Engineering Physics 1
Dr. M. K. Srivastava
Department of Physics
Indian Institute of Technology-Roorkee

Module-08
Lecture-02
Image Formation by Lenses

I make formation by lenses by M. K. Srivastava, Department of Physics, Indian Institute of Technology, Roorkee, Uttarkhand.

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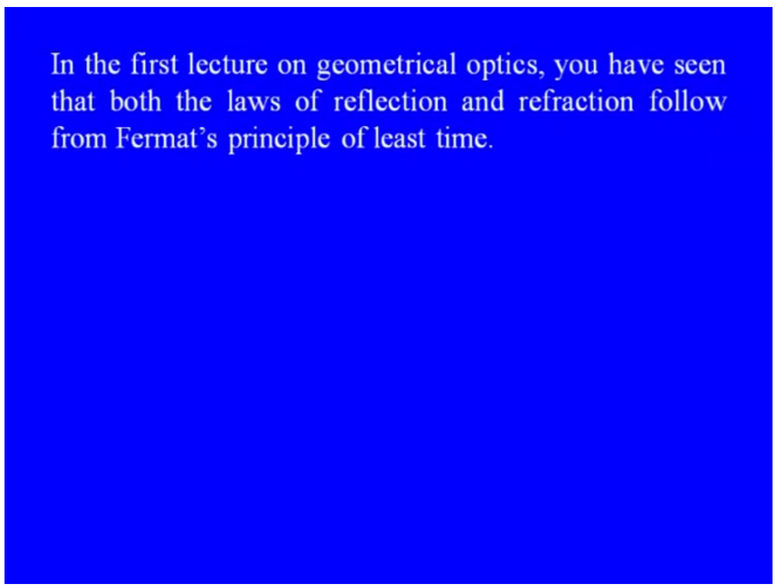

See in the first lecture geometrical optics you have seen that both the laws of reflection and diffraction follow from Fermat's principle of least time. Now in this lecture we shall review diffraction and image formation by thin lenses.

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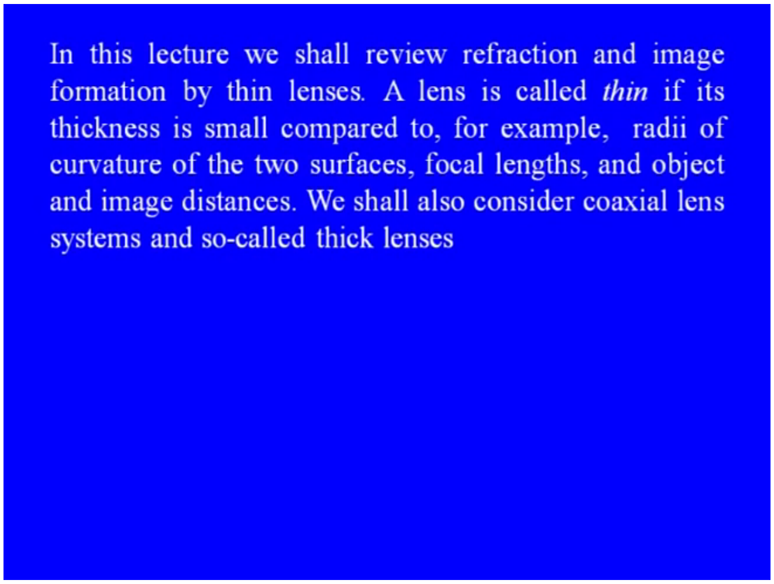

A lens is called ten if the thickness is small compared to for example radii of curvature of the two surfaces, focal lengths and imagine object distances, we shall also consider coaxial and systems and so called thick lenses.

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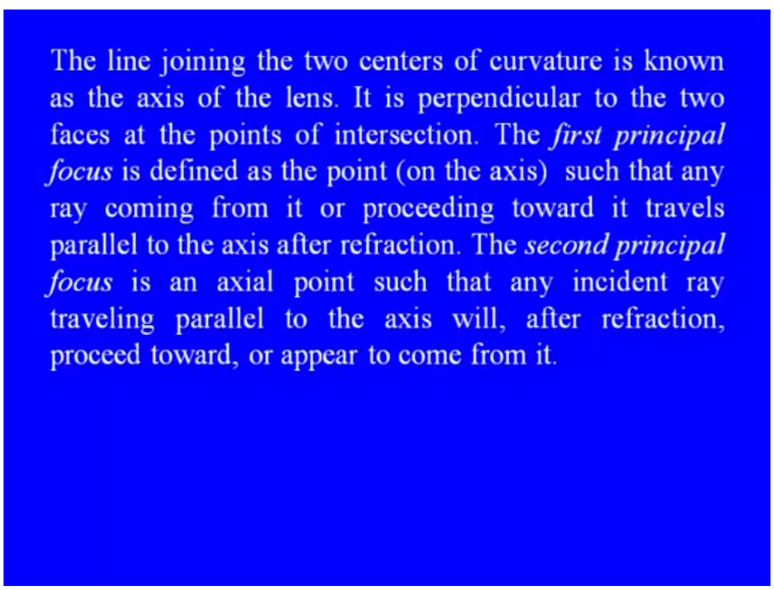

Now the line joining the two centers of curvature of the two surfaces of the lens is known as the axis of the lens which is perpendicular to the two faces at the points of intersection. The first principal focus is defined as the point on the axis such that any ray coming from it or proceeding towards it travels parallel to the axis after reflection. The second principal focus is an axial point such that any incident ray traveling parallel to the axis will after reflection proceed towards are appear to come from it.

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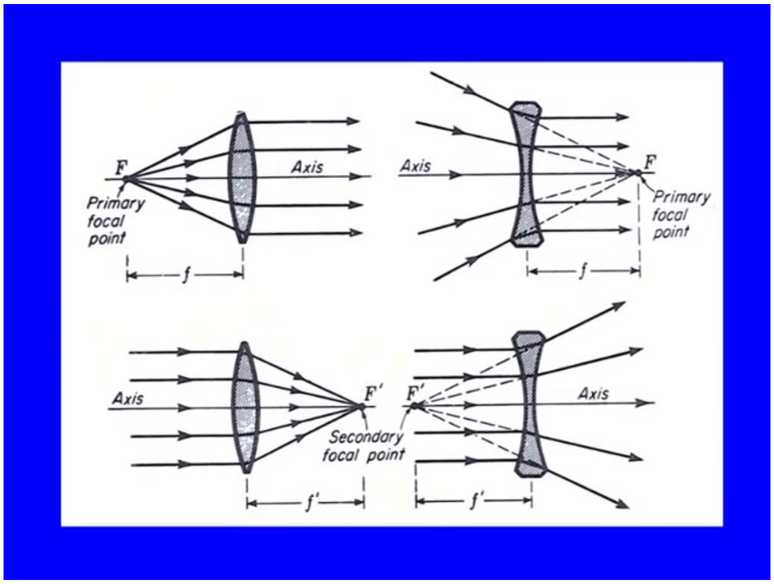

So, this picture shows these features primary may be called as first secondary may be called a second. In the primary from F the old diverging rays they become parallel after reflection or if the lens is a concave lens all these rays which was about to converge to F after refraction again become parallel to the axis. For the second focal point the rays which are parallel to begin with after reflection they converge to a point called us second difficult point or second focal point second focus. And similarly for the rays which after refraction appear to come from the focal point.

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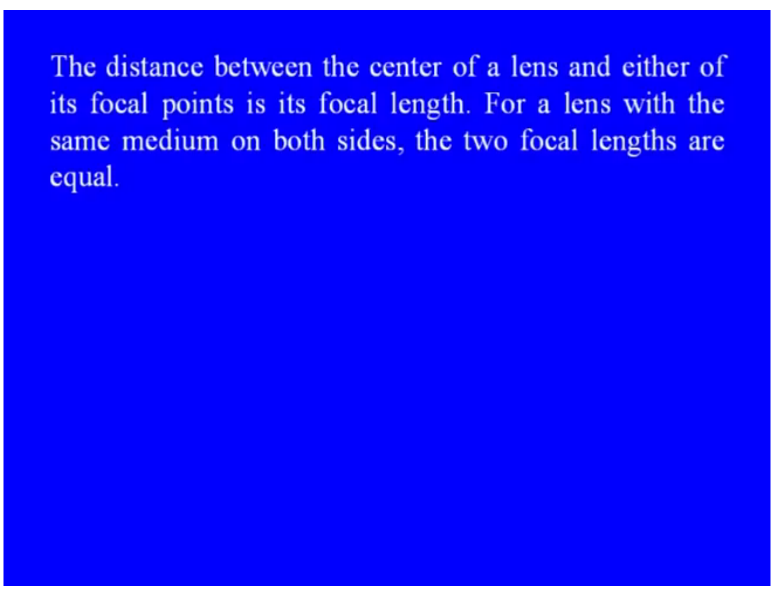

Now the distance between the center of a lens and either of its focal points is its focal length. For a lens with the same medium on both sides the two focal lengths are equal.

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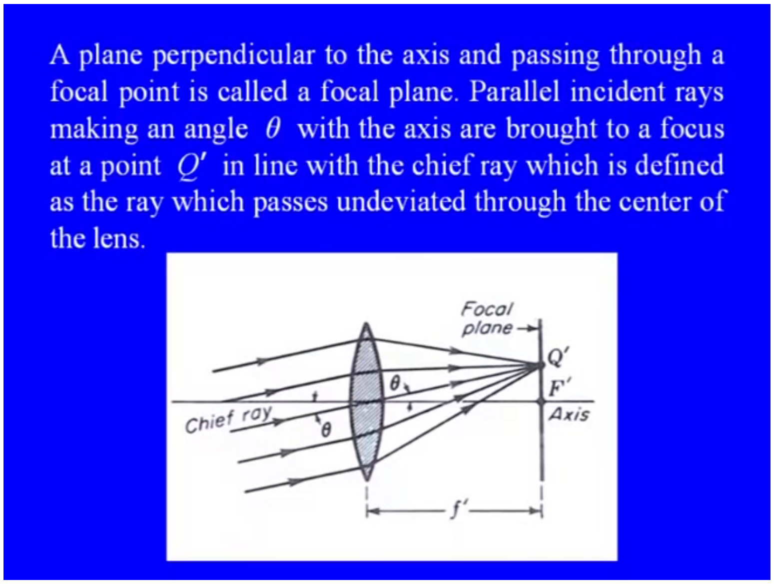

Now a plane perpendicular to the axis and passing through a focal point is called a focal plane. Parallel incident ray's making an angle theta with the axis or brought to focus at a point Q Prime in line with the chief ray which is defined the chief ray is defined as the ray which passes undeviated to the center of the lens. The object and image are called conjugate points.

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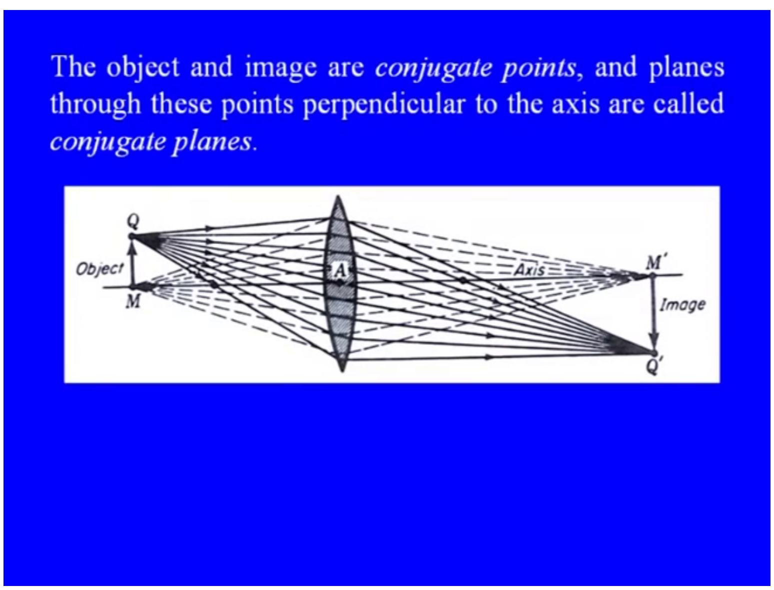

And the planes through these perpendiculars to the axis are called conjugate planes. The plane MQ and the plane M prime Q prime are conjugate planes. M prime is the image corresponding to the object point M, Q prime used image corresponding to the object point Q.

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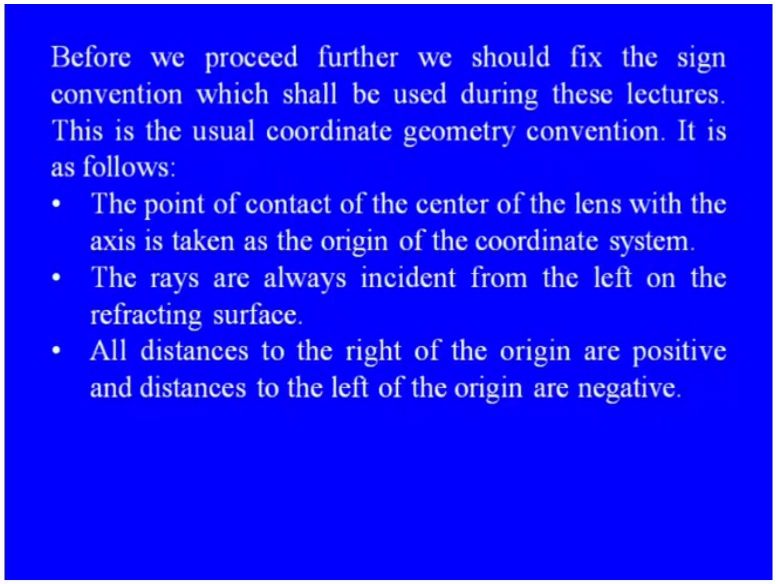

Now before we proceed further we should fix the sign convention which shall be used during these lectures for finding out where the image will be formed for a given position of the object. This is the usual coordinate geometric convention it is as follows; the point of contact of the center of the lens with the axis is taken as the origin of the coordinate system. Remember we are talking about thin lenses.

Now the rays are always incident from the left on the refracting surface the incident from the left going towards right. All distances to the right of the origin are positive and distances to the left of the origin are negative usual coordinate geometry arrangement.

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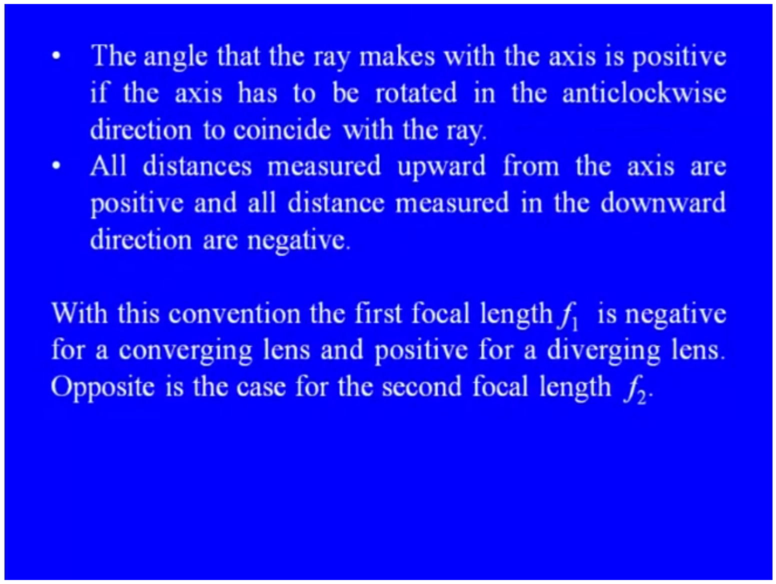

The angle that the ray makes with the axis is positive if the axis has to be rotated in the anti-clockwise direction to coincide with the ray and all distances major upwards from the axis are positive and all distances measured downward direction or negative I mean the same convention should be pretty familiar. Now with this convention the first focal length F1 is negative for a converging lens because this point comes from the left of the lens.

And positive for a diverging lens which is a double concave lens and a positive the case for the second focal length F2.

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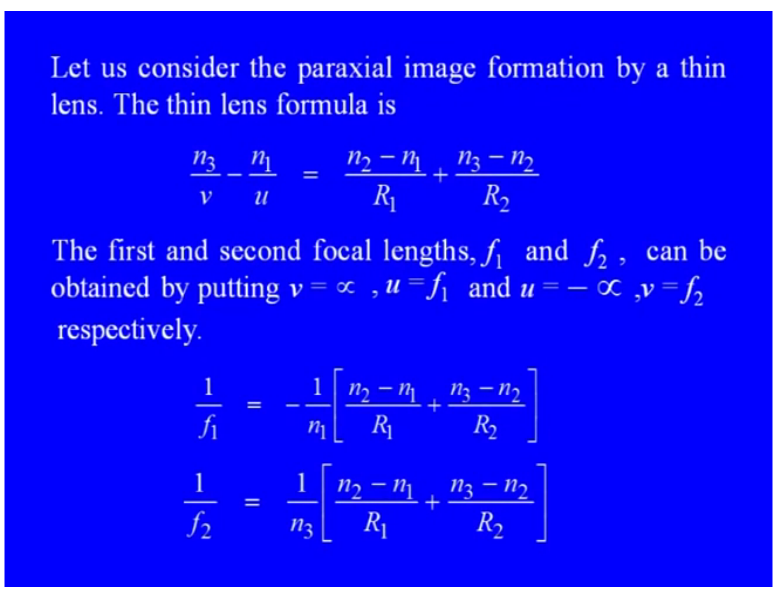

Let us consider this paraxial image formation by a thin lens but axial means they are very close to the axis they are not making a large angle with the axis. The thin lens formula is n3 by v - n1 upon u = n2 - n1 upon R1 + n3 - n2 upon R2. You see n1, n2, n3 are the refractive indices n1 is for the medium on the left of the lens and n2 is for the material of the lens and n3 is for the medium on the right of the lens.

Now the first in the second focal length f1 and f2 can be obtained from this expression by putting v equal to infinity and u becomes equal to f1 or u = - infinity and v = f2 respectively. And this gives 1 upon f1 = - 1 upon n1 n 2 - n1 upon R 1 + n3 - n2 upon R2. And for the second one 1 upon f2 = 1 upon n3 n2 - n1 upon R1 + n3 - n2 upon R2. You see this capital R1 and R2 they are the Ray of radii of curvature of the two curved surfaces of the lens.

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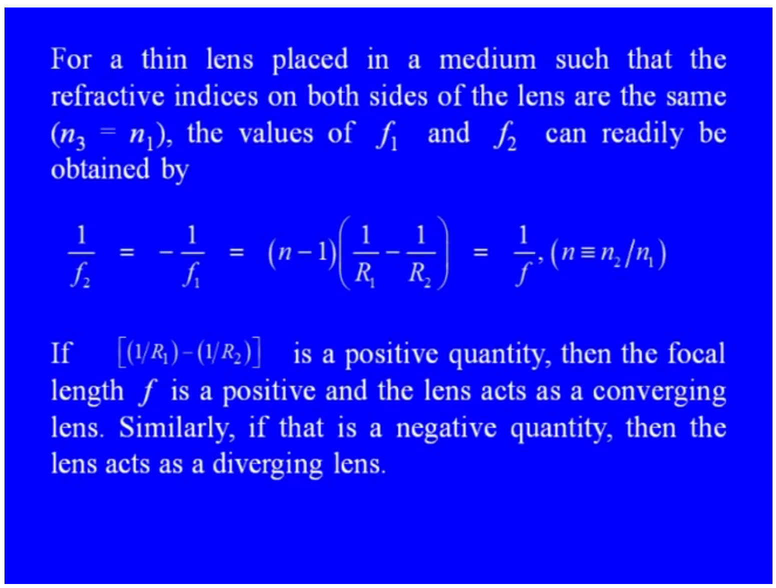

For a thin lens placed in a medium such that the refractive indices on both sides of the lens are the same that is n3 = n1 the values of f1 and f2 can readily be obtained by this relation 1 upon f2 naturally in this case it is = -1 upon f1, I mean they are equal in magnitude there is a sign difference because one focal point is on the left that is on the right. So, this is = n – 1, n is the refractive index of the material of the lens relative to the medium outside multiplied by the factor 1 upon R1 - 1 upon R2.

Now if this factor 1 upon R1 - 1 upon R2 is a positive quantity then the focal length f which is = f2 which is the second focal length is a positive quantity. And the lens acts as a converging lens this is a situation for a by convex lens, similarly if that is a negative quantity then the lengths acts the diverging lens.

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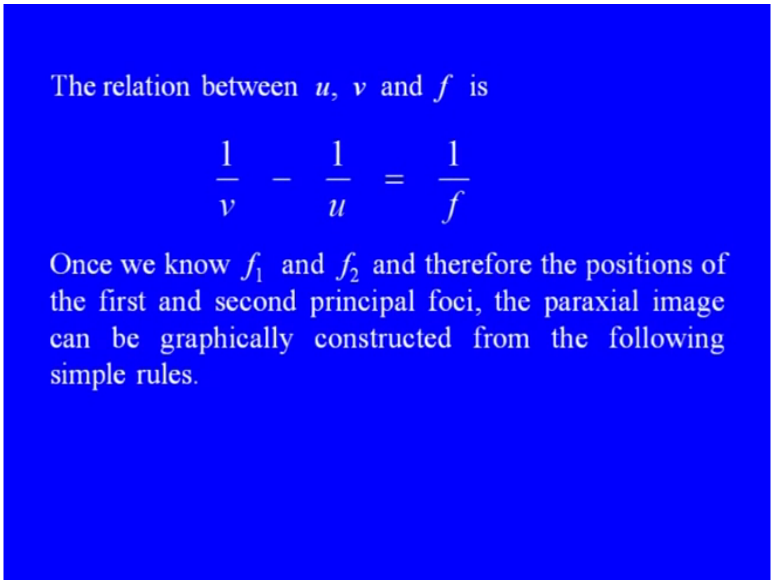

Now the relation between u, v and f is 1 upon v - 1 upon u = 1 upon f the solution is very popular when most commonly used for finding out the distance of the image when the object is that distance u and f in the focal length of lens. Now once we know f1 and f2 and therefore the positions of the first and the second principle for side. The paraxial image can be graphically constructed from the following simple rules.

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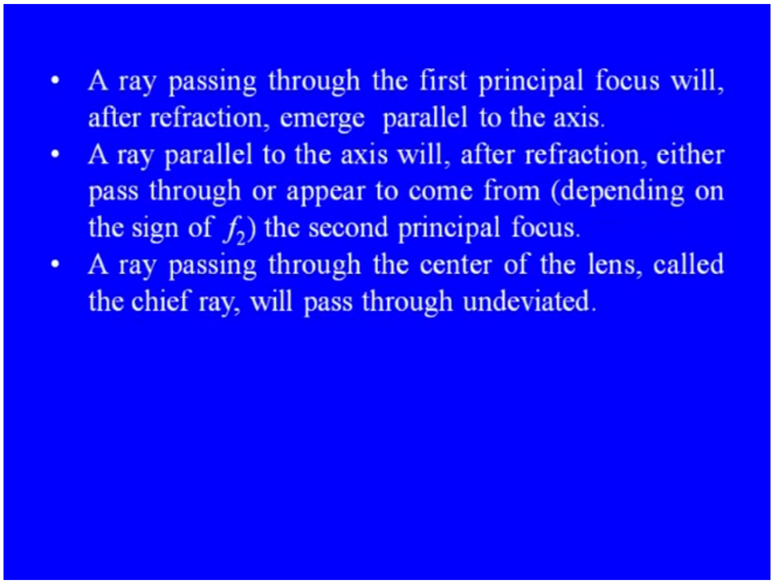

A ray passing through the first principal focus will up the refraction image parallel to the axis that is characteristic of the focal point, ray parallel to the axis will after refraction either pass through or happier to come from depending on the sign of f2 depending on whether the lens is a converging lens or diverging lens coming from the second principal focus. A ray passing through the center of the lens called the chief ray will pass through undeviated.

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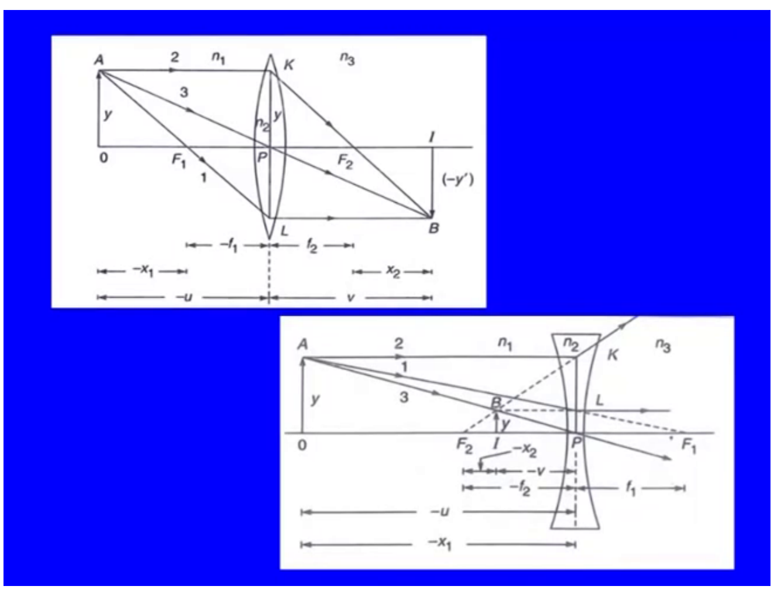

Consider these figures you see from the point A considered that the ray two which is parallel to the axis after refraction pass it through f2 which is the focal point then a ray from A going through the focal point f1 after refraction becomes parallel to the axis they meet at the point me which is the image of the point A. If you consider the chief ray now this should also reach the point B, APB.

Similarly for the case of the concave lens parallel ray starting from A after refraction appears to come from f2. A ray because going towards f1 after refraction becomes parallel at the point L becomes parallel to the axis and these two meet at the point B which is the image of the point A.

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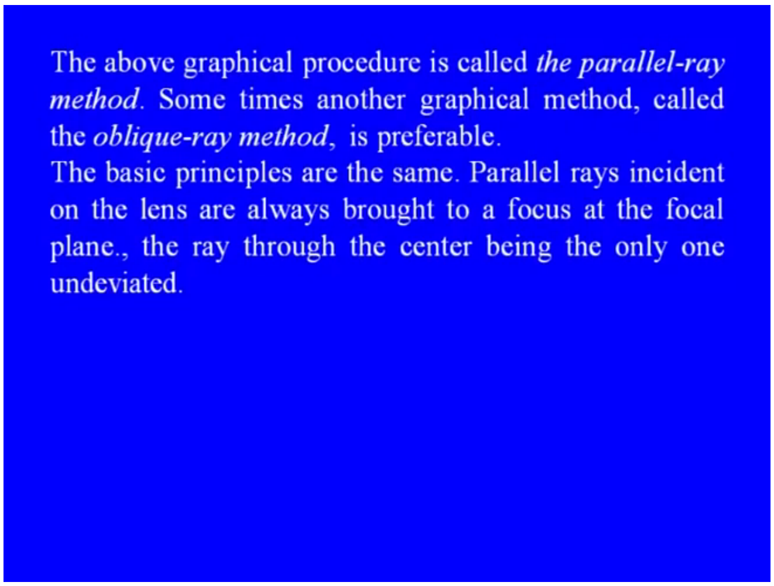

The above graphical procedure is called the parallel ray method. Sometimes another graphical method called the Oblique ray method. That is preferable the basic principles are the same parallel rays incident on the lens are always brought to a focus at the focal plane. And the rays of the center being the only one undeviated that is the chief ray.

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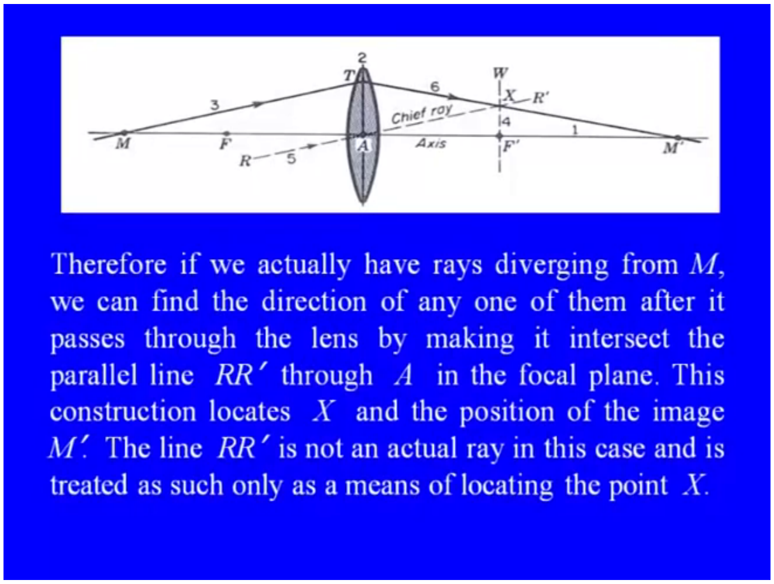

Let us consider the picture here you consider the rays starting from the point M if we actually have real diverging from M we can find the direction of any one of them after it passes through the lens by making it intersect the parallel line R R prime through A. Let us consider this ray MT this is ray. Now we wish to find out the reflected ray corresponding to this. So, consider a parallel chief ray and consider the focal plane through f prime.

Then one can construct the rate TX because the two parallel rays they must meet in the focal plane. The Ray RR prime is not an actual ray in this case and stated as such only the means of locating the point X and once we have located the point X we extend this TX to M prime and M prime is the image of the object at the point M.

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Newton's Formula

Let x_1 be the distance of the object from the first principal focus (x_1 will be positive if the object point is on the right of F_1 and conversely) and let x_2 be the distance of the image from the second principal focus F_2 .

Newton's formula you see this is another relation which is used to relate the object and image distances with the focal length of the lens. The difference here is the distances are not measured from the lens. You see let X1 be the distance of the object. From the first principal focus not from the lens f1 will be positive if the object point is on the right of f1 that is as per sign convention. And let X2 be the distance of the image from the second principal focus.

So, object distance to be measured from first principle focus image distance to be measured from the second principal focus.

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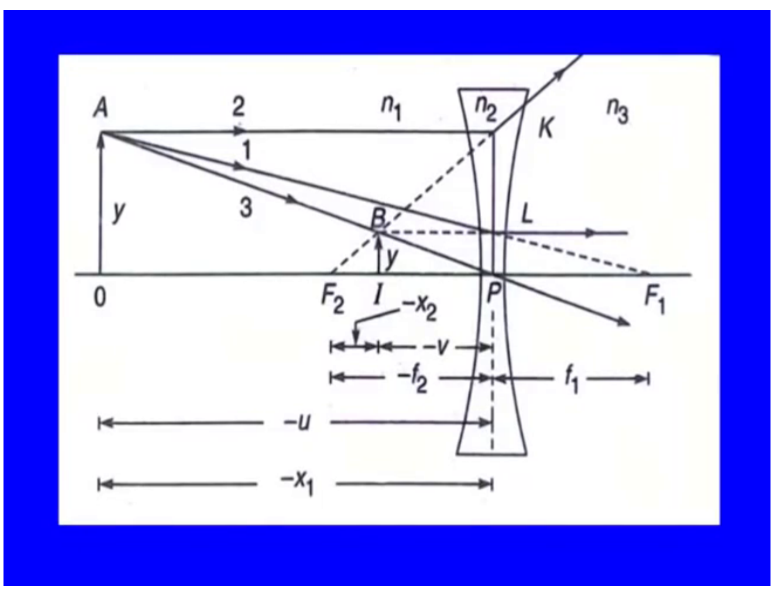

Considered this picture I mean to obtain a relationship in this case. Let us first fix up the distances you see they consider the distance for example X2 which is the distance of image from the focal point f2. Usually your object distance U which is the distance of the object from the center of the lens. The distance of the image I from P is again from the center. To obtain a relationship in this case we shall make use of the similar triangles.

Consider the triangle for example AOP and the triangle BIP this they will give information some sort of a relationship between the various quantities involved here.

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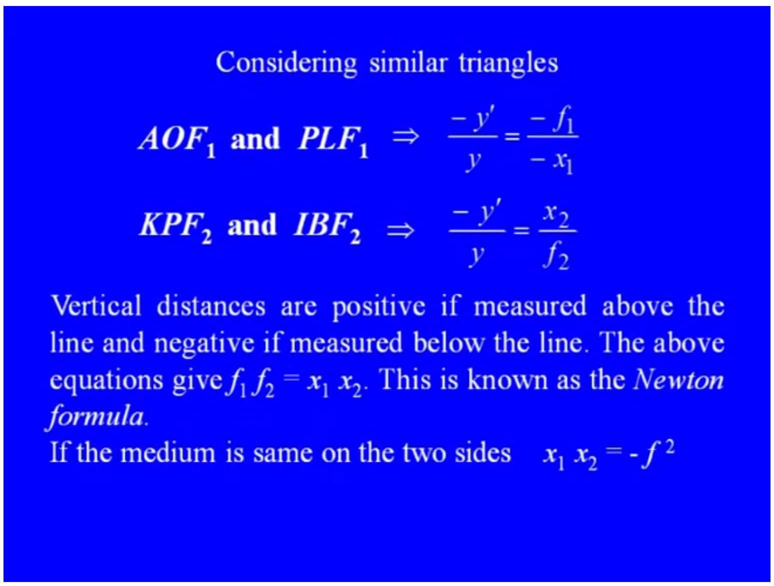

So, as I said considering the similar triangles AOF1 and PLF1 you have a relationship like this -y prime upon y these are the heights of the object points is = - f1 upon -X1. The minus and plus signs are S per sign convention. Similarly from another set of similar triangles the ratio -y prime to y = X2 to f2 the above equation gives the relationship f1 f2 = X1 into X2, this is known as the Newton's formula.

The medium is same on the two sides has is the situation in most cases. The formula basically now is X1 X2 = - f square.

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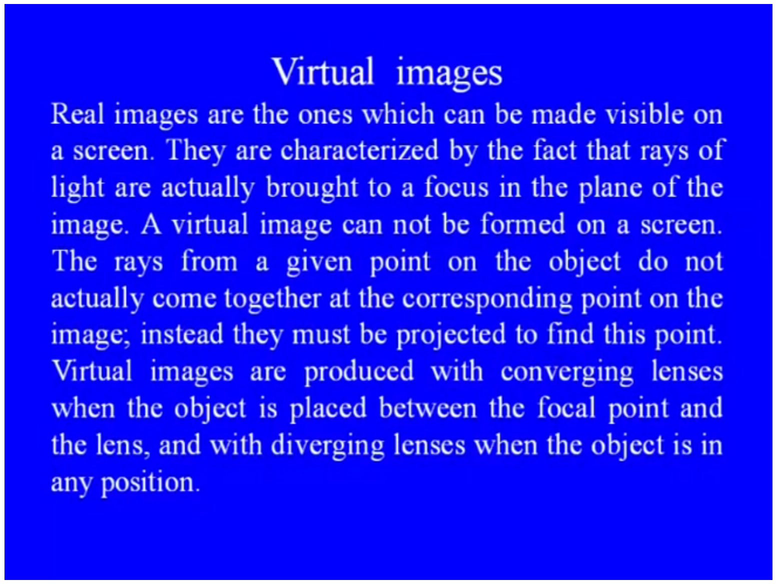

Let us consider virtual image real images are the ones which can be made visible on a screen. They are characterized by the fact the rays of light are actually brought to a focus in the plane of the image. A virtual image on the other hand cannot be formed on a screen. The rays from a given point on the object do not actually come together at the corresponding point on the image instead they must be projected to find this point.

These images cannot be taken on a screen virtual images are produced with converging lenses when the object is placed between the focal point and the lens. And with diverging lenses this is the situation and the object is in any position. Diverging lenses always lead to virtual images.

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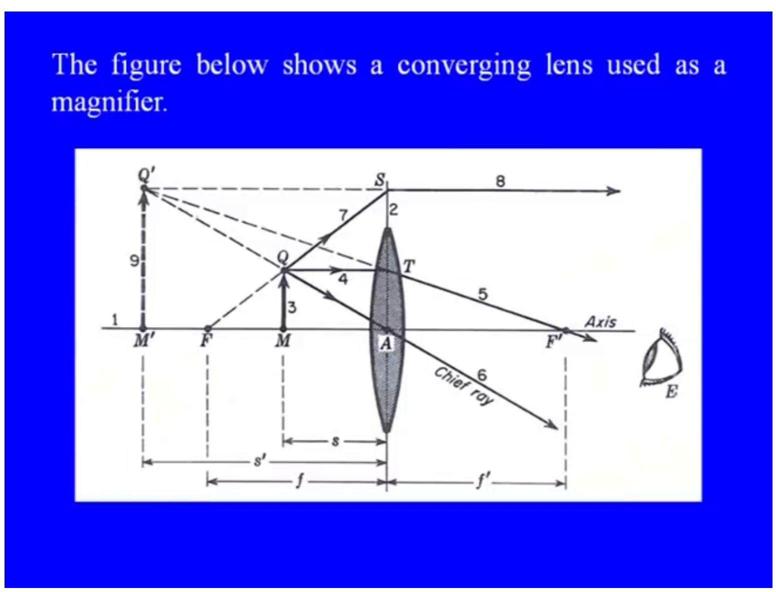

Is this is a very typical example here the object is very close to the lens between the lens and the focal point if you consider a parallel ray starting from the point Q then naturally after reflection it passes through the F Prime and it should then you consider the chief ray from starting from Q goes through undeviated. And if we extend them backwards they meet at the point Q prime which is the image of the point Q.

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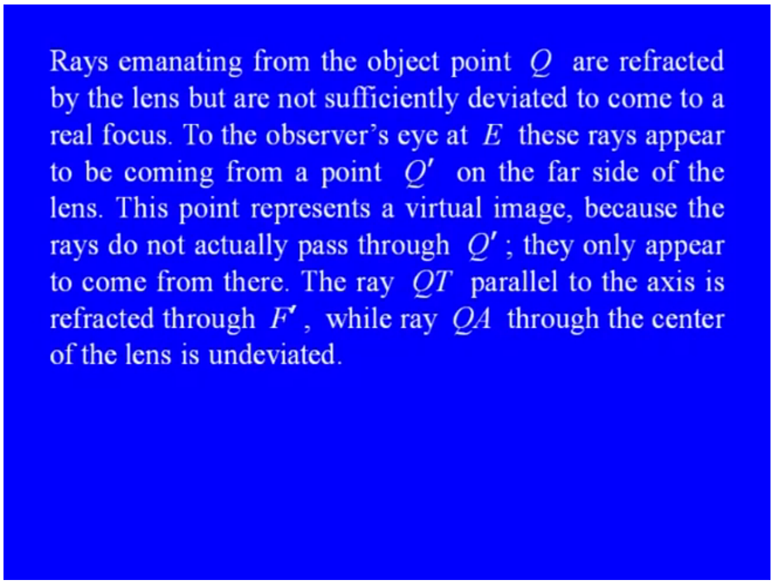

So, that what it says here rays emanating from the object point Q are reflected by the lens but are not sufficiently deviated. To come to a real focus to the observer's eye at these rays appear to be coming from a point Q prime on far side of the lens. The distance of Q prime from the lens is more than the distance of Q. This point represents a virtual image because the rays do not actually pass through Q prime, they only happier to come from there.

The ray QT parallel to the axis is reflected through F prime while there really QA to the center of the lens which is the chief ray is undeviated.

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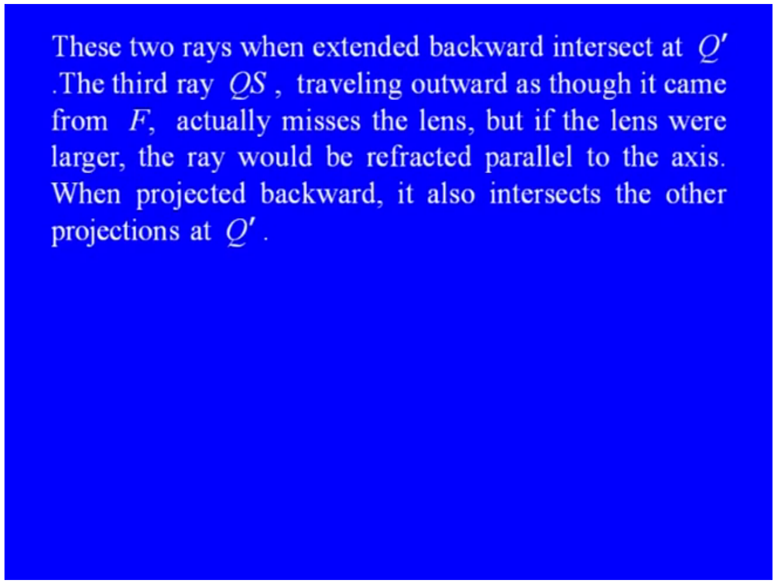

Now these two rays when extended backwards as I said earlier we have seen in the figure. The intersect at Q prime. The third ray picture QS traveling outward as though it came from F actually misses the lens in this case but it will ends for larger. The day would be refracted parallel to the axis as it should when projected backward it also intersects the other projections at Q prime.

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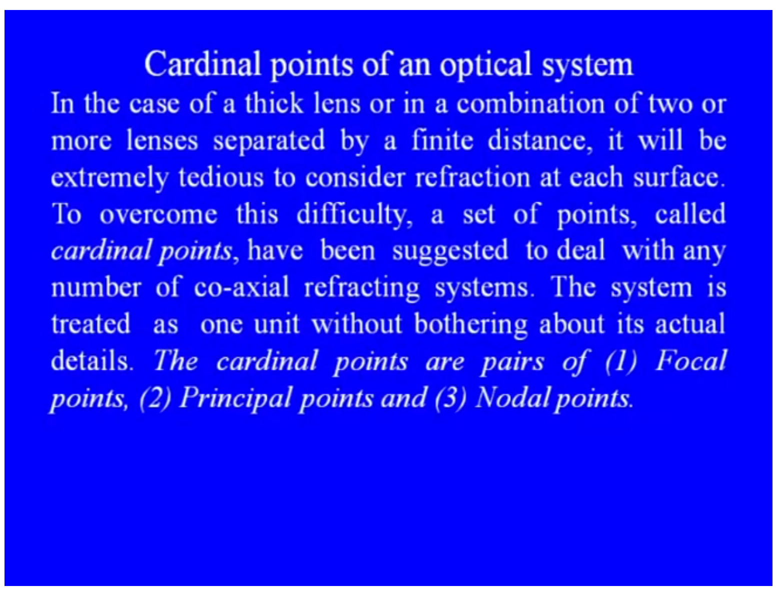

Cardinal points of an optical system, in the case of at a thick lens l in a combination of two or more lenses separated by a finite distance. It will be extremely tedious to consider refraction at each surface. To overcome this difficulty a set of points called cardinal points have been suggested to deal with any number of coaxial refracting systems. The system is treated as one unit without bothering about its actual details.

The cardinal points are pairs of focal points, principal points and nodal points. We have already discussed focal points first principal focus and the second principal focus. The planes passing through the principal foci perpendicular to the axis are called focal planes. The main property of the focal planes is that the ray is starting from a point in the focal plane in object space they correspond to a set of conjugate parallel rays in the image space.

We have seen that similarly they set a parallel rays in the object space they correspond to a set of rays intersecting at a point the focal plane in the image space.

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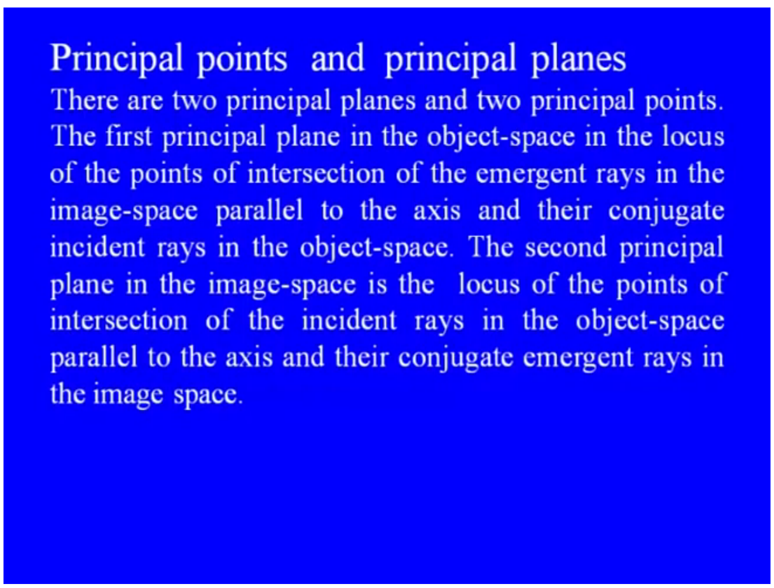

Now the principal points and the principal piece let us see what they signify. There are two principal planes and two principal points. The principal plane in the object space the locus of the points of intersection of the emergent trace in the major space parallel to the axis and their conjugate incident rays in the object space. The second principal plane in the image space is the locus of the points of intersection of the incident rays in the object space parallel to the axis and there conjugate emergent rays in the image space.

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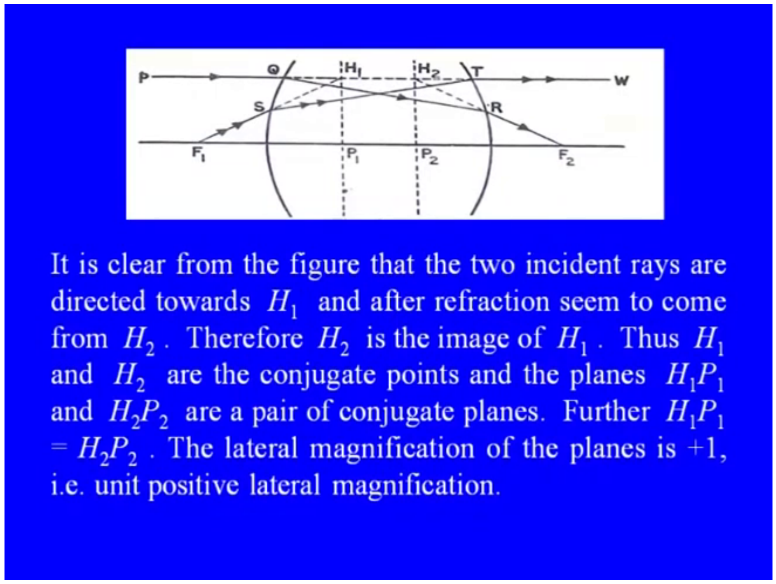

We can see it from here it is clear from the figure then the two incident rays are directed towards H1 and after refraction seem to come from H2. Therefore H2 is the image of H1 thus H1 and H2 are the conjugate points and the planes H1 P1 and H2 P2 are a pair of conjugate planes. Further H1 P1 = H2 P2. So, the lateral magnification of the plane says +1 unit positive little magnification and that characterizes the principal points and the principal planes.

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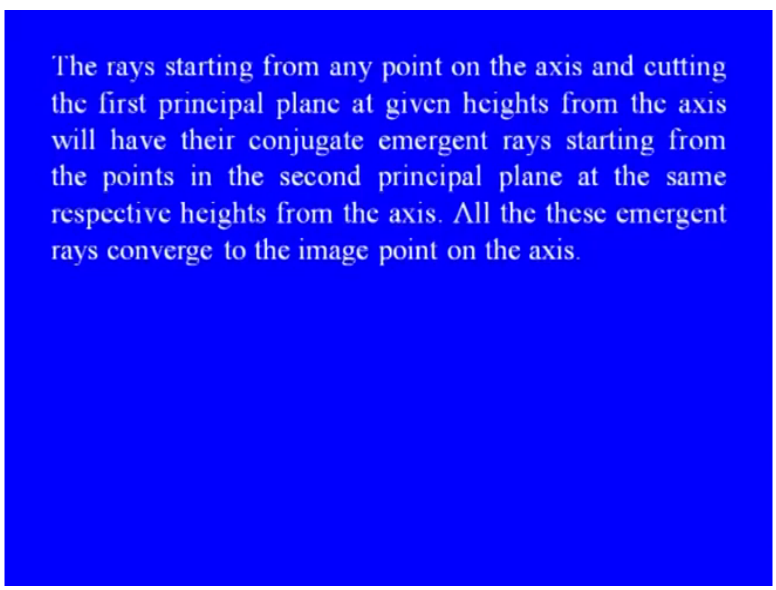

The range is starting from any point on the axis and cutting the principal plane had given heights from the axis will have their conjugate emergent rays starting from the points in the second principal plane and the same respective height that is the main thing that the same respective heights from the access. All these emergent rays converge to the image point on the axis.

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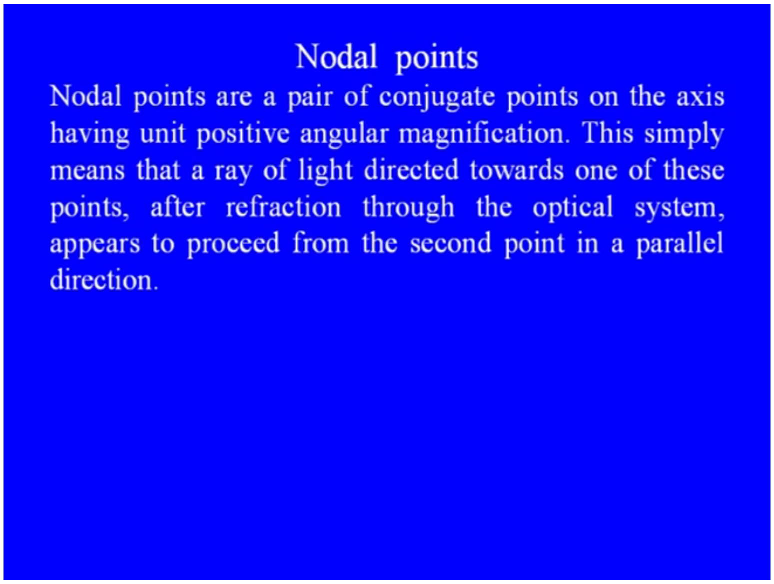

Let is consider the nodal points these points are a pair of conjugate points on the axis having unit positive angular magnification. Principal points were having unit positive lateral magnification nodal points have a unit positive angular magnification. This simply means that a ray of light directed towards one of these points after refraction through the optical system appears to proceed from the second point in a parallel direction.

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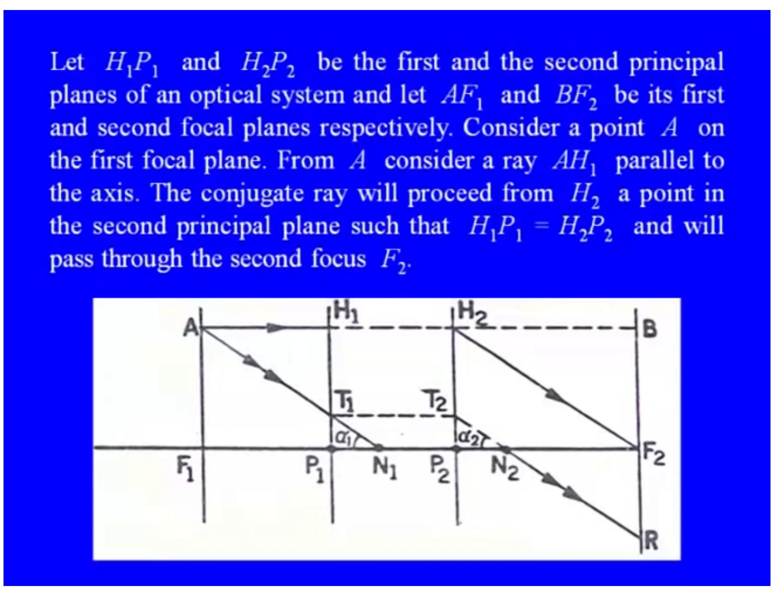

Let's consider Let H1 P1 and H2 P2 the first and the second principal planes of an optical system. And let F1 AF1 and BF2 be the first and second focal planes respectively. Consider a point AF1 the first focal plane from A, consider a ray AH1 parallel to the axis the conjugate ray will proceed from H2 a point in the second principal plane such that H1 P1 = H2 P2 and will pass through the second focus F2.

Take another ray AT1 parallel to the emergent ray H2 F2 and striking the first principal plane at T1 it will emerge from T2 point on the second principal plane such that T2 P2 = T1 P1 and we will proceed parallel to the ray H2 F2 as the two rays originate at A, a point on the first focal plane. Then the points of intersection of the incident ray AT1 and conjugate emergent ray T2 are with the axis give the position of the two nodal points N1 and N2.

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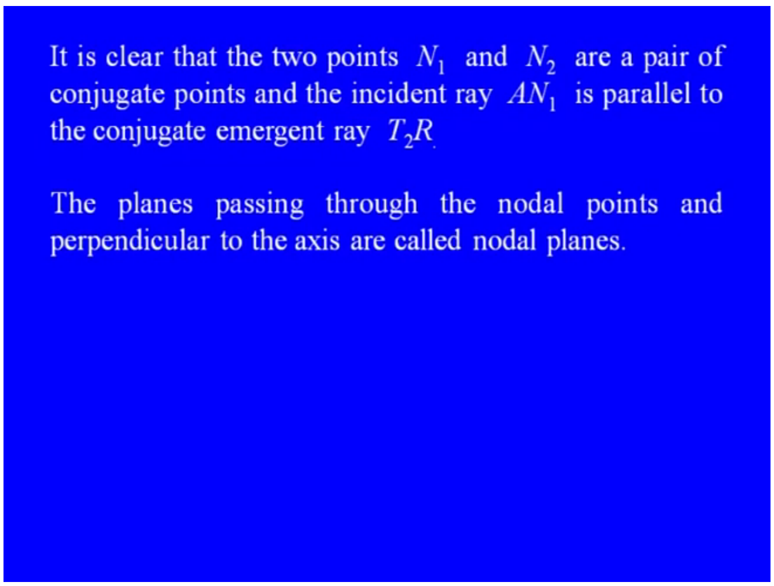

It is clear that the two points N1 and N2 are a pair of conjugate points an incident ray A N1 is parallel to the conjugate emergent ray T2R. The planes passing through the nodal points and perpendicular to the axis are called nodal planes.

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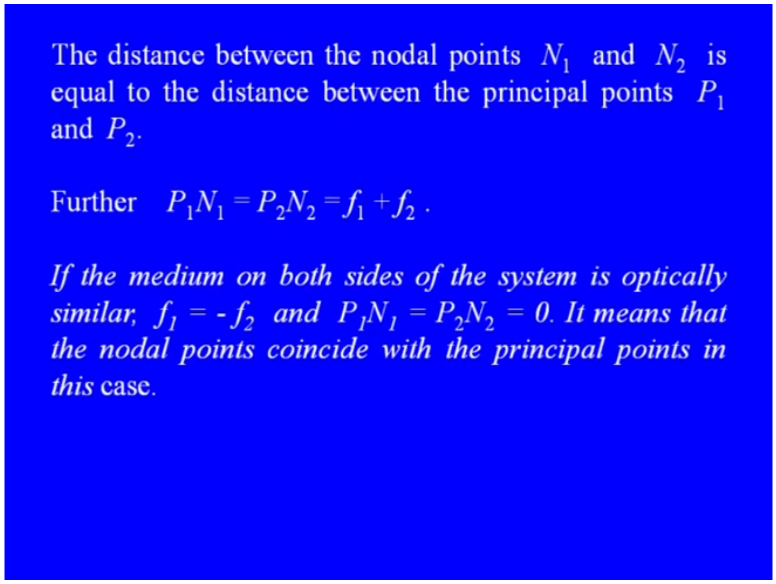

Now the distance between the nodal points N1 and N2 is equal to the distance between the principal points P1 and P2 further P1 N1 = P2 N2 which is really equal to f1 + f2. Now if the medium on both sides of the system is optically similar f1 will be equal to -f2 and this means P1 N1 = P2 N2 = 0. It means that the nodal points coincide with the principal points, so, in this situation if you have the same medium on both sides. These pair of points have equal lateral magnification and wholesome equal angular magnification.

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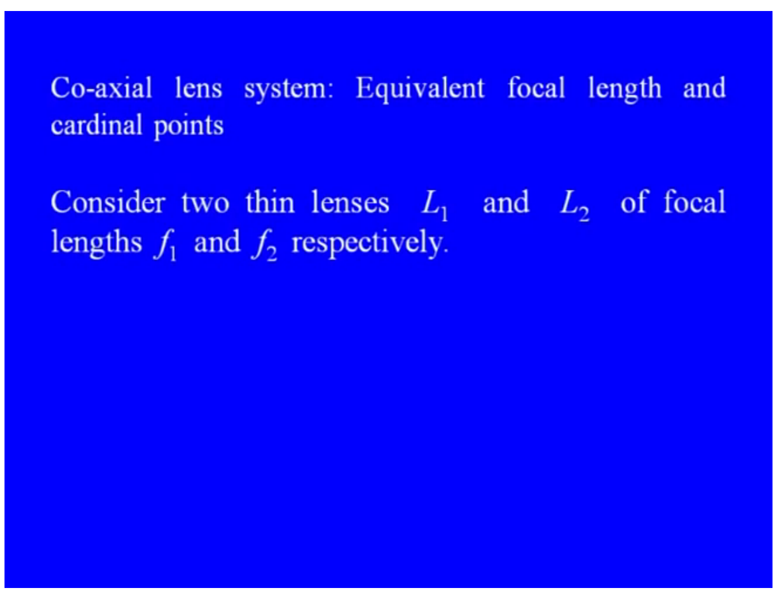

Let us consider the question and system its equivalent focal length and it is cardinal points. We consider now here to thin lenses L1 and L2 or focal lengths f1 and f2 respectively.

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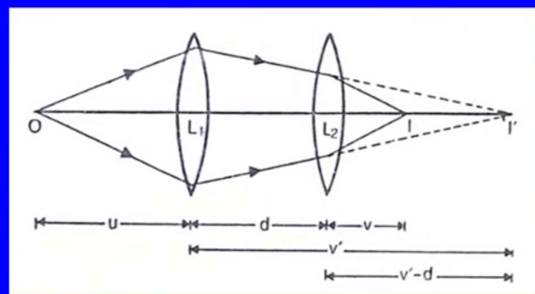
A point-object O is placed at a distance u from the first lens and the final image is formed at I at a distance of v from the second lens.



A point objects O is placed at a distance u from the first lens and if final image is formed at I a distance of v from the second lens. You see after the reflections of the first lens an intermediate image can be thought of at the point I prime which then serves as a virtual object for the second lens and the final image is formed at the point r. The d is the distance between the lenses use the object distance from the first lens v prime is the distance of that intermediate image which is formed.

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The first image due to the first lens is formed at I' at a distance of v' from it.

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}, \qquad or \qquad \frac{1}{v'} = \frac{u+f_1}{u f_1}$$

The image I' behaves as an object for the second lens. The object distance for the second lens is $v' - d$ and the final image is formed at I.

$$\frac{1}{v} - \frac{1}{v'-d} = \frac{1}{f_2}, \qquad or \qquad \frac{1}{v'-d} = \frac{f_2 - v}{f_2 v}$$

Eliminating v' from these, we get

$$uv(d - f_1 - f_2) + u(f_1 f_2 - d f_2) + v(d f_1 - f_1 f_2) - d f_1 f_2 = 0$$

So, all the distances are given here the first image due to the first lens is formed by primers I pointed out earlier at a distance of v prime from it. So, we have the standard relation 1 upon v prime - 1 upon u = 1 upon f1, so this can be written like this 1 upon v prime = u + f1 divided by uf1. Now, image i prime that an object for the second lens the object distance for the second lens is v prime - d and the final image is formed at I, so here we have the relation 1 upon v, v the major distance from the second lens.

We are writing this relation for the second lens, so 1 upon v - 1 upon v prime - d = 1 upon f2, f2 is the focal length of the second lens or this can be written as 1 upon v prime - d = f2 – v divided by f2v. Now from these relations we eliminate v prime and we get a relationship like this uv multiplied by d - f1 - f2 + u multiplied by f1 f2 - d f2 + v multiplied by df1 - f1 f2 - d f1 f2 = 0.

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If α represents the distance of the first lens from the first principal plane and β represents the distance of the second lens from the second principal plane, then the reduced object-distance and the reduced image-distance are given by $U = u - \alpha$ and $V = v - \beta$. If f is the focal length of the equivalent lens, then

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}, \qquad or \qquad \frac{1}{v-\beta} - \frac{1}{u-\alpha} = \frac{1}{f}$$

On rearranging, this leads to

$$uv + u\left(-\beta - f\right) + v\left(-\alpha + f\right) + \left(\alpha\beta - \beta f + \alpha f\right) = 0$$

Now we treat this problem in an alternative way. If alpha represents the distance of the first lens from the first principal plane and beta represents the distance of the second lens from the second principal plane. Then the reduce object distances and the reduced image distances are u capital U is given by u - alpha remember you were the distance of the object from the first lens else y is the distance of the first principal point from the first lens.

So, capital u is the object distance from the first principal point. Similarly v capital V small v - beta is the distance of the image from the second principal point. If f is the focal length of this of the combination of the equivalent lens we have the standard relation like this 1 upon capital V minus 1 upon capital U = 1 upon f. We put the values of capital V and capital U, so write 1 upon v - beta - 1 upon u - alpha = 1 upon half.

On rearranging this again we have a equation having terms like uv, u multiplied by - beta – f, v multiplied by - alpha + f and alpha beta - beta + alpha =0. We can compare this equation with the only a similar equation.

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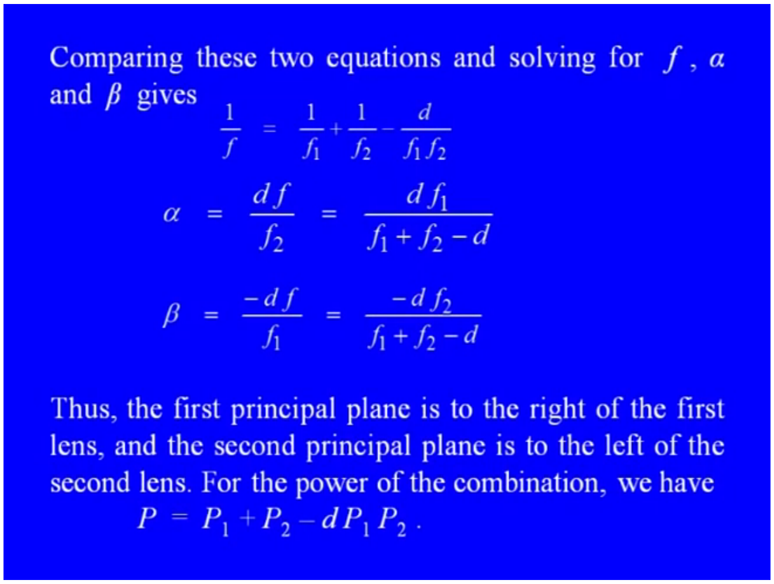

And that gives us these relationships 1 upon f comes out to be equal to 1 upon f1 +s 1 upon f2 - d upon f1 f2, so this is the focal length of the combination. And alpha, beta, alpha is df upon f2 df 1 upon have 1 + f2 - d and beta is - df upon f1, so -df2 –f1 f2 – d. So, now the idea is the two lenses are not to be treated separately anymore. We have just one optical system, object image distances are to be measured from the principal points.

And we have the focal length equivalent focal length of the combination for the power of the combination naturally we have this relationship the total power is equal to P1 + P2 - d times, d the distance between those two thin lenses, so d times P1 P2.

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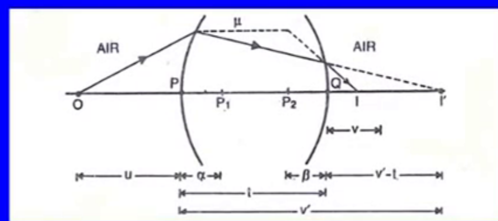
Refraction through a Thick Lens

Consider a lens of thickness t and made from a material of refractive index μ and placed in sir. The radii of curvature are R_1 and R_2.



A luminous point object O is situated on the axis at a distance u from the first refracting surface and forms an image I' at a distance v' from P.

Now we are in a position to consider the thickness. Consider lens of thickness t and made from a material of refractive index Mu and placed in sir. Radii of curvature R1 and R2, consider look at the thickness between the points P and PQ that is t a luminous point object O is situated on the axis at a distance u from the first reflecting surface and forms an image at I prime at a distance v prime from P. This image will serve as a virtual object the second surface.

So, for the reflection from the first surface one can write Mu upon v prime - 1 upon u which is equal to Mu - 1 upon R1 that is tendered relation for reflection from a spherical surface. The image I prime formed by the first surface acts as the object for the second surface and he finally made it formed a I, so here you have the relationship 1 upon Mu divided by v - 1 upon v prime -t whose P is coming because it is a thick lens is equal to 1 upon Mu – 1divided by R2, R2 the radius of curvature of the second surface.

Now v prime is to be eliminated between these two equations to obtain an equation in UV, R1 and R2.

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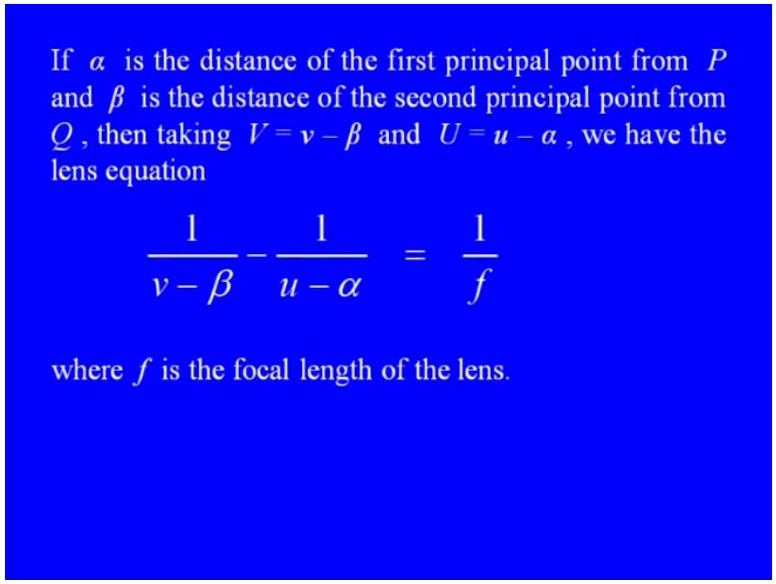

And you now if alpha is the distance of the first principal point from P and Beta in the distance of the second principal point from Q then taking the capital V = v - beta capital U = u - l, we have the lens equation as before 1 upon v - beta - 1 upon u - alpha = 1 upon m again substituting those relations as before.

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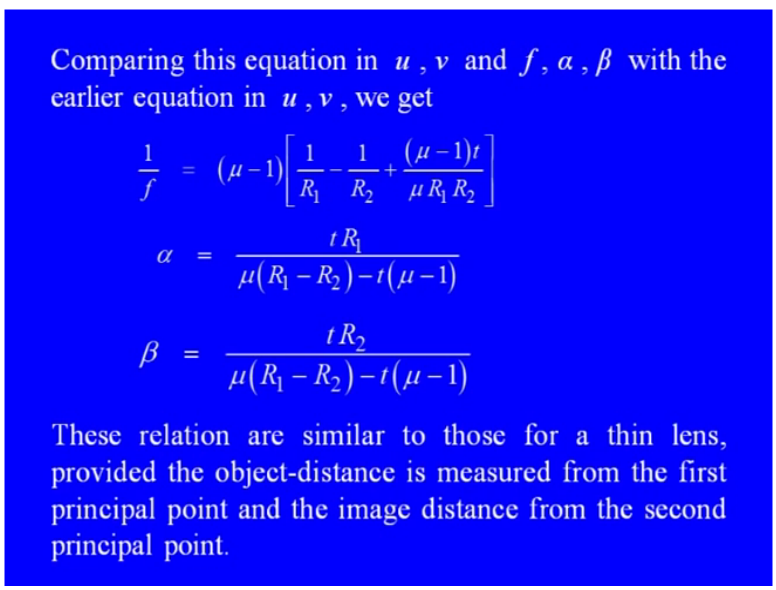

And comparing this equation with the earlier equation in u, v, I have alpha beta simpler, as we did for the combination of two thin lenses, we get these results. So, 1 upon f = Mu - 1 R1 - 1 upon R1 - 1 upon R2 + Mu - 1 into t divided by Mu R1 R2 that is the and defining the focal length. Remember if t =0, if it is the thin lens then this term will not be there you will have a simple relationship Mu - 1 into 1 upon R1- 1 upon R2.

For the thick lens this is the additional term which is coming up. Similarly for the distances of the principal points alpha and beta if t is 0, you do not have to bother about them but the center of the lens is the only point but now for the thick lens alpha is given by this relationship theta is given by the next one similar to those for the thin lens.

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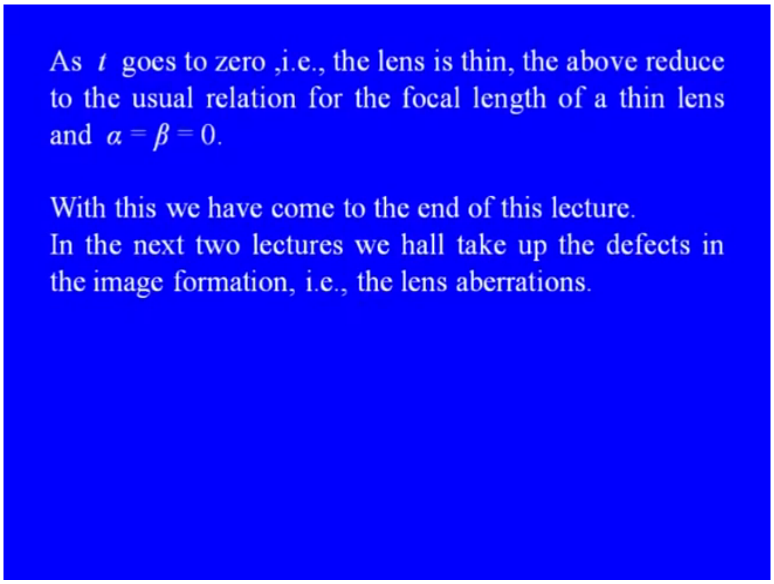

Now as t goes to 0 as I pointed out the lens thin the above reduce to the usual relation for the focal length of a thin lens okay. With this we have come to the end of this lecture now in the next two lectures we shall take up the defects in the image formation that is the lens aberrations, thank you.