

Engineering Physics 1
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Module-08
Lecture-04
Lens Aberrations Part II

Lens aberrations one by M. K. Srivastava, Department of Physics, Indian Institute of Technology Roorkee, Uttarakhand.

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In the last lecture we considered chromatic aberration and spherical aberration. In this lecture we shall take up the remaining monochromatic aberrations. These are (i) Coma, (ii) Astigmatism, (iii) Curvature of the field, and (iv) Distortion.

Let us begin with coma.

In the last lecture we considered chromatic aberration and spherical aberration. In this lecture we shall take up the remaining monochromatic aberrations and these are number 1. Coma, Number 2. Astigmatism, Number 3. Curvature of the field and the fourth one Distortion. Let us begin with coma.

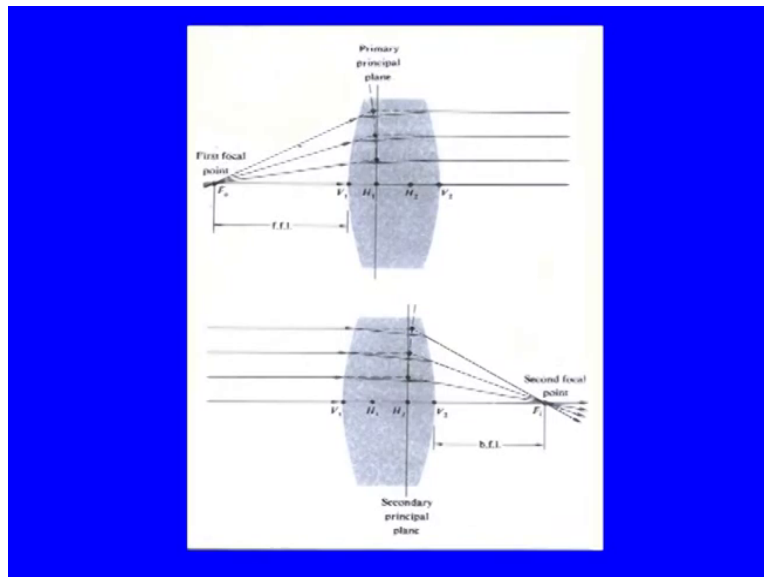
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Coma

Coma or *comatic* aberration is a primary aberration associated with an object point even a short distance away from the axis. Its origin lies in the fact that the principal “planes” can actually be treated as planes only in the paraxial region. They are, in fact, principal curved surfaces.

The coma or comatic aberration is a primary aberration associated with an object point even a short distance away from the axis away from the axis the main thing. Its origin lies in the fact that the principal planes can actually be treated as planes only in the paraxial region. They are in fact principle curved surfaces.

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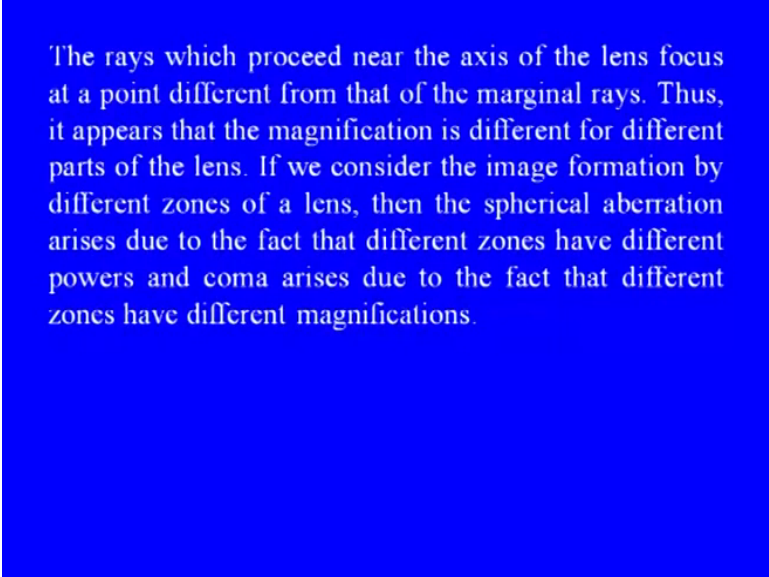


Look at this figure showing grace coming from first principle focal point the emergent ray is naturally our parallel. But the point of intersection of the race diverging from the first principle focal point and the emergent parallel rays and their point of intersection does not lie on a plane. It is really curved particularly mainly actually for those days which are not paraxial. Similarly in

the second figure the rays parallel rays is after refraction they are focused to the second focal point.

But the point of intersection of the incident parallel rays and the refracted rays which are getting focused at the circle focal point their intersection does not lie on a plane surface that surface is curved. This curvature appears really for those rays which are not paraxial for the marginal rays.

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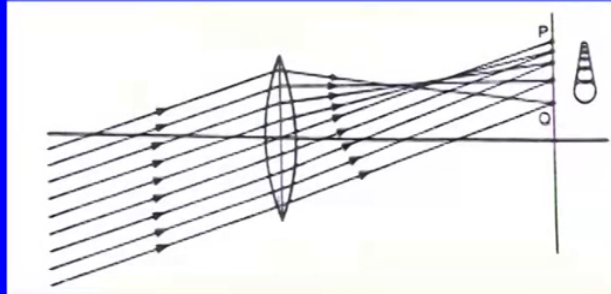


The rays which proceed near the axis of the lens focus at a point different from that of the marginal rays. Thus, it appears that the magnification is different for different parts of the lens. If we consider the image formation by different zones of a lens, then the spherical aberration arises due to the fact that different zones have different powers and coma arises due to the fact that different zones have different magnifications.

The rays we proceed near the axis of the lens they focus at a point different from that of the marginal rays. Thus it appears that the magnification is different, for different parts of the lens. If you consider the image formation by different zones of a lens the paraxial zone or the marginal zone. Then the spherical aberration arises due to the fact the different zones have different powers and coma it arises due to the fact the different zones have different magnifications.

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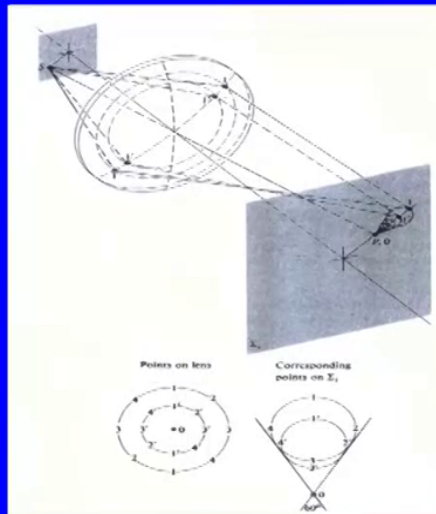
The figure illustrates the effect of coma. The resultant image of a distant point off the axis is shown in the side figure. The rays in the tangent plane are shown.



The figure illustrates the effect of a coma. The resultant image of distant point object of the axis is shown in the side figure. The rays in the tangent plane are shown you see the chief ray which passes through the center of lens goes undeviating and you see the marginal rays these are the ones which are which give rise to coma, a broader circle near the axis and the vertex close to the point P.

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In this figure several skew rays are shown from an extra-axial object point. Note that each circular cone of rays whose end points (1-2-3-4-1-2-3-4) form a ring on the lens in a comatic circle.



Let us consider in this figure several skew rays are shown from an extra axial point. Note that the each circular cone of rays whose end points 1, 2, 3, 4 that a circular zone of the lens from where the rays are coming they are focused in a in a chromatic circle. You see at the radius of the circular zone decreases as we come towards the center of the lens.

This cone on the image the comatic circle this radius goes also goes on becoming smaller and smaller and it centered moves towards the vertex of the cone in the limit when we consider the rays passing through the center of the lens that gives rise to the vertex of the cone.

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This case corresponds to positive coma, so the larger the ring on the lens, the more distant its comatic circle from the axis.

Now this case corresponds to a positive gamma so that larger the ring on the lens the more distant its comatic circle from the axis.

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Coma can be eliminated if a lens satisfies the Abbe's sine condition:

$$n_1 y_1 \sin \theta_1 = n_2 y_2 \sin \theta_2 ,$$

where n_1 , y_1 , θ_1 refer to the refractive index, height of the object above the axis and the slope angle of the incident ray of light. Similarly n_2 , y_2 , θ_2 refer to the corresponding quantities in the image medium. The magnification of the image is given by

$$y_2 / y_1 = n_1 \sin \theta_1 / n_2 \sin \theta_2 .$$

Now gamma can be eliminated if a lens satisfies the Abbe's sine condition $n_1 y_1 \sin \theta_1 = n_2 y_2 \sin \theta_2$, $n_1 y_1 \sin \theta_1$ they refer to the refractive index, height of the object above the axis and the slope angle of the incident ray of light. Similarly $n_2 y_2 \sin \theta_2$ they refer to the

corresponding quantities in the image medium they made site. Now the magnification of the image is given by y_2 divided by y_1 which according to this condition should be equal to $n_1 \sin \theta_1$ divided by $n_2 \sin \theta_2$.

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The elimination of coma is possible if the lateral magnification y_2 / y_1 is same for all the rays, irrespective of the slope angles. Thus coma can be eliminated if $\sin \theta_1 / \sin \theta_2$ is a constant. A lens that satisfies this condition is called an *aplanatic lens*. Such a lens is mostly used as a front lens of a high power microscope objective called the *oil immersion objective*.

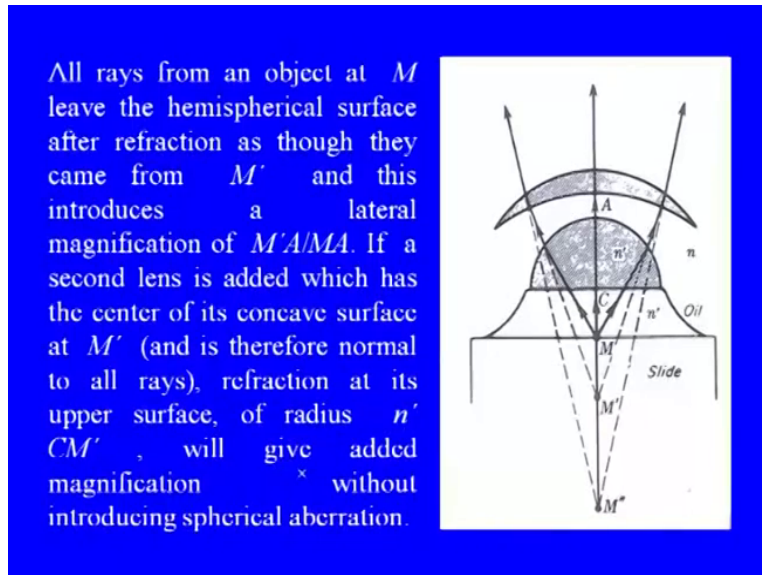
Now the elimination of coma is possible if the lateral magnification y_2 upon y_1 is same for all the rays irrespective of the slope angle the respective of θ_1 and θ_2 . Thus coma can be eliminated if $\sin \theta_1$ divided by $\sin \theta_2$ is a constant. A lens that satisfies this condition is called an aplanatic lens. Such a lens is mostly used at the front lens of a high powered microscope objective called a oil immersion objective.

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The oil chosen is naturally such that it has the same refractive index as that of the lens. The oil is placed on the microscope slide and the lens is lowered into contact.

It is the oil chosen is naturally such that it has the same refractive index as that of the lens. The oil is placed on a microscope slide which is being observed and then the lenses lowered into contact with the oil fill.

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In this figure shows the aplanatic lens is all raised from an object that M leave the hemispherical surface after refraction as they though they came from the point M' which is the first image on the spherical surface and this introduces a lateral magnification and prime MA divided by MA . Now if the second lens is added which has the center of its curvature called of its concave surface at M' Prime.

And therefore is the surface is normal to all rays. The reflection at its upper surface of ready of radius M' prime times CM' prime will give an additional magnification without introducing a spherical aberration and the final image is formed at M double Prime.

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This property of the upper lens, however holds strictly only for rays from the single point M , and not for points adjacent to it.

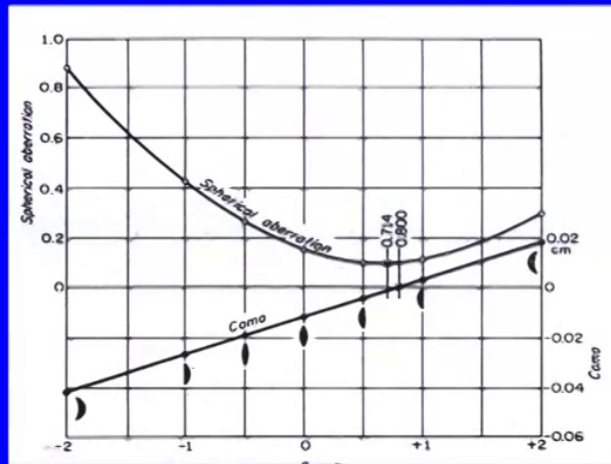
Now this property of the upper lens however holds strictly only for rays from a single point M and not for points adjacent to it that is a limitation of this arrangement.

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Like spherical aberration, coma is dependent on the shape of the lens. The interesting thing is the fact that it can be made exactly zero for a single lens *with a given object distance*. The particular shape it then has is almost convex-planar and this shape is also nearly the shape for minimum spherical aberration.

Now like it is very collaboration coma is dependent on the shape of the lens. But the interesting thing is the fact that it can be made exactly zero for the single lens with a given object distance. This is important with a given object the particular shape it then has is almost convex planar one side convex other side almost plane. And this shape if you remember it also nearly the shape for minimum spherical aberration.

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As we have seen in the last lecture this figure shows the variation of the spherical aberration and also the variation of coma for different shape factors. If you remember for a shape factor about +.5 the spherical aberration is minimum and that is also the region where the coma passes through the 0 axis. So, that is a very interesting thing at the same time you can eliminate this coma and minimize spherical aberration by proper choice of the radii R_1 and R_2 .

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Astigmatism

If the first two Seidel sums vanish, all rays from points on or very close to the axis of a lens will form point images and there will be no spherical aberration or coma.

Now we come to Astigmatism, now if the first two Seidel sums vanish all rays from points on are very close to the axis of a lens will form point images and there will be no spherical aberration or coma.

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When an object point lies an appreciable distance from the optical axis, the incident cone of rays will strike the lens asymmetrically, giving rise to a third primary aberration known as astigmatism if the third Seidel sum S_3 is not zero. In the case of an axial point, the cone of rays is symmetrical with respect to the spherical surface of a lens. There is no need to make a distinction between meridional and sagittal planes. The ray configurations in all planes containing the optical axis are identical.

Now when an object point lies an appreciable distance from the optical axis the incident cone of rays will strike the lens symmetrically giving rise to a third primary aberration known as Astigmatism the third Seidel sum S_3 is not there. In the case of an axial point the cone rays is symmetrical with respect to the spherical surface of the lens there is no need to make a distinction between Meridional and Sagittal planes.

The ray configurations in all planes containing the optical axis or identical there, in contrast here the configuration of an oblique parallel ray will be different in the Meridional and Sagittal planes as a result before Collins in this principle in these planes will be different. In effect here the Meridional rays are tilted more with respect to the lens than or the Sagittal planes rays and they have therefore the shorter focal length.

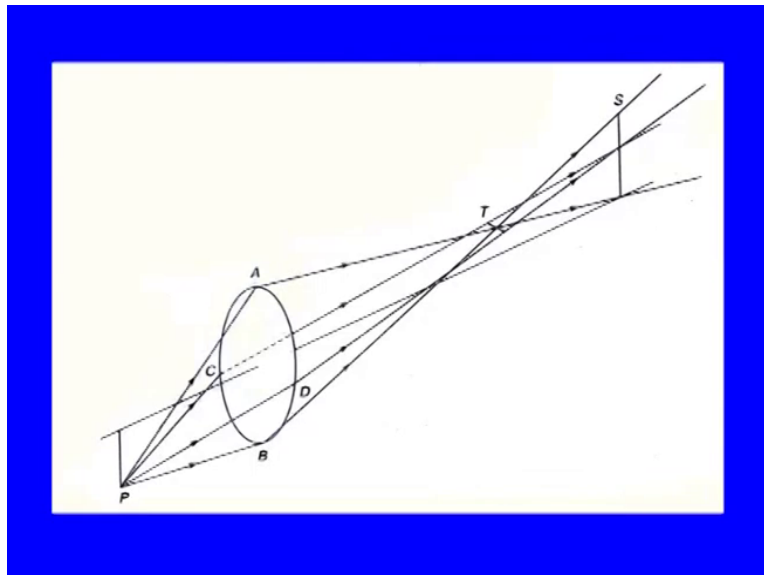
This Astigmatic difference increases rapidly as the rays become more and more oblique that is as the object point moves further of the axis and is naturally 0 when the object is on the axis.

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Having two distinct focal lengths, the incident conical bundle of rays, takes on a considerably altered form after refraction. The rays in the meridional plane converge at a different point as compared to those in the sagittal plane.

Having two different focal lenses the incident conical bundle of rays diverging from the object takes on a considerably altered form after reflection. The rays in the Meridional plane converged at a different point as compared to those in this Astigmatic plane.

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Now this figure shows rays diverging from the point P in the Meridional plane constituted by rays PA and PB they are focused at the point T while the rays for the Sagittal plane constituted by rays PC and PD are focused at the point S their focus is a little farther away at the point S, the rays PA and PB in the figure focus at the point T and rays PC and PD focus at a point S which is different from T as shown in the figure.

Now since at the point T the rays in the Sagittal plane have not still focused. One in fact has a focal line which is normal to the Meridional plane this focal line T is called the initial focus, Meridional focus are the primary image.

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Beyond this point, the beam's cross section rapidly opens out until it is again circular. At this location, the image is a circular blur, known as the circle of least confusion.

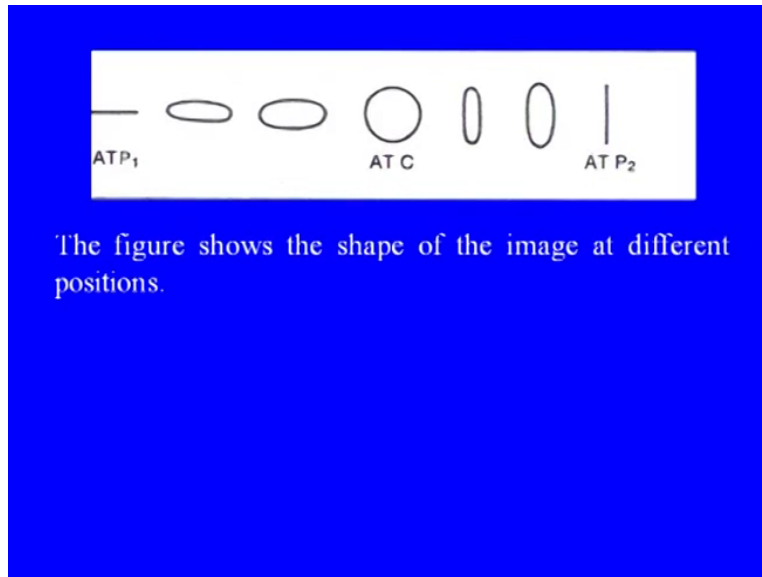
Now beyond this point the beams cross section rapidly opens out until it is again almost a circle. At this location image in a circular blur not very well focused naturally and known as the circle of least confusion.

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Moving further at S , the rays in the meridional plane have defocused, one again obtains a focal line lying in the tangential plane; this is called the *sagittal focal line or the secondary image*. It is in the meridional plane. The distance between S and T is a measure of astigmatism.

Moving further at S rays in the Maridional plane have now defocused one again obtains a focal line lying in the tangential plane and this is called the Sagittal focal line or the secondary image. It is in the Maridional plane the distance between S and T is a measure of astigmatic.

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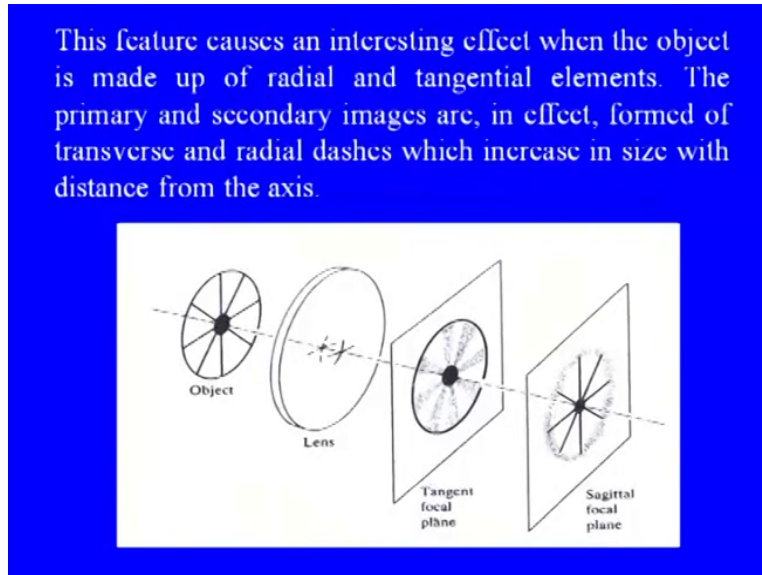
This figure shows that variation at P_1 this is what in the figure was the point T the tangential image where the rays in the Maridional plane are focused. Then as we move away from it away from the lens the beam bundle of beam opens up a little more becomes circular then again begins to close in a perpendicular direction close it further and at the point P_2 . You get the image where the Sagittal rays are in focus.

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One thing should be noted. The secondary “line” image will change in orientation with changes in the object position, but it will always point toward the optical axis; that is, it will be radial. Similarly, the primary “line” image will vary in orientation, but it will remain normal to the secondary image.

One thing should be noted the secondary line image will change in orientation, the changes in the object position but it will always point towards the optical axis that is it will be radial that is the interesting thing. Similarly the primary image where the Maridional rays came in focus this line image will vary in orientation but it will remain normal to the secondary image.

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You see this features causes a very interesting effect when object is made up of radial and tangential elements. The primary and secondary majors are in fact formed of the transverse and radial dashes. Let us look at this little carefully this is where we are interested in forming the image of an object which has got a rim and radial spokes okay. After passing through the lens if we consider here in the tangential focal plane T this is where the Maridional rays have come in focus.

This is the transverse image, so the rim is getting focused the spokes they are very blurred but then as we move it to the Sagittal focal plane now in this case we are getting the radial spokes are focused and the tangential rim has gone out of focus.

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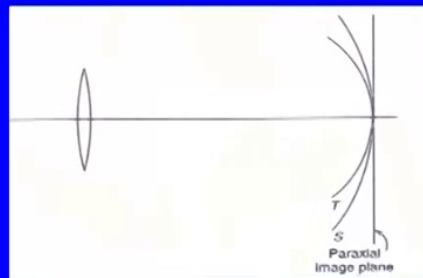
To analyse the origin of astigmatism one finds that for a point on the axis (when the lens is free from other aberrations) the wavefront emerging from the lens is spherical and thus, as the wavefront progresses, it converges to a single point. But when the object point is non-axial, then the emerging wavefront is not spherical and as the wavefront converges, it does not focus to a point but to two lines, which are normal to each other. Somewhere between the two line, the image is circular and is called the circle of least confusion.

Now to analyze the origin of astigmatic 1 points that for a point on the axis and the lens is free from other aberrations. The wave fronts emerging from the lens are the spherical and us as the wave front progresses that is moves it converges to a single point. But when the object point is non-axial quite away from the axis then the emerging wave front is not a spherical anymore. And as the wave front converges it does not focus to a point.

But to two lines which are normal each other the Maridional line and Sagittal line. Now somewhere between these two lines image is circular and is called the circle of least confusion. But is it is still not too very well focus there.

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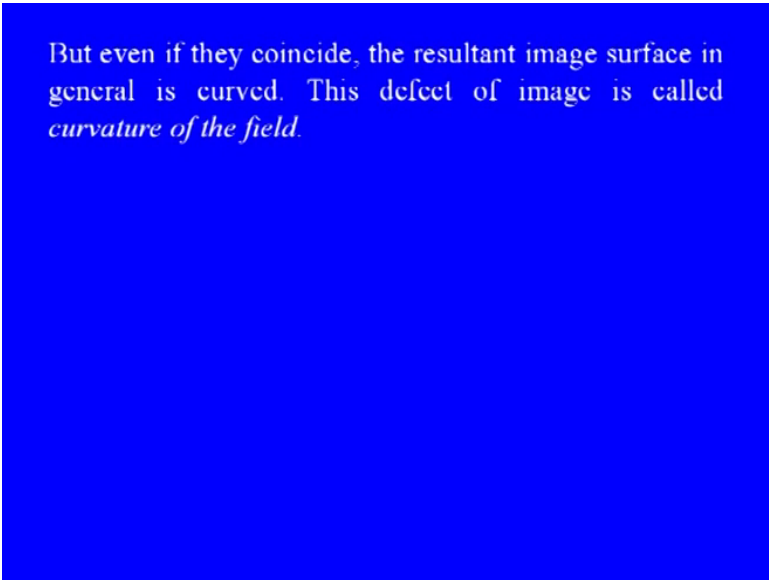
The distance between the tangential and sagittal foci increase as the object point moves away from the axis. Thus the tangential foci and the sagittal foci of points at different distances from the axis lie on two surfaces. The optical system is said to be free from astigmatism when the two surface coincide.



Now the distance between the tangential foci and Sagittal foci increase as the object point moves away from the axis. Thus the tangential foci and Sagittal foci of the points at different distances from the axis lie on two surfaces. The optical system is said to be free from astigmatism when the two surfaces coincide they will still be curved. But we shall come to that a little later. One thing we know these two surfaces they are coinciding at the optical axis of the system.

But naturally that the axial point this is the problem of astigmatism is there as we move away from the axis.

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But even if they coincide, the resultant image surface in general is curved. This defect of image is called *curvature of the field*.

And as I pointed out earlier even if we surfaces coincide okay the SD committal will not be there but the resultant image surface is in general curve and this defect of image is called curvature of the field.

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The shape of the image surface depends on the shape of the lens and the position of the stops. If the primary surface is to the left of the secondary image surface, astigmatism is said to be positive, otherwise negative. By using a convex and a concave lens of suitable focal lengths and separated by a distance, it is possible to minimize the astigmatic difference. Such a lens combination is called an *anastigmat*.

Now the shape of the image surface depends on the shape of the lens and the position of the stops. If the primary surface is to the left of the secondary image astigmatism is said to be positive otherwise negative. Now by using a convex and concave lens of suitable focal lengths and separated by a distance it is possible to minimize the astigmatic difference. Such a lens combination is called astigmatism. Now we come to another defect and the reparation called curvature.

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Curvature

The image of an extended plane object due to a single lens is not a flat one, but is a curved surface. The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. This defect is called *the curvature of the field*.

The image of an extended plane object due to a single lens is not a flat one but is a curved surface the central portion of the image nearer the axis is in focus but the outer region of the

image away from the axis those regions are blurred this defect is called the curvature of the field. It is due to the defect that the paraxial focal length is greater than the marginal focal length.

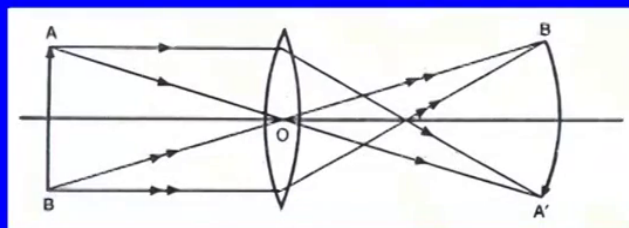
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The defect is due to the fact that the paraxial focal length is greater than the marginal focal length. This aberration is present even if the aperture of the lens is reduced by a suitable stop, usually employed to reduce spherical aberration, coma and astigmatism.

Marginal focal length means the focal length for the marginal rays this separation is present even if the aperture of the lens is reduced by a suitable stop usually employed to reduce a spherical aberration coma and astigmatism.

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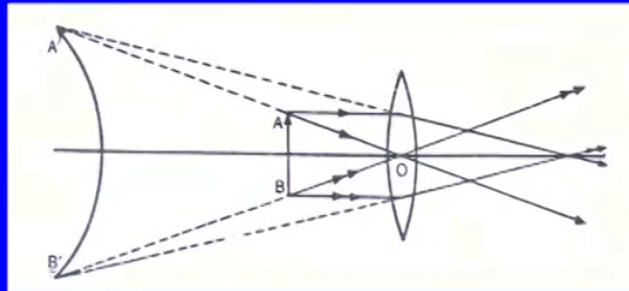
A real image formed by a convex lens curves towards the lens.



You see a real image formed by a convex lens curves toward the lens if the points A and B are mirrored curves towards the lens.

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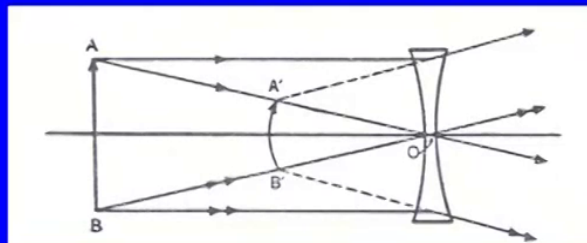
A virtual image curves away from the lens.



For the case of a virtual image which is formed at a distance larger than that of the object A prime and B prime these are curved away from the lens.

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This figure shows curvature of the field present in the image formed by a concave lens.



This figure shows the curvature of the field in the case of a concave lens here again the images formed nearer the ends and the ends A prime and B prime or curved toward lens.

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For a system of thin lenses, to get a flat image

$$\sum_i \frac{1}{n_i f_i} = 0$$

For two lenses, this condition reduces to

$$\frac{1}{n_1 f_1} + \frac{1}{n_2 f_2} = 0$$

This is known as Petzval's condition for no curvature.

Now for the system of thin lenses to get a flat image this is the condition summation over $\frac{1}{n_i f_i}$ summed over the various lens components. For two lenses this condition reduces to $\frac{1}{n_1 f_1} + \frac{1}{n_2 f_2} = 0$ this is known as Petzval's condition for no curvature.

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This condition holds good whether the lenses are separated by a distance or placed in contact. As the refractive indices are positive, the above condition will be satisfied if the lenses are of opposite sign. If one lens is convex, the other must be concave.

This condition holds good whether the lenses are separated by a distance or placed in contact and the refractive indices are positive. The above condition will be satisfied if you lenses are of opposite sign. If one lens is convex the other must be concave.

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In visual instruments a certain amount of curvature can be tolerated, because the eye can accommodate for it. However, in photographic lenses field curvature is most undesirable, since it has the effect of rapidly blurring the off-axis image.

Curvature of the field can be corrected for a single lens by means of a stop

Now individual instruments a certain amount of curvature can be tolerated is not very serious. The reason is the eye can accommodate for it however in photographic lenses field curvature most undesirable since it has the effect of rapidly blurring the off axis image. The curvature of the field can be corrected for single lens by means of a stop.

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Distortion

The variation in the magnification produced by a lens for different axial distances results in an aberration called *distortion*. To be free of distortion a system must have uniform lateral magnification over its entire field. A pin-hole camera is ideal in this respect, for it shows no distortion, all straight lines connecting each pair of conjugate points in the object and image planes pass through the opening.

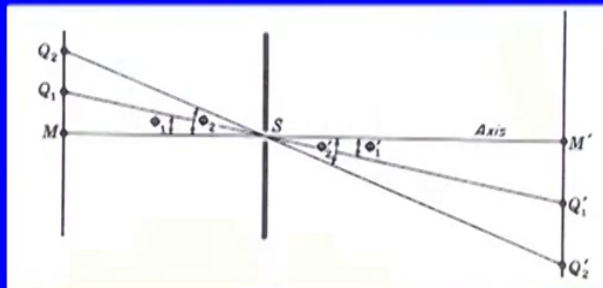
Now we come to the last one of mono chromatic aberrations and that is distortion. The variation in the magnification produced by a lens for different actual distances results in an aberration called distortion. To be free of distortion the system must have uniform lateral magnification over its entire field. A pinhole camera is ideal in this respect what it shows no distortion at all. All the

state lines connecting each pair of conjugate points in the object image planes they pass through the opening.

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Constant magnification for a pin-hole camera as well as for a lens implies

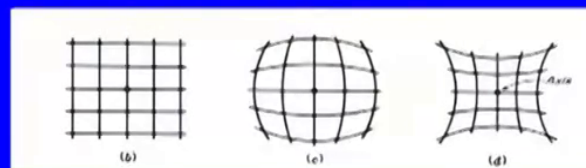
$$\frac{\tan \varphi'}{\tan \varphi} = \text{const.}$$



The constant magnification for a pinhole camera as well as for a sim lens implies then Phi prime upon tan Phi should be a constant. If this figure shows several ways MM prime is the actual ray then for the ray Q1 upon Q2 Q1 Q1 prime, the angle is Phi1Phi1 prime, this ratio is same as for the ray Q2 Q2 prime and the angles are Phi2 and Phi2 prime.

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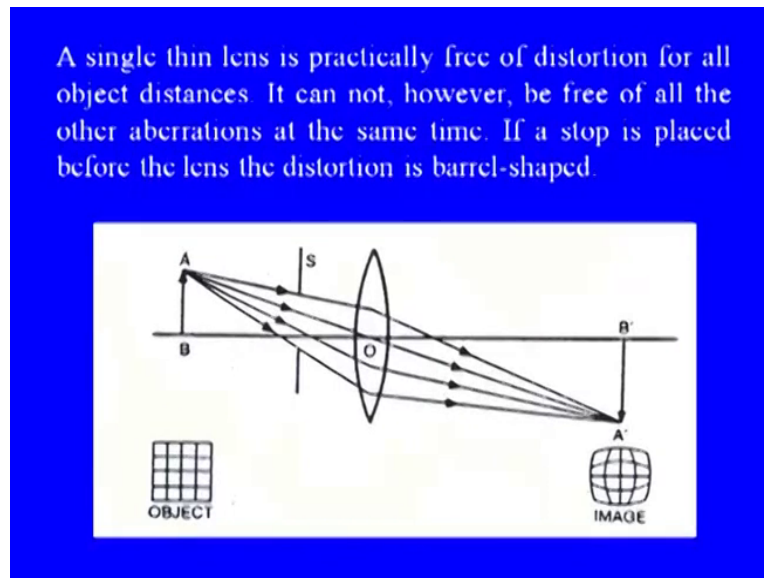
The common forms of image distortion produced by lenses are shown in the figure. First one represents the undistorted image of an object consisting of a regular wire mesh. Next diagram shows *barrel* distortion which arises when the magnification decreases towards the edge of the field. The third represents *pin cushion* distortion, corresponding to a greater magnification at the borders.



Common forms of image distortion in an actual lens system are shown in the figure here. First one represents the undistorted image of an object consisting of a regular wire mesh. The next

diagram shows the barrel distortion this arises when the magnification decreases towards the edge of the field it is more near the center of the field. And the third figure represents a pincushion distortion corresponding to a greater magnification at the borders leading to looks like a stretched configuration.

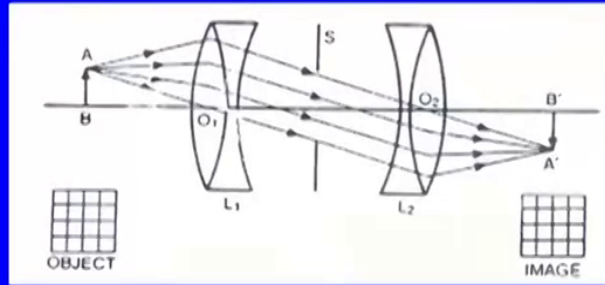
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A single thin lens is particularly free of distortion for all object distances it cannot however be free for all the other aberrations at the same time. Now if a stop is placed before the lens that is in the object space the distortion is barrel shaped, more magnification near the axis and less towards the edges? And if I stop is placed after the lens that is in the image region the distortion is pincushion type stretching at the edges.

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To eliminate distortion a stop is usually placed in between two symmetrical lenses, so that the pin cushion distortion produced by the first lens is compensated by the barrel-shaped distortion produced by the second lens. Projection and camera lenses are constructed in this way.



So, that immediately gives us an idea to eliminate distortion is stop is usually placed in between two symmetrical lenses so that the pincushion distortion produced by the first lens compensated by the barrel shape distortion produced by the second lens. Projection and camera lenses are constructed in this way.

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Summarizing very briefly various methods of correcting for monochromatic aberrations, spherical aberration and coma can be corrected by using a contact doublet of the proper shape, astigmatism and curvature of field require for their correction the use of several separated components, and distortion can be minimized by the proper placement of a stop.

Now let us summarize these things briefly all the aberrations we have gone through the various methods of correcting for on a chromatic aberrations, spherical aberration and coma we have seen can be corrected by using a contact doublet of the proper shape. Astigmatism and curvature of the field require for the correction the use of several separated components the distortion can

be minimized by the proper placement of a stop in between the components of a lens system.

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Let us see this problem in totality and keeping in mind different types of optical instruments. The lens may be affected by as many as seven primary aberrations --- five monochromatic aberrations and two chromatic aberrations. One might therefore wonder how it is possible to make a good lens at all when rarely can a single aberration be eliminated completely, much less all of them simultaneously.

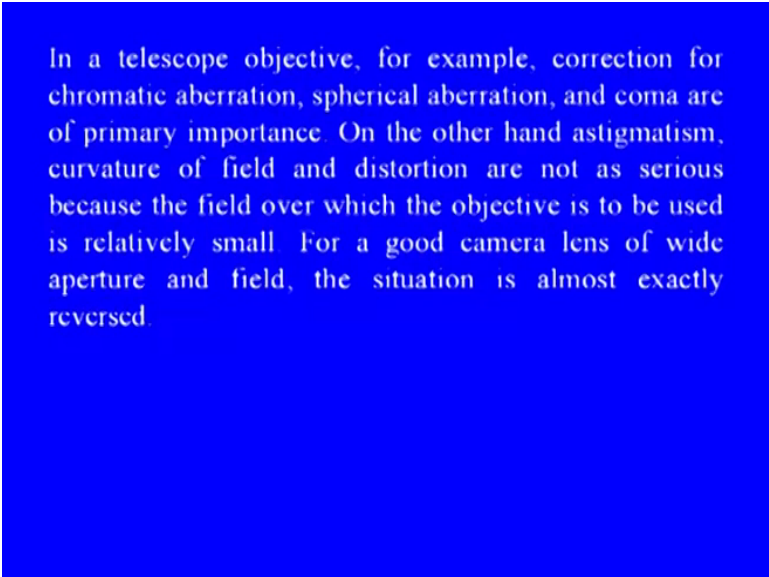
Now let us see this problem in totality and keeping in mind different types of optical instruments the lens may be affected by as many as 7 primary aberrations, 5 monochromatic aberrations as we have seen and 2 chromatic aberrations longitudinal and lateral. Now one might therefore wonder how it is possible to make a good lens at all when rarely can a single aberration be eliminated completely much less all of them at the same time simultaneously.

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Good usable lenses are nevertheless made by the proper balancing of the various aberrations. The design is guided by the purpose for which the lens is to be used.

This is the good usable insular nevertheless made by the proper balancing of the various aberrations. You see the design is guided by the purpose for which the lens is to paint. You see that is the main thing where the lenses to be used then guides which aberrations are to be eliminated or minimize or properly to be taken care of.

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In a telescope objective, for example, correction for chromatic aberration, spherical aberration, and coma are of primary importance. On the other hand astigmatism, curvature of field and distortion are not as serious because the field over which the objective is to be used is relatively small. For a good camera lens of wide aperture and field, the situation is almost exactly reversed.

In a telescope objective for example correction for chromatic aberration, spherical aberration and coma are of primary importance. On the other hand astigmatism, curvature of the field and distortion are not a serious because the field over which the objective is to be used is relatively small. For a good camera lens of wide of aperture wide angle the field situation is almost exactly reverse. In case I think this we have come to the end of this lecture series hope you have liked it, thank you.