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Module-08 Lecture-03 Lens Aberrations Part I

Lens aberrations 1 by M. K. Srivastava, Department of Physics, Indian Institute of Technology Roorkee, Uttharkand.

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In this in the next lecture we shall study and analyze the defects in the image formation by lenses, the lens aberrations.

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In studying the formation of images by refracting surfaces and lenses, we make the assumption that the object point does not lie far away from the axis of the optical system and that the rays taking part in image formation are essentially those which make small angles with the axis of the system. To the extent, these assumptions are true, the images of the objects will be perfect, i.e. all rays emanating from a single object point will converge to a single image point. And the magnification of the system will be a constant, independent of the particular ray under consideration.

In studying the formation of images by reflecting surfaces and lenses, we make the assumption that the object point does not lie far away from the axis of the optical system and then the ray is taking part in image formation or essentially those which make small angles with the axis of the system. Sometimes we may not say this phenomenally but this is the implied assumption all the time.

To the extent these assumptions are true the images of the objects will be perfect always emanating from a single object point will converge to a single image point. And the magnification of the system will be a constant independent of the particular ray under consideration.

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In real systems *nonparaxial* rays also take part (some times even dominate) in image formation. The result is that the actual images are not ideal images.

In real systems over non paraxial rays also take part these are the marginal rays making a larger, larger angle with the axis they also take part sometime even dominate in the image formation. The result is the actual images are not ideal images.

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This deviation is essentially due to $\sin\theta$ not being equal to θ for non-paraxial rays where θ is not small. In the next approximation, the so-called third-order theory, $\sin\theta$ is replaced by $\theta = \theta^3/3!$. In this theory the aberration of any ray ,i.e., the deviation from the prescribed path for the para-axial rays, is expressed in terms of, S_1 to S_5 , called the *Seidel sums*.

This deviation is essentially due to sine theta not being equal to theta for non paraxial rays where theta is not small. In the next approximation the so called third order theory sine theta is replaced by theta - theta cube divided by factorial 3. In this theory the aberration of any ray the deviation from the prescribed path or the paraxial race is expressed in terms of S1, S2, S3, S4, S5 called the Seidel sums.

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If a lens were to be free of all defects in its ability to form images, all five of these sums would have to be equal to zero. No optical system can be made to satisfy all these conditions at once. Therefore one considers cach sum separately, and its vanishing corresponds to the absence of certain aberration.

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Thus, if for a given axial point the Seidel sum $S_1 = 0$, there is no *spherical aberration* at the corresponding image point. If $S_2 = 0$, the system will be free of *coma*. The sums $S_3 = 0$ and $S_4 = 0$ correspond to no *astigmatism* and no *curvature of the field* respectively. If S_5 could be made to vanish, there would be no *distortion* of the image. These aberrations together are called *monochromatic aberrations* as they exist for any specified colour and refractive index.

Thus if for a given axial point the Seidel sum S1 is 0 there is no spherical aberration and the corresponding image point. If S2 = 0 the system will be free of coma. The sums S3 = 0 and S4 = 0 correspond to no astigmatism and no curvature of the field respectively. If S5 could be made to vanish there would be no distortion of the image. You see these abrasions together are called mono chromatic aberrations as they exist for any specified color for any refractive index.

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In addition if the light used for image formation is polychromatic (like white light which is used in many optical instruments) then, in general, the images will be coloured. This is due to the dependence of the refractive index of the material of the lens on wavelength of the light used. Different wavelength components in the light proceed along different directions after refraction and form images at different points. This is called *chromatic aberration*. Let us study chromatic aberration first.

In addition if the light used for image formation is polychromatic like white light which is used in almost all optical instruments. Then in general the images will be colored this is due to the dependence of the refractive index of the material of the lens on wavelength of the light used. Different wavelength components in the light proceed along different directions after reflection and naturally form images at different points. This is called chromatic aberration.

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Let us study chromatic aberration first consider a parallel beam of white light incident on a thin lens.

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The incident beam is a white light after reflection it colors they appear see the refractive index of all the transparent media varies with color. And blue light gets reflected more than the red light the single lens formed not one image of an object but a series of images one for each color of the light present in the beam.

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The prismatic action of the lens, which increases toward its edge, is such as to bring the violet light to a focus nearest to the lens. As a consequence of the variation of focal length of a lens with colour, *the lateral magnification must vary as well*.

You see the prismatic action of the lens which increases towards its edges is such as to bring the white light to a focus nearest to the lens. As a consequence of the variation of the focal length of a lens with colors the lateral magnification must vary as well, so that is an additional aberration coming up.

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See let us see this situation the figure shows red and violet image heights of an off axis point Q. If you consider the point M which is on the axis the images violet image and the red image they are not at the same point. Similarly if you consider an off axis point here again the corresponding Q prime R and Q prime V they are not at the same point leading to chromatic aberration in the lateral sides, the lateral and longitudinal both of these features will be there.

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The horizontal distance between the axial images is called *axial* or *longitudinal chromatic aberration*, while the vertical differences in height is called *lateral chromatic aberration*. Because these aberration are often comparable in magnitude with the Seidel aberrations, correction for both lateral and longitudinal colour is of considerable importance.

The horizontal distance between the axial images is called axial or longitudinal chromatic aberration as we have seen in the last picture. While the vertical differences in height called the lateral chromatic aberration. Because these aberration are often comparable in magnitude with the Seidel aberration corrections for both lateral and longitudinal color is of considerable importance.

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The expression for longitudinal chromatic aberration can be obtained from $\frac{1}{f} - (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $\Rightarrow -\frac{\delta f}{f^2} = \delta n \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\delta n}{n-1} \frac{1}{f}$ $\Rightarrow \quad \delta f = -f \frac{\delta n}{n-1}$ $\Rightarrow \quad f_r - f_b = f \left(\frac{n_b - n_r}{n-1}\right)$ This represents its magnitude.

Now the expression for the longitudinal chromatic aberration can be obtained from the lens formula. You see the length formula is 1 upon f = n - 1 multiplied by 1 upon R1 - 1 upon R2, see we are assuming the same medium on both sides of the lens. If you differentiate it, we get - Delta f upon f square Delta n times this factor 1 upon R1 - 1 upon R2. We substitute its value so this becomes equal to Delta n upon n - 1 times 1 upon f.

So, Delta f = -f times Delta n upon n - 1 so this means that the difference between the focal lengths a red and a blue fr - fb = f times Delta n means refractive index for blue - refractive index for red divided by n - 1.

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While there are several general methods for correcting chromatic aberration, the method employing a combination of two thin lenses made of different materials (say, crown glass and flint glass), one converging and the other diverging, placed in contact with each other is the commonest.

So, now let us look at possible methods to make achromatic combination at least to minimize chromatic aberration or to try to eliminate it. While there are several general methods for correcting chromatic aberration the method employing a combination of two thin lenses made of different materials say crown glass and flint glass one converting lens the other diverging lens and placed in contact with each other is the commonest.

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If $\omega = \frac{n_b - n_r}{n-1}$ and $\omega' = \frac{n'_b - n'_r}{n'-1}$ are the dispersive powers of crown and flint glasses respectively, and f and fare respectively the focal lengths of the convering and diverging lenses, then the condition for achromatism is $\frac{\omega}{f} + \frac{\omega'}{f'} = 0$

and the focal length of the doublet is given by

$$\frac{1}{F} = \frac{1}{f} - \frac{1}{f'}$$

The convex lens and a concave lens the method will be if this combination nb - nr upon n - 1 for the material of one lens and similarly you make a prime the corresponding for the material of the other lens these quantities are called dispersive powers. So, suppose we have two lenses of

crown and flint losses of dispersive powers to Omega and Omega prime and f and f prime are respectively the focal lengths of the converging and diverging lenses.

Then the condition for achromatism is given by this, this actually follows from the last relation we obtain because for the longitudinal difference between the two foresights for blue and red that is really given by Omega by f, so Omega by fr1 is compensated by Omega by f by the other. The focal length of the combination naturally will be given by this relationship.

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Although the lens in our above example would appear to have been corrected for longitudinal chromatic aberration. It has actually been corrected for lateral chromatic aberration. Equal focal lengths for different colours will produce equal magnification, but the different coloured images along the axis will coincide only if the principal points also coincide. Practically speaking, the principal points of a thin lens are so close together that both types of chromatic aberration can be assumed to have been corrected by the above arrangement.

Although the lens in our above example would appear to have been corrected for longitudinal chromatic aberration, it has actually been corrected for lateral chromatic aberration as well. You should have equal focal lengths for different colors will produce equal magnification. But the different colored images along the axis will coincide believe the principal points also coincide; focal lengths will remain becoming equal is not enough.

The focal point should also consign and that will be the situation if the principal points also. But practically speaking the principal points of a thin lens are so close together that both types of chromatic aberrations can be assumed to have been corrected by the above arrangement which you have two thin lenses one concave the convex of different materials satisfying that relationship of Omega by f Omega prime by f prime = z.

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In a thick lens, however, longitudinal aberration is absent if the colours corrected for come together at the same axial image point.

In a tip lens however the longitudinal aberration is absent if the colors connected for come together and the same axial image point like this for both the colors red and blue

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But see as the principal point the thickness is the principal point for the blue and red do not coincide the focal lengths are not equal and the magnification is different for different colors. Consequently the majors formed in different colors will have different sizes and the system will suffer from lateral chromatic aberration.

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Chromatic aberration can also be removed by taking a combination of two converging lenses of the same material separated by a distance t equal to the mean of their focal lengths. The focal length of the combination is given by $\frac{1}{F} = \frac{1}{f} + \frac{1}{f'} - \frac{t}{ff'} \quad \text{with} \quad t = \frac{f+f'}{2}$

The chromatic aberration can also be removed by taking a combination of two converging lenses of the same material separated by a distance t equal to the mean of their focal lens. Focal length of the combination is given by they have obtained this relation or layer 1 upon capital F is equal to 1 upon f + 1 upon f prime - t upon ff prime and the thickness is kept as mean value f + f prime by 2.

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This arrangement has the same focal length for all colours near those for which f and f' are calculated For visual instruments this colour is chosen to be at the peak of the visual-brightness. Such spaced doublets are used as oculars in many optical instruments because the lateral chromatic aberration is highly corrected through constancy of the focal length.

This arrangement has the same focal length for all colors near those for which f and f prime are calculated. For visual instruments this color is chosen to be at the peak of the visual brightness such a spaced doublets are used as oculars in many optical instruments because the little chromatic aberration is highly corrected to the constancy of the focal length.

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The longitudinal aberration higher but is relatively large due to white differences in the principal points and therefore white differences between focal points. But because the focal lengths are equal, the image sizes are equal, lateral chromatic aberration has been taken care of.

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An illustration of a system that has no longitudinal chromatic aberration is shown here. It is a combination of convex lens and concave lens. You have got the red and blue images at the same position remember the focal lengths are not same because the principal points are not additional or not item you are not at the same point for blue and red. So, here the images are formed the same point but they are of different size.

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Now we come to the mono chromatic aberrations, spherical aberration. Considered a beam of light parallel to the axis incident on a thin lens, the light rays after passing the lens they bend towards the axis and cross it at some point.

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Now if one does not restrict to the paraxial region then in general rays which are incident different heights on the lens hit the axis at different points.

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For example, for a convex lens, the marginal rays (which are incident near the periphery of the lens) focus at a point closer than the focal point of paraxial rays.



For example for a convex lens the marginal rays which are incident near the periphery of the lens they focus at a point closer the focal point of paraxial rays. FP is the focal point of paraxial rays FM is difficult point of the marginal rays. This figure also shows the difference between the longitudinal, spherical and lateral cycle aberration. Similarly for a concave lens rays which are incident farther from the axis they appear to be emerging from a point which is nearer to the lens. FM is nearer to the lens compared to FP which is the focal point for paraxial rays.

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The point, F_P , at which the paraxial rays strike the axis is called the paraxial focus and the point, F_M , at which the rays near the periphery strike is called the marginal focus. The distance between the two foci is a measure of longitudinal spherical aberration in the lens. Similarly, the distance between the paraxial image point and the point at which the marginal ray strikes the paraxial image plane is called the lateral spherical aberration.

Now the point FP at which paraxial rays the strike the axis is called the paraxial focus and the point FM at which the rays near the periphery strike is called the marginal focus. The distance between the two is a measure of longitudinal spherical aberration in the lens. Similarly the

distance between the paraxial image point and the point at which the marginal rays strike the paraxial image plane that is called the lateral and spherical aberration.

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Spherical aberration can be minimized by using *stops* which reduce the effective aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, as the amount of light passing through the lens is reduced, correspondingly the image appears less bright.

Spherical aberration can be minimized as appears naturally by using stops which reduce the effective aperture the stop used can be such as to permit either actual rays of light or the marginal rays of light not both. But however in the amount of light passing through the lens gets reduced correspondingly the image appears less bright.

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Let us label the lens by a number

$$q = \frac{R_1 - R_2}{R_1 + R_2}$$

called its *shape factor*. A series of positive lenses of the same diameter and paraxial focal length but of different shape is shown. Each lens is labeled by a q number. The alteration of shape is known as *bending* of the lens. The reason for this is to find the shape for which the spherical aberration is a minimum.

Now let us label the lens by a number this is called the shape factor R1 - R2 upon R1 + R2. A series of positive lenses of the same diameter I mean same aperture same size and same paraxial

focal length. We are making a comparison of lenses of different shape vectors, different combinations of R1 and R2 of all these lenses of the same size and of the same paraxial focal length but of different shapes.

We consider the each lens is labeled by some value of Q that we show now in the next figure the alteration of shape is known as the bending of the lens.





Let us consider 1, 2, 3, 4 several lenses different shape factors are given here for a by convex lens that is a number 4 shear factor is 0. In the bottom figure on the vertical axis we have got the add height of the ray, 0 means along the axis and from 0.5 going up to 2.5 that is the height of the ray. And on the horizontal thing shows how much they the difference of their how much is the cycle is the magnitude of the longitudinal aberration.

Difference between FM and FP and we find here that for the shift lens of a shape factor of + 0.5 this variation is minimal. So, this shape corresponds to a shape which will lead to a minimum spherical aberration.

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We find that by choosing the proper radii for the two surfaces of a lens the spherical aberration can be reduced to a minimum, but can not be made to vanish.

We find that by choosing the proper radii for the two surfaces of a lens the spherical aberration can be reduced to a to a minimum but naturally it cannot be made to vanish.

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The spherical aberration is the result of marginal rays undergoing a large deviation. We should expect the spherical aberration to be minimum when the angle of deviation is minimum. This would occur, as in the case of a prism, when the deviations suffered at each of the refracting surfaces are exactly equal. Such an equal division of refraction will yield the smallest spherical aberration.

This is let us see that basically why the spherical aberration is coming up and can we do something better than this. The spherical aberration is the result of marginal rays undergoing large deviation. So, we should expect this spherical aberration to be minimum, when the angle of deviation is minimal and happens. This is would occur as in the case of a balloon, when the deviation suffered at each of the refracting surfaces are exactly equal. Such an equal division of refraction will yield smallest spherical aberration.

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And see for the shape factor remember we have seen for the same factor of 0.5 this is again we are showing the same be on the vertical axis is spherical aberration and the same fact shape factor on the horizontal axis at the point too close to 0.5 that is the minimum of this curve. The reason basically is that the total deviation caused by the lens is almost equally shared by the two surfaces.

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Although spherical aberration can not be entirely climinated for a single spherical lens, it is possible to do so for a combination of two or more lenses of opposite sign. The amount of spherical aberration introduced by one lens of such a combination must be equal and opposite to that introduced by the other.

Now although the spherical aberration cannot be entirely eliminated for a single spherical lens, it is possible to do so for a combination of two or more lenses of opposite sign. We want to cancel out the effect. The amount of spherical aberration introduced by one lens of such a combination must be equal and opposite to that introduced by the other.

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If, for example, the doublet is to have a positive power and no spherical aberration, the positive lens should have greater power and its shape should be at or near that for minimum spherical aberration, while the negative lens should have a smaller power and its shape should not be near that for the minimum. Neutralization of spherical aberration by such an arrangement is possible because spherical aberration varies as the cube of the focal length.

If for example the doublet is to have a positive power and no spherical aberration the positive lens should have a greater power positive lens means the converging lens. And its shape should be at or near that for the minimum is spherical aberration. Well the negative lens which is the concave lens should have a smaller power and its shape should not be near that for the minimum, utilization of spherical aberration by such an arrangement as possible because the spherical aberration varies roughly as the cube of the focal length.

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In a cemented lens of two elements, the two interfaces should have the same radii. The other two may then be varied and used to correct for spherical aberration. *With four radii* to *manipulate, other aberrations like chromatic aberration can be reduced at the same time.*

In a cementing lens of two elements the two interfaces should have the same radii naturally the other two may then be varied and used to correct for a spherical aberration. So, does with before

radii to manipulate two for each lens. Other aberration like chromatic aberration can be reduced at the same time.

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Now using this criteria of equal deviation for a combination of two thin lenses one finds that the spherical aberration is minimum when their separation is equal to the difference in their focal lengths have this combination.

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The curved surface of the Plano convex lenses faces the incident light. This is to divide the total deviation over the surfaces. So, in this arrangement the total deviation is equally shared by the two lenses.

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With this we have come to the end of this lecture. In the next one we shall take up other monochromatice aberrations. These are: (i) Coma, (ii) Astigmatism, (ii) Curvature of the field, and (iv) Distortion.

And with this friend we have come to the end of this lecture we have studied chromatic aberration and spherical aberration in the next one we shall take up other mono chromatic aberrations and these are coma, astigmatism, curvature of the field and distortion, thank you.