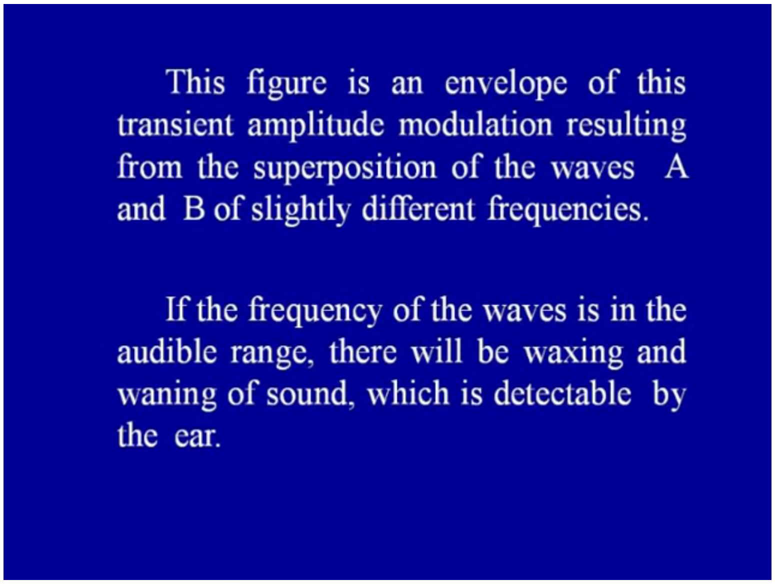

The frequency of the waves is in the audible range. One can hear it. There will be waxing and waning of sound which is detectable by the ear. So this is a very interesting phenomena whenever we have two sources of slightly different frequencies and they are sounded together. For example, if the two tuning forks, say a frequency of 500 vibrations per second and 502 vibrations per second, a difference of 2 if they are sounded together, we expect two beats per second.

This is means in a second, the sound will be maximum at two instants and minimum at two instants. It is a very interesting and easy and can project a simple experiment, can be done in any lab. Let us consider the stationary waves.

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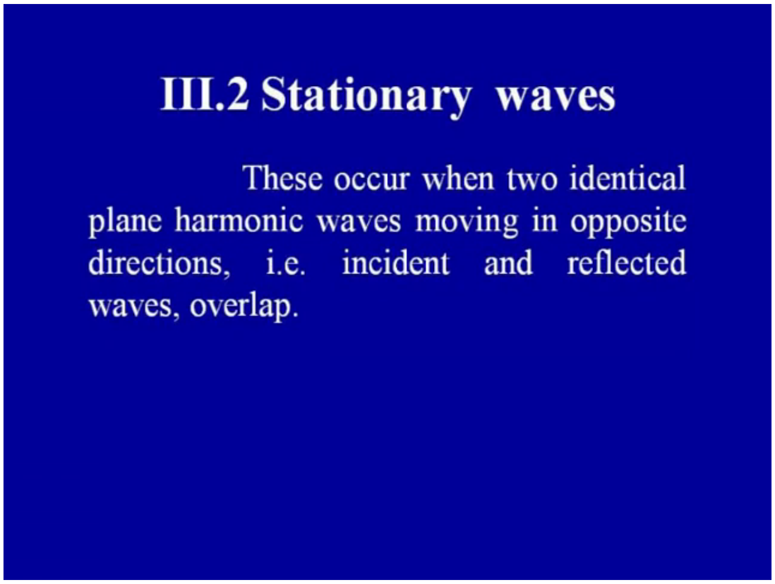

These occur when two identical plane harmonic waves moving in opposite direction. Remember, earlier we considered superposition of two waves in the same direction. Now we are considering two plain harmonic waves moving in opposite direction. Incident and reflected waves and they now they overlap.

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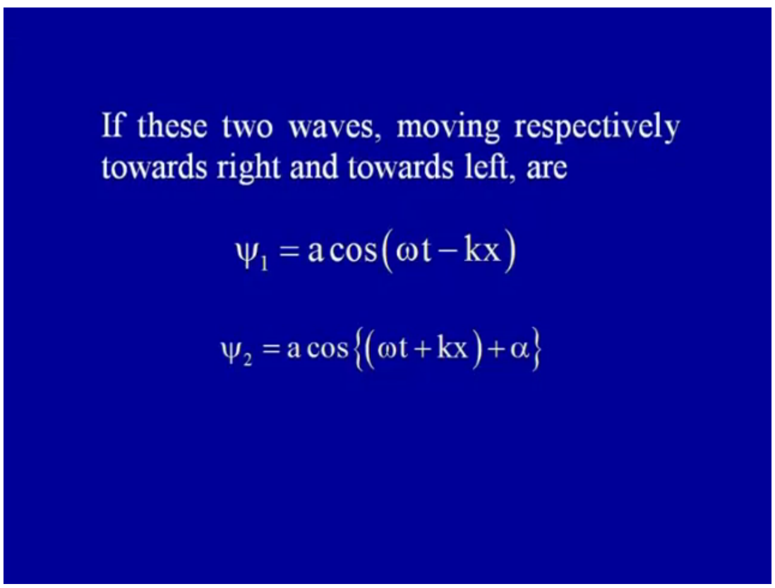

These two waves moving respectively towards right and left are Psi 1 = a cos Omega t - kx moving towards right. That is positive direction of x axis and the other one Psi 2 a cos of Omega t + kx + alpha moving towards the other direction, negative direction of the x axis,

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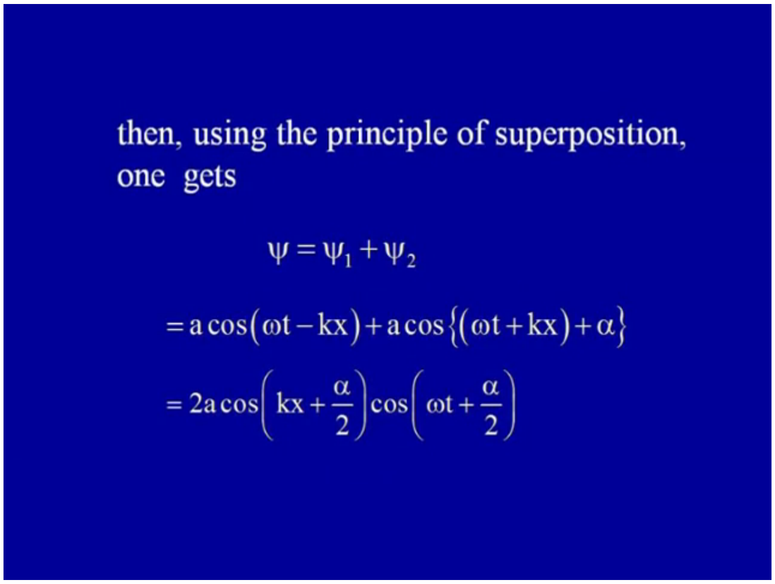

Then, using the principle of superposition as before the total displacement, Psi is given by, Psi 1 + Psi 2, remember, these displacements by themselves can be along the direction of x axis if these waves are longitudinal or they can be in the transverse plane, if these waves are transverse, alright. Psi is = Psi 1 + Psi 2 it means a cos of Omega t - kx + a cos Omega t kx + + kx + alpha.

It means to a cos of kx + alpha by 2 multiplied by cos of Omega t + alpha by 2. This equation corresponds to what is called a stationary wave.

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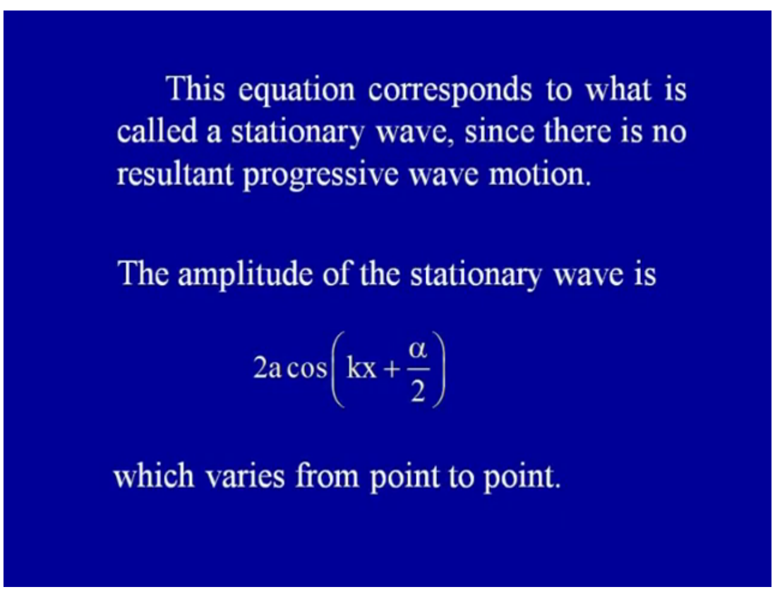

Since there is no resultant progressive motion, there is no energy transfer to the right or left. That is why these waves are called stationary waves as against progressive waves with progress in some direction. Now, the amplitude of this stationary wave is 2a cos of kx + alpha by 2 which varies from point to point. Remember again, the amplitude of a progressive wave is constant. It does not change just not vary from point, point to point.

But here, for a stationary wave, we find the wave is a stationary but we find that the amplitude of such a wave is not same for all values of x.

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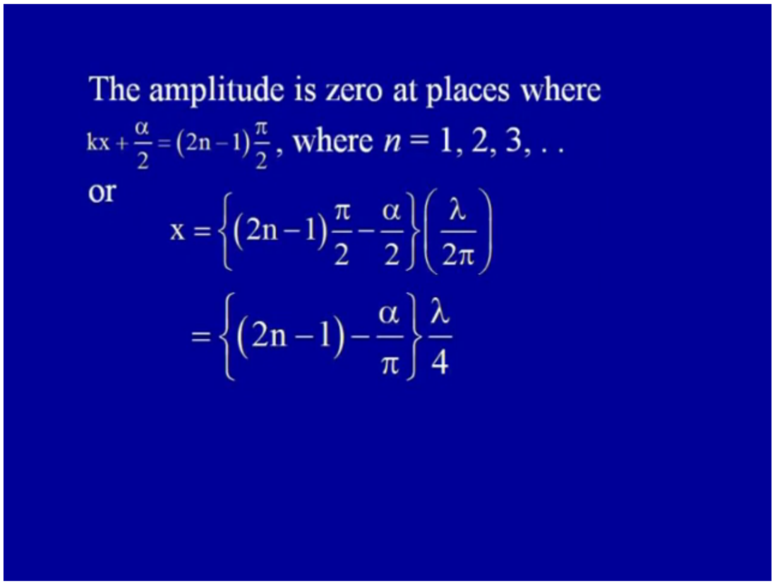

The amplitude is 0 at places where this factor is 0 that is where kx + alpha by 2 is = an odd multiple of Pi by 2 taking n as integer values 1, 2, 3 which means for x = 2n - 1 PI by 2 - alpha by 2 times lambda by 2Pi which gives 2 n - 1 - alpha by PI times lambda by 4.

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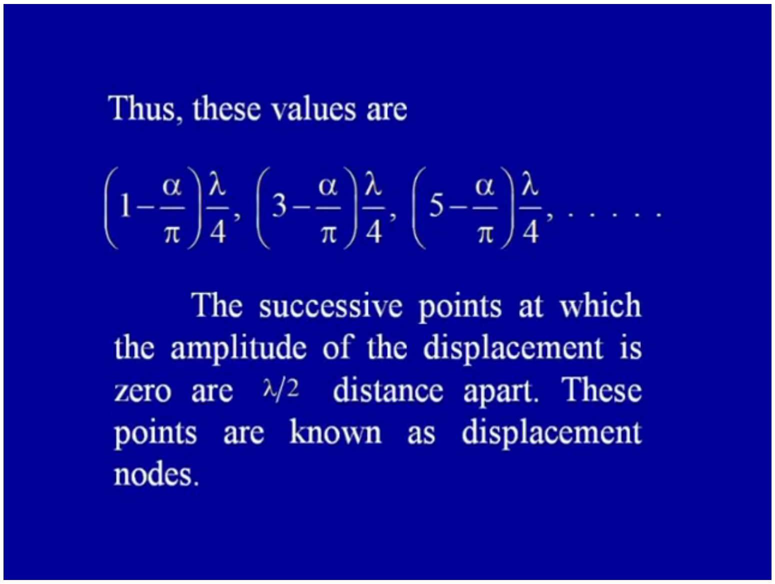

Well, these values are values of x. 1 - alpha by Pi times Pi by 4, 3 - alpha by PI times Pi by 4, 5 - alpha by Pi, Pi by 4, like this. The successive points we see at which the amplitude of the displacement is 0 or lambda by 2 distance apart. These points are known as displacement notes. The particles at these points remain permanently at rest, they just do not move. It is just very interesting phenomena.

The waves are there in the region as a result of superposition of a direct and the reflected wave and we will find these are the points which remain permanently at rest. The displacement is 0 for all time.

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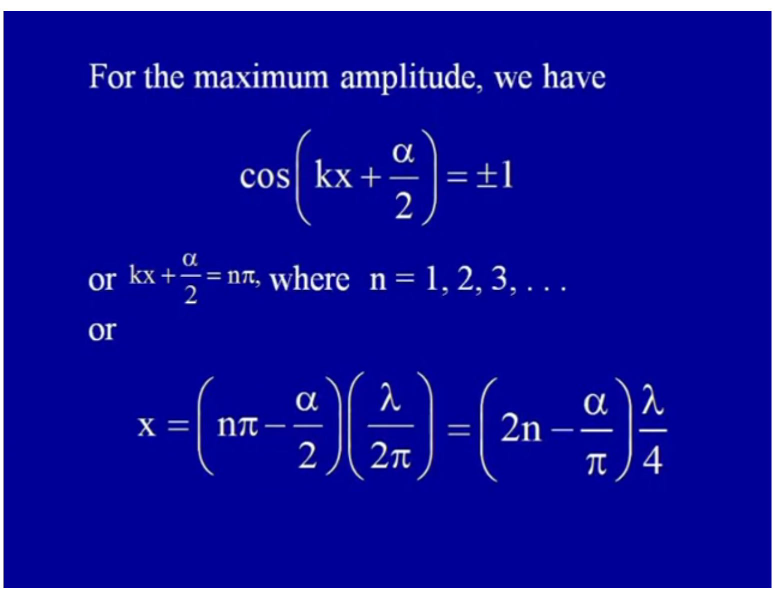

You have a maximum amplitude where the cos factor is = + - 1 that is kx + alpha by 2 is = n Pi, n is n Pi, 2 Pi, 3 Pi and this gives x is = n Pi - alpha by 2 times lambda by 2 Pi which gives 2n - alpha by PI times lambda by 4.

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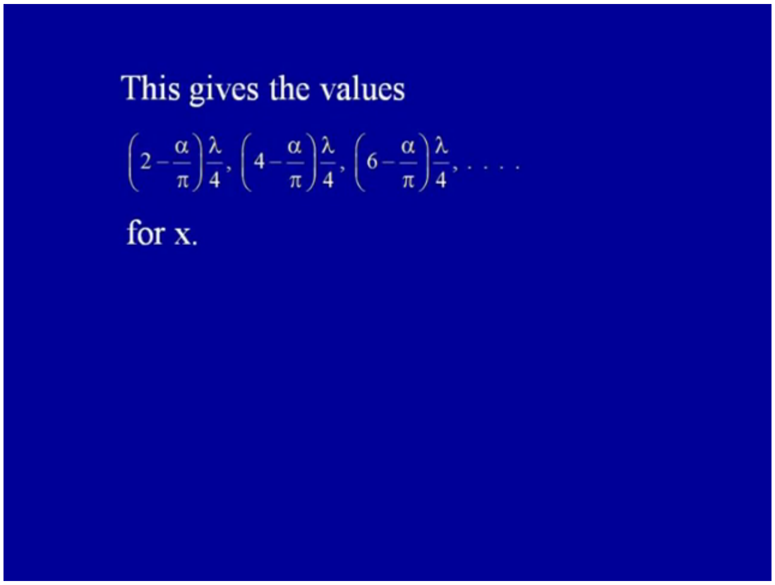

So for x, this gives the values 2 - alpha by Pi times lambda by 4, 4 - alpha by Pi lambda by 4, 6 - alpha by Pi lambda by 4, like this.

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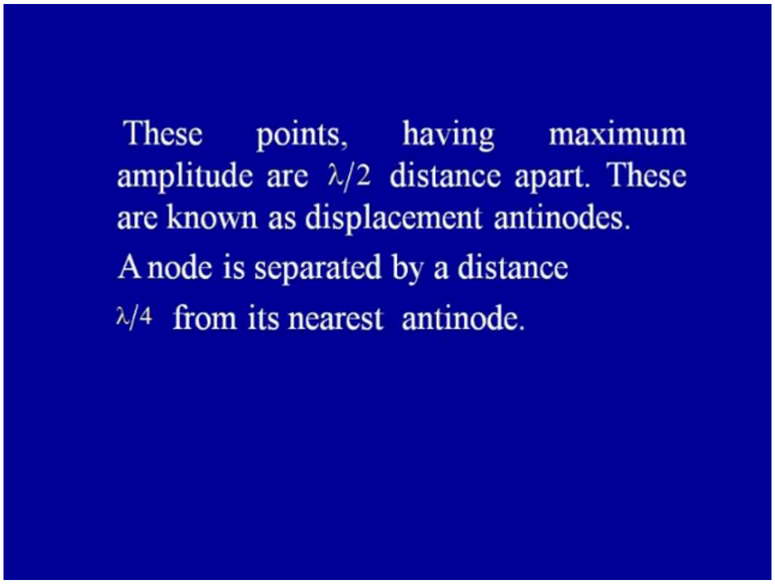

These points having maximum amplitude or again lambda by 2 distance apart; these are known as displacement anti nodes. These are the points where the amplitude is maximum, it is = 2a. Remember a, is the amplitude in the individual waves and at antinodes, the amplitude is 2 wave and two consecutive anti-nodes, as I said, are separated by lambda by 2. A node separated by the distance lambda by 4 from its nearest antinodes; between 2 anti nodes there is a node. Similarly, between two nodes there is an antinode.

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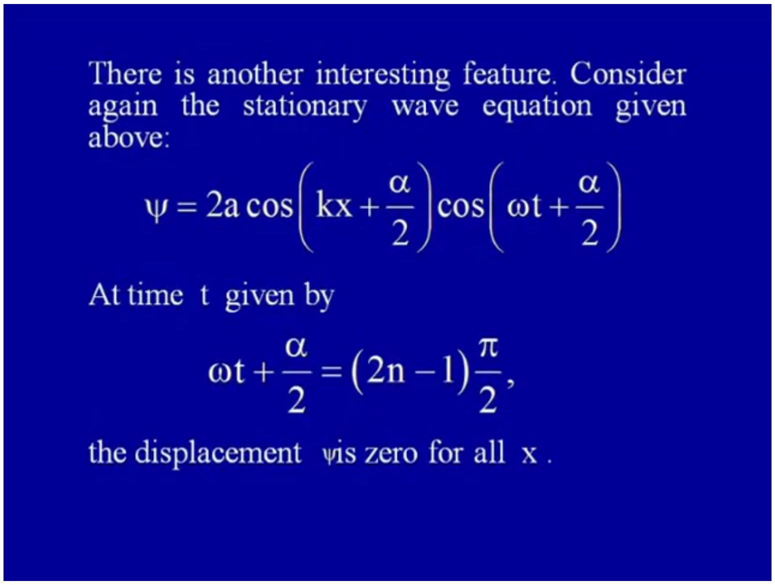

Now, there is another interesting feature. Consider again the stationary wave equation given above: Psi is = 2a cos of kx + alpha by 2 times cos of Omega t + alpha by 2, we had this earlier. The interesting thing is at time t given by this expression Omega t + alpha by 2 is = an odd multiple of Pi by 2. The time-dependent cos factor is 0. And therefore Psi is 0 for all x, all through for all values of all the particles are passing through the mean position simultaneously.

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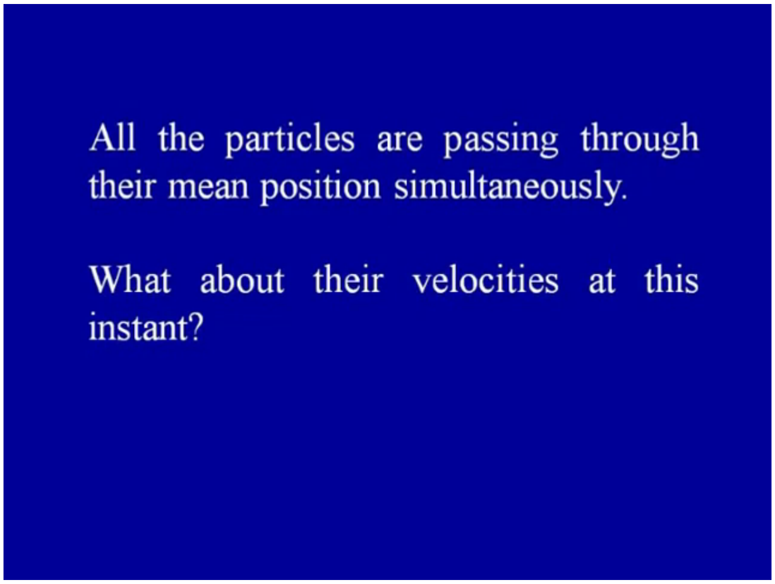

For all values of x, Psi is 0. What about their velocities at this incident? We find that.

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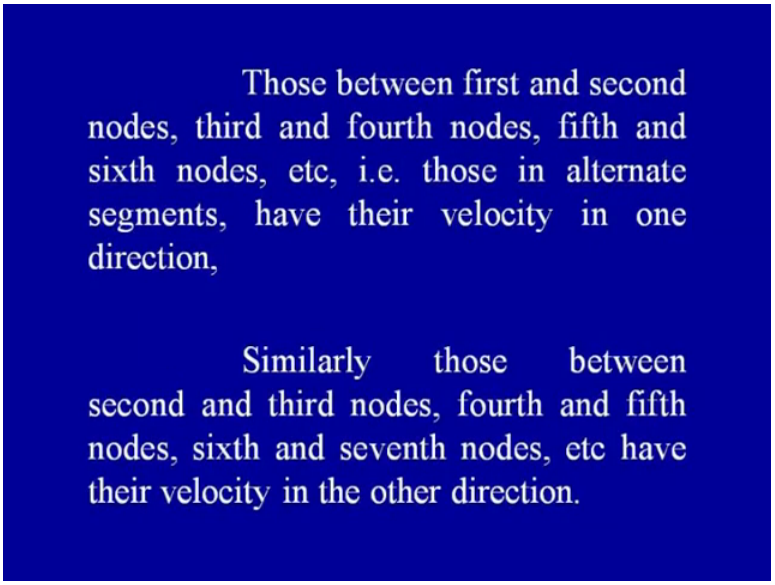

Those between first and second nodes I am leaving between second and third, for first and second nodes, third and fourth node, fifth and the sixth nodes etcetera. Those in the alternate segments have their velocity in one direction. Similarly, those between second and third nodes, fourth and fifth nodes, sixth and seventh nodes, they all have their velocities in the opposite direction. It appears something like this.

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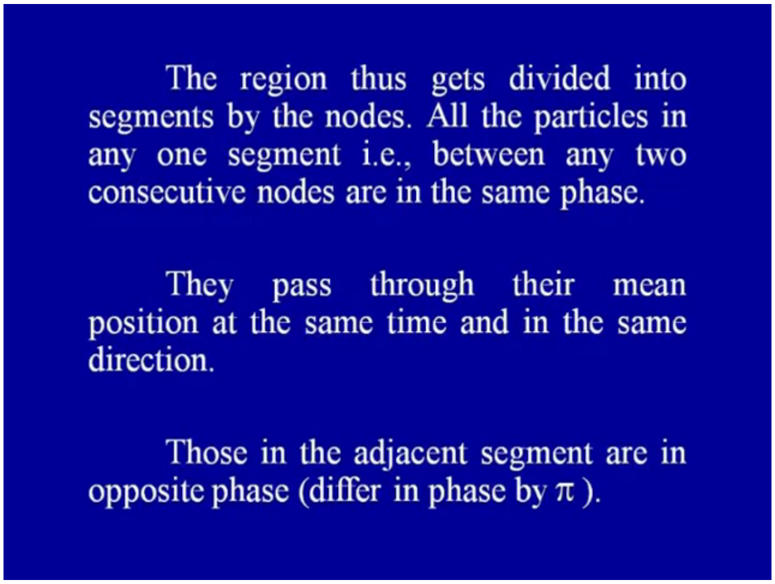

The region is thus gets divided into segments by the nodes. All the particles in any one segment between any two consecutive nodes are in the same phase. All of them passes through the mean positions at the same time and have their velocities in the same direction. They pass through their mean positions at the same time and in the same direction. All of them are in the same phase. Then, those in the adjacent segments are in the opposite phase.

This means if the particles in one segment are going towards right, all those in the adjacent segment are going towards left, at the same time, okay.

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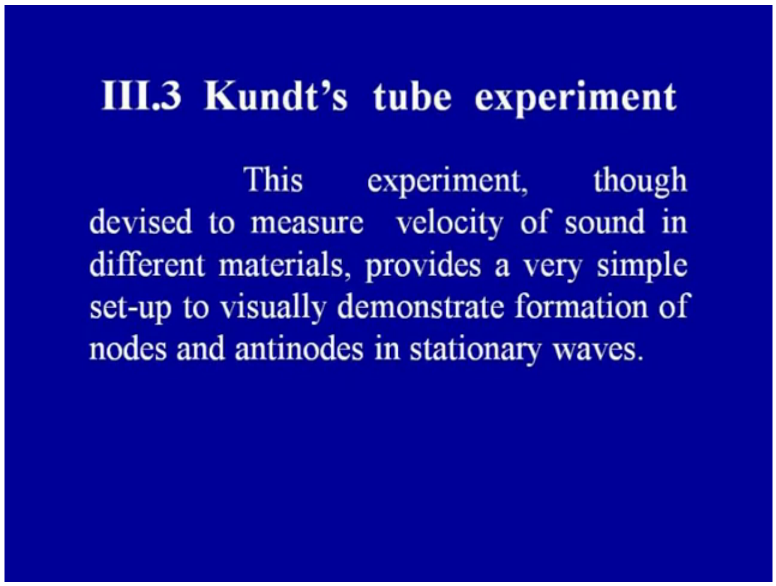

Now, if we consider another interesting thing, experimental setup Kundt's tube experiment. This experiment early in the beginning it was devised to measure velocity of sound in different materials, but we are using it for a different purpose. It provides a very simple setup to visually demonstrate, formation of nodes and antinodes in the stationary waves.

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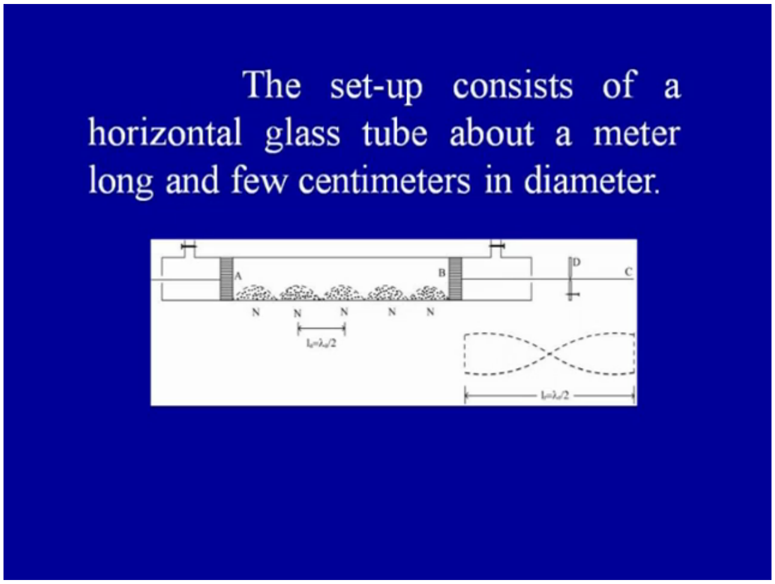

The setup consists of a horizontal glass tube, about a meter long and a few centimeters in diameter.

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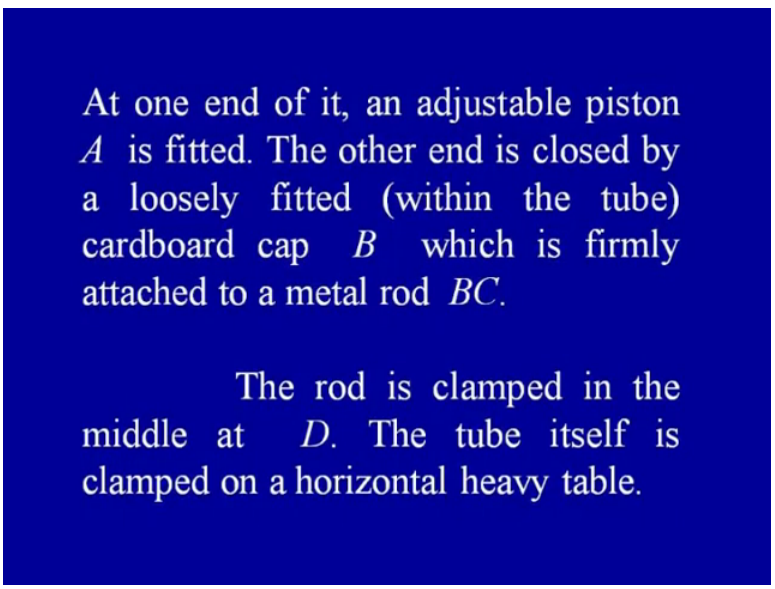

At one end of it, adjustable Piston A is fitted. The other end is closed by a loosely fitted cardboard cap B within the tube which is firmly attached to the metal rod BC. They are all just clamped in the middle at the point D. The tube itself is clamped on horizontal heavy table.

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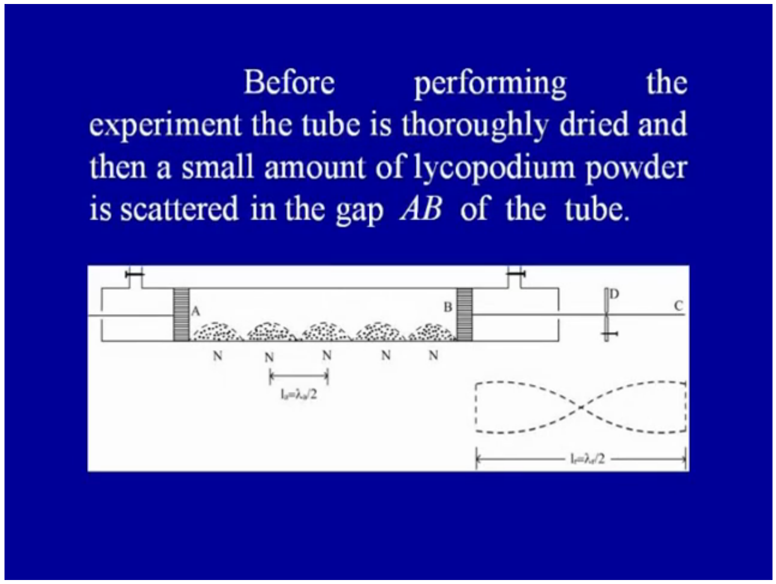

Not before performing the experiment, the tube is thoroughly dried. And then, a small amount of lycopodium powder it was scattered in the gap AB of the tube.

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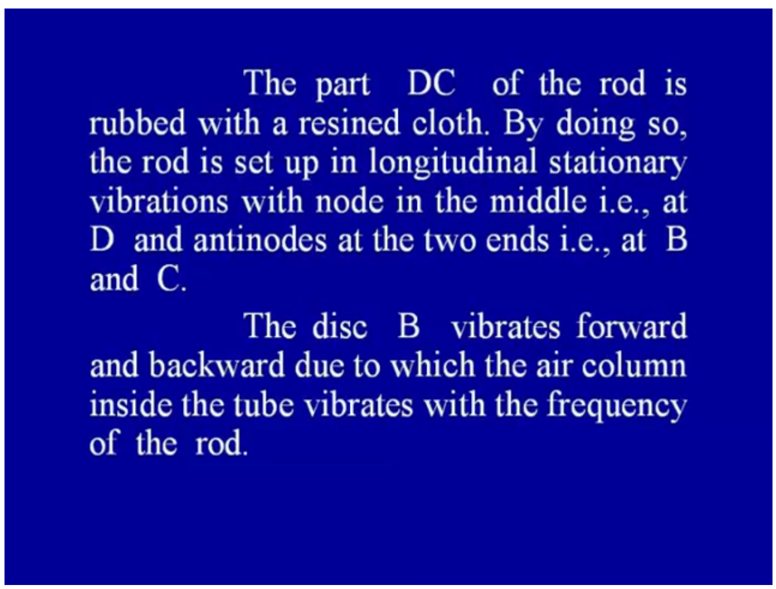

The part DC of the rod is now rubbed with the resin cloth rubbed along the length. By doing so, the rod is set up in longitudinally stationary vibrations, with node in the middle at the point E, which is clamped and antinodes at the two free ends at B and C. The disc B now vibrates forward and backward you see, the rod is a longitudinal motion. So, the disc B vibrates forward and backward due to wind, the air column inside the tube also vibrates in the same frequency, the frequency of the rod.

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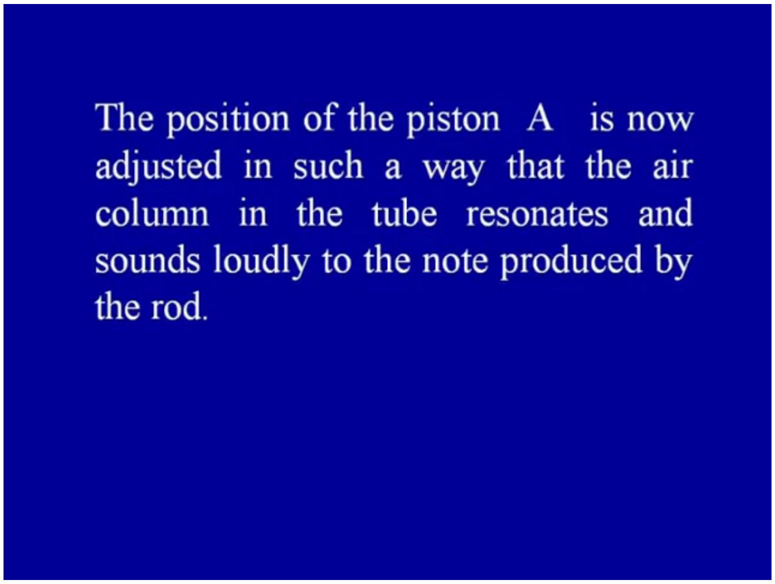

Now, the position of the Piston A is adjusted in such a way that the air column in the tube resonates. Resonates means the natural frequency of the air column in the tube is same now as the frequency with which the rod is vibrating, the natural frequency of the air column can be adjusted by adjusting the position of the Piston A. And this is the experiment.

So, the position of the Piston is now adjusted in such a way that the natural frequency of the air column becomes = the frequency of the rod. And there is no net and the air column sounds loudly to the node produced by the rod.

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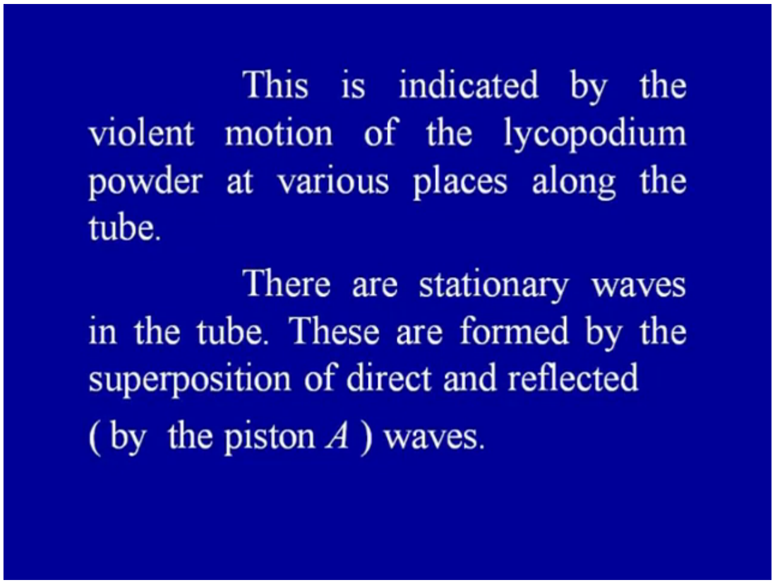

This is indicated by the violent motion of the Lycopodium powder which is there in the tube at various places along the tube. Now, we are, they are the stationary waves in the tube. These waves are formed by the superposition of direct and reflected waves. They are reflected by the Piston A since you have incident and direct waves of the same frequency, one traveling in one direction, the other traveling in the opposite direction and stationary waves are formed.

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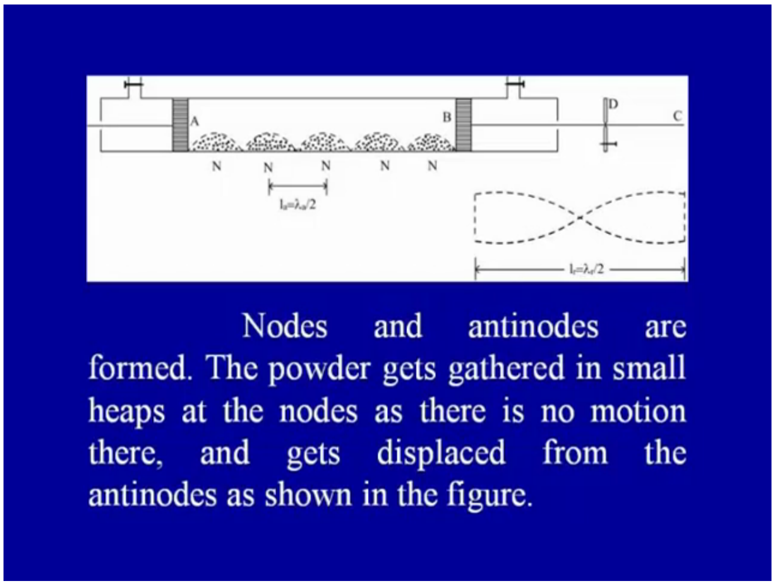

Nodes and Internodes are formed. The powder gets gathered in small heaps at the nodes as there is no motion there and gets displaced from the antinodes as shown in the figure. It is a very clear simple demonstration of the formation of nodes and antinodes alternately and also the property that there is no motion at the nodes.

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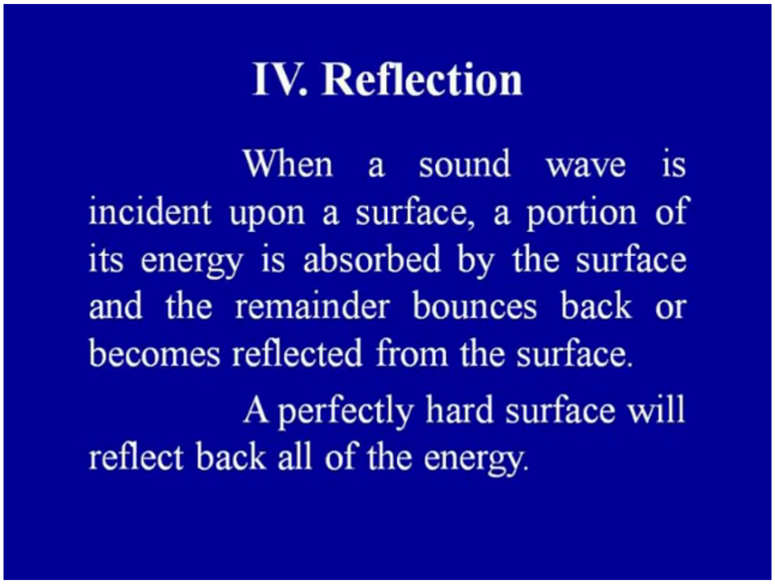

Now, we come the study of this phenomena of reflection, refraction and diffraction of sound waves. When a sound wave is incident upon a surface, a portion of its energy is absorbed by the surface and the remainder bounces back have becomes reflected around the surface. A perfectly hard surface reflect back all of the energy. Perfectly hard subtle really does not exist. But the idea is, in this the harder surfaces, more will be the reflection coefficient.

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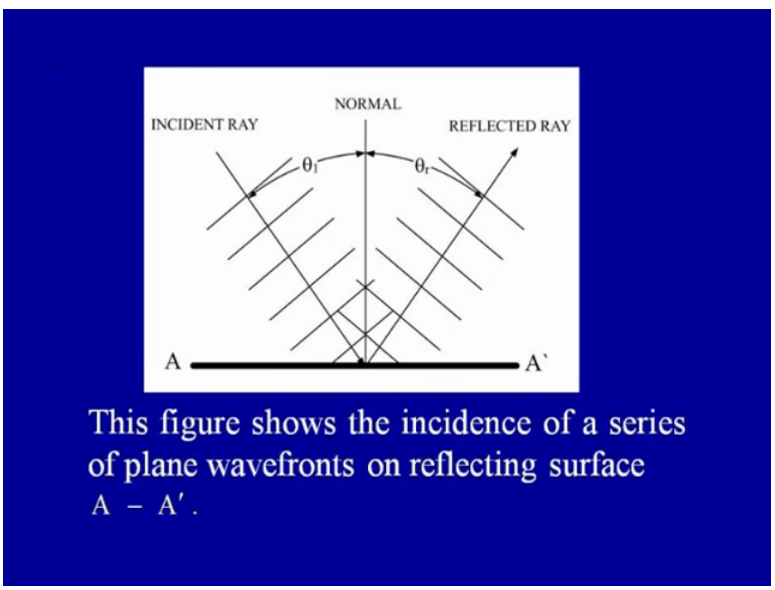

This figure shows the incidence of a series of plane wave fronts, on the reflecting surface, A, A prime. The arrows normal to the wave fronts or rays which represent the direction of propagation are drawn to represent the incidence and the consequent reflection of the wavefront.

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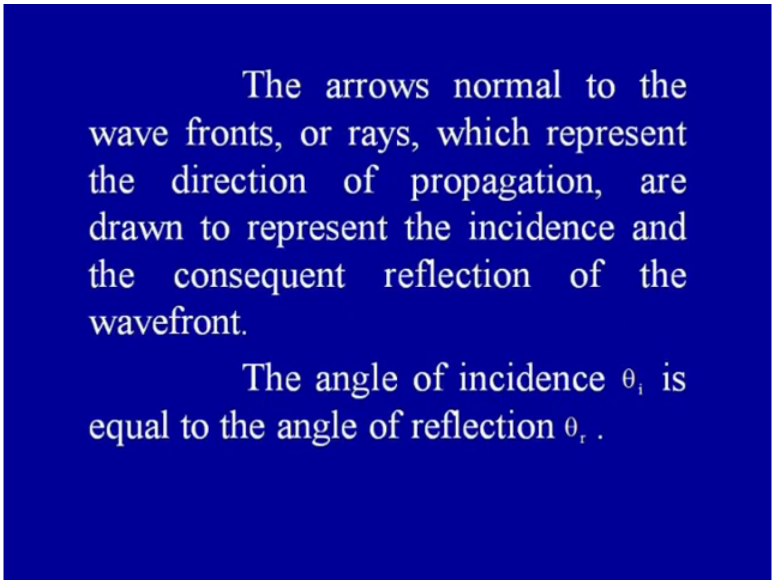

The angle of incidence theta i is = the angle of reflection theta r.

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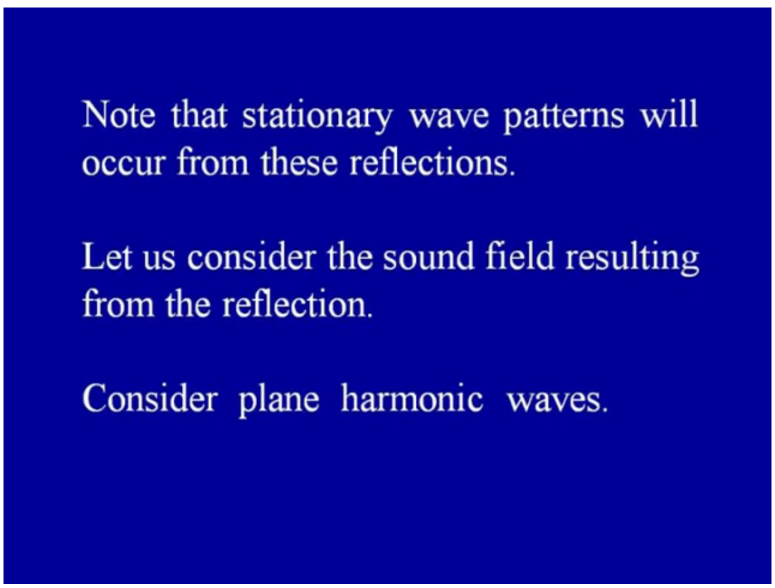

Note that a stationary wave patterns will occur from these reflections. Let us consider the sound field resulting from the reflection considered plain harmonic waves.

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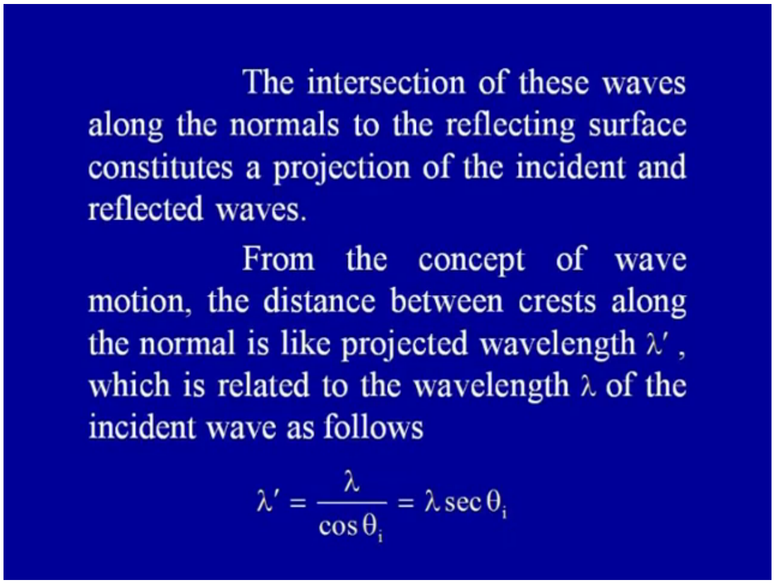

The intersection of these waves along the normal's to the reflecting surface constitutes a projection of the incident and reflected wave. From the concept of wave motion, the distance between crests, I mean, between consecutive crests or consecutive compressions or consecutive rarefactions, along the normal is like projected wavelength lambda prime which is related to the wavelength lambda of the incident wave as follows: Lambda prime is = lambda divided by cos of theta i which is = lambda sec of theta i.

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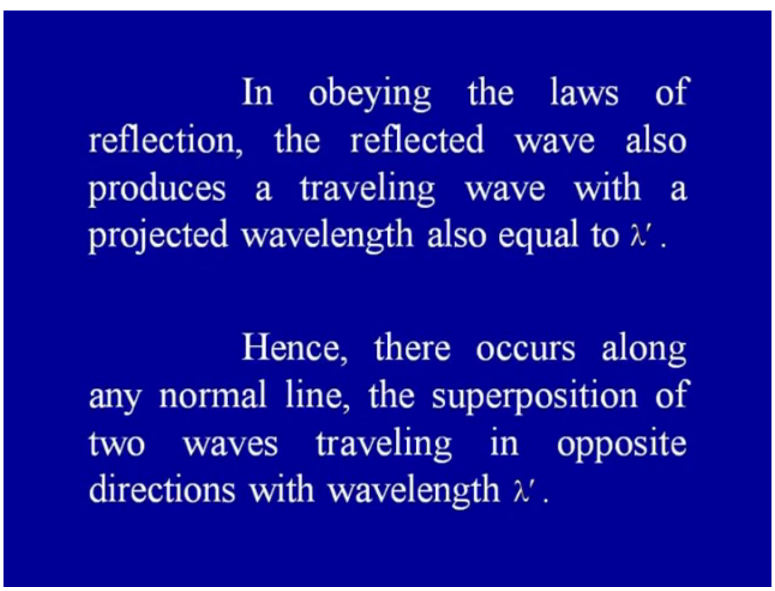

In obeying the laws of reflection, the reflected wave also produces a traveling wave the projected wavelength also = lambda Prime. Hence, there occurs along any normal line, the superposition of two waves traveling in opposite directions, with wavelength lambda Prime.

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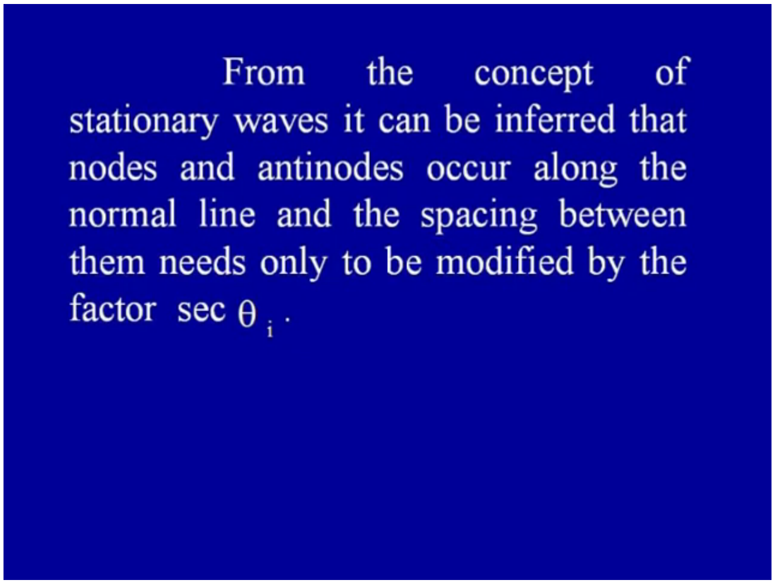

From the concept of stationary waves it can be inferred straight away that nodes and antinodes occur along the normal line and i spacing between them is only to be modified by the factor sec theta i.

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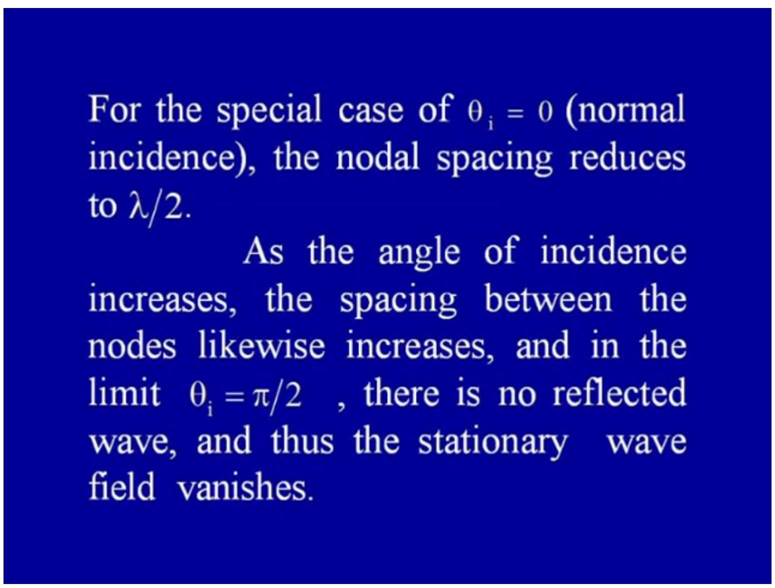

For the special case of theta i = 0 which means the normal incidence, the nodal spacing reduces to the standard value lambda by 2, between any 2 nodes or any 2 antinodes. As the angle of incidence increases the spacing between the nodes likewise increases and in the limit theta i = Pi by 2, there is no reflected wave and thus the stationary wave field vanishes.

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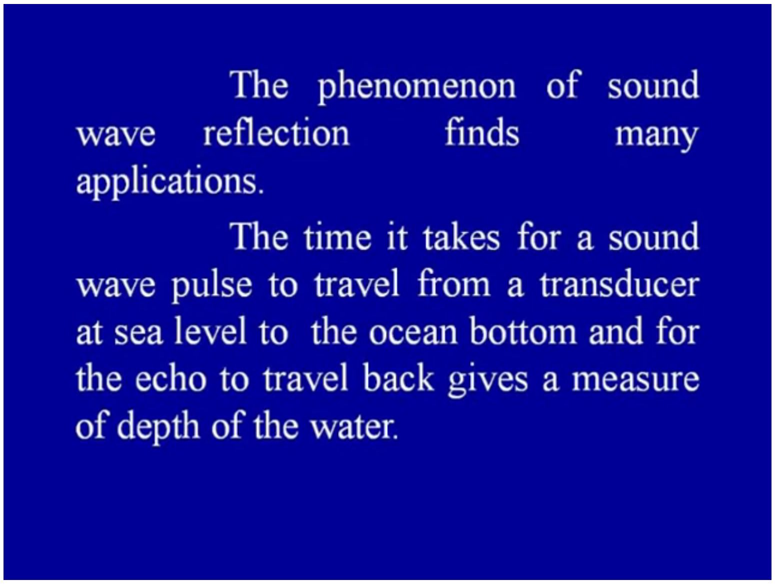

The phenomenal sound wave reflection finds many applications. See, the time it takes for the sound wave pulse to travel from a transducer at sea level to the ocean bottom and for the Eco to travel back gives a measure of depth of the water.

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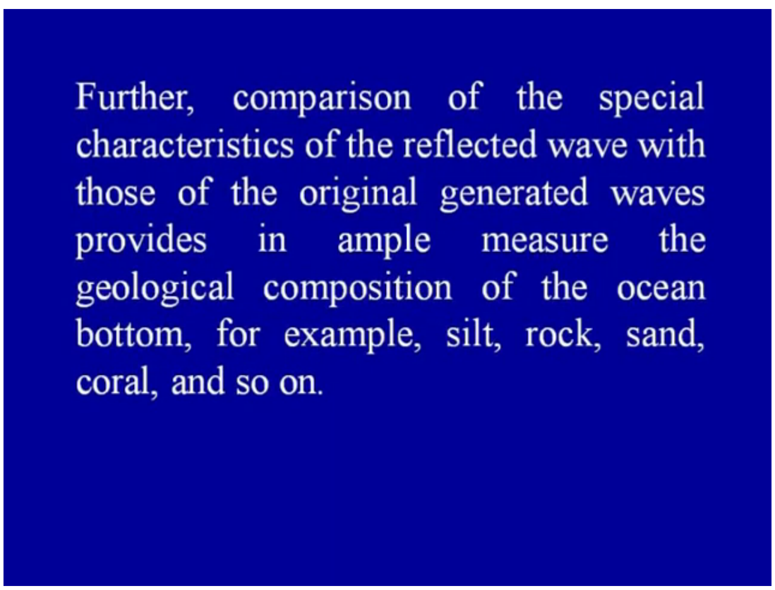

Further, comparison of the spatial characteristics of the reflected wave, with those of the original generated waves but provides in ample measure, the geological composition of the ocean bottom, for example, the occurrence of silt, or rock, or sand, or coral and so on.

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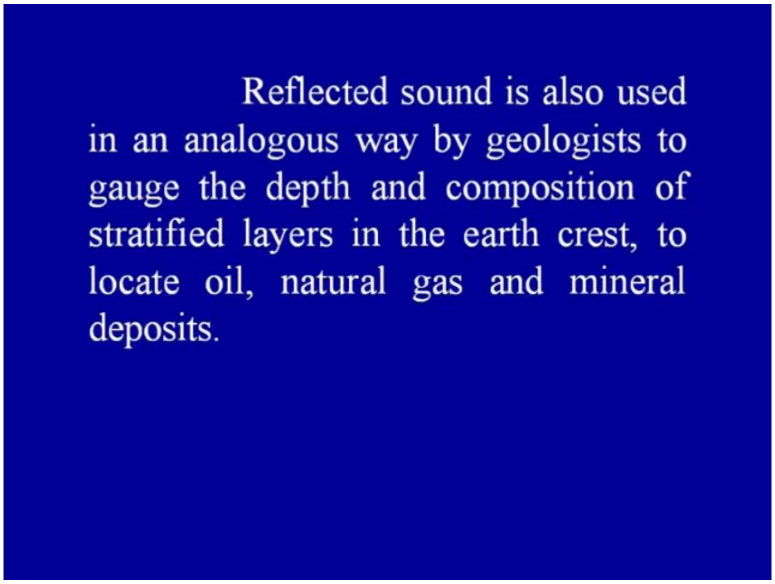

Reflected sound is also used in an analogous way by geologist to gauge the depth and composition of stratified layers in the earth crest to locate the occurrence of oil, natural gas or mineral deposits.

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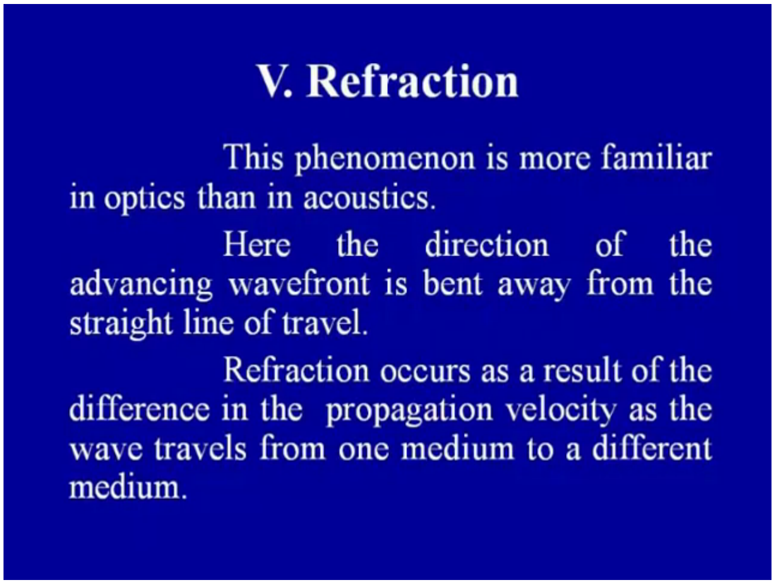

Let us now consider refraction. This phenomena is more familiar in optics than in acoustics. Here the direction of the advancing wavefront is bent away from the straight line of travel. Refraction occurs as a result of the difference in the propagation velocity as the wave travels for one medium to a different medium.

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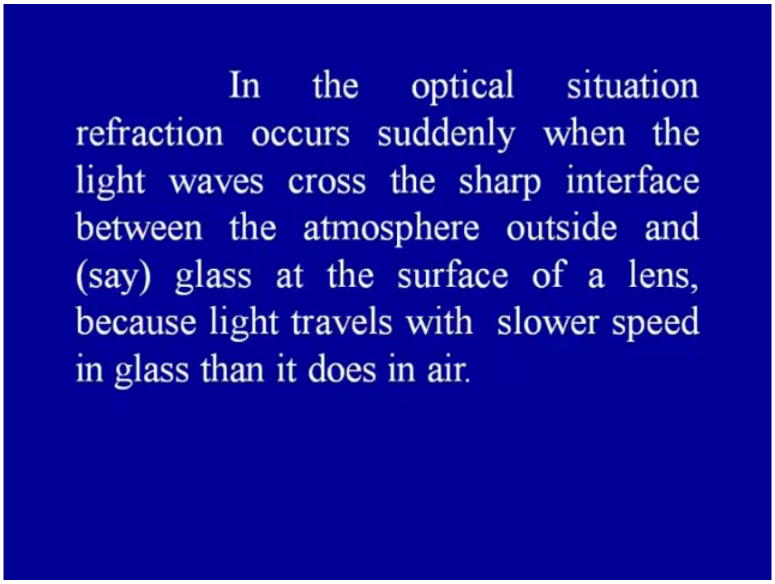

In the optical situation, refraction occurs rather suddenly. See, the wavelengths are very small when the Light waves cross the sharp interface between the atmosphere outside and say glass, at the surface of a lens, because light travels with slower speed in glass than what it does is in air.

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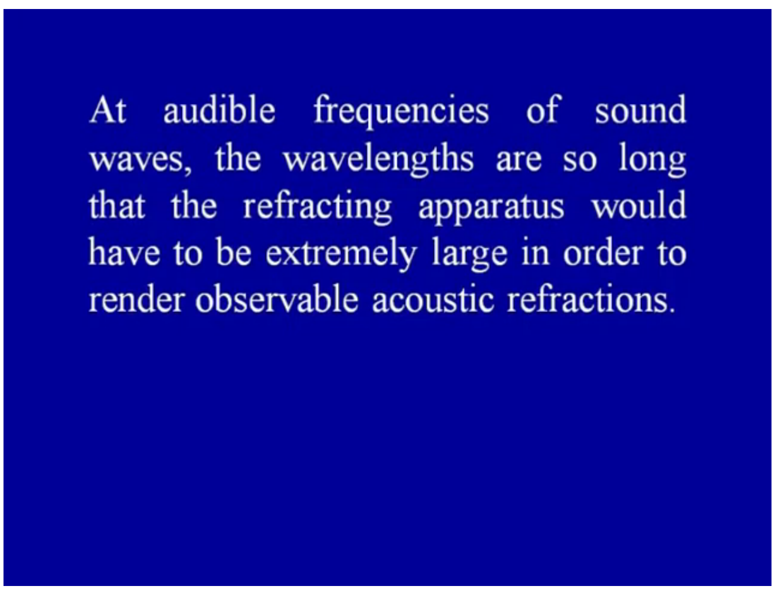

At audible frequencies of sound waves, the wavelengths are so long that reflecting apparatus would have to be extremely large in order to render observable acoustic reflections.

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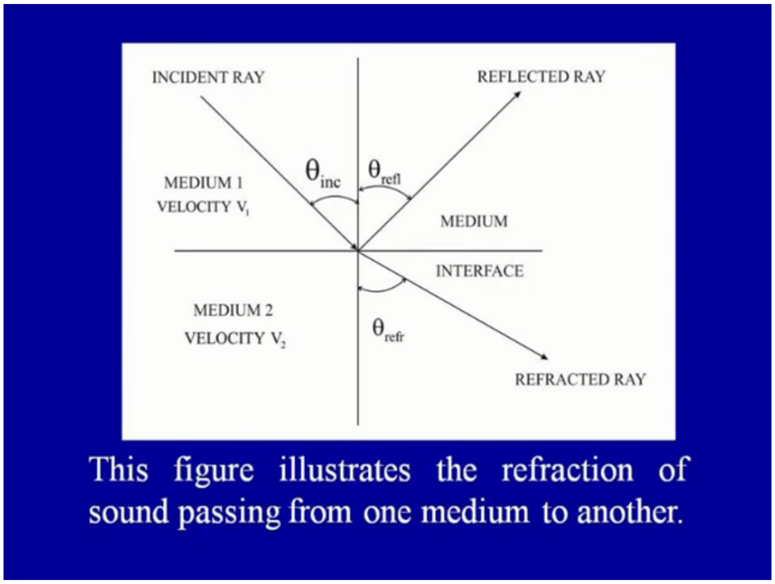

This Picture is really very similar to what one has in optics. The propagation is from medium 1 to medium 2. Velocity medium is v1 and velocity in the second medium is v2. The refracted ray moves away from the normal or towards the normal. It is really determined whether the velocity v2 is more or less relatively. Basically the basic structure is essentially the same.

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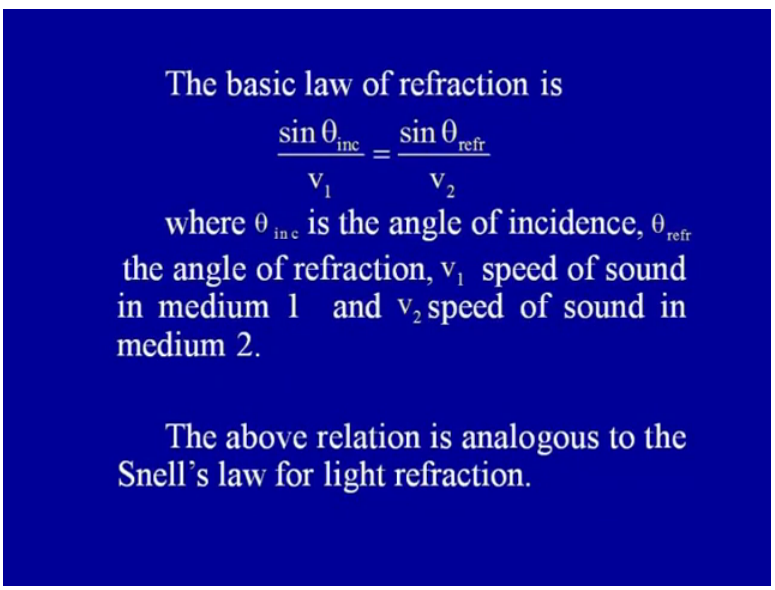

The basic law of refraction sine theta incident divided by v1 is = sine theta refraction divided by v 2. theta incidence is the angle of incidence, theta area refract is the angle of refraction even at the speed of sound in medium 1 v2 the speed of sound in medium 2. The above relation is analogous to the Snell's law for light refraction.

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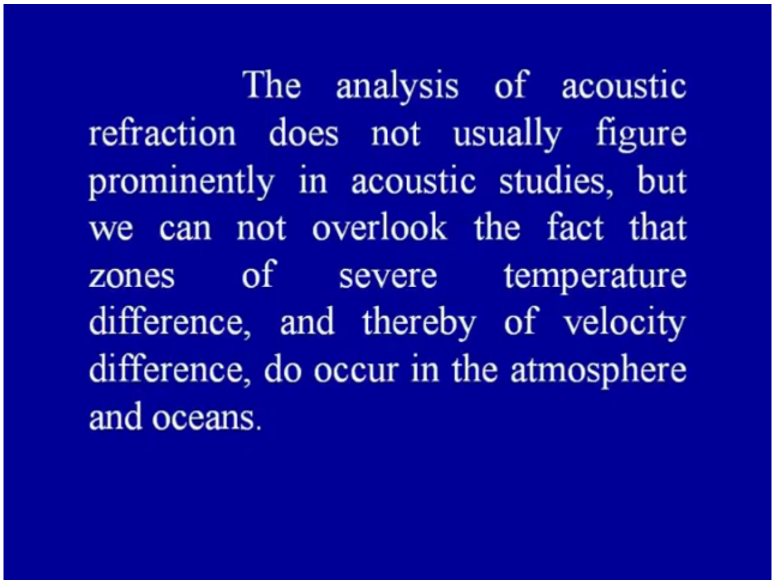

See the analysis of caustic diffraction does not usually figure prominently most of the time we really do not bother very much about it in acoustic studies. But we cannot over look the fact that the zones of severe temperature difference and thereby severe velocity difference do occur in the atmosphere and oceans.

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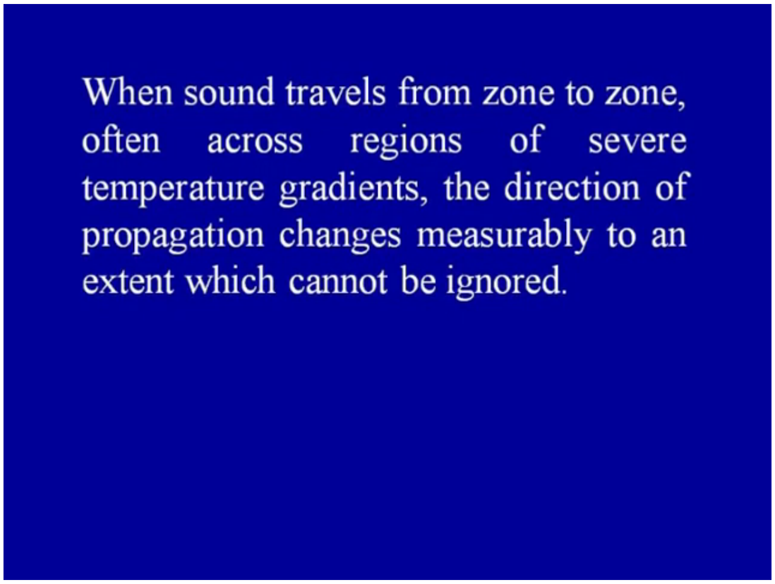

When sound travels from zone to zone, often across regions of severe temperature gradients, the direction of propagation changes measurably, to an extent which cannot be ignored.

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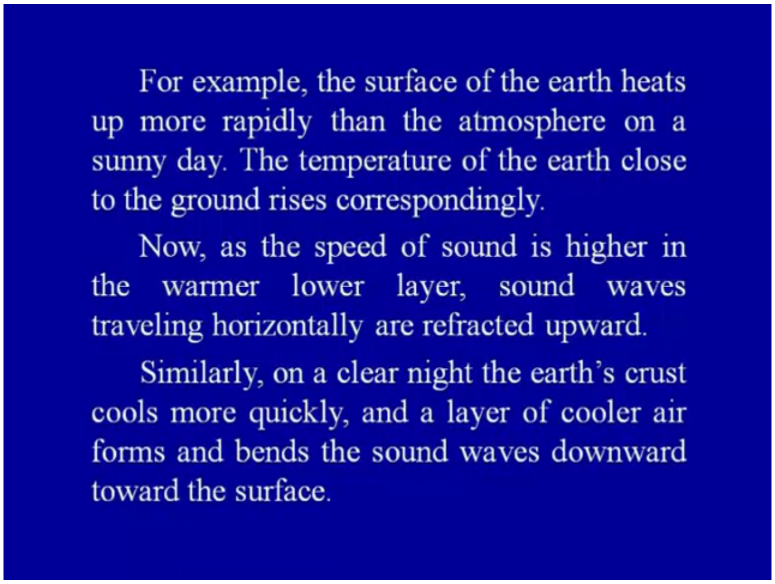

For example surface of the earth heats up more rapidly than the atmosphere on a sunny day. The temperature of the earth close to the ground rises correspondingly. Now, as the speed of sound is higher in the warmer lower sound waves traveling horizontally are refracted upwards. Similarly, on a clear night the Earth's crest cools more quickly and a layer of cooler air forms and whence the sound waves downwards towards the surface towards the Earth's surface.

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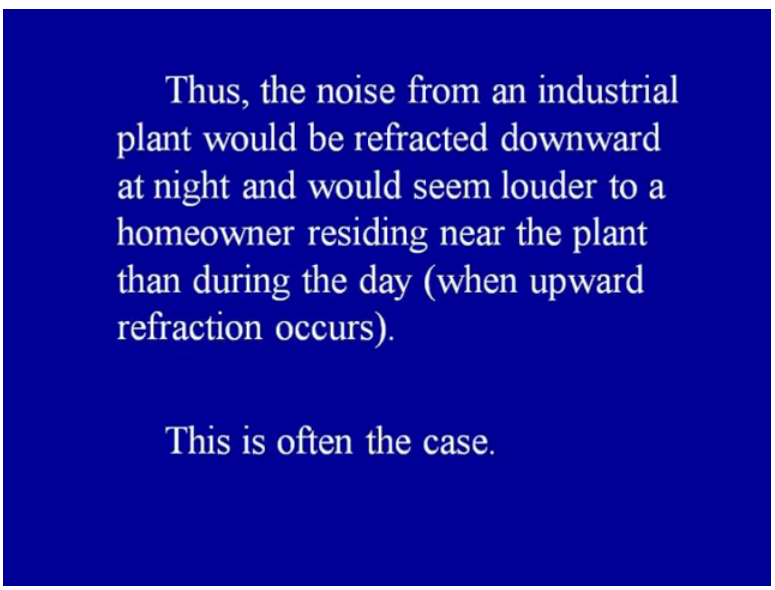

Thus noise from industrial plant for example, would be refracted downwards at night and which seem louder to a homeowner residing near the plant than during the day. When the upward refraction occurs this is quite often the case.

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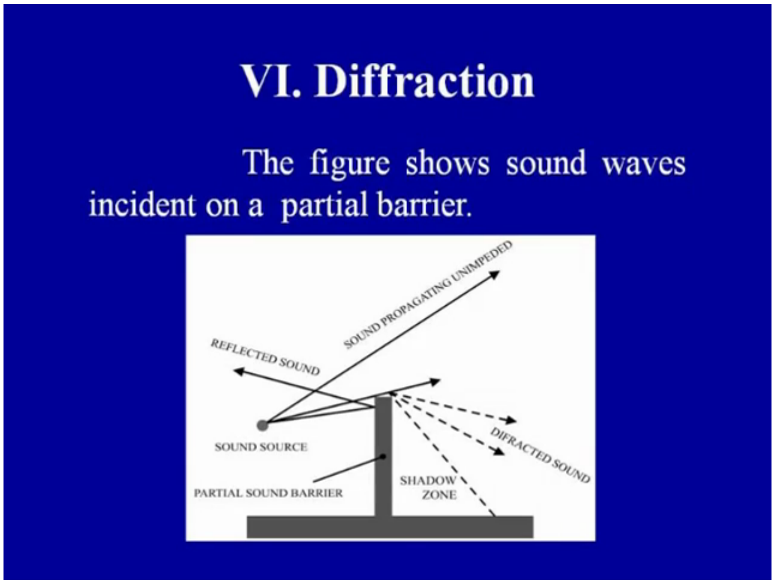

Let us now consider the Diffraction of the case. This figure shows sound waves incident on a partial barrier.

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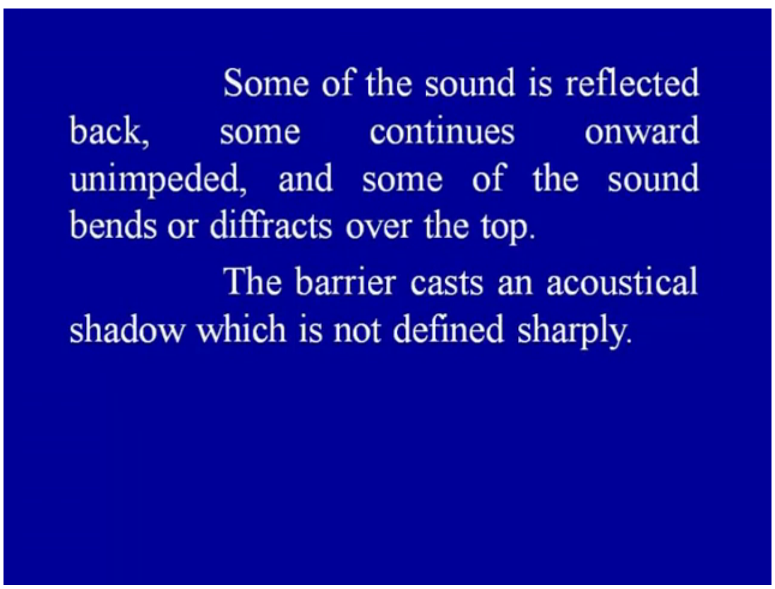

Some of the sound is reflected back, some continues onwards unimpeded and some of the sound bends diffracts over the top. The barrier casts an acoustical shadow which is not defined sharply.

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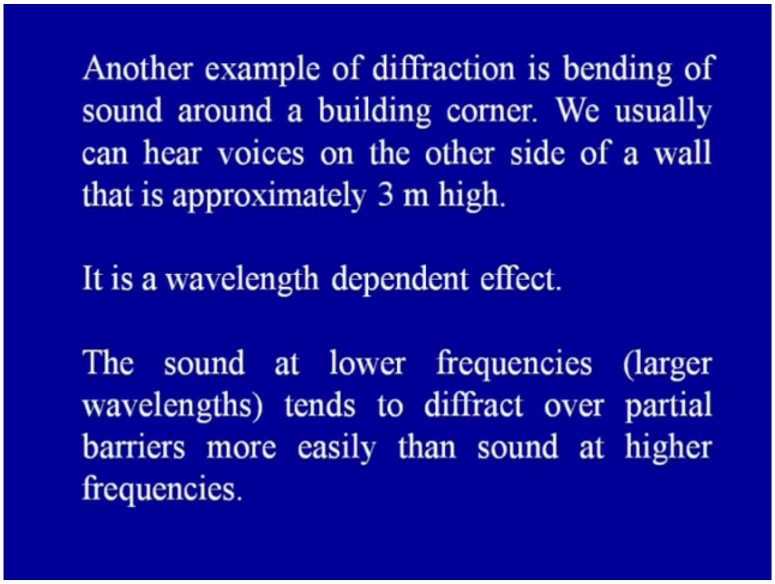

Another example of diffraction is bending of sound around the building corner, usually can hear voices on the other side of a wall that is approximately 3 meters or so high. It is the wavelength dependent affect. The sound at lower frequencies larger wavelengths tend to diffract over partial barriers more easily than the sound at higher frequencies.

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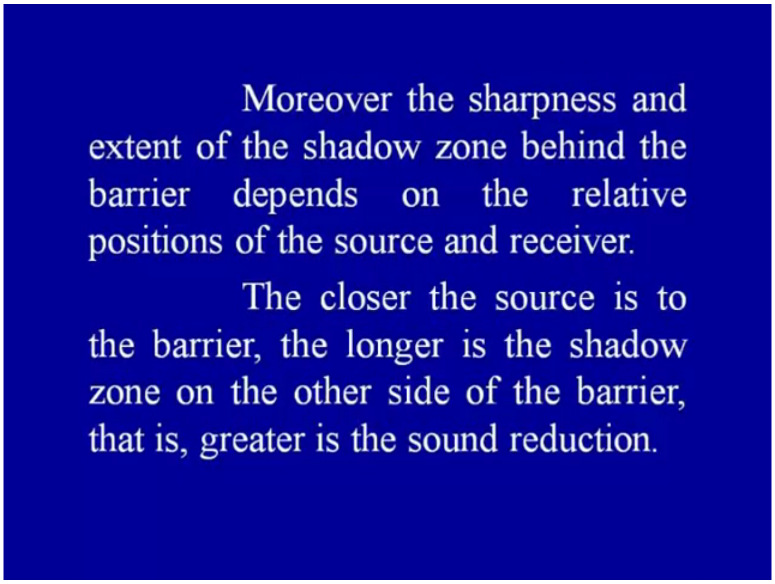

Moreover the sharpness and extent of the sound zone behind the barrier depends on the relative positions of the source and the receiver. The closer the source is to the barrier, longer in the shadow zone, on the other side of the barrier and that is greater is the sound reduction. That is all we need to know about some diffraction. So, we have come to the end of this lecture.