Engineering Physics 1

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Module-01
Lecture-04

Interference of Polarized Light

This is the fourth lecture of these five lecture series on polarization. In the last lecture which was the third one we considered phenomena of double refraction, looked the dichroism and then studied the working of a Nicol prism. So, double refraction is a very typical phenomena of an isotopic crystals. We considered crystals of calcite and quartz.

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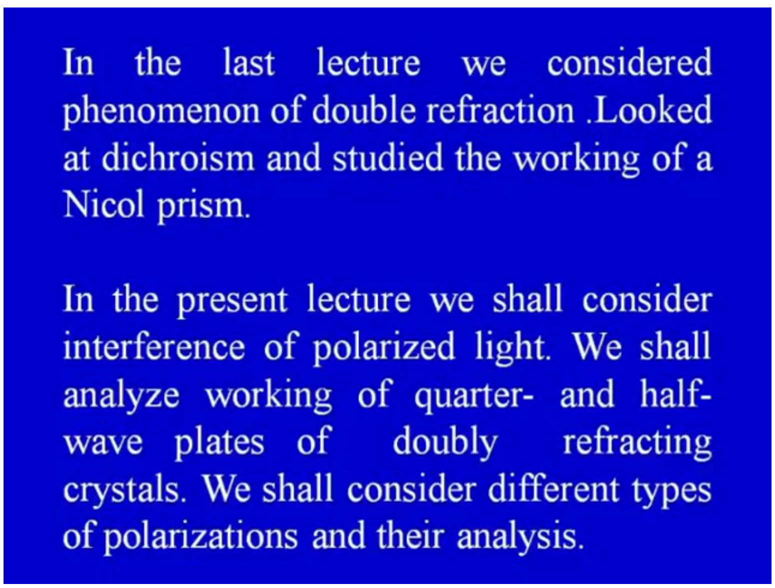

What happens in these when a light beam is incident, there are two refracted rays one the so called ordinary; ordinary in the sense that, it obeys Snell's laws of refraction, the usual one. The speed is same in all directions, the phase front is a spherical that it ends the refracted beam called the extraordinary beam where the direction of propagation is different it moves with different speeds in different directions.

The result is that the phase fronts are ellipsoid of revolution about the optic axis. Optic axis is the axis of symmetry; the two beams ordinary and extraordinary, both of them are plane polarized with vibrations in perpendicular directions. In the case of ordinary, the direction of vibration is perpendicular to the plane containing the propagation direction and the optic axis. In the case of extraordinary beam, these vibrations are in the same plane. Both of them these beams are plane polarized.

We also studied dichroism you see dichoric core crystals like tourmaline. They are also doubly refracting with the additional property that they have a very markedly unequal absorption of the ordinary and extraordinary beams, with the result that the merchant beam is plane polarized to a petty good degree. Then, we considered Nicole prism. Here the ordinary beam is totally reflected things are arranged in such a way and the result is, the emergent beam is plane polarized and we have got a polarizing device.

In the present lecture which is the fourth one, we shall consider interference of polarized light. We shall analyze working of quarter and half wave plates of doubly refracting crystals. We shall consider different types of polarization and their analysis, Interference of polarized light.

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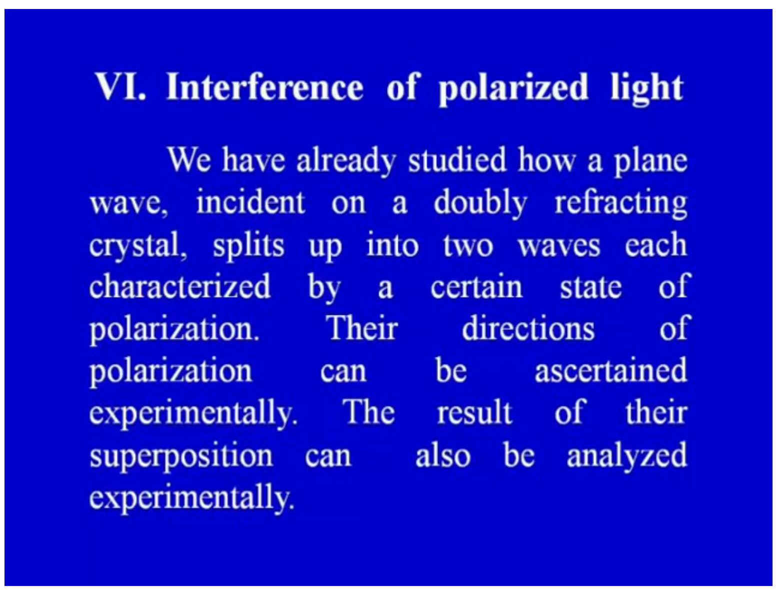

We have already studied how a plane wave incident on a doubly refracting crystal, splits up into two waves within the crystal, each characterized by a resultant state of polarization. Their directions of polarization can be ascertained experimentally. The result of their superposition can also be analyzed experimentally.

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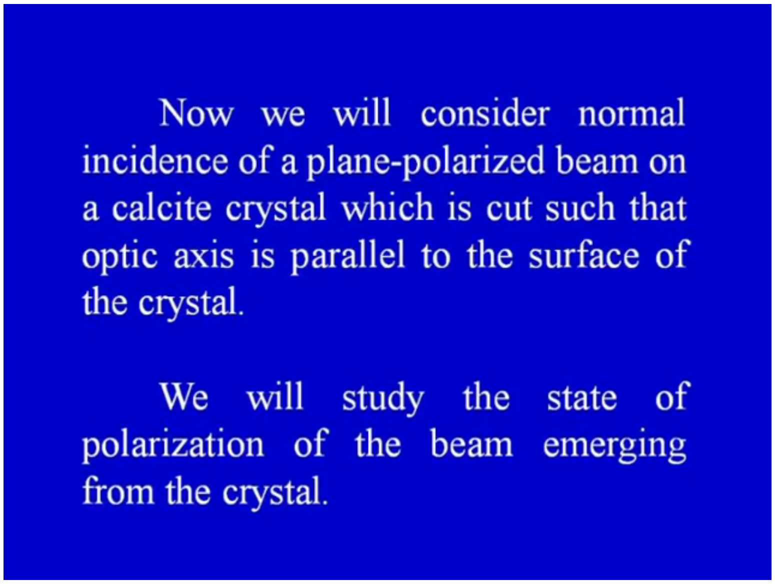

Now, we will consider normal incidence of a plane polarized beam on a calcite crystal which is cut such that the optic axis is parallel to the surface of the crystal. The propagation here is perpendicular to the optic axis.

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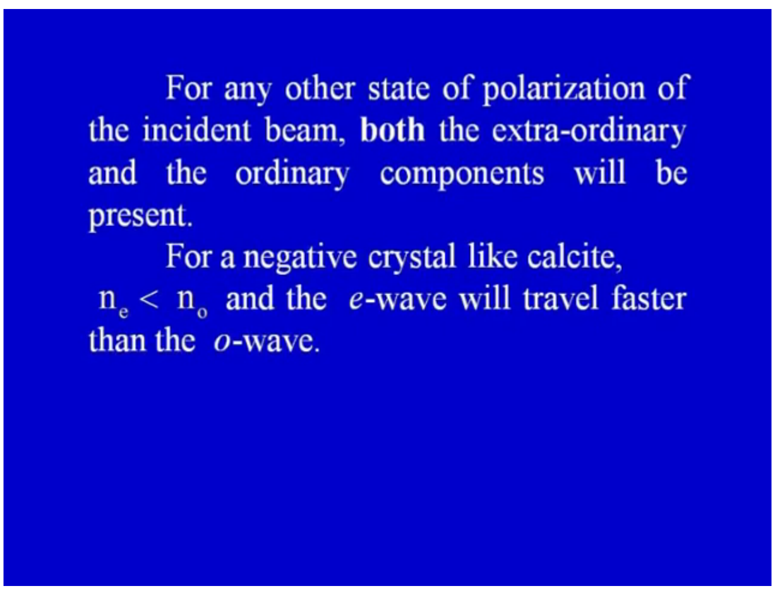

We will study the state of polarization of the beam emerging from the crystal.

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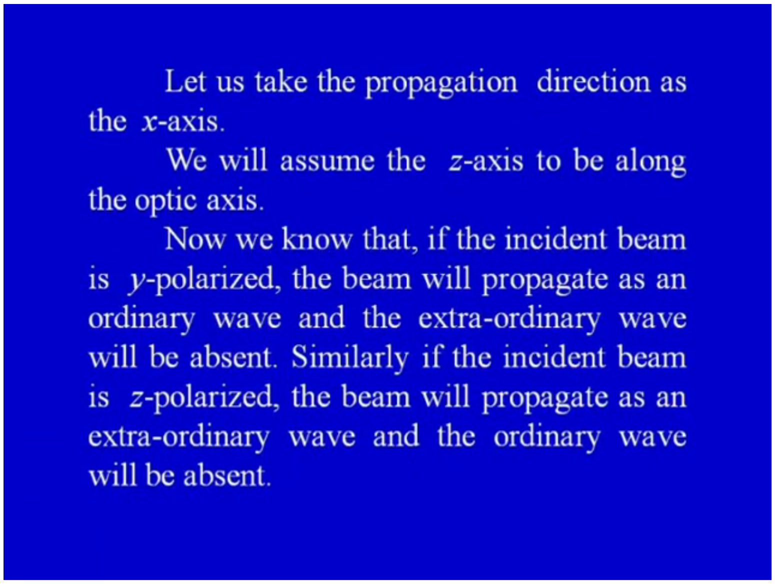

Let us make the propagation direction as the x-axis, just choose it like that. We will assume the z axis to be along the optic axis which is the axis of symmetry in the crystal. Now, we know that if the incident beam is y polarized, the beam will propagate as an ordinary wave and the extraordinary wave will be absent. In this case the vibrations are in that explained. Similarly, if the beam is z polarized the beam will propagate as an extraordinary wave and the ordinary wave in this case will be absent.

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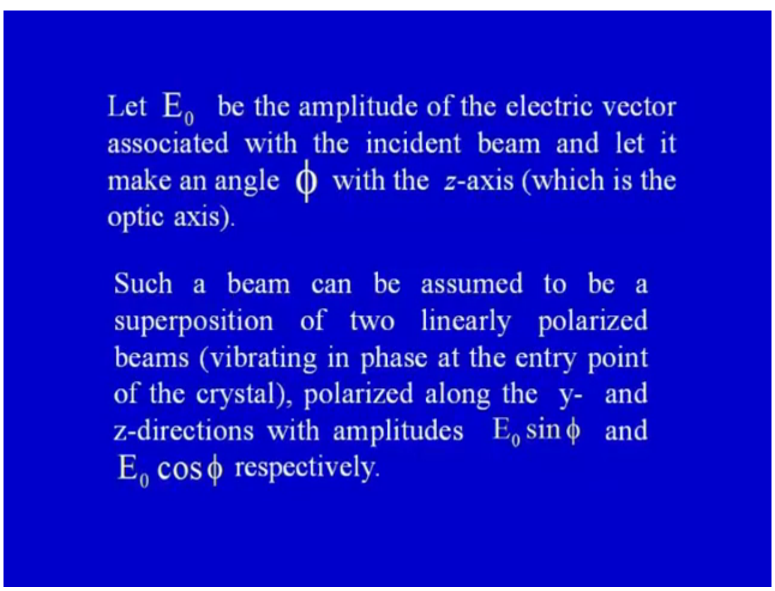

Let E naught the amplitude of electric vector associated with the incident beam and let make an angle Phi with z axis which is the optic axis, as we have chosen. Such a beam can be assumed to be the superposition of two linearly polarized beams vibrating in phase at the entry point of the crystal and polarized along the y and z directions with amplitudes E naught sine Phi and E naught cos Phi respectively.

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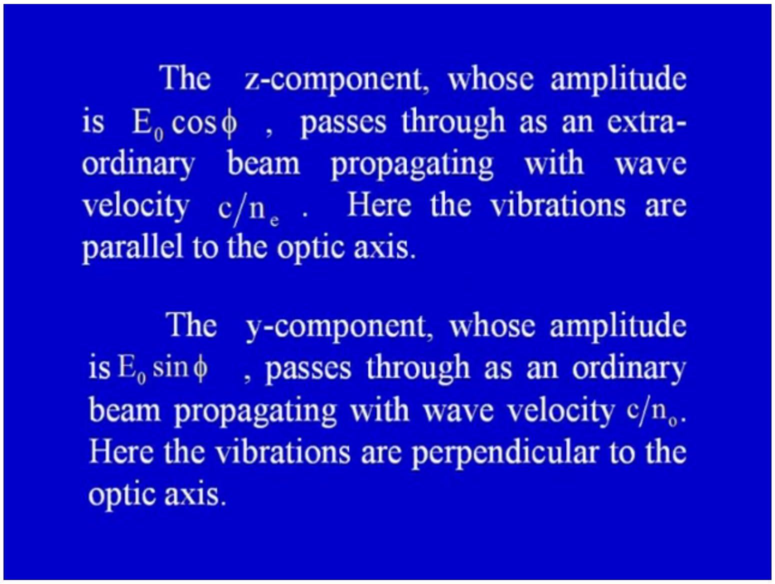

The z component whose amplitude is E naught cos Phi passes through as an extraordinary beam propagating with the wave velocity c by ne, here the vibrations are parallel to the optic axis. The y component whose amplitude E naught sine Phi, passes through as an ordinary beam propagating with the wave velocity c over no. Here the vibrations are perpendicular to the optic axis, also perpendicular to the propagation direction.

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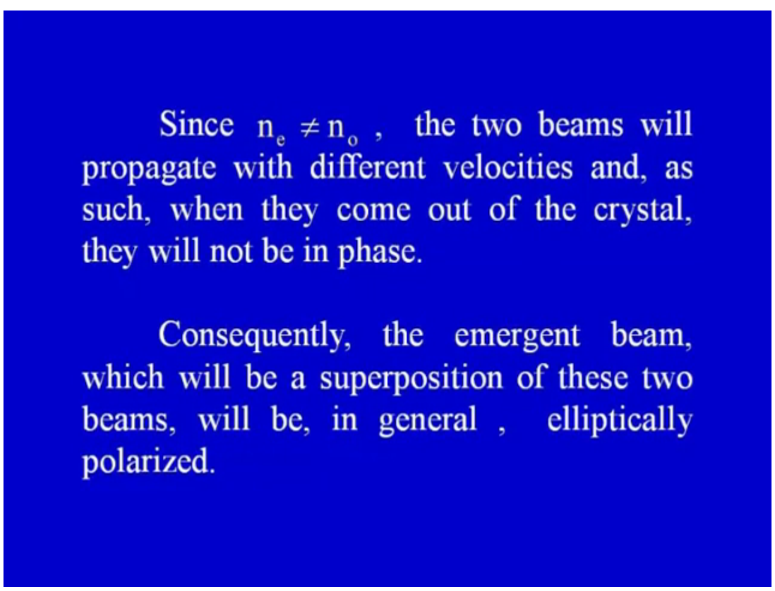

Since n e is not = no, the two beams will propagate with different velocities but there would not be any separation in this case because they are incident normally and as such when they come out of the crystal naturally, they will not be in phase. Consequently, the emergent beam which will be a superposition of these two beams will be in general elliptically polarized. We have discussed this case in the earlier lecture as well. The arrangement thus provides an experimental setup and that is the main thing we are discussing the experimental setup.

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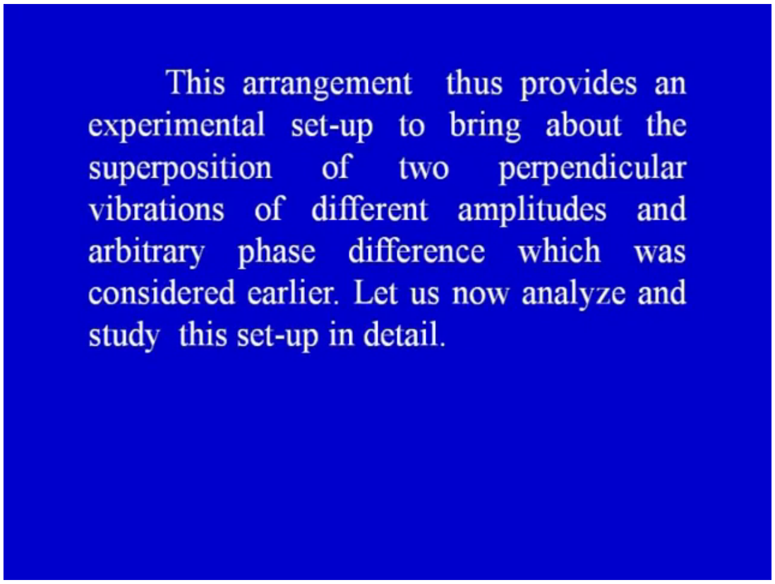

Now to bring about the superposition of two perpendicular vibrations of different amplitudes and arbitrary phase difference which was considered earlier, as I pointed out? You see, the different amplitudes can be obtained by varying the angle Phi and the phase difference can be varied by varying the thickness of the crystal. Let us now analyze and study this setup in detail.

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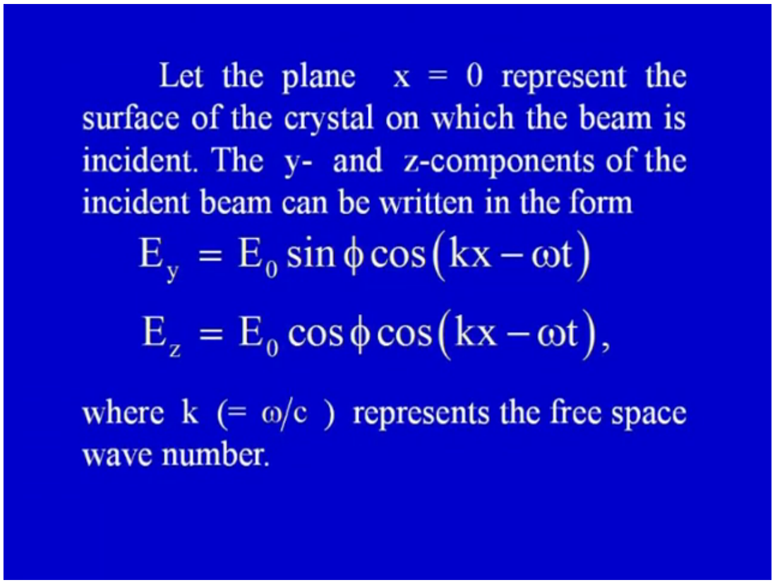

Let the plane x = 0 represent the surface of the crystal on which the beam is incident. x = 0 plane is the incident. The y and z components of the incident beam now can be written in the form Ey = E naught sine Phi. That is the amplitude cos kx - Omega t Ez = E naught cos Phi and cos of kx - Omega t. The propagation is along the x axis k = Omega by c. It represents the free space wave number.

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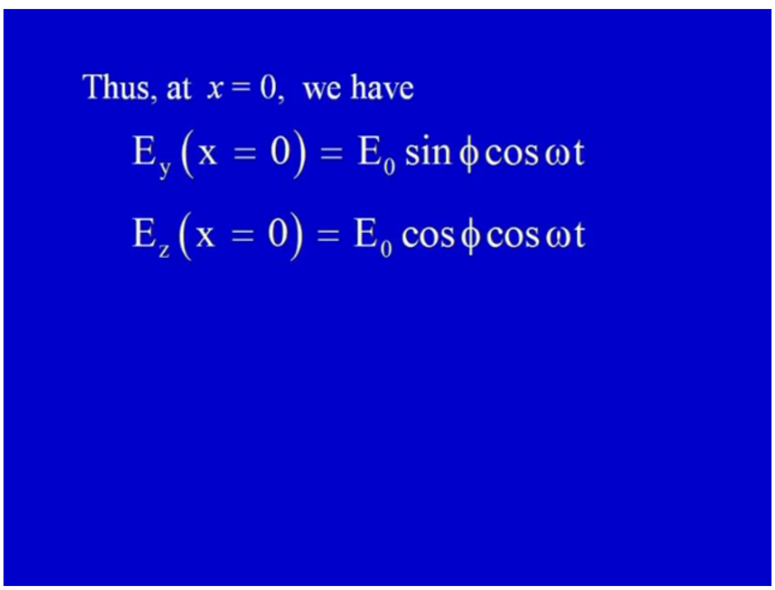

At x = =which was the entry point, we have E y at x = 0 = E sine Phi cos Omega t and Ez at x = 0 = E naught cos Phi cos Omega t.

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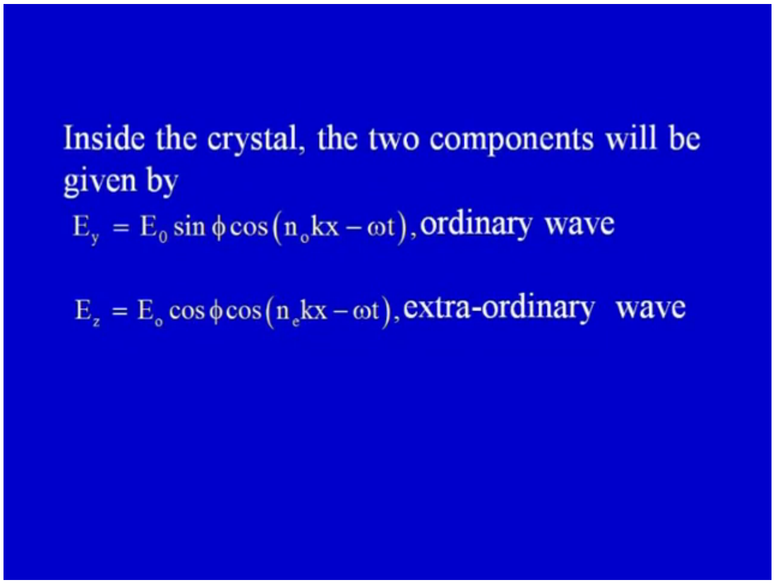

Inside the crystal, the two components will be given by E y = E naught sine Phi cos no and kx - Omega t is for the ordinary, no is the refractive index for the ordinary. And E z = E naught cos Phi and ek cos of ne kx - Omega t. ne the refractive index for the extraordinary. So, this is for the extraordinary width.

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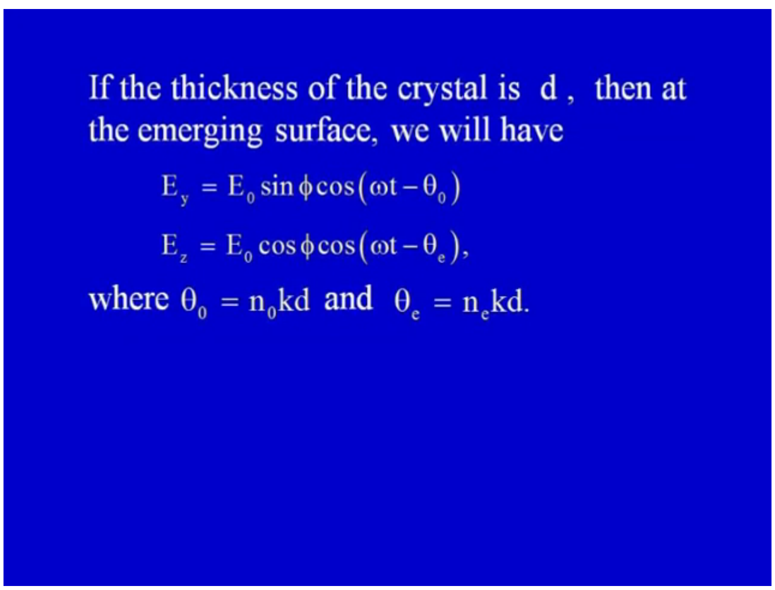

If the thickness of the crystal is d then, after traversing a distance = d, they were at the emerging surface, we will have, Ey = E naught sine Phi cos Omega t - theta 0. Ez = E naught cos Phi cos Omega t - theta e where theta o = n0 kd and theta e = ne kd.

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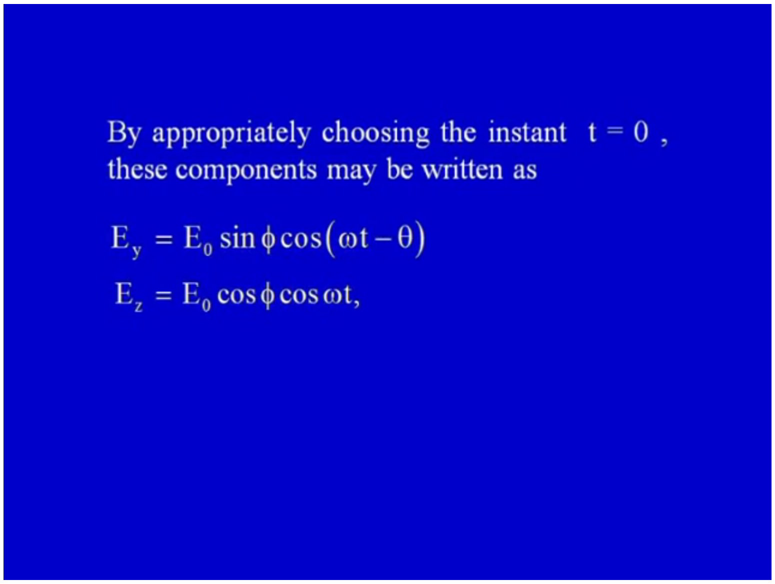

By appropriately choosing the instant t = 0, these components may also be written as in a simpler way Ey = E naught sine Phi cos Omega t - theta and Ez = E naught cos Phi cos of Omega t.

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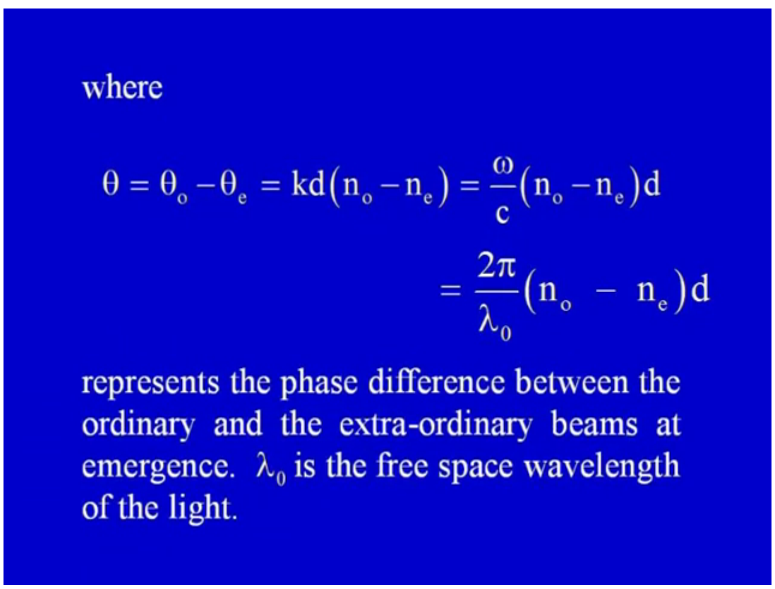

Where theta is the difference between theta o theta e which is = kd times no - ne which = Omega by c times no - ne times d which = 2 Pi by lambda naught into ne - no times d, is represented the phase difference at the emergence, between the ordinary and the extraordinary beams. Lambda naught with the free space wavelength of the light used.

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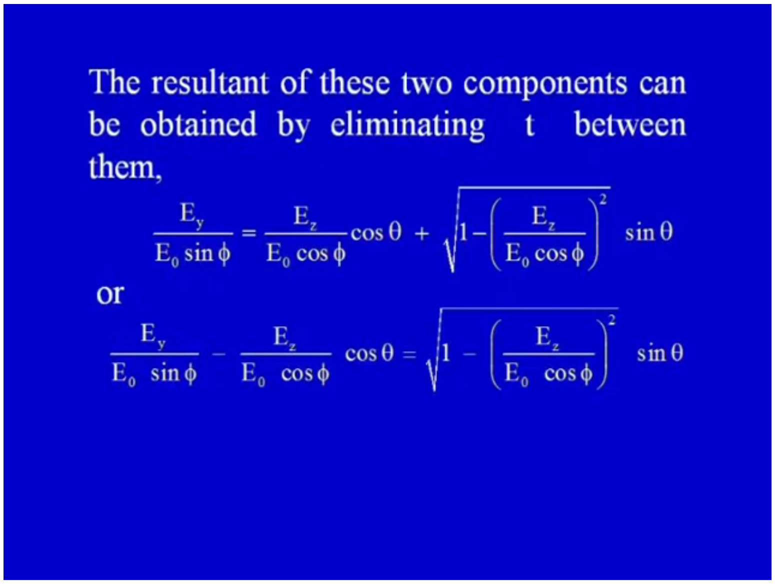

The resultant of these two components can be obtained by eliminating t between them and that gives Ey upon E naught sine Phi, = Ez upon E naught cos Phi times cos theta + the square root of 1 - Ez upon E naught cos Phi squared times sine theta. We can rearrange that and that gives Ey upon E naught sine Phi - Ez upon E naught cos Phi times cos theta which is = the square root of 1 - Ey upon E naught cos Phi squared times sine theta

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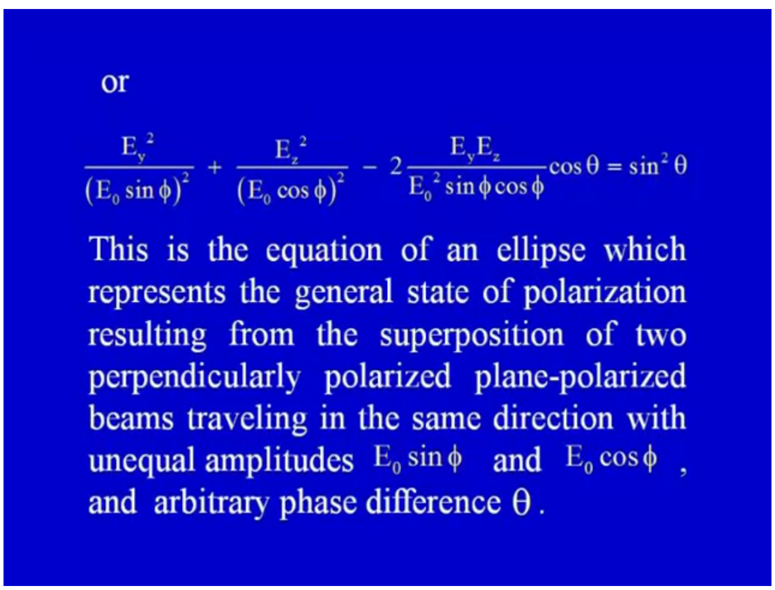

or Ey square upon E naught sine Phi square + E z square upon E 0 cos Phi squared - twice of Ey Ez upon E 0 square sin Phi cos Phi times cos theta = sine square theta. This is the equation of an ellipse which represents the general state of polarization resulting from the superposition of two perpendicularly polarized, plane polarized beams, traveling in the same direction, with an equal amplitudes E naught sine Phi and E naught cos Phi and an arbitrary phase difference theta.

Various cases corresponding to different values of theta were in a general way discussed in our in our earlier lecture. We shall now, in this lecture consider in detail, especial situations where theta = 2Pi or Pi or Pi by 2 and Phi = 45 degrees Pi by 4 and how to obtain them how to realize them.

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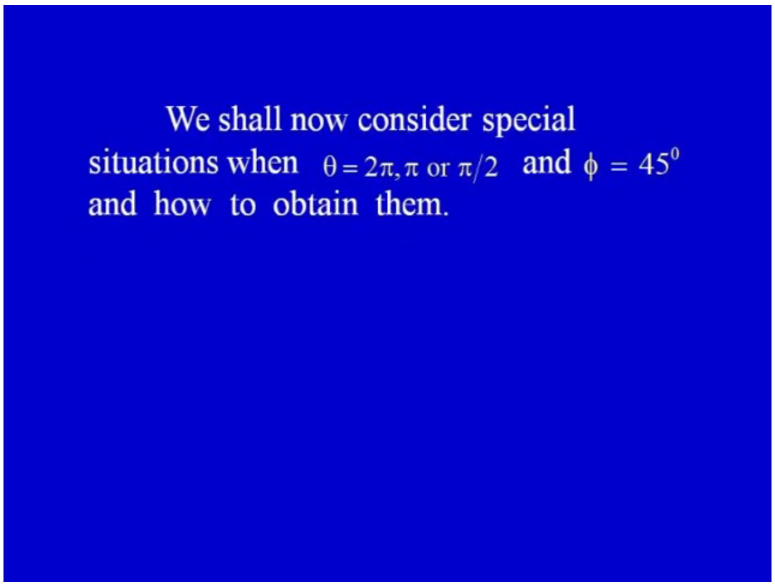

In the lab performing the experiment that is the main idea of today's lecture, let us consider these full half and quarter wave plates.

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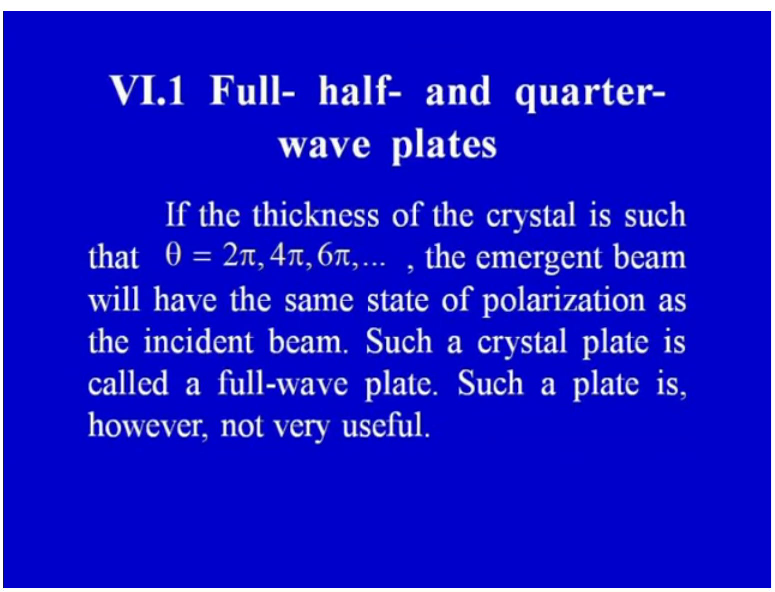

Remember, we are talking about a crystal plate where the optic axis is in the plane of the crystal. And the propagation is perpendicular to the optic axis and the incidence is normal under those conditions. If the thickness of the crystal is such, the theta = 2 PI, 4Pi 6Pi, I mean one complete vibration, 2 complete vibration, 3 complete vibrations, the emergent beam will have the same state of polarization as the incident beam. Such a crystal plate is called a full wave plate. Such a plate however is not very useful.

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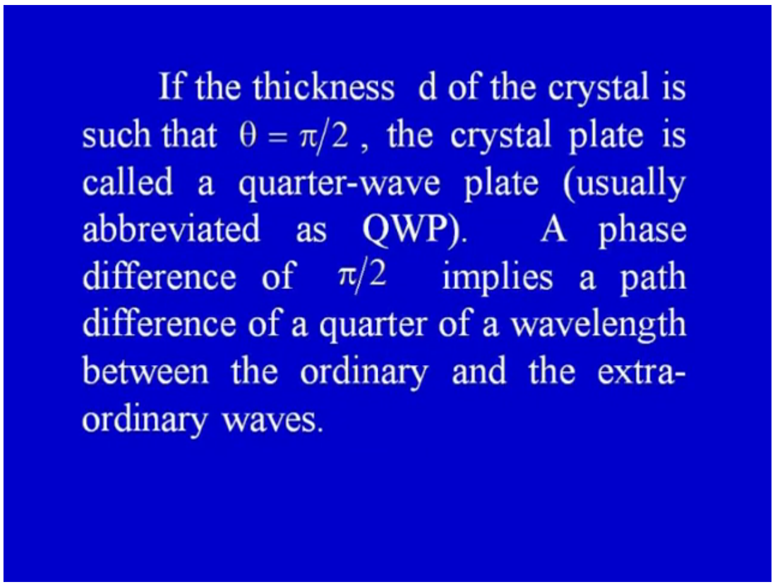

If the thickness d of the crystal is such that theta = Pi by 2, a quarter vibration, the crystal plate is called a quarter wave plate. Usually abbreviated as QWP; Phase difference of Pi by 2 implies a path difference of a quarter of wavelength that is lambda by 4, between the ordinary and the extraordinary waves.

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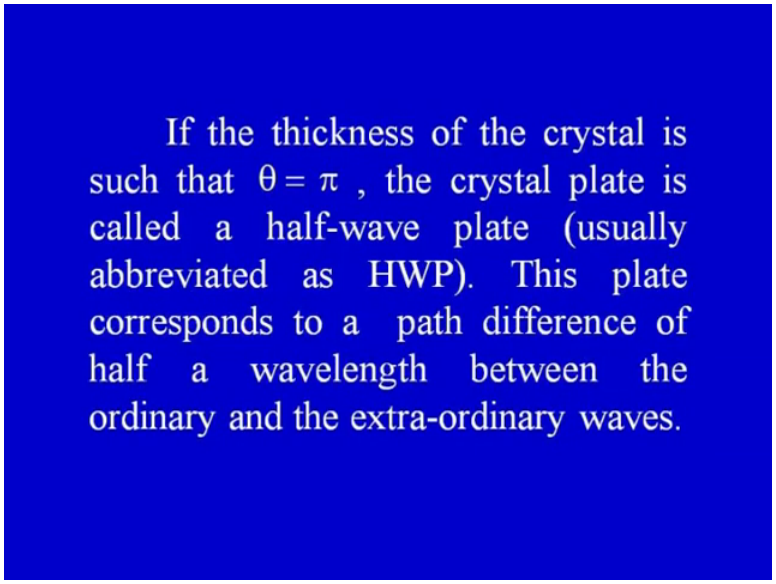

Similarly, if the thickness of the crystal is such the theta = Pi, the crystal plate is called a half wave plate, usually abbreviated as HWP. This plate corresponds to a path difference of half a wavelength that is lambda by2, between the ordinary and the extraordinary waves. Let us consider in detail the working of a quarter wave plate.

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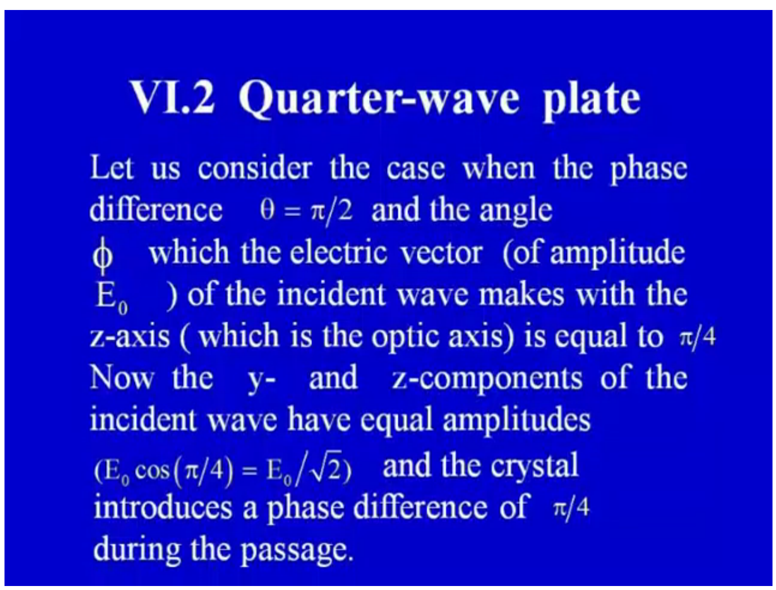

Let us consider the case when theta = Pi by 2 and the angle Phi with the electric vector of amplitude E naught of the incident wave, makes with the z axis, which is the optic axis. Remember, the optic axis is the symmetry axis and the propagation here is perpendicular to the optic axis. So, the angle Phi = Pi by 4. Now the Pi and z components of the incident wave have equal amplitudes in hard course of Pi by 4 or E naught sine of Pi by 4.

Both of them are = E naught upon a square root of 2 and the crystal introduces a phase difference of Pi by 4 during the passage.

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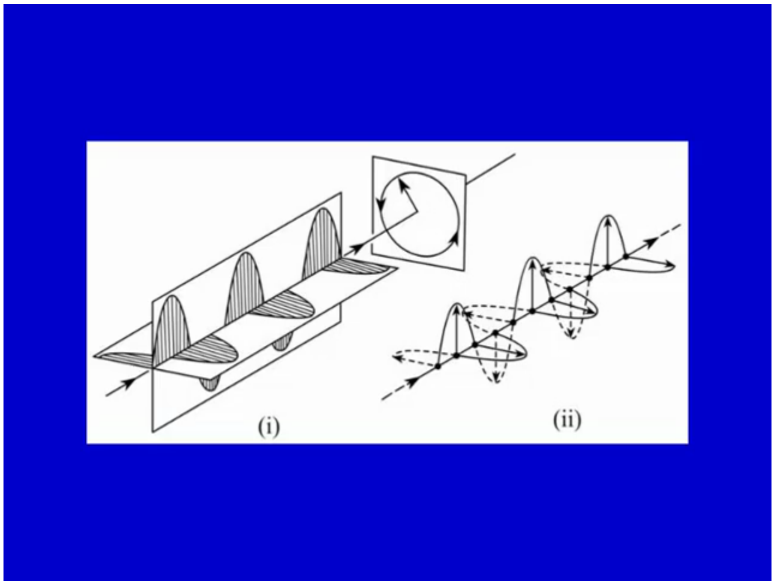

The figure shows the progressive phase difference between the ordinary and the extraordinary beams.

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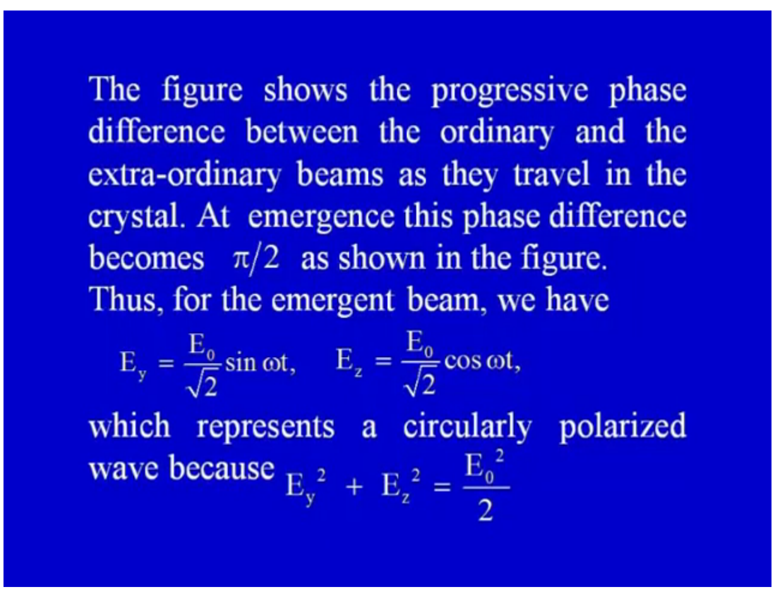

As they travel in the crystal in the beginning they are in the same phase at the entry point. At emergence at the other end of the crystal, after traveling a distance d, this phase difference in this case becomes Pi by 4 as shown in the figure. Just for the emergent beam, we have, Ey = E naught upon a square root of 2 times sine omega t and E z = E naught upon the square root of 2 cos Omega t which represents circularly polarized wave. Because Ey square + Ez square = E naught square by 2 which is the equation of a circle.

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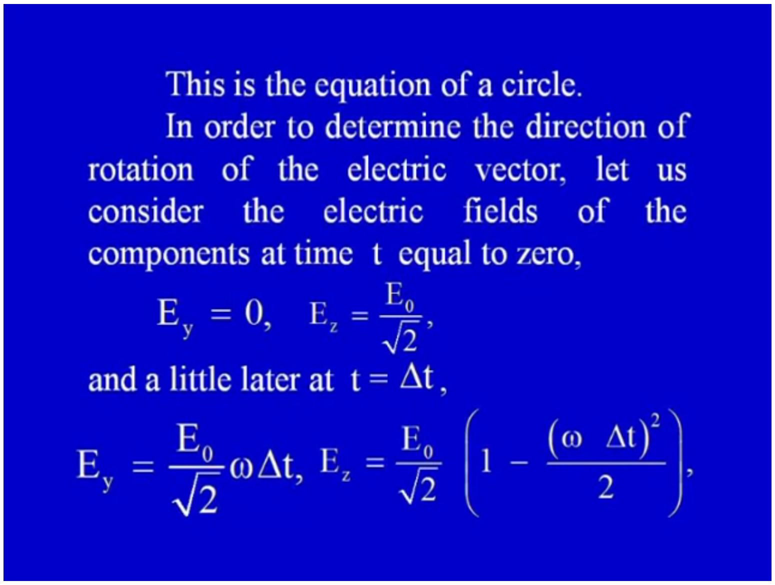

In order to determine the direction of a rotation, I mean the sense of rotation of the electric vector. Let us consider the electric fields of the components initially at time t = 0 Ey = 0 and Ez = E naught upon square root of 2. And a little time later at t = delta t, Ey becomes = E naught upon the square root of 2 times omega times delta t and Ez = E naught upon square root of 2 times 1 - omega delta 2 squared by 2.

The y component has slightly increased and the z component has slightly decreased. That is why the above values show that as the time increases, the electric vector rotates in the anti clockwise direction.

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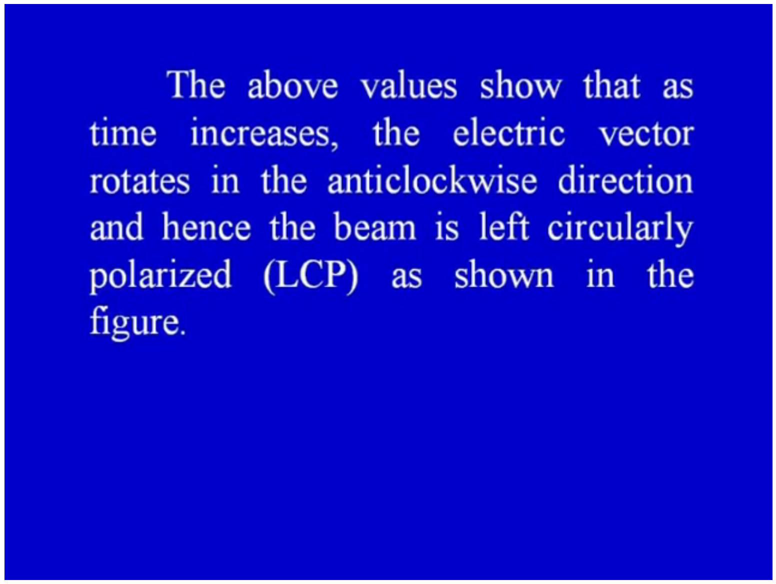

And hence the beam is left circularly polarized LCP as shown in the figure.

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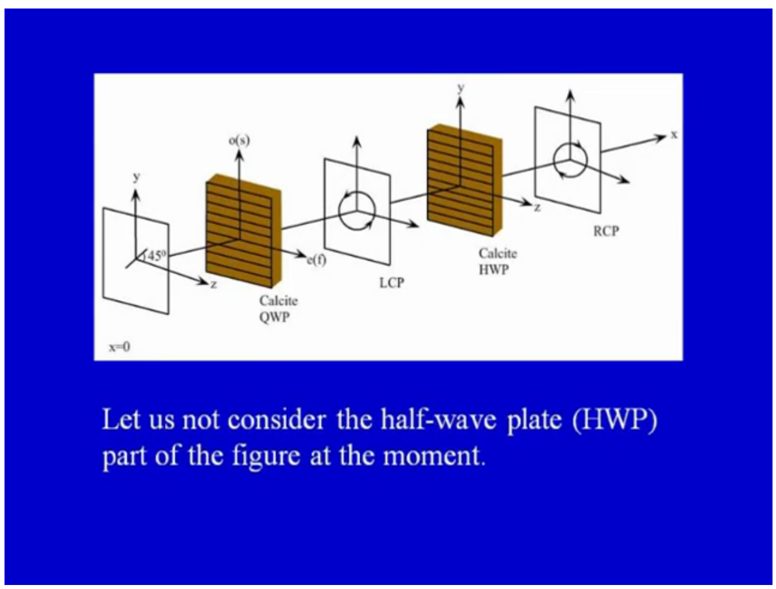

The passage through the quarter wave plate makes the initially plane polarized light. With equal amplitudes, the quarter wave plate introduces a phase difference of Pi by 2, of quarter vibration the result is we get a left circularly polarized light.

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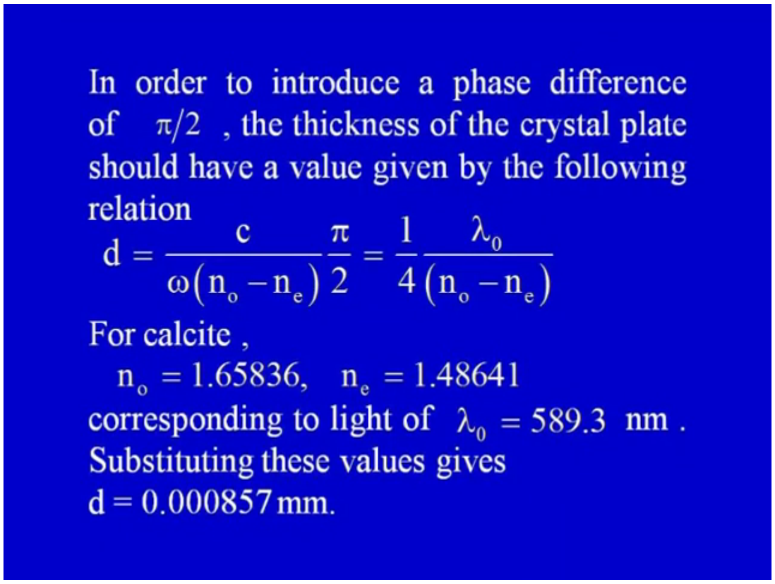

In order to introduce a phase difference of Pi by 2, the thickness of the crystal plate should have a value given by the following relation d = c upon Omega into no - ne times Pi by 2 which = lambda naught by 4 divided by no - ne. For calcite, no was given by 1.65836 and ne = 1.48641. This is corresponding to a light of wavelength 589.3 nanometers. And substituting these values gives d = 0.000857 millimeters.

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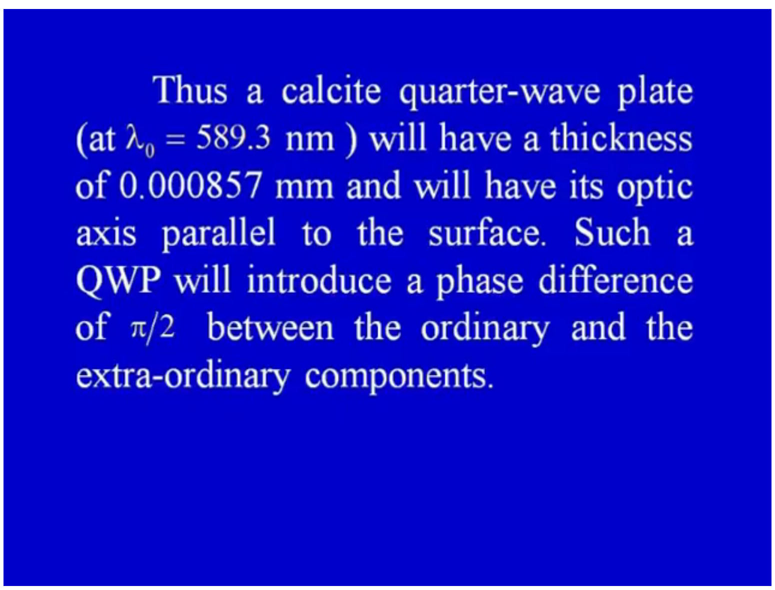

Thus a calcite quarter wave plate at lambda = 589.3 nanometers, will have a thickness of 0.000857 millimeters and will have its optic axis parallel to the surface. Remember, all the time the propagation is perpendicular to the optic axis and there is no separation between the two beams. Such a quarter wave plate will introduce a phase difference of Pi by 2 between the ordinary and extraordinary components.

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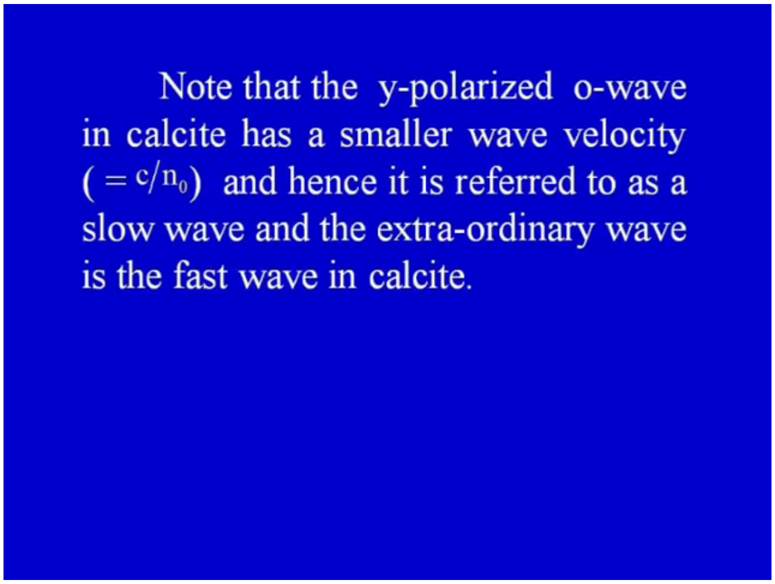

Note that y polarized ordinary wave in calcite has a smaller wave velocity co no and hence it is referred to a slow wave and the extraordinary wave is the fast one in calcite.

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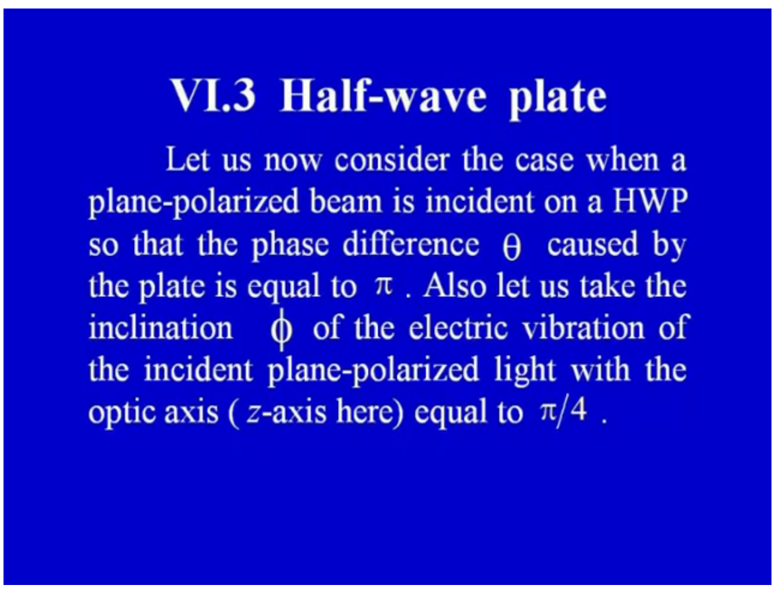

Let us consider now a half wave plate. It is the case when a plane polarized beam with incident on HWP, half wave plate so that the phase difference theta caused by the plate = Pi half a vibration, path difference of lambda by 2. Again let us take the inclination Pi of the electric vibrations of the incident plane polarized beam with the optic axis which we have taken it z axis here again = Pi by 4.

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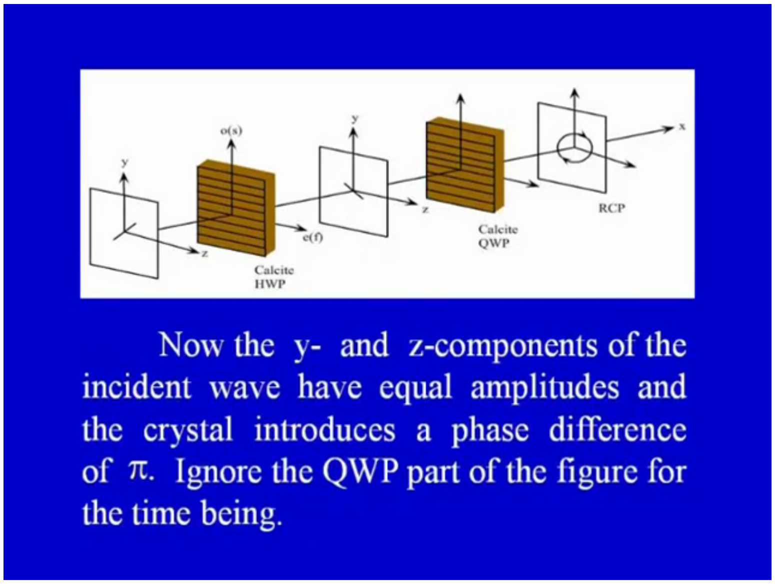

So, the ordinary and extraordinary again have equal amplitudes. The crystal introduces the phase difference of PI. Let us not consider the quarter wave plate part at the moment. Just for the emergent light beam in this case we have,

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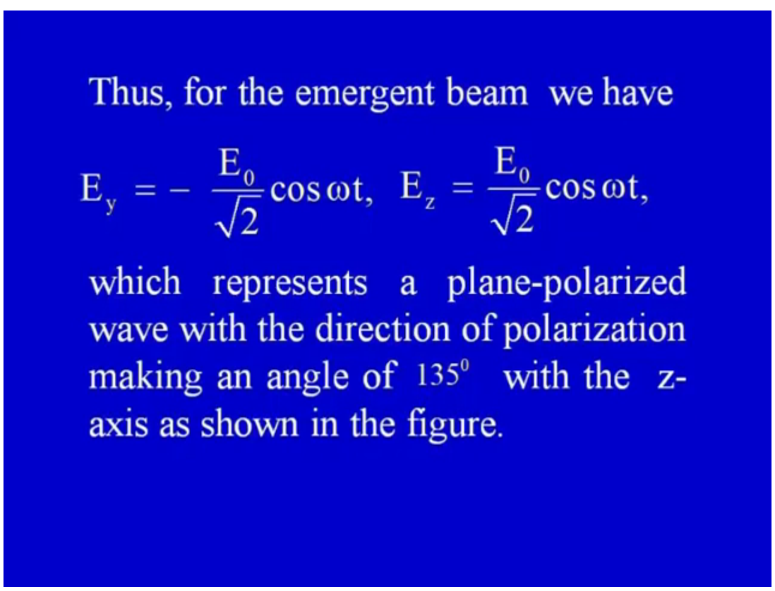

Ey = - E0 upon a square root of t cos Omega t and Ez = E naught upon the square root of 2 cos Omega t. Note the - sign which is a appearing in the expression for Ey. This represents a plane polarized wave with the direction of polarization making an angle of 135 with the z axis. Remember, in the beginning, at the entry point the angle was 45 degrees. Now it is 135 degrees.

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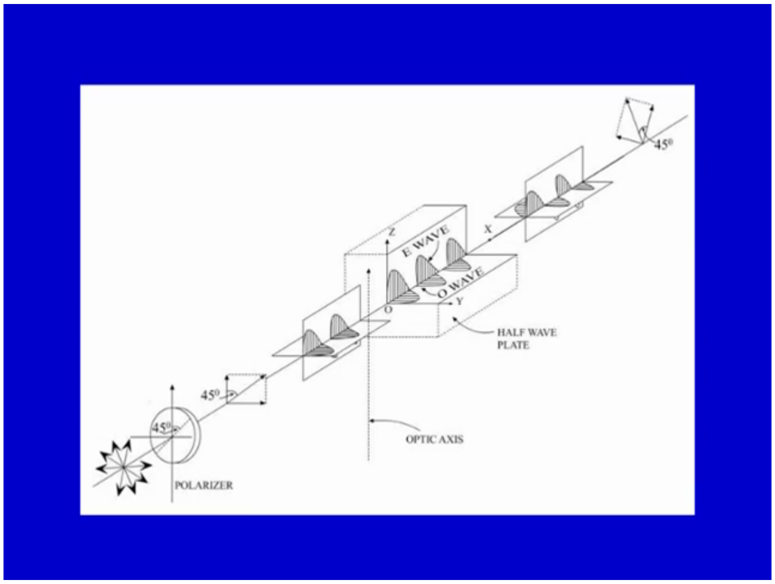

In the beginning okay, remember they are in the same phase.

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This figure again shows the build up of the phase difference between the ordinary and the extra-ordinary. This phase difference ultimately becomes equal to 135°.

If we now pass this beam through a calcite QWP, the emergent beam will be right circularly polarized (RCP) as shown in the figure.

This figure again shows the buildup of the phase difference between the ordinary and extraordinary. The phase difference ultimately becomes = 135 degrees. If we now pass this beam through a calcite quarter wave plate, the quarter wave plate introduces a phase difference of Pi by 2. The emergent beam will be right circularly plane polarized, right circularly polarized as shown in the figure.

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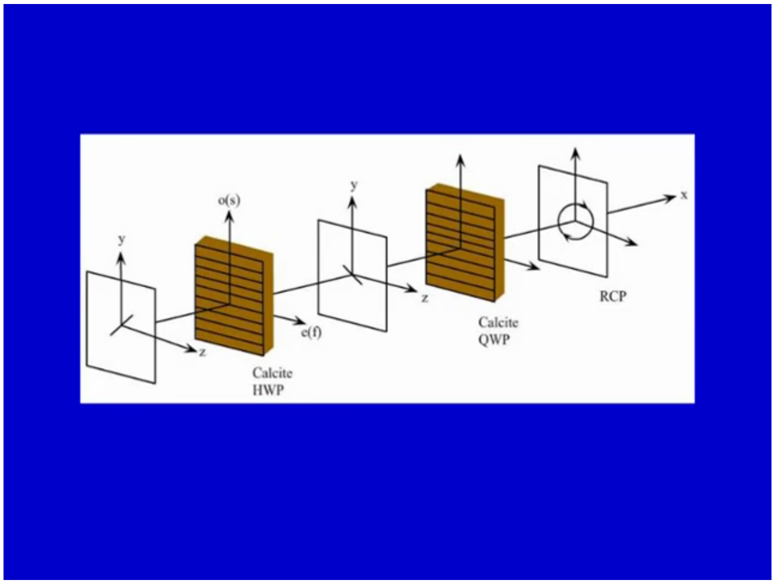

The incident light with a plane polarized with angle of 45 degrees passes through a half wave plate. The phase difference becomes 135 degrees passes through quarter wave plate which introduces another phase difference of Pi by 2. And the result is the light wave right circularly polarized.

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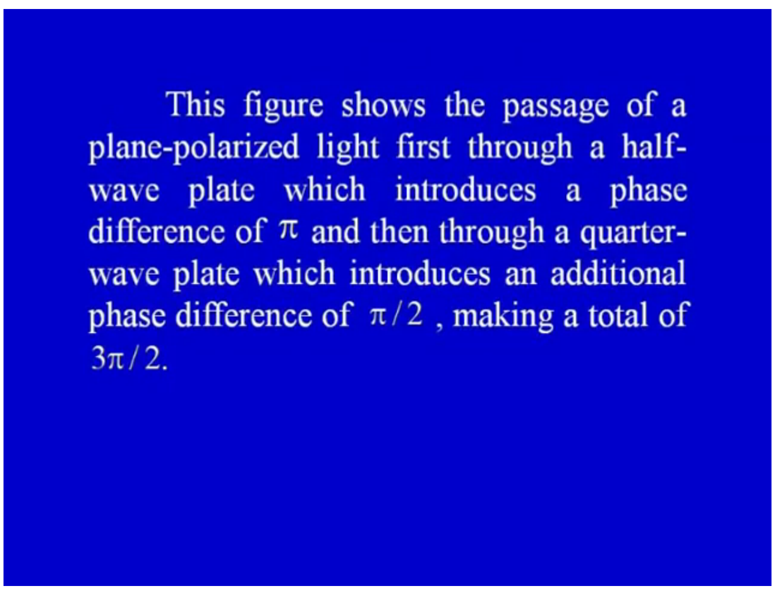

So, this is what this figure shows passage of a plane polarized light first through a half way plate which as I said introduces a phase difference of PI. And then, through a quarter wave plate which introduces an additional phase difference of Pi by 2 making a total of 3 Pi by 2.

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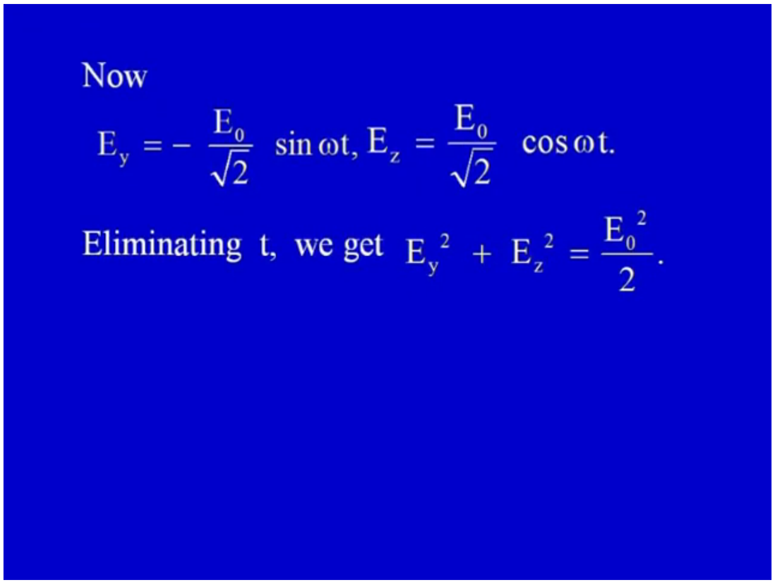

Now finally at this stage Ey is given by - E naught upon square root of 2 sine Omega t and Ez = E naught upon square root of 2 cos Omega t, eliminating t. We again get the equation of a circle Ey square + Ez square = E naught square upon 2 again as we did it earlier. In order to determine the sense of rotation of the electric vector E,

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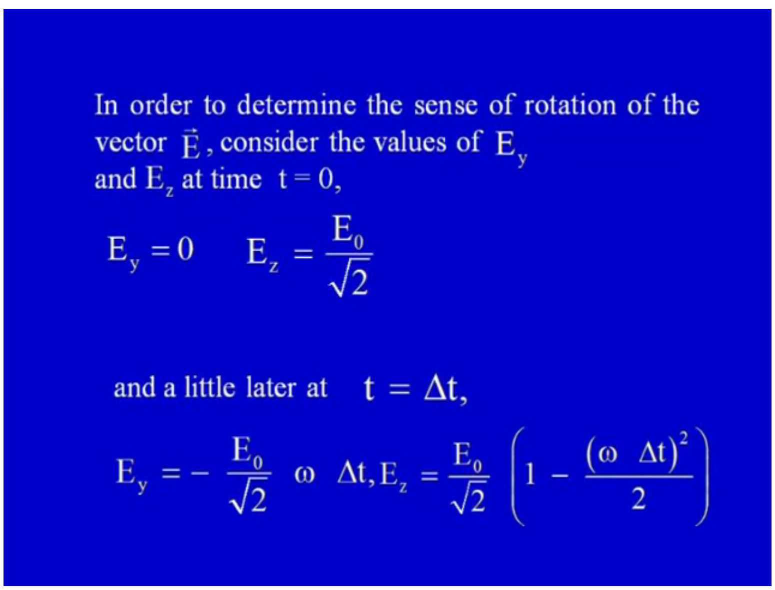

Considere the values of Ey and Ez at time t = 0. So, at time t = 0 Ey = 0 and Ez = E naught upon square root of 2. A little time later, at t = delta t, Ey = - E naught upon square root of 2 times Omega delta t and Ez = E naught upon square root of 2 times 1 - Omega delta t square upon 2.

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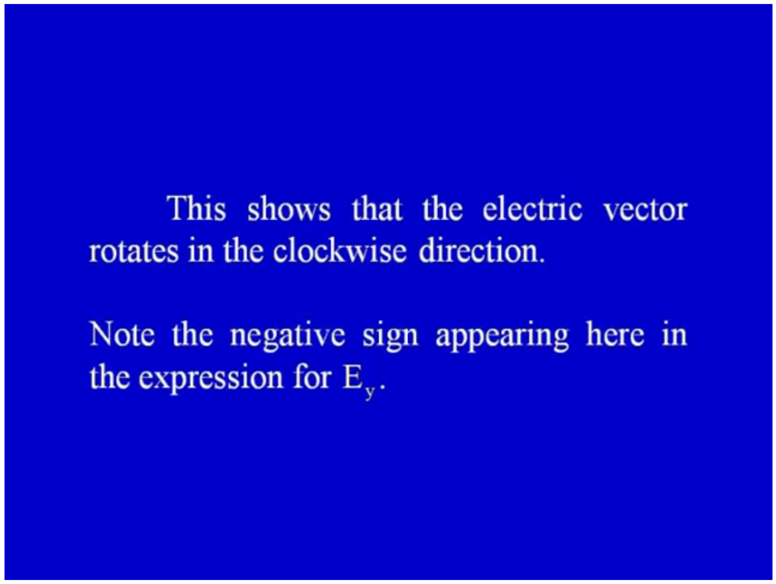

The electric vector E has again increased, but in the other direction and Ez component has slightly decreased. And this shows the electric vector now in this case, rotates in the clockwise direction.

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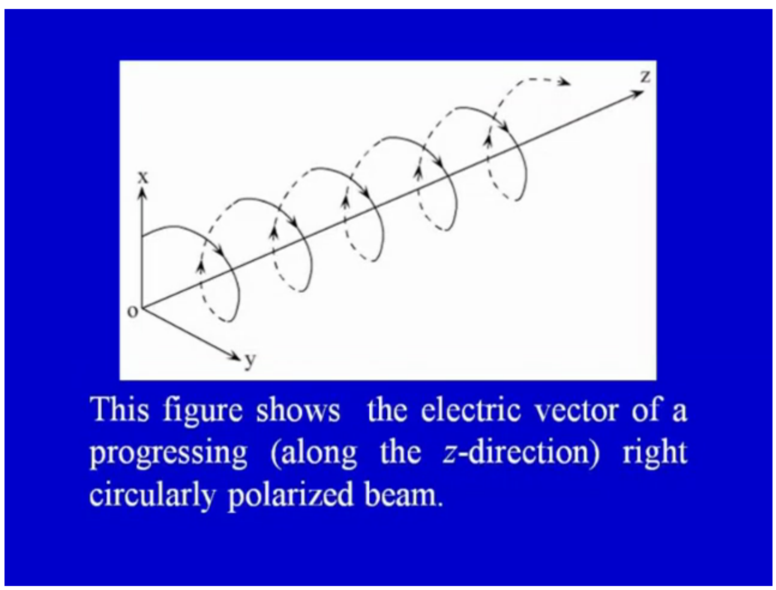

This figure shows the electric vector of a progressing right circularly polarized beam.

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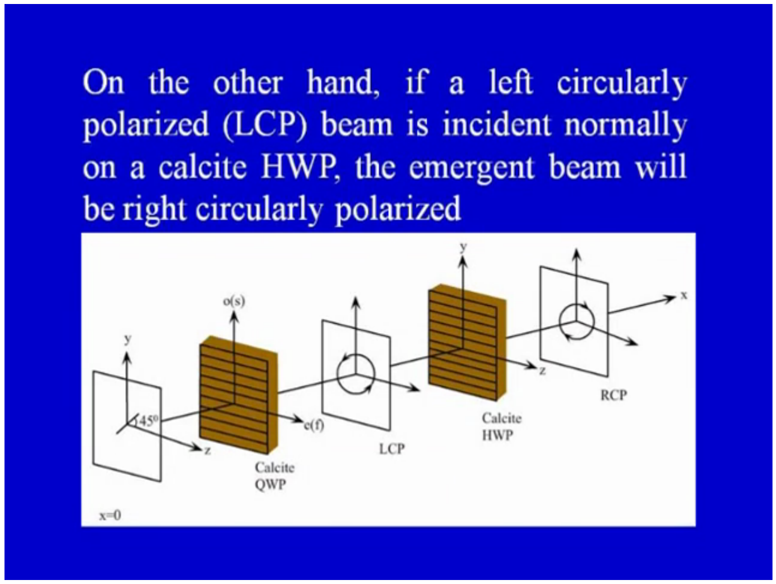

Now, on the other hand, if your left circularly polarized beam is incident normally on a half wave plate, the emergent beam will be right circularly polarized; initial plane polarized light passes through a quarter wave plate becomes a left circularly polarized light passes through a half wave plate becomes right circularly polarized light.

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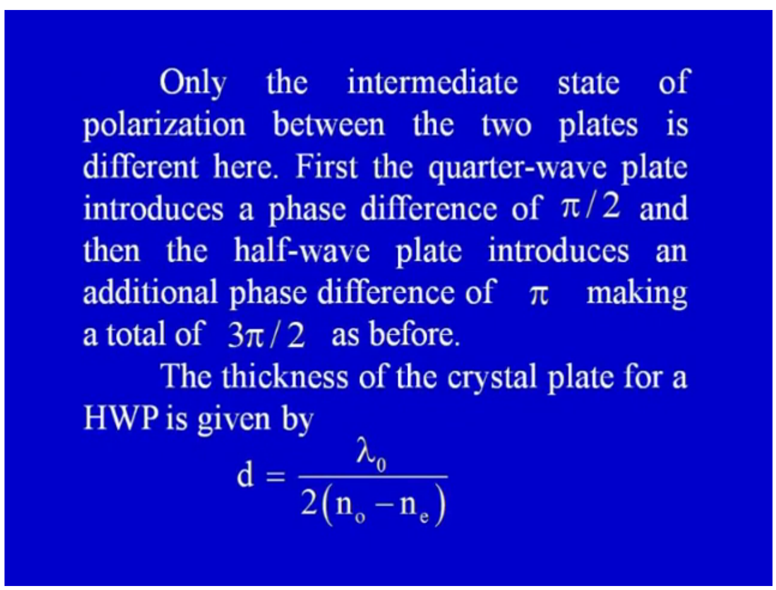

Only the intermediate state of polarization between the two plates is different here. First as I said, the quarter wave plate introduces the phase difference of Pi by 2, makes the light beam left circularly polarized. And then, the half wave plate introduces an additional phase difference of Pi making a total of 3 Pi by 2 as before. And making a light beam right circularly polarized, the thickness of the crystal plate for half a plate is given by, in a very natural way, d is given by lambda naught by 2 divided by 2 no - ne.

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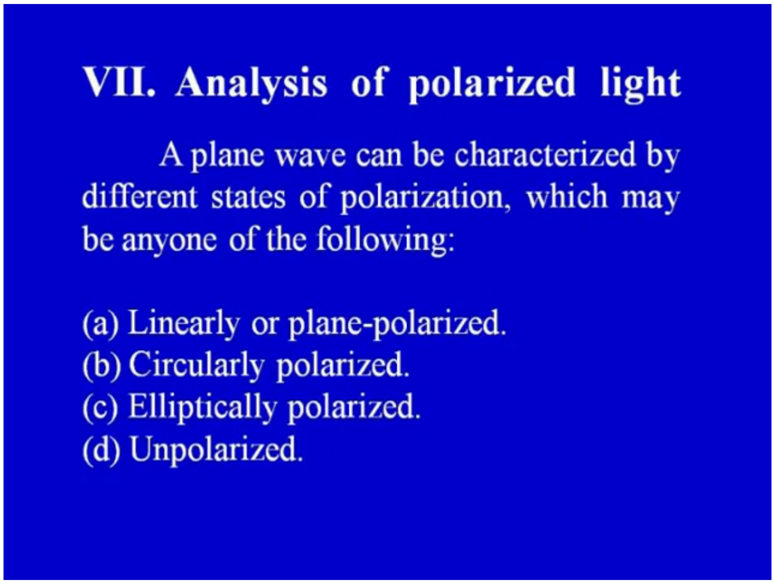

Let us now consider the analysis of polarized light. The plane waves can be cut by different states of polarization as we have seen during the course of this lecture. It may be any one of the following. It could be a linearly polarized light, b. it could be a circularly polarized light, c. it could be elliptically polarized light and d. it will be just unpolarized.

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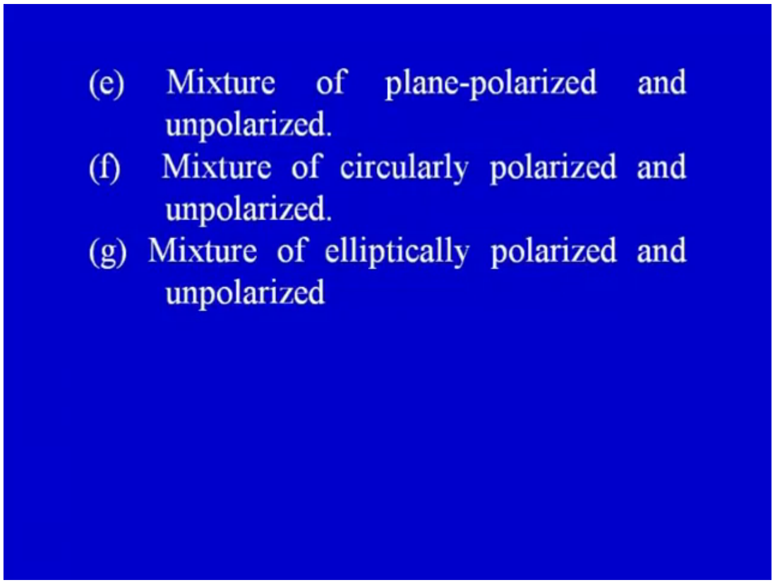

Or in addition to all these 4 cases, e. it could be a mixture of plane polarized light and unpolarized light, it could be a mixture of circularly polarized light and unpolarized light or it could be a mixture of elliptically polarized light and unpolarized light. So, we have 7 different possibilities. One thing should be noted.

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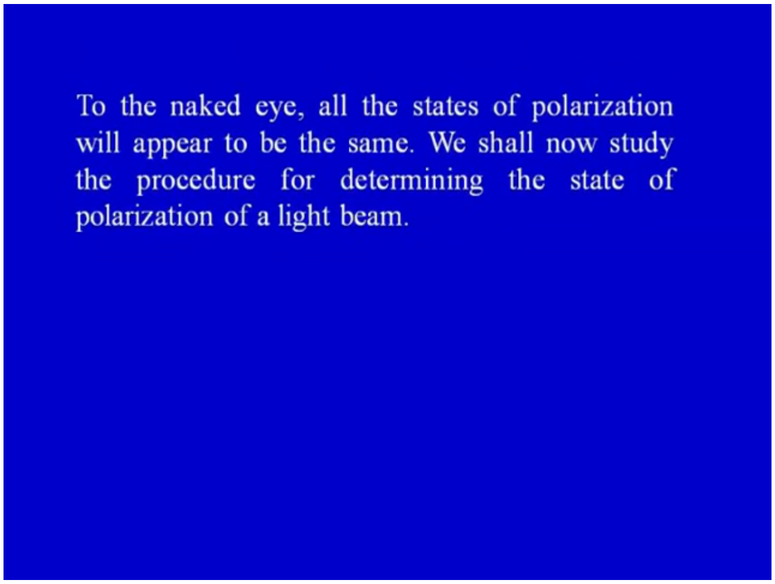

To the naked eye, all these states of polarization will appear to be the same, absolutely no difference. We shall now study the procedure, what are the steps are to be taken, for determining the state of polarization of a light beam.

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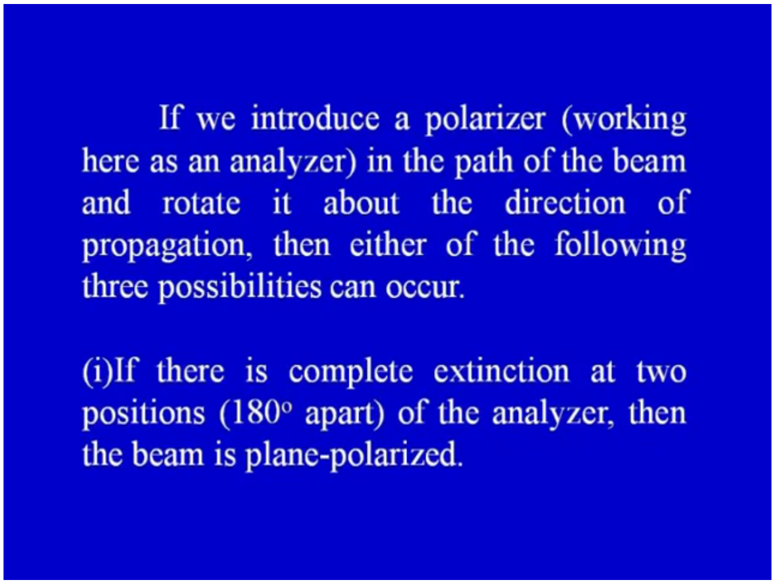

If we introduce a polarizer, working here as an analyzer, in the path of the beam and we rotate it about the direction of propagation, then either of the following three possibilities can occur. You see, the incident light beam with the air which is to be analyzed. We pass it through a polarizer and we rotate the polarizer about the direction of the beam and watch what happens.

First case if there is complete extension at 2 positions, which are 180 degrees apart of the analyzer then, the beam is plane polarized. This means we rotate it we get 0 intensity at some position; rotate it 90 degrees, becomes maximum for the rotation of 90 degrees, again the intensity becomes 0. So, in a complete rotation, there are two positions 180 degrees apart, where there is a complete extension. Remember, Malus law. And that shows that incident beam is plane polarized.

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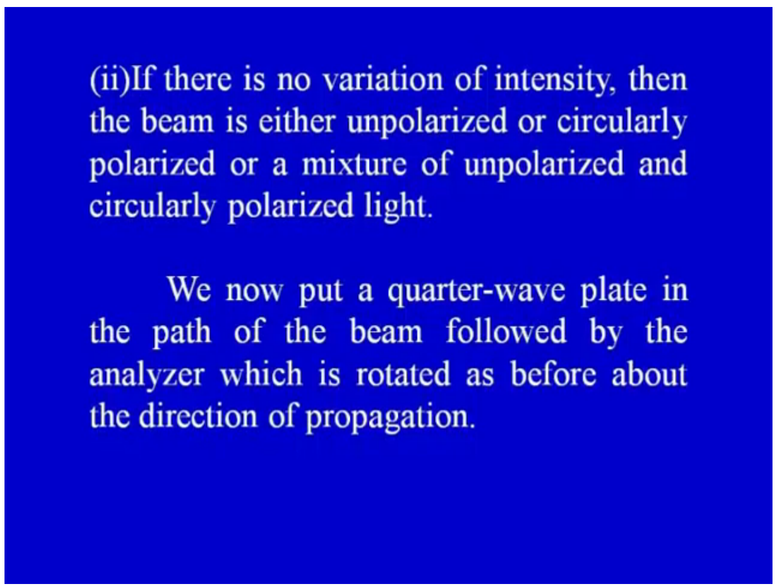

Number two, if there is no variation of intensity then the beam is either unpolarized again we have seen this thing earlier. If the beam is unpolarized and you pass it through a polarizer, let it be call it as an analyzer here; and rotate it, there is no variation in the intensity. So, the beam could be unpolarized but we also know that the same thing will happen if the beam is circularly polarized or is a mixture of unpolarized and circularly polarized light.

To analyze the differentiate between these two possibilities that the light beam is unpolarized circled upon a mixture of unpolarized and circularly polarized. We now put a quarter wave plate in the path of the beam. So, the incoming beam we allow it to pass through a water wave plate and again by an analyzer, which is rotated as before about the direction of propagation.

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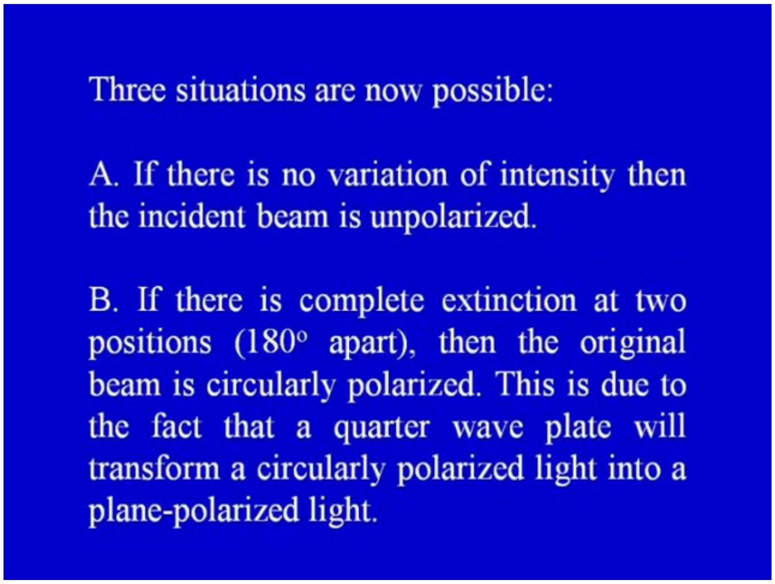

When we do this, three situations are possible: A. Now, there is no variation of intensity. The result is this means the incident beam is unpolarized. The reason is a quarter wave plate does essentially nothing to an unpolarized beam. It was showing no variation when analyzer is rotated even after passes through a quarter wave plate, the same situation will prevail. There will be no variation in intensity so that so that incident beam is unpolarized.

B. If there is a complete extension at two positions 180 degrees apart, this means the original beam is circularly polarized this is due to the fact that a quarter wave plate will transform circularly polarized light into a plane polarized light. You see, the incident incident beam is circularly polarized means, a combination of two perpendicular plane polarized light with a phase difference of Pi by 2. The quarter wave plate introduces an additional phase difference of Pi by 2 making a total of Pi R0.

In both the cases, the emergent light becomes a plane polarized and there will be complete extension right to the 2 positions following Malu's law, okay. So, in case B clearly shows that the beam is circularly polarized.

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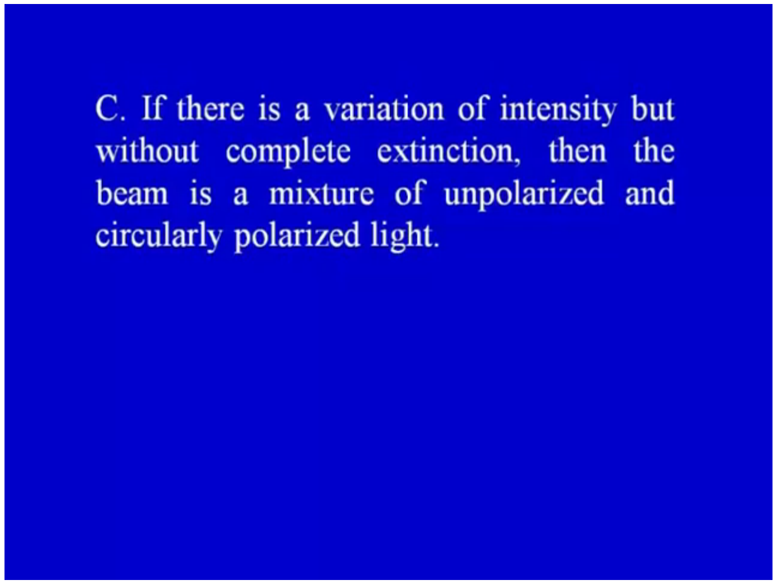

C.A third possibility C, C is if there is a variation of intensity but without complete extension naturally in this case at the incident beam is a mixture of unpolarized and circularly polarized passage to the quarter of a plate does nothing to the unpolarized part of the beam, part of the mixture, circularly polarized light becomes plane polarized. So, the result after passage through the quarter wave plate is something like a mixture of unpolarized and plane polarized. When we pass it to an analyzer and rotate it, the variation in intensity but complete extinction is not there.

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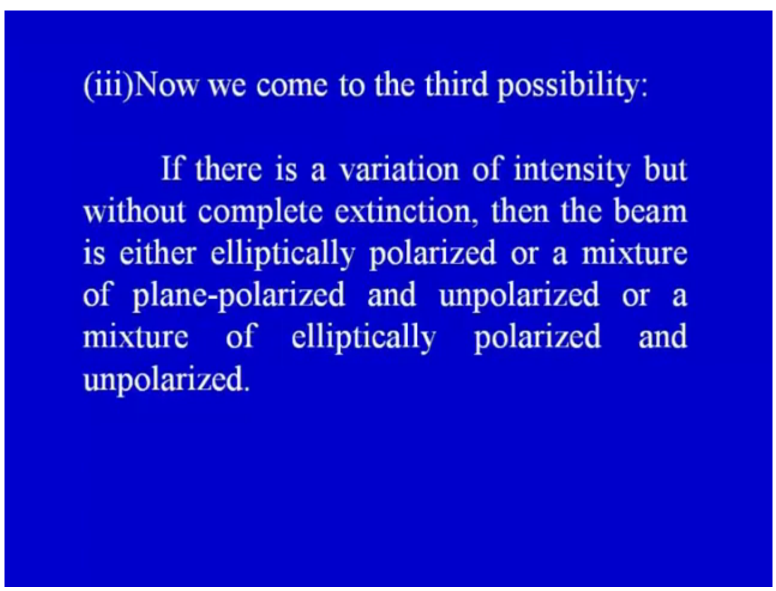

Now, we come to the third possibility. If there is a variation of intensity, but without complete extinction, the incident beam could be either elliptically polarized, given the same variation or a mixture of plane polarized and unpolarized, but it could be a mixture of elliptically polarized and unpolarized. So, here again, its a quarter wave plate is a very useful set, very useful piece.

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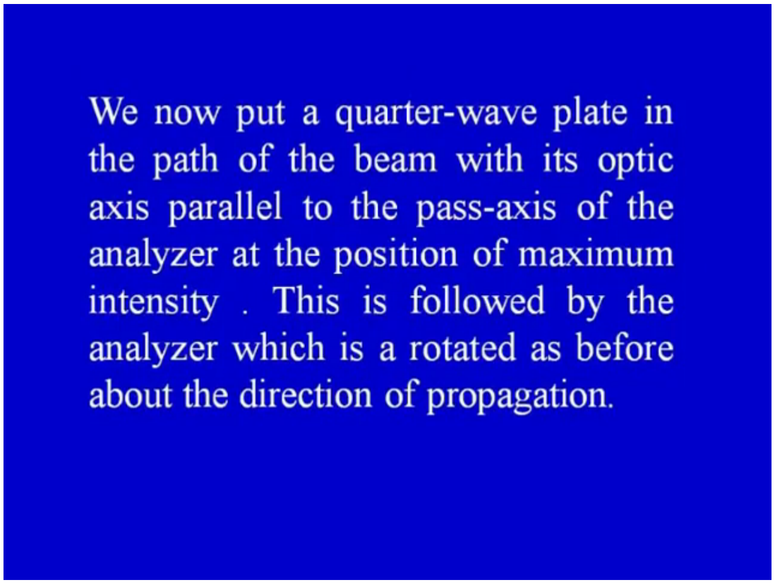

We again put a quarter wave plates in the path of the beam with its optic axis parallel to the pass axis of the analyzer, at the position of maximum intensity. This is a change from the earlier case we put the quarter wave plate, all right deep in the path of the beam. There is optic axis parallel to the pass success of the analyzer, led the position of maximum intensity.

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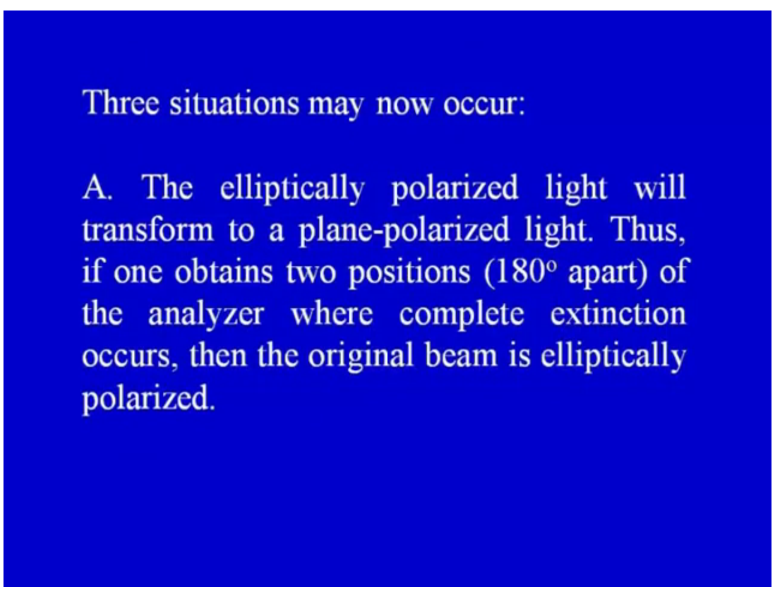

This is followed by the analyzer which is rotated again as before about the direction of propagation. Three situations may occur: A. The elliptically polarized light will transform to a plane polarized light. Remember, elliptically polarized light, there is also a path difference of a quarter vibration. Quarter wave plate will introduce an additional quarter vibration. The result is the emergent light will become plane polarized.

And one will get two positions and the 180 degree apart of the analyzer where the complete extinction occurs, the Malus Law. Then, the original beam is elliptically polarized.

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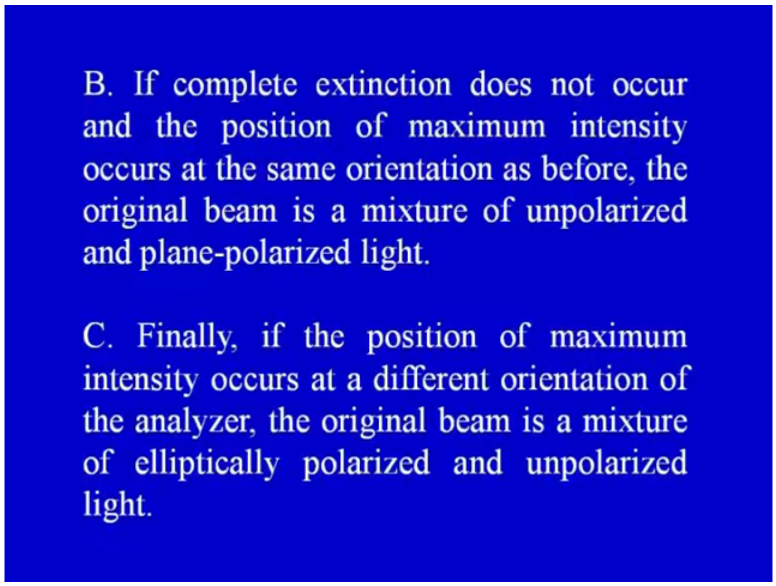

B. If complete extension does not occur and the position of maximum intensity occurs at the same orientation as before, before putting the quarter wave plate, same position as before, the original beam is a mixture of unpolarized and plane polarized light. The case C is finally the position of maximum intensity. Again there is a variation but, the position of maximum intensity occurs at a different orientation of the analyzer. In this case, the original beam is a mixture of elliptically polarized and unpolarized light.

So, these are the various steps which are needed to analyze the polarization, various polarization of the light. The 7 cases which we have considered, the unpolarized plane polarized, circularly polarized, elliptically polarized or the mixture when you have unpolarized along with circularly polarized or unpolarized along with plane polarized or unpolarized along with elliptically polarized case. With this, we have come to the end of the present lecture.